JEE Main

Joint Entrance Examination

Question Booklet Code **P**

Practice Set 4

Duration: 3 Hours *Max. Marks: 360*

Read the Following Instructions Carefully

1. *Immediately fill the particulars on this page of the test booklet with blue / black ball point pen. Use of pencil is strictly prohibited.*
2. *The test is of 3 hours duration.*
3. *The test booklet consists of 90 questions. The maximum marks are 360.*
4. *There are three parts in the question paper A, B, C consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage. Each question is allotted 4 (four) marks for correct response.*
5. *Candidates will be awarded marks as stated in above instructions for correct response of each question. ¼ (one fourth) marks will be deducted for indicating incorrect response of each question. There is no negative marking for unattampted questions.*
6. *There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted accordingly as per above instructions.*
7. *No candidate is allowed to carry any textual material, printed or written, bits of papers, paper, mobile phone, any electronic device, etc., except the Admit Card inside the examination hall/room.*
8. *Rough work is to be done on the space provided for this purpose in the test booklet only. This space is given at the bottom of pages.*

Name of the Candidate (in Capital Letters)_____

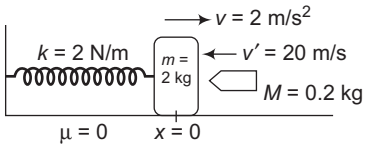
Roll Number (in Figures) | | | | | | | | | in Words _____

PART A Physics

1. A cubical block of mass m and edge a slides down a rough inclined plane of inclination θ with a uniform velocity. The torque of the normal force on the block about its centre has a magnitude

(a) zero (b) mga (c) $mga\sin\theta$ (d) $\frac{mga\sin\theta}{2}$

2. A block is executing SHM on a horizontal floor. At an instant when it was at its equilibrium position and moving towards right a bullet hits it and embedded in it. The new amplitude of oscillation A will be



(a) $A > 2m$ (b) $A = 4m$

(c) $A = 0$ (d) $0 < A < 2m$

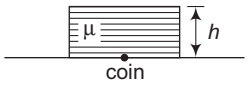
3. In a room, where the temperature is 30°C, a body cools from 61°C to 59°C in 4 min. The time taken by the body to cool from 51°C to 49°C will be

(a) 4 min (b) 6 min (c) 5 min (d) 8 min

4. When an AC voltage of variable frequency is applied across the L-C-R circuit, the current in the circuit was found to be same at 8 MHz and 18 MHz. Then, the frequency at which current will be maximum is

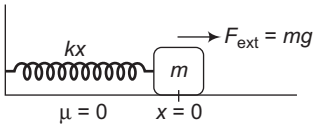
(a) 24 MHz (b) 13 MHz (c) 10 MHz (d) 12 MHz

5. A slab of refractive index μ is placed on a coin. If the slab is lifted upwards by a distance d, then the apparent depth of the coin will be



(a) h/μ (b) $(h+d)/\mu$ (c) d/μ (d) $\frac{(h-d)}{\mu}$

6. A force start acting on a spring block system. At a stretch when $x = \frac{mg}{k}$



(a) $W_{F_{ext}} + W_{F_s} = 0$ (b) $W_{F_{ext}} = \frac{1}{2}kx^2$

(c) $W_{F_s} + W_{F_{ext}} > 0$ (d) None of these

7. An electron with KE 2.0 eV is incident on a H-atom which is in excited state n = 2. Then, the collision

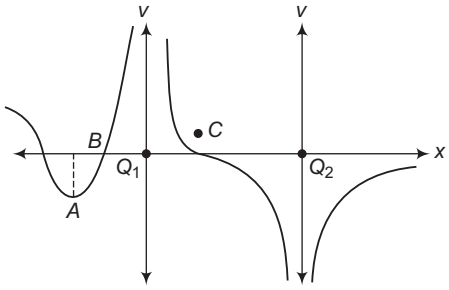
(a) must be elastic (b) must be inelastic

(c) can be either elastic or inelastic (d) can be either elastic or inelastic or super elastic

8. If the signal is transmitted from an optical fibre core of Refractive Index (RI) $\sqrt{\frac{12}{5}}$ to an another optical fibre with RI of core and cladding as 1.8 and 1.2 respectively, then the maximum angle of acceptance for 2nd optical fibre is

(a) 30° (b) 45° (c) 60° (d) $\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$

9. The curve represents the distribution of potential energy along the straight line joining two charges Q_1 and Q_2 separated by distance r. Then, which of the following statement are correct?



1. $|Q_1| < |Q_2|$
2. Q_1 is positive and Q_2 is negative
3. A is a point of stable equilibrium
4. B and C are points of unstable equilibrium

(a) 1 and 2 (b) 1, 2, 3 and 4
(c) 1, 2, and 3 (d) 1 and 3

10. Which of the following statements is true about magnetic susceptibility X_m of paramagnetic substance?

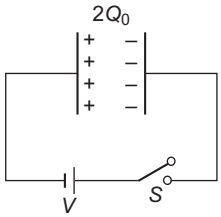
(a) X_m increases exponentially with increase in temperature
(b) X_m decreases exponentially with temperature
(c) X_m value can be positive, negative as well as zero.
(d) X_m is always positive

11. Statement I If two non-ideal, unidentical batteries are connected in parallel with positive terminals are connected together, then the equivalent emf is smaller than either of two emf's.

Statement II The equivalent resistance is smaller than either of the two internal resistances.

(a) Both Statement I and Statement II are true and the Statement II is the correct explanation of the Statement I
(b) Both Statement I and Statement II are true but the Statement II is not the correct explanation of the Statement I
(c) Statement I is false but Statement II is true
(d) Both Statement I and Statement II are false

12. A capacitor of capacitance C having initial charge $2Q_0$, is connected to a battery of potential difference $V = \frac{Q_0}{C}$ as shown, then work done by the battery is

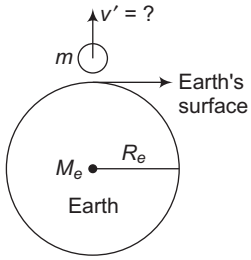


(a) $\frac{Q_0^2}{2C}$ (b) $\frac{3Q_0^2}{2C}$
(c) $\frac{3Q_0^2}{C}$ (d) $\frac{2Q_0^2}{3C}$

13. Which of the following statements is correct?

(a) All the elastic materials follow Hooke's law over considerable range of stress.
(b) A material with high value of Young's modulus can be stretched less as compared to other material having low value of Young's modulus by applying same force.
(c) Gap between ultimate point and breaking point on stress-strain graph decides whether material is brittle or ductile.
(d) In an elastic material, Hooke's law is followed upto yield point.

14. **What should be the speed by which we should throw a ball so that it could reach up to a height $R_e/2$?**



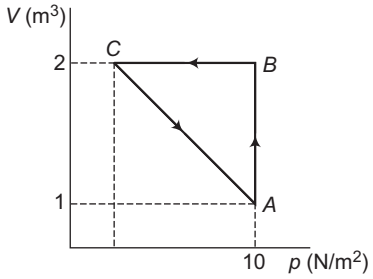
(a) $\frac{v_e}{\sqrt{3}}$ (b) v_e

(c) $2v_e$ (d) $\frac{v_e}{\sqrt{2}}$

15. **Statement I If the centre of mass of a system is at rest, the KE of the system must be zero.**
Statement II The change in velocity depends on the external force.

(a) Both Statement I and Statement II are true and the Statement II is the correct explanation of the Statement I
(b) Both Statement I and Statement II are true but the Statement II is not the correct explanation of the Statement I
(c) Statement I is false but Statement II is true
(d) Both Statement I and Statement II are false

16. **An ideal gas is taken through a cycle A → B → C → A, as shown. If the net heat supplied to the gas in the cycle is 5 J, then work done by the gas in process C → A is**



(a) – 5 J (b) – 10 J (c) – 15 J (d) – 20 J

17. **Two beams of light have intensities of I_0 and $4I_0$ in double slit experiment. The phase difference between the beams is $\frac{\pi}{3}$ at point A and $\frac{\pi}{2}$ at point B. Then, the difference between the resultant intensities at A and B is**

(a) $2I_0$ (b) I_0 (c) $4I_0$ (d) $5I_0$

18. **A and B are two radioactive substances whose half lives are 1 and 2 years respectively. Initially, 100 g of A and 10 g of B is taken. The time (approx) after which they will have same quantity remaining is**

(a) 3.33 yr (b) 6.6 yr (c) 10 yr (d) 5 yr

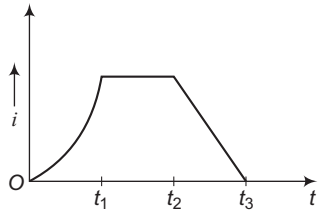
19. **Consider a transistor with $\alpha = 0.98$. This transistor is 1st used as common base amplifier and then as common emitter amplifier. Then, the ratio of power gain in two arrangements shall be (Given $\frac{R_{out}}{R_{in}}$ is same in both cases)**

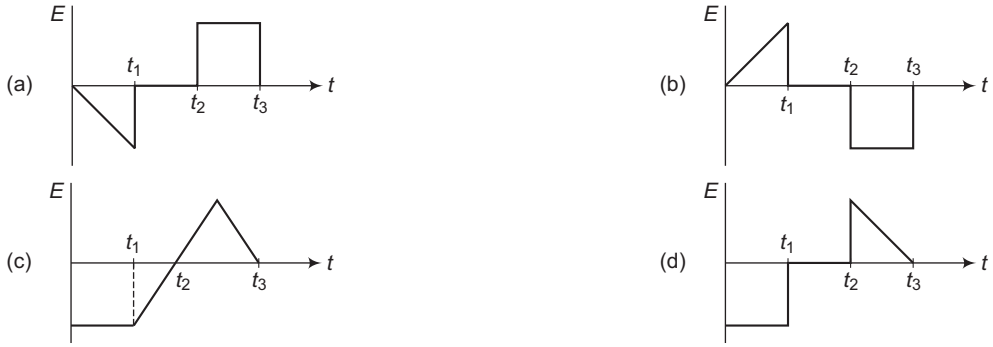
(a) 1 : 2500 (b) 1 : 500 (c) 500 : 1 (d) 2500 : 1

20. **A parallel beam of light has intensity 53.1 W/m^2. Then, its electric and magnetic field would be**

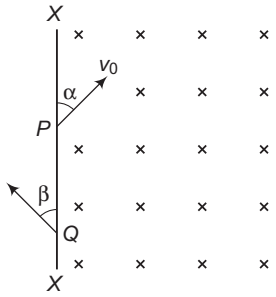
(a) 144 N/C and 4.8×10^{-7} T (b) 144 N/C and 6.7×10^{-7} T
(c) 200 N/C and 4.8×10^{-7} T (d) 200 N/C and 6.7×10^{-7} T

21. **The current i in induction coil varies with time t according to the graph shown below. Which of the following graph shows the induced emf?**





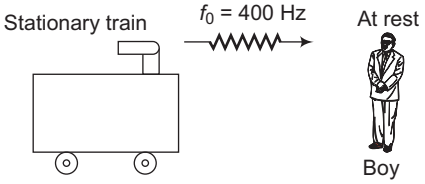
22. **A particle of charge – q and mass m enters in a uniform magnetic field B (directed inwards) at P with a speed v_0 at angle α and leaves the field at Q at angle β. Then, the time for which particle remained in the field is**



(a) $\frac{2m(\beta-\alpha)}{Bq}$ (b) $\frac{m[2\pi-(\alpha+\beta)]}{Bq}$

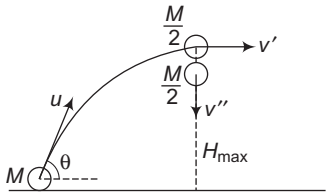
(c) $\frac{2m(\pi-\alpha)}{Bq}$ (d) $\frac{2m(\pi-\beta)}{Bq}$

23. **A boy is standing in front of stationary train. The train blow the horn of frequency 400 Hz. If the air is blowing from train to boy at the speed of 30 m/s, the apparent frequency of sound heard by the boy will be**



(a) 400 Hz (b) 440 Hz (c) 360 Hz (d) 480 Hz

24. **A ball of mass M is projected with a velocity u as shown in the figure. At the highest point it breaks into two equal parts. One part starts moving horizontally with a velocity v′ and the other part starts falling vertically downward with a velocity v′′. Choose the correct option.**



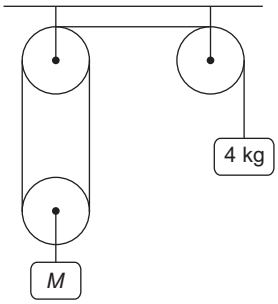
(a) $v' = 2u\cos\theta$ and $v'' = 2u\cos\theta$

(b) $v' = u\cos\theta$ and $v'' = u\cos\theta$

(c) $v' = 2u\cos\theta$ and $v'' = 0$

(d) $v' = u\cos\theta$ and $v'' = 0$

25. **All the pulleys are light and frictionless and string is light. Choose the correct option.**



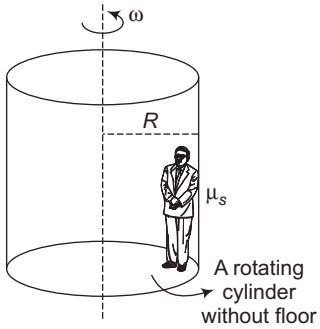
(a) For any value of M, then 4 kg block will move upwards

(b) If $M > 4$ kg, then 4 kg block must move upwards

(c) If $M = 8$ kg, then 4 kg block will move upwards

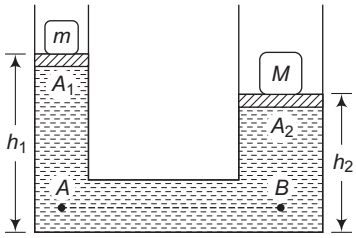
(d) None of the above

26. **A boy of mass m is in touch with the wall of a rotating cylinder. The boy will stay in touch with wall and rotate if**



(a) $\omega \le \sqrt{\frac{g}{\mu_s R}}$ (b) $\omega \le \sqrt{\frac{\mu_s R}{g}}$ (c) $N \le \frac{mg}{\mu_s}$ (d) $N \ge \frac{mg}{\mu_s}$

27. **For the given diagram if the system is in equilibrium and M = 2 m, then relation between two points A and B which are at the same level**



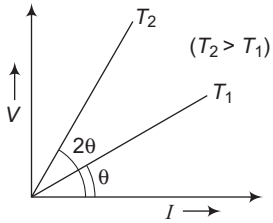
(a) $p_A > p_B$ (b) $p_A = \frac{p_B}{4}$

(c) $p_A = p_B$ (d) $p_A = \frac{p_B}{2}$

28. **The surface of some metal is radiated by waves of $\lambda_1 = 3.5 \times 10^{-7}$m and $\lambda_2 = 6.2 \times 10^{-7}$m respectively. The ratio of the stopping potential in two cases is 2 : 1. The work function of the material is**

(a) 1.05 eV (b) 0.5 eV (c) 1.5 eV (d) 1 eV

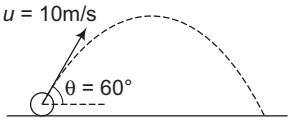
29. **The figure shows variation of current I with voltage of a resistor at different temperatures T_1 and T_2.**
Then, $(T_2 - T_1)$ is proportional to



(a) $\sec^2 2\theta$ (b) $\sec^2 \theta$

(c) $\tan^2 2\theta$ (d) $\operatorname{cosec}^2 \theta$

30. **For the projectile motion, the speed of the projectile when the angle between the initial velocity vector and the velocity vector at that instant become 90° will be**



(a) $10/\sqrt{3}$ m/s (b) $10\sqrt{3}$ m/s

(c) 10 m/s (d) 5 m/s

PART B Chemistry

31. **Arrange the following in decreasing order of the sweetening capacity.**
I. Saccharin, II. Aspartame, III. Alitame

(a) I > II > III (b) III > II > I (c) III > I > II (d) II > I > III

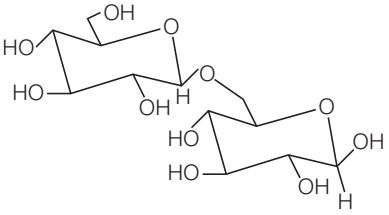
32. **When N_2 is converted into N_2^+ and O_2 is converted into O_2^+, these two processes lead to ..... and ..... bond strength respectively.**

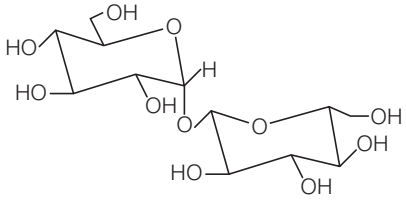
(a) increases, increases (b) decreases, decreases
(c) increases, decreases (d) decreases, increases

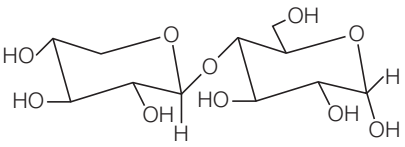
33. **A rigid and insulated tank of 6 m^3 volume is divided into the two compartments. One compartment of volume 5 m^3 contains an ideal gas at 0.8314 MPa and 400 K while the second compartment of volume 1 m^3 contains the same gas at 8.314 MPa and 500 K. If the partition between the two compartments is ruptured, the final temperature of the gas will be**

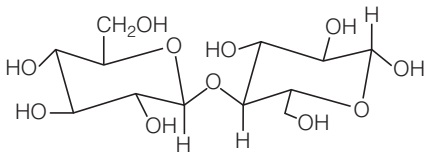
(a) 417 K (b) 523 K (c) 323 K (d) 517 K

34. **β anomer of 4-O-(β D glucopyranosyl) D-glucopyranose is**

(a) 

(b) 

(c) 

(d) 

35. **When a mixture of NO and NO_2 is passed through an aqueous solution of ammonium sulphate, we get**

(a) A dibasic acid whose anhydride in solid state forms a cyclic trimer.
(b) A dibasic acid which has no dehydrating property
(c) A diatomic gas which on reaction with acetylene (C_2H_2) under electric spark gives a monobasic acid whose ion is pseudo halide.
(d) A dibasic acid whose negative ion (di) has all bonds equal with bond angle (90°)

36. **Reaction A + B $\longrightarrow$ C + D follows rate law $R = K[A]^{1/2}[B]^{1/2}$ starting with 1 M of A and B. What is time taken for concentration of A become 0.1 M? [Given, $K = 4.606 \times 10^{-4}\ s^{-1}$]**

(a) 1000 s (b) 100.5 s (c) 2000 s (d) 5000 s

37. **The correct order of ionic radii of Ce, La, Pm and Yb in + 3 oxidation state is**

(a) $Yb^{3+} < Pm^{3+} < Ce^{3+} < La^{3+}$ (b) $La^{3+} < Pm^{3+} < Ce^{3+} < Yb^{3+}$
(c) $La^{3+} < Ce^{3+} < Pm^{3+} < Yb^{3+}$ (d) $Yb^{3+} < Ce^{3+} < Pm^{3+} < La^{3+}$

38. **For a hypothetical hydrogen like atom, the potential energy of the system is given by $U(r) = \frac{-ke^2}{r^4}$, where r is the distance between the two particles. If Bohr's model of quantisation of angular momentum is applicable, then velocity of particle is given by**

(a) $\frac{nh}{16ke\pi^2 m^{3/2}}$ (b) $\frac{n^2h^{2\backprime}}{8k^2e^2\pi^4m^3}$ (c) $\frac{n^3h^3}{2k^2e^3\pi^3m^4}$ (d) $\frac{n^2h^2}{4\sqrt{2}ke\pi^2m^{3/2}}$

39. **The product obtained by condensation polymerisation reaction between A and B is [X]. A can be prepared by reaction of 1, 6-dicyano hexane with water in presence of acid and B can be prepared by the reduction of the same compound.**

[X] is

(a) Nylon 6 (b) Nylon 6 6 (c) Nylon 6 6 6 (d) Buna-*N*

40. **(A) $\xrightarrow{KOH}$ (B) (gas turns red litmus blue)**

(C) $\xrightarrow{Zn + KOH}$ (B) (gas)

(A) $\xrightarrow{Heat}$ Gas (does not support combustion)

(a) $A = NH_4NO_3$, $B = N_2$, $C = KNO_3$ (b) $A = NH_4NO_2$, $B = NH_3$, $C = KNO_2$

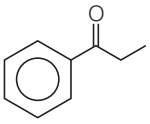
(c) $A = (NH_4)_2SO_4$, $B = NH_3$, $C = KNO_2$ (d) $A = NH_4Cl$, $B = N_2$, $C = KNO_3$

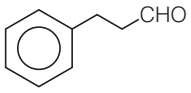
41. **During electro-osmosis of $Al(OH)_3$ solution**

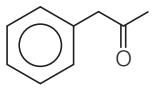
(a) the solution particles move towards anode (b) the dispersion medium moves towards cathode

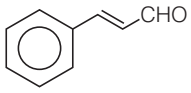
(c) the solution particles move towards cathode (d) the dispersion medium moves towards anode

42. **An organic compound having molecular formula $C_9H_{10}O$ is supplied to a student to determine the structural formula of compound, he observe that compound forms an orange red precipitate with 2,4 DNP reagent and yellow precipitate on heating with I_2 / NaOH. It neither reduces tollens reagent or Fehling solution nor it decolorises bromine water solution. On drastic oxidation with CrO_3, it produces a carboxylic acid having molecular formula $C_7H_6O_2$. Identify the correct compound A.**

(a) 

(b) 

(c) 

(b) 

43. **The vapour pressure of two pure liquids A and B which form an ideal solution are 1000 and 1600 torr respectively at 400 K. A liquid solution of A and B for which the mole fraction of A is 0.60 is contained in a cylinder by a piston on which the pressure can be varied. The solution use slowly vapourised at 400 K by decreasing the applied pressure. What is the composition of last droplet of liquid remaining in equilibrium with vapour?**

(a) $X_A = 0.30$, $X_B = 0.70$ (b) $X_A = 0.40$, $X_B = 0.60$

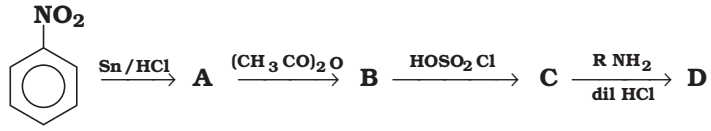
(c) $X_A = 0.70$, $X_B = 0.30$ (d) $X_A = 0.50$, $X_B = 0.50$

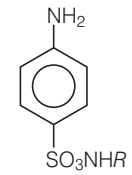
44. **When an inorganic compound X having 3 centre-2-electron as well as $2e^-$—$2e^-$ bonds react with NH_3 gas at a certain temperature, gives a compound Y, isostructural with benzene. Compound X with ammonia at a high temperature produces a substance Z, then**

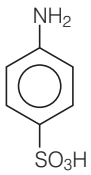
(a) *X* is BH_3, *Y* is $B_2N_2H_3$, *Z* is inorganic benzene (b) *X* is B_2H_6, *Y* is $B_3N_3H_6$, *Z* is in organic graphite

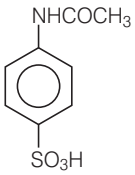
(c) *X* is borax, *Y* is B_2O_3, *Z* is inorganic benzene (d) reactions insufficient to predict

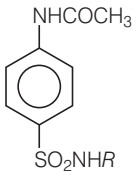
45. **Identify the final product and comment the true statement regarding product**



(a) 

(b) 

(c) 

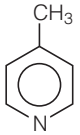
(d) 

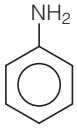
46. **What is the potential of an electrode which originally contained 0.1M NO_3^- and 0.4 M H^+ and which has been treated by 60% of the cadmium necessary to reduce all the NO_3^- to NO(g) at 1 atm**

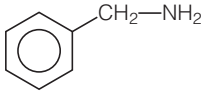
Given, $NO_3^- + 4H^- + 3e^- \longrightarrow NO + 2H_2O$, $E^\circ = 0.95$ V and $\log 2 = 0.3010$

(a) 0.52 V (b) 0.44 V (c) 0.86 V (d) 0.78 V

47. **Molecular formula of an organic compound is C_6H_7N. The degree of unsaturation of the compound is 4. Which of the following may be correct molecular structure of organic compound?**

(i) 

(ii) 

(iii) 

(a) (I) and (II) (b) (II) and (III) (c) (I) and (III) (d) (II), (I) and (III)

48. **Select the correct statement among following.**

(a) Al forms AlF_6^{3-}, an octahedral complex
(b) Be and Al show diagonal relationship
(c) Be forms tetrahedral complex $[Be(C_2O_4)_2]^{2-}$
(d) All of these

49. **Nitric acid can be produced from NH_3 in three step process**

I. $4NH_3(g) + 5O_2(g) \longrightarrow 4NO(g) + 6H_2O(g)$

II. $2NO(g) + O_2(g) \longrightarrow 2NO_2(g)$

III. $3NO_2(g) + H_2O(l) \longrightarrow 2HNO_3(aq) + NO(g)$

% yield of Ist, IInd and IIIrd are respectively 40%, 50% and 70% respectively, then what volume of NH_3 (g) at 1 atm and 0°C required to produce 1075 g of HNO_3?

(a) 3413 L (b) 3500 L (c) 6826 L (d) 1750 L

50. **Two liquids A and B are made of same elements and are diamagnetic. Liquid A on treatment with KI and starch gives blue coloured solution, however liquid B is neutral to litmus and does not give any response to starch iodide paper, then A and B will be**

(a) $A \longrightarrow H_2O_2$, $B \longrightarrow H_2O$
(b) $A \longrightarrow H_2O$, $B \longrightarrow H_2O_2$
(c) $A \longrightarrow H_2O$, $B \longrightarrow H_2O$
(d) $A \longrightarrow H_2O_2$, $B \longrightarrow D_2O$

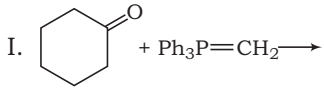
51. **What is the reaction intermediate involved in the preparation of salicylaldehyde from chloroform?**

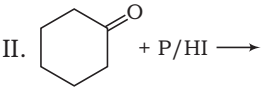
(a) carbocation (b) carbanion (c) carbene (d) None of these

52. **It is known that atoms have proton, neutron, electron and nucleus. If the mass of neutron assumed to be half of its original value whereas that of proton is assumed to be twice the original value, then the atomic mass of $_6C^{14}$ will be**

(a) 14.3% less than original value
(b) 14.3% greater than original value
(c) 28.6% less than original value
(d) 28.6% greater than original value

53. Which of the following processes is used to form alkene?

I. 

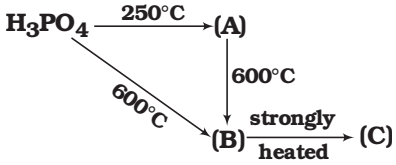
II. 

III. $2CH_3I + Na \longrightarrow$

IV. $CH_3CH_2Cl \xrightarrow{\text{alc. KOH}}$

(a) II and IV (b) only IV (c) I, II and IV (d) II, III and IV

54. Then, A, B and C are



(a) HPO_3, $H_4P_2O_7$ and P_4O_6
(b) $H_4P_2O_7$, HPO_3 and P_4O_6
(c) $H_4P_2O_7$, HPO_3 and P_4O_{10}
(d) HPO_3, $H_4P_2O_7$ and P_4O_{10}

55. What will be the correct relation between product when 2-methyl cyclohexane is treated with
(a) B_2H_6 in presence of H_2O_2 / $OH^{\ominus}$ and (b) H_2O / H_2SO_4 (also consider stereochemistry of product)?

(a) They are metamers
(b) They are tautomers
(c) They are functional isomer
(d) They are positional isomer

56. Which of the following have three (isomeric) forms?

(i) $K_3[CoCl_2(C_2O_4)_2]$
(ii) $[CoBrCl(en)_2]$
(iii) $[Cr(NO_3)_3(NH_3)_3]$
(iv) $K_3[Co(C_2O_4)_3]$

(a) (i), (ii) and (iii)
(b) (i) and (ii)
(c) (ii) and (iv)
(d) (iii) and (iv)

57. Which of the following is correct regarding solubility of hydroxide of group II elements?

(a) $Be(OH)_2 < Mg(OH)_2 < Ca(OH)_2 < Sr(OH)_2 < Ba(OH)_2$
(b) $Be(OH)_2 > Mg(OH)_2 > Ca(OH)_2 > Sr(OH)_2 > Ba(OH)_2$
(c) $Mg(OH)_2 > Be(OH)_2 > Ca(OH)_2 > Sr(OH)_2 > Ba(OH)_2$
(d) $Be(OH)_2 < Mg(OH)_2 > Ca(OH)_2 < Sr(OH)_2 < Ba(OH)_2$

58. The oxidation states of S-atoms in Marshell's and Caro's acid are

(a) + 6, + 7
(b) + 6, + 6
(c) + 7, + 7
(d) + 4, + 4

59. Statement I Diethyl malonate undergo nucleophilic substitutions reaction on reaction with alkyl halide in presence of base.
Statement II Base causes formation of enolate anion.

(a) Both Statement I and Statement II are true and the Statement II is the correct explanation of the Statement I
(b) Both Statement I and Statement II are true but the Statement II is not the correct explanation of the Statement I
(c) Statement I is true but Statement II is false
(d) Both Statement I and Statement II are false

60. **Statement I The dissociation constants of a polyprotic acid are in the order $k_1 > k_2 > k_3$.**

Statement II The $[H^+]$ furnished in first step of dissociation exerts common ion effect to reduce the second dissociation and so on.

(a) Both Statement I and Statement II are true and the Statement II is the correct explanation of the Statement I
(b) Both Statement I and Statement II are true but the Statement II is not the correct explanation of the Statement I
(c) Statement I is true but Statement II is false
(d) Both Statement I and Statement II are false

PART C Mathematics

61. **If $(x + y)^2 \frac{dy}{dx} = a^2$, $y = 0$ when $x = 0$, then $y = a$ if $\frac{x}{a}$ is equal to**

(a) 2 (b) tan 2 (c) tan 2 + 1 (d) tan 1 – 1

62. **The perpendicular distance of point P(0, – 1, 3) from the straight line passing through A (1, – 3, 2) and B(2, – 1, 4) is**

(a) 5/3 unit (b) 5/2 unit (c) 5/4 unit (d) 5 unit

63. **If z lie on the circle $|z - 2i| = 2\sqrt{2}$, then the value of $\arg\left[\frac{z-2}{z+2}\right]$ is equal to**

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

64. **OPQR is a square and M, N are the mid points of the sides PQ and OR respectively. If the ratio of the areas of the square and the triangle OMN is $\lambda : 6$, then λ is equal to**

(a) 2 (b) 4 (c) 12 (d) 16

65. **If $f(x) = \min\{2\sin x, 1 - \cos x, 1\}$, then $\int_0^{\pi} f(x)dx =$**

(a) $\sqrt{3} - 1 + \frac{5\pi}{6}$ (b) $\sqrt{3} - 1 + \frac{2\pi}{3}$ (c) $1 - \sqrt{3} + \frac{2\pi}{3}$ (d) $1 - \sqrt{3} + \frac{5\pi}{6}$

66. **If range of $f(x) = \cos x$, $x \in \left(\frac{-\pi}{3}, \frac{\pi}{6}\right]$ is (a, b], then**

(a) $a + b = \frac{3}{2}$ (b) $b - a = 1 - \frac{\sqrt{3}}{2}$ (c) $a^2 + b^2 = \frac{5}{6}$ (d) $a^2 + b^2 = \frac{7}{4}$

67. **If marks scored by students of a class are 1, 2, 4, ... 2^{10} with frequency 1, 2, 3, 4 ..... 11, then median is**

(a) 128 (b) 64 (c) 32 (d) 16

68. **Let a matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, then it will satisfy the equation**

(a) $A^2 - 4A + I = 0$ (b) $A^2 + 4A + I = 0$ (c) $A^2 - 4A + 5I = 0$ (d) $A^2 - 4A - 5I = 0$

69. **For a random variable X, E(X) = 3 and $E(X^2) = 11$. Then, variable X is**

(a) 8 (b) 5 (c) 2 (d) 1

70. **Consider the function**

$$f(x) = \begin{cases} x \sin(\log x^2), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Then,

(a) f is continuous and differentiable at $x = 0$ (b) f is not continuous but differentiable at $x = 0$

(c) f is neither continuous nor differentiable at $x = 0$ (d) f is continuous but not differentiable at $x = 0$

71. **The number of divisor of 3630, which have a remainder of 1 when divided by 4 is**

(a) 12 (b) 6 (c) 4 (d) None of these

72. **If A, B, C and D are four points in space, then $|AB \times CD + BC \times AD + CA \times BD| = k$. The area of ΔABC, where k is equal to**

(a) 4 (b) 3 (c) 2 (d) 5

73. **If a, b and c are in AP, then $a^3 + c^3 - 8b^3$ is equal to**

(a) $2abc - 8$ (b) $-2b(a + c)$ (c) $-ac^2 - 5a^2c$ (d) $-6abc$

74. **The sum of the squares of the lengths of the chords intercepted by the line $x + y = n$, $n \in N$ on the circle $x^2 + y^2 = 4$ is 11 k, where k is equal to**

(a) 2 (b) 0 (c) Cannot say (d) None of these

75. **A condition that $x^3 + ax^2 + bx + c$ may have no extremum is**

(a) $a^2 \geq 3b$ (b) $b^2 < 3a$ (c) $a^2 < 2b$ (d) None of these

76. **If the equation $k(6x^2 + 3) + rx + 2x^2 - 1 = 0$ and $6k(2x^2 + 1) + p(x) + 4x^2 - 2 = 0$ have both the roots common, then $2r - p$ is equal to**

(a) 2 (b) 1 (c) 0 (d) k

77. **The equation of the plane through the line of intersection of the planes $ax + by + cz + d = 0$ and $\alpha x + \beta y + \gamma z + e = 0$ and perpendicular to xy-plane is**

(a) $(a\gamma - c\alpha)x + (b\gamma - c\beta)y + (d\gamma - ce) = 0$ (b) $(a\gamma + c\alpha)x + (b\gamma - c\beta)y + e = 0$

(c) $(a\gamma - c\alpha)x + (b\gamma - c\beta)y + d = 0$ (d) None of these

78. $\int_0^{10\pi} [\tan^{-} x]dx$, **where [ ] represent greatest integer function**

(a) $10\pi - \tan 1$ (b) $10 - \tan 1$ (c) $10 - \frac{\pi}{4}$ (d) None of these

79. **If coefficient of x^{101} in**

$1 + (1 + x) + (1 + x)^2 + \ldots + (1 + x)^n$

(if $n \geq 100$) is $^{201}C_{101}$, then value of n equals

(a) 202 (b) 100 (c) 200 (d) 201

80. **Consider the three points $A(2\hat{i}+3\hat{j}+5\hat{k})$, $B(-\hat{i}+3\hat{j}+2\hat{k})$ and $C(\lambda\hat{i}+5\hat{j}+\mu\hat{k})$ are vertices of a triangle and its median through A is equally inclined to the positive direction of the axis. The value of $2\lambda-\mu$ is equal to**

(a) 3 (b) 4 (c) 2 (d) 1

81. **If $a > b > c > 0$, then**

$$\cot^{-1}\left(\frac{ab+1}{a-b}\right)+\cot^{-1}\left(\frac{bc+1}{b-c}\right)+\cot^{-1}\left(\frac{ca+1}{c-a}\right) \text{ is}$$

(a) $\frac{\pi}{2}$ (b) π (c) $-\frac{\pi}{2}$ (d) $-\pi$

82. **If $A=\begin{bmatrix}\frac{-1+i\sqrt{3}}{2i} & \frac{-1-i\sqrt{3}}{2i}\\ \frac{1+i\sqrt{3}}{2i} & \frac{1-i\sqrt{3}}{2i}\end{bmatrix}$, $i=\sqrt{-1}$ and $f(x)=x^2+2$. Then, f(A) is equal to**

(a) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ (b) $\begin{bmatrix}\frac{3-i\sqrt{3}}{2}\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ (c) $\begin{bmatrix}\frac{5-i\sqrt{3}}{2}\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ (d) $(2+i\sqrt{3})\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

83. **A 2×2 matrix is formed with entries from the set $\{0, 1\}$. The probability that it is singular is**

(a) $\frac{5}{8}$ (b) $\frac{1}{8}$ (c) $\frac{5}{16}$ (d) $\frac{3}{16}$

84. **The condition for the two tangents drawn from a point not lying in the y-axis to the parabola $y^2=4ax$, to become normals to the parabola $x^2=4by$ is that**

(a) $ab>4$ (b) $b^2>a^2$ (c) $ab^2\geq 8$ (d) $a^2>8b^2$

85. **Which among the following statements is false?**

(a) The statement $p\leftrightarrow q$ has its truth value true (T) when both the statements p and q are false.
(b) The statement $p\leftrightarrow q$ has its truth value true (T) when both the statements p and q are true.
(c) The statement $p\rightarrow q$ has its truth value true (T) when statement q is either true or false.
(d) The statement $p\rightarrow q$ has its truth value false (F) when among the statements p and q only p is true.

86. **If $y=\cos^{-1}\left[\frac{\cos x+4\sin x}{\sqrt{17}}\right]$, then $\frac{dy}{dx}$ is**

(a) 1 (b) $\sqrt{17}$ (c) 4 (d) None of these

87. **Consider the equation**

$$4^{\sin x}+3^{\sec y}=1 \quad \text{and} \quad 5(16)^{\sin x}-2(3)^{\sec y}=2$$

Then, the values of x and y respectively are

(a) $x=n\pi+\frac{\pi}{6}, y=2m\pi+\frac{\pi}{3}: m,n\in z$ (b) $x=n\pi-\frac{\pi}{6}, y=2m\pi+\frac{\pi}{3}: m,n\in z$

(c) $x=n\pi+(-1)^n\frac{\pi}{6}, y=2m\pi\pm\frac{\pi}{3}: m,n\in z$ (d) $x=n\pi+(-1)^n\frac{\pi}{6}, y=2m\pi-\frac{\pi}{3}: m,n\in z$

88. **If in a triangle ABC, $A=(1, 10)$, circumcentre $O=\left(-\frac{1}{3},\frac{2}{3}\right)$ and orthocentre $H=\left(\frac{11}{3},\frac{4}{3}\right)$, then the coordinates of mid-point of side opposite to A is**

(a) $\left(1,\frac{-11}{3}\right)$ (b) (1, 5) (c) (1, – 3) (d) (1, 6)

89. **Statement I** Let X and Y be two sets then, $X \cap (Y \cup X)' = \phi$

Statement II If $X \cup Y$ has m elements and $X \cap Y$ has n elements, then the symmetrical difference $X \Delta Y$ has $(m - n)$ elements.

(a) Statement I is true, Statement II is also true and Statement II is the correct explanation of the Statement I

(b) Statement I is true, Statement II is also true and Statement II is not the correct explanation of the Statement I

(c) Statement I is true but Statement II is false

(d) Statement I is false but Statement II is true

90. **Statement I** $x \sin x \frac{dy}{dx} + (x + x \cos x + \sin x)y = \sin x, y\left(\frac{\pi}{2}\right) = 1 - \frac{2}{\pi} \Rightarrow \lim_{x \to 0} y(x) = \frac{1}{3}$

Statement II The differential equation is linear with integrating factor $x(1 - \cos x)$.

(a) Statement I is true, Statement II is also true and Statement II is the correct explanation of the Statement I

(b) Statement I is true, Statement II is also true and Statement II is not the correct explanation of the Statement I

(c) Statement I is true but Statement II is false

(d) Statement I is false but Statement II is true

Analytical Explanations

Physics

1. (*d*) **Idea** Here rigid body is moving with constant velocity *i.e.,* it is in mechanical equilibrium.

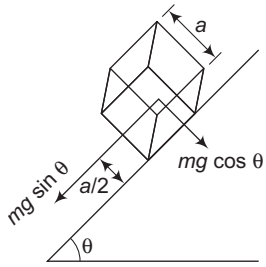
$\Rightarrow \quad \Sigma F_{ex} = 0$, *i.e.* $\Sigma F_x = 0$, $\Sigma F_y = 0, \Sigma F_z = 0$

and $\Sigma \tau_x = 0, \Sigma \tau_y = 0, \Sigma \tau_z = 0$

Because the cubical block slides with a uniform velocity and does not topple.

Hence, torque produced by weight

= torque due to normal force on the block



∴ Torque due to normal force

= torque due to weight

= component of weight parallel to plane

× perpendicular distance from lower face

$= (mg \sin\theta)\frac{a}{2}$

TEST Edge Other questions in which external forces and torque may not be zero can be asked so students are advised to remember important relation of rotational and translational motion *i.e.,*

$$\frac{d(\mathbf{L})}{dt} = \tau_{ext}, \frac{d\mathbf{p}}{dt} = F_{ext}, \mathbf{L} = \Sigma(\mathbf{r} \times \mathbf{p}) \text{ etc.}$$

2. (*c*) At the instant of collision we can apply conservation of linear momentum for the system of block and bullet.

$\Rightarrow \quad mv + Mv' = (2 \times 2) - (0.2 \times 20)$

$= 0$ [before collision]

So, the system of block and bullet will come to rest just after collision.

So, amplitude $A = 0$

3. (*b*) **Idea** Here Newton's law of cooling will be used *i.e.,* rate of cooling $\frac{dT}{dt} = -k(T - T_0)$ or (rate of cooling) ∝ (Difference in temperature)

From the Newton's law of cooling,

Rate of cooling ∝ difference in temperature

$$\frac{dT}{dt} \propto \Delta\theta$$

$\Rightarrow \quad \frac{dT}{dt} = k\Delta\theta$

In first case, $dT = 61 - 59 = 2$

$\Delta\theta = 60 - 30 = 30$

$dt = 4 \text{ min}$

$\therefore \quad k = \frac{1}{\Delta\theta}\left(\frac{dT}{dt}\right) = \frac{2}{30 \times 4} = \frac{1}{60}$

For second case,

$dT = 2$

$\Delta\theta = 50 - 30 = 20$

$\therefore \quad dt = \frac{1}{k}\left(\frac{dT}{d\theta}\right) = \frac{2}{\frac{1}{60} \times 20} = 6 \text{ min}$

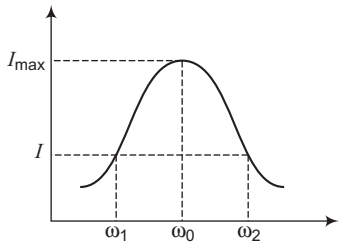
TEST Edge Question based on Stefan's law, Wien's displacement law etc., are frequently asked and are very important, so students must remember all important laws.

i.e., Stefan's law $\Rightarrow [H = \sigma_e A T^4]$

Wien's displacement law $\Rightarrow [\lambda_m T = b]$ etc.

4. (*d*) **Idea** In a given *L-C-R* circuit, current will be same at different frequencies if impedance Z is same at difference frequency also if current in the circuit will be maximum if circuit is in resonance.

Typical graph of current *versus* frequency of *L-C-R* circuit is



Now, as resistance is independent of frequency. So, current will same when Z at two frequencies is same. At lower frequency capacitive reactance is more but at higher frequencies inductive reactance dominates.

So, $\quad Z_{1 \text{ at } \omega_1} = \frac{1}{\omega_1 C} - \omega_1 L$

$$Z_{2 \text{ at } \omega_2} = \omega_2 L - \frac{1}{\omega_2 C}$$

For current to be same

$Z_1 = Z_2$

$\Rightarrow \quad \frac{1}{\omega_1 C} - \omega_1 L = \omega_2 L - \frac{1}{\omega_2 C}$

$$\Rightarrow \quad \left(\frac{1}{\omega_1}+\frac{1}{\omega_2}\right)\frac{1}{C}=(\omega_1+\omega_2)L$$

$$\Rightarrow \quad \frac{\omega_1+\omega_2}{\omega_1\omega_2}=(\omega_1+\omega_2)LC$$

Now, current is maximum at $\omega_0=\dfrac{1}{\sqrt{LC}}$

$\therefore$ Resonant frequency, $\omega_0=\sqrt{\omega_1\omega_2}$

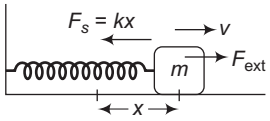
For given problem, $\omega_0=\sqrt{8\times 18}=12$ MHz

TEST Edge Resonance in series *L-C-R* circuit is asked almost every year so students must know all important result in these types of circuits.

i.e., at resonance Z is minimum, i is maximum, resonant frequency $f_0=\dfrac{1}{2\pi\sqrt{LC}}$ etc.

5. (*a*) There will be no change due to the position of the slab. So, apparent depth $h'=\dfrac{h}{\mu}$.

6. (*c*) $F_s=kx$



At $x=\dfrac{mg}{k}$

$$F_s=kx=k\frac{mg}{k}=mg$$

So, $F_s=F_{ext}$

But at this point, the block must have gain some KE as its motion was accelerated.

So, $W_{F_{ext}}+W_{F_s}=\Delta KE$

[from work energy theorem]

$$W_{F_{ext}}+W_{F_s}>0$$

7. (*d*) Initially electron is in excited state $n=2$.

Now, $E_3-E_2=\dfrac{-13.6}{9}+\dfrac{13.6}{4}=1.9$ eV

Incident electron energy is $>(E_3-E_2)$

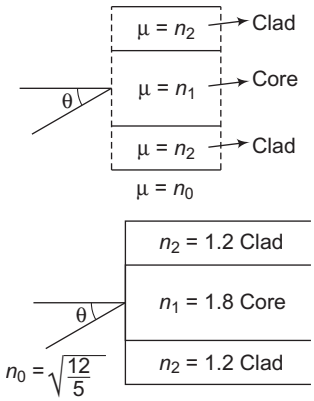
So, its capable of exciting it to $n=3$ state. In that case the KE of incident electron would be converted into KE. Thus, collision would be inelastic [as KE is not conserved]. Also the incident electron can completely transfer the KE to H-atom without exciting it further. So, the collision can be elastic as well. And there exist a third possibility also in which, the excited atom may de-excite to $n=1$ state and thus transfer its extra energy to incident electron. Which would increase its KE further. In this case collision would be super-elastic.

i.e., KE of particles after collision > KE of particles before collision

Note If H-atom were in ground state and incident electron energy were less than (E_2-E_1) which is equal to 10.2 eV. Then, collision would always be elastic.

8. (*c*) **Idea** In optical fibre maximum angle of acceptance is given by

$$\sin\theta=\frac{\sqrt{n_1^2-n_2^2}}{n_0}$$



Maximum angle of acceptance is given by

$$\sin\theta=\frac{\sqrt{n_1^2-n_2^2}}{n_0}=\frac{\sqrt{1.8^2-1.2^2}}{\sqrt{12/5}}$$

$$=\sqrt{(1.8-1.2)(1.8+1.2)}\times\sqrt{\frac{5}{12}}$$

$$\Rightarrow \quad \sin\theta=\sqrt{0.6\times 3}\times\sqrt{\frac{5}{12}}=\frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad \theta=\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow \quad \theta=60^\circ$$

TEST Edge Questions based on optical are frequently asked in examination, students are advised to study important terms like cladding, principle of optical fibre etc.

9. (*a*) Clearly Q_1 is +ve and Q_2 is –ve because potential energy due to +ve charge is + ve and –ve charge is –ve.

Secondly $|Q_2|$ is more than $|Q_1|$ as seen from graph. Also potential energy at point A would be $U=qV$.

For stable equilibrium, U should be minimum and U would be minimum only when charge placed at A is + ve. Thus, stable equilibrium would depend on the sign of charge placed at A and thus statement 3 is not always true.

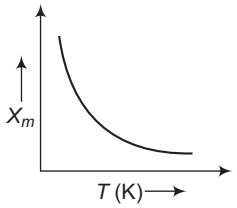
10. (*d*) **Idea** For paramagnetic substances,

$$X_m=\frac{C}{T}\left[\begin{matrix}C: & \text{Curie constant}\\ T: & \text{absolute temperature}\end{matrix}\right] \text{ and } X_m>0$$

X_m for paramagnetic is always + ve. It's related to temperature by Curie's law as given below

$$X_m=\frac{C}{T}$$

So, X_m decreases as T increases but not exponentially



TEST Edge Question related to different magnetic materials *i.e.,* diamagnetic materials, paramagnetic materials and ferromagnetic materials are asked frequently students are advised to study these materials and their properties.

11. (*d*) $E_{eq} = \dfrac{\dfrac{E_1}{r_1} + \dfrac{E_2}{r_2}}{1/r_1 + 1/r_2} \Rightarrow E_{eq} = \dfrac{E_1 r_2 + E_2 r_1}{r_1 + r_2}$

or $E_{eq} = E_1 \left[\dfrac{r_2 + r_1 (E_2/E_1)}{r_1 + r_2}\right]$

So, E_{eq} can be greater than, less than or equal to E_1 depending upon the condition whether

$$E_2 > E_1, E_2 = E_1 \text{ or } E_2 < E_1$$

Also $r_{eq} = \dfrac{r_1 r_2}{r_1 + r_2}$ which is less than both of r_1, r_2.

12. (*c*) **Idea** Here, initial charge on capacitor is $2Q_0$, battery is attached but with opposite polarity also potential difference is $\dfrac{Q_0}{L}$. So, total charge through battery = $Q_0 - (-2Q_0) = 3Q_0$ and work done by battery = (charge supplied) × (potential)

Charge through the battery = $Q_0 - (-2Q_0)$

$$= 3Q_0$$

Work done by battery = $\dfrac{(3Q_0)\, Q_0}{C} = \dfrac{3Q_0^2}{C}$

TEST Edge Question related to work done by battery, force between plates of a perallel plates capacitor etc., may be asked in examination. It will be convenient for student to memorise basic concept related to these topics.

13. (*c*) Not all the elastic materials follow Hooke's law over a considerable range for *e.g.*, elastomers don't follow Hooke's law.

$$\Delta L = \frac{\text{Force}}{\text{Area} \times Y} \times L$$

So, ΔL depend on length and area of material as well.

If ultimate point and breaking point is far away, then material is ductile otherwise it would be brittle.

Hooke's law is followed till elastic point in elastic materials.

14. (*a*) From mechanical energy conservation,

$$\left[\frac{-GmM_e}{R_e} + \frac{1}{2}mv'^2\right]_i = \left[-\frac{GmM_e}{r} + (\text{KE})_f\right]$$

$$\frac{-GmM_e}{R_e} + \frac{1}{2}mv'^2 = \frac{-GmM_e \times 2}{3R_e} + 0 \quad \left[r = \frac{3R_e}{2}\right]$$

$$\frac{1}{2}mv'^2 = \frac{-GmM_e \times 2}{3R_e} + \frac{GmM_e}{R_e}$$

$$\frac{1}{2}mv'^2 = \frac{GmM_e}{R_e}\left[\frac{1}{3}\right]$$

$$v'^2 = \frac{2GM_e}{3R_e} \quad \text{or} \quad v' = \frac{v_e}{\sqrt{3}}$$

15. (*c*) Statement I is false but the Statement II is true.

If a nucleus at rest explodes into two parts, due to internal force its CM will be at rest but the KE of the system is non-zero.

$$\Rightarrow \quad \mathbf{F}_{ext} = m\mathbf{a}$$

16. (*a*) **Idea** In a cyclic process $\Delta U = 0$

$$\Rightarrow \quad \Delta Q = \Delta W \quad [\text{by } \Delta Q = \Delta W + \Delta U]$$

$$\Delta W_{AB} = p\Delta V = (10)(2-1) = 10\,\text{J}$$

$$\Delta W_{BC} = 0 \quad (\text{as } V = \text{constant})$$

From first law of thermodynamics,

$$\Delta Q = \Delta W + \Delta U$$

$\Delta U = 0$ (process $ABCA$ is cyclic)

$$\therefore \quad \Delta Q = \Delta W_{AB} + \Delta W_{BC} + \Delta W_{CA}$$

$$5 = 10 + 0 + \Delta W_{CA}$$

$$\Delta W_{CA} = -5\,\text{J}$$

TEST Edge Question related to work done in p-V curve are frequently asked in examination also conversion of p-V graph to p-T graph can be asked. Students are advised to study various graphs in thermodynamics thoroughly.

17. (*a*) **Idea** Two coherent sources having intensity I_1 and I_2 when undergoes superposition produce resultant wave of intensity given by,

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

Given, $I_1 = I_0$, $I_2 = 4I_0$

Then, $I_A = I_0 + 4I_0 + 2 \times \sqrt{I_0 \times 4I_0} \times \cos\dfrac{\pi}{6}$

$$= 5I_0 + 2I_0 = 7I_0$$

and $I_B = I_0 + 4I_0 + 2\sqrt{4I_0^2} \times \cos\pi/2$

$$= 5I_0 + 0 = 5I_0$$

$\therefore I_A - I_B = 7I_0 - 5I_0 = 2I_0$

TEST Edge Question related to resultant wave amplitude can also be asked. Resultant wave amplitude is given by $A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\phi}$. Students are also advised to learn formulae for condition of constructive interference and destructive interference.

18. (*b*) $N = N_0 \times \left(\frac{1}{2}\right)^{t/T}$

where T is half-life of radioactive substance.

$\therefore \quad N_A = 100\left(\frac{1}{2}\right)^{t/1}$ and $N_B = 10\left(\frac{1}{2}\right)^{t/2}$

For required condition, $N_A = N_B$

$\Rightarrow \quad 100\,(1/2)^t = 10\,(1/2)^{t/2}$

$\Rightarrow \quad 10 = \left(\frac{1}{2}\right)^{-t/2}$

Taking log on both sides,

$\Rightarrow \quad \log_{10} 10 = +\, t/2 \log_{10} 2$

$\Rightarrow \quad 1 = \frac{t}{2} \times 0.3010 \Rightarrow t = \frac{2}{0.3010} = 6.6$ yr

19. (*a*) **Idea** In a junction transistor $\beta = \frac{\alpha}{1-\alpha}$ and power gain for common box $= \alpha^2 \frac{R_{out}}{R_{in}}$ and power gain in common emitter $= \beta^2 \frac{R_{out}}{R_{in}}$

Since, $\alpha = 0.98$

Then, $\beta = \frac{\alpha}{1-\alpha} = \frac{0.98}{0.02} = 49$

Now, power gain $= \alpha^2 \frac{R_{out}}{R_{in}}$ (for common base)

Power gain $= \beta^2 \cdot \frac{R_{out}}{R_{in}}$ (for common emitter)

$\therefore \quad$ Ratio $= \frac{\alpha^2}{\beta^2} = \frac{0.98 \times 0.98}{49 \times 49} = \frac{1}{2500}$

TEST Edge The junction transistor is an important topic in electronic devices and question may come to find base current (I_B), collector current (I_C) and emitter current (I_E), students are advised to memorise all important formulae like $I_E = I_B + I_C$, $\alpha = \frac{I_C}{I_E}$, $\beta = \frac{I_C}{I_B}$ etc.

20. (*d*) Intensity $I = \frac{1}{2} \times \varepsilon_0 E_0^2 C$

$\Rightarrow \quad 53.1 = \frac{1}{2} \times 8.85 \times 10^{-12} \times E_0^2 \times 3 \times 10^8$

$E_0 = \sqrt{\frac{2 \times 53.1}{8.85 \times 3 \times 10^{-4}}} = \sqrt{\frac{2 \times 53.1}{26.55 \times 10^{-4}}}$

$\Rightarrow \quad E_0 = \sqrt{4 \times 10^{+4}} = 2 \times 10^2$

$= 200$ N/C

Further $B_0 = \frac{E_0}{C} = \frac{200}{3 \times 10^8}$

$= 6.67 \times 10^{-7}$ T

21. (*b*) Induced emf, $E = -L\, di/dt$

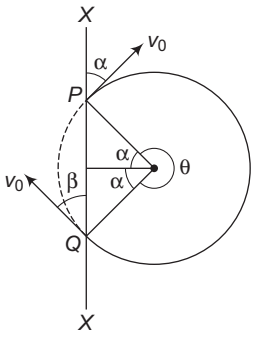
i.e., $E \propto -di/dt$ *i.e.*, $E \propto -$(slope)

Now from 0 to t_1 slope is increasing (+ ve) with time starting with zero slope at $t = 0$

Form t_1 to t_2 slope is zero and from t_2 and t_3 slope is constant and negative.

22. (*c*) **Idea** Whenever a charged particle enters a uniform magnetic field with its direction of velocity perpendicular to magnetic field then it will describe a circular path.

The trajectory of the particle shall be



v_0 will be tangent to the circle when particle enters and leaves the field.

Now, distance travelled by particle $= \frac{\theta}{2\pi} \times 2\pi r$

As $\theta = 2\pi - 2\alpha = 2(\pi - \alpha)$

Distance travelled $= 2(\pi - \alpha) r$

$\therefore$ Time spent by particle in field is

$T = \frac{2(\pi - \alpha) r}{v_0}$

Now, $\frac{mv_0^2}{r} = Bqv_0$

[$\because$ Magnetic force will provide the necessary centripetal force]

$\therefore \quad r = \frac{mv_0}{Bq}$

$\therefore \quad T = \frac{2m(\pi - \alpha)}{Bq}$

TEST Edge Questions involving above concept are frequently asked in various ways like what will radius of circle traced $\left(r = \frac{mv}{qB}\right)$ etc.

23. (*a*) **Idea** If there is no relative motion between source and observer then frequency heard by observer will not change.

Here, the relative motion between the train and boy is zero so the apparent frequency to boy will not change (= 400 Hz).

If air blow, it will just change the speed of sound, it will not create any relative motion between source and observer.

TEST Edge Question based on change in frequency due to motion of source and observer are frequently asked students are advised to go through all cases of Doppler effect to solve such problems.

24. (*c*) **Idea** There is no external force on the block internal force break the block in two parts so linear momentum will be conserved.

v'' has to zero to keep the Y component of momentum to be zero.

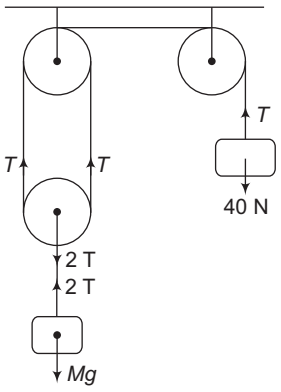
$\Rightarrow$ Conservation of momentum along x-axis

$$M \times u\cos\theta = \frac{M}{2}v'$$

$$v' = 2u\cos\theta$$

TEST Edge Questions related to linear momentum conservation in the form of collision are asked frequently, Students are advised to learn concept which involve change in energy, change in velocity etc before and after collision.

25. (*d*) (i) If $M = 8$ kg



$$T = 40\text{ N} \quad \ldots(i)$$

$$2T = 8 \times 10 = 8\text{ N} \quad \ldots(ii)$$

$\Rightarrow$ $a = 0$

(ii) If $M > 8$ kg, then the 4 kg block will move upwards.

26. (*d*) **Idea** When a body moves along a circular path, a centripetal force will start to act which is directed towards centre.

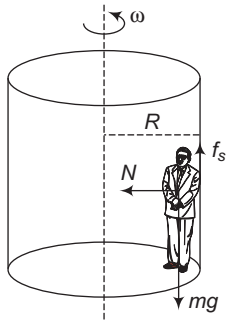
$N = mr\omega^2$ …(i)

Here, N will provide required F_c.

But $(f_s)_{max} = \mu_s N$ …(ii)

f_s will balance the weight.

$\Rightarrow$ $mg \le (f_s)_{max}$ …(iii)



From Eqs. (ii) and (iii), we get

$$mg \le \mu_s N \quad \ldots(iv)$$

From Eqs. (i) and (iv), we get

$$mg \le \mu_s mr\omega^2$$

$\Rightarrow$ $$\omega \ge \sqrt{\frac{g}{\mu_s r}} \quad \ldots(v)$$

From Eq. (iv), $N \ge \dfrac{mg}{\mu_s}$

TEST Edge Question related to uniform and non-uniform circular motion are frequently asked, some questions based on (work-energy theorem + circular motion) are in trend so students are advised to study these concept thoroughly.

27. (*c*) From Pascal's law, the pressure at same depth must be same for static liquid.

$\Rightarrow$ $p_A = p_B$

28. (*b*) **Idea** Stopping potential,

$$V = \frac{\text{maximum KE of fastest electron}}{\text{charge on electron}}$$

$$\Rightarrow V = \frac{\text{Incident photon energy} - \text{work function of metal}}{\text{charge on electron}}$$

$$\text{Stopping potential} = \frac{\text{Incident photon energy} - \text{work function of metal}}{e\ (i.e., \text{charge of electron})}$$

$\Rightarrow$ $$\frac{V_1}{V_2} = \frac{\frac{hc}{\lambda_1} - W}{\frac{hc}{\lambda_2} - W}$$

$\Rightarrow$ $$\frac{2}{1} = \frac{\frac{12375}{3500} - W}{\frac{12375}{6200} - W} \qquad [\because hc = 12375\text{ eV \& Å}]$$

$\Rightarrow$ $$\frac{2}{1} = \frac{3.5 - W}{2 - W}$$

$\Rightarrow$ $4 - 2W = 3.5 - W$

$\Rightarrow$ $W = 0.5$ eV

TEST Edge Question related to Einstein's photoelectric equation *i.e.*, $\frac{1}{2}mv_{max}^2 = h(\nu - \nu_0)$ is asked almost every year also laws of photoelectric emission can also be asked so students are advised to go through these topics thoroughly.

29. (*b*) **Idea** Slope of voltage *versus* current graph will give resistance and resistance increases with increasing temperature according to relation.

$R_T = R_0(1 + \alpha T)$.

$R_{T_2} = R_{T_1}(1 + \alpha \Delta T)$

So, $\Delta T \propto \frac{R_{T_2} - R_{T_1}}{R_{T_1}}$

i.e., $T_2 - T_1 \propto \frac{R_{T_2} - R_{T_1}}{R_{T_1}}$

Now, resistance $R = \frac{V}{I}$ = slope

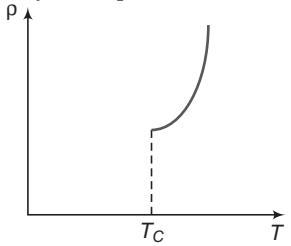
Hence, $R_{T_2} = \tan 2\theta, R_{T_1} = \tan\theta$

$\Rightarrow \quad T_2 - T_1 \propto \frac{\tan 2\theta - \tan\theta}{\tan\theta}$

$= \frac{\frac{2\tan\theta}{1-\tan^2\theta} - \tan\theta}{\tan\theta}$

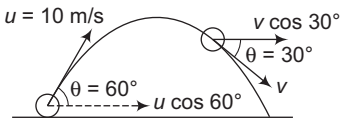
$\Rightarrow \quad (T_2 - T_1) \propto (1 + \tan^2\theta) = \sec^2\theta$

TEST Edge Graphical variation of resistivity (ρ) with absolute temperature (T) for various substance can also be asked including superconductors. So, students are advised to pay proper attention to graphs. *e.g.*, variation of resistivity for superconductor.



Critical temperature

30. (*a*)



As, we know that horizontal component always remains constant.

$u\cos 60° = v\cos 30°$

$v = \frac{u(1/2)}{(\sqrt{3})/2} = \frac{u}{\sqrt{3}}$

$= 10/\sqrt{3}$ m/s

Chemistry

31. (*c*) This problem includes conceptual mixing of sweetening agent and their sweetening capacity

Sweetening agent The substances other than sugar which causes the sweetening of food and enhances the odour and flavour are known as sweetening agent. Commonly used sweetening agent are saccharin, Aspartame, Alitame etc.

Comparative study of sweetening agent Comparative study of sweetening agent can be done with respect to sugar and are as follows

Sweetening capacity of saccharin

= 500 × sweetening capacity of sugar

Sweetening capacity of aspartame

= 180 × sweetening capacity of sugar

Sweetening capacity of alitame

= 2000 × sweetening capacity of sugar

Hence, the correct order of sweetening capacity is represented by choice (c).

32. (*d*) **Idea** This problem involves conceptual mixing of molecular orbital electronic configuration of diatomic molecule and bonds strength.

While solving this problem students are advised to go through knowledge of MOT including their determination of bond order.

Write the molecular orbital electronic configuration and then calculate bond order using formula

$$\text{B.O.} = \frac{N_b - N_a}{2}$$

Now compare bond strength of given species

MOEC of N_2

$= \sigma_{1s^2}\ \sigma^*_{1s^2}\ \sigma_{2s^2}\ \sigma^*_{2s^2}\ \pi_{2p_x^2} \equiv \pi_{2p_y^2}\ \sigma_{2p_z^2}$

Bond order $= \frac{10-4}{2} = 3$

MOEC of $N_2^- = \sigma_{1s^2}\ \sigma^*_{1s^2}\ \sigma_{2s^2}\ \sigma^*_{2s^2}\ \pi_{2p_x^2} = \pi_{2p_y^2}\ \sigma_{2p_z^1}$

Bond order $= \frac{9-4}{2} = 2.5$

Here, on converting N_2 to N_2^+ bond order decreases.

MOEC of $O_2 = \sigma_{1s^2}\ \sigma^*_{1s^2}\ \sigma_{2s^2}\ \sigma^*_{2s^2}\ \sigma_{2p_z^2}$

$\pi_{2p_x^2} \equiv \pi_{2p_y^2}\ \pi^*_{2p^1{}_x} \equiv \pi^*_{2py^1}$

Bond order $= \frac{10-6}{2} = 2$

MOEC of $O_2^+ = \sigma_{1s^2}\ \sigma^*_{1s^2}\ \sigma_{2s^2}\ \sigma^*_{2s^2}\ \sigma_{2p_z^2}$

$\pi_{2p_x^2} \equiv \pi_{2p_y^2}\ \pi^*_{2p^1{}_x} \equiv \pi^*_{2p_y}$

$$\text{Bond order} = \frac{10-5}{2} = 2.5$$

On conversion of O_2 to O_2^+ leads to increase in bond order hence bond strength also increases.

$\because$ Bond strength $\propto$ Bond order

TEST Edge Students are advised to understand the clear concept of MOT and their application on determination of bond order, bond strength along with study of magnetic properties and colour of diatomic molecule which may also be asked.

33. (*a*) **Idea** While solving this problem students are adivsed to go through concept of ideal gas equation and internal energy determine the number of moles of each components, then calculate the value of final temperature using following equation

$$\Delta U = nC_p \Delta T$$

Mole of the gas in the first compartment

$$n_1 = \frac{p_1V_1}{RT_1} = \frac{0.8314 \times 10^6 \times 4}{8.314 \times 400}$$

$$= \frac{8.314 \times 10^5 \times 4}{8.314 \times 400} = \frac{10^5}{100} = 1000$$

and $$n_2 = \frac{0.8314 \times 10^6 \times 1}{8.314 \times 500}$$

$$= \frac{8.314 \times 10^5}{8.314 \times 500} = \frac{10^5}{5 \times 100}$$

$$= \frac{1000}{5} = 200$$

The tank is rigid and insulated, hence $W = 0$, $q = 0$, therefore $\Delta U = 0$

$$\Delta U = nC_V \Delta T$$

$$\Rightarrow \quad 1000\,(T_f - 400) + 200\,(t_f - 500) = 0$$

$$1200\,T_f = 1000 \times 400 + 500 \times 200$$

$$T_f = \frac{1000 \times 4 + 500 \times 2}{12}$$

$$= \frac{5000}{12}$$

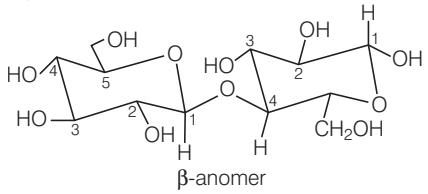
$$T_f = 417\,\text{K}$$

TEST Edge These type of question are asked in JEE Main generally, students are advised to study similar question including conceptual mixing of enthalpy change, entropy change with it.

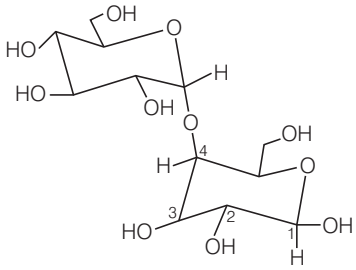
34. (*d*) **Idea** This problem includes conceptual mixing of structure of disacharides and anomer.

Problem related to structure of carbohydrates are solved by using the pyranose structure and looking towards the substituents present in axial or equitorial position.

Structure of 4-0 (D-glucopyranosyl-D -glucopyranose) is written by linking the OH of 4th carbon of β D-glucopyranose with 1st carbon of another D-glucopyranose as shown



β-anomer



Anomer The isomers of disaccarides differing in configuration of C – 1 is known as **anomer**.

β-anomer has OH present at equatorial position.

TEST Edge Similar type of question including structure of amino acids and nucleic acid may also be asked in JEE Main .

35. (*c*) $NO + NO_2 + (NH_4)_2SO_4 + H_2O \longrightarrow$

$$2NH_4NO_2 + H_2SO_4 \longrightarrow 2N_2 + 4H_2O$$

$$N_2 + C_2H_2 \xrightarrow[\text{arc}]{\text{Electric}} 2HCN$$

CN^- is a pseudo halide.

36. (*d*) **Idea** This problem can be solved by using the concept of rate of reaction in first order reaction. Student are advised to write the chemical reaction and the rate law expression for the chemical reaction followed by calculating the required parameter using equation

$$K = \frac{2.303}{t} \log \frac{a}{(a-x)}$$

	A	+	B	$\longrightarrow$	C	+	D
$t = 0$	1		1		0		0
$t = t$	$1-x$		$1-x$		x		x

$$R = K[A]^{1/2}\,[B]^{1/2}$$

$$\frac{dx}{dt} = K\,(1-x)^{1/2}\,(1-x)^{1/2}$$

$$\frac{dx}{dt} = K(1-x)$$

Integrating on both sides

$$K = \frac{2.303}{t} \log \frac{1}{1-x}$$

$$4.606\times10^{-4}=\frac{2.303}{t}\log\frac{10}{0.1}$$

$$2\times10^{-4}=\frac{1}{t}\text{ or }t=\frac{10^4}{2}=\frac{10000}{2}=5000\text{ s}$$

TEST Edge These types of questions are generally asked in JEE Main students are advised to go through study of rate constant determination for various order reaction, which may also be asked.

37. (*a*) **Idea** This problem is based on periodic trend of atomic radii of lanthanide. Students are advised to know the clear concept of atomic size of lanthanides.

Lanthanide contraction decreases atomic sizes from La^{3+} to Lu^{3+}, hence (a) will be correct order.

TEST Edge Students are advised to understand the basic trend of complexing ability, basic character of the oxides of lanthanide which may be asked in JEE Main.

38. (*d*) $\frac{d[U(r)]}{dr}=\frac{4ke^2}{r^5}=\text{force}$

$$\frac{4ke^2}{r^5}=\frac{mv^2}{r}\text{ and }mvr=\frac{nh}{2\pi}$$

or $$r=\frac{nh}{2\pi mv}\Rightarrow\frac{1}{r}=\frac{2\pi mv}{nh}$$

$$4ke^2\times\frac{1}{r^5}=\frac{mv^2}{r}$$

$$2ke^2\times\frac{1}{r^4}=mv^2$$

$$2ke^2\times\frac{16\,\pi^4m^4v^4}{n^4h^4}=mv^2$$

$$v^2=n^4h^4/32\,ke^2\pi^4m^3$$

$$v=\frac{n^2h^2}{4\sqrt{2}\,k\,e\,\pi^2m^{3/2}}$$

39. (*b*) This problem is a conceptual mixing of hydrolysis and reduction of dicyano compound and the condensation polymerisation.

Firstly complete the reaction by using information supplied in the question then analyse the product whose formation will occur.

(i) Hydrolysis and reduction of dicyano compound 1, 6 dicyanohexane produces adepic acid and hexamethylene diamine respectively as follows

$$N\equiv C-CH_2-CH_2-CH_2-CH_2-C\equiv N\xrightarrow{H_3O^+}HOOC-(CH_2)_4-COOH$$

(*A*) adepic acid

$$\xrightarrow{[H]}H_2N-(CH_2)_6-NH_2$$

(*B*) Hexamethylene diamine

Condensation polymerisation between *A* and *B* occur as follows

$$HOOC-\underset{A}{(CH_2)_4}-COOH+NH_2-\underset{B}{(CH_2)_6}-NH_2\downarrow$$

$$\left[-NH-(CH_2)_6-NHCO(CH_2)_4-CO-\right]_n$$

Nylon 6 6

TEST Edge The students are advised to study the information relating preparation of starting material involved in various polymerization reaction because question are asked from preparation of starting material in trend form. Such as preparation of bakelite, orlon, nylon 4, 6 etc.

40. (*b*) $\underset{(A)}{NH_4NO_2}+KOH\longrightarrow\underset{(B)}{NH_3}\uparrow+\underset{(C)}{KNO_2}+H_2O$

$$NO_2^-+3\,Zn+5\,OH^-+5H_2O\longrightarrow 3\,[Zn(OH)_4]^{2-}+\underset{(B)}{NH_3}\uparrow$$

$$\underset{(A)}{NH_4NO_2}\xrightarrow{\text{Heat}}N_2\uparrow+2H_2O$$

(Does not support combustion)

41. (*d*) **Idea** This problem includes concept of electroosmosis. So, student must have clear understanding of electroosmosis.

In electro osmosis the sol particles are prevented from migration where as the dispersion medium migrates in the direction opposite to those of particle. Here the medium is negatively charged.

TEST Edge In JEE Main question relating properties of colloidal solution are asked very easily. Student are advised to go through the understanding of Brownian movement, tyndall effect, electrophoresis etc.

42. (*c*) **Idea** This problem includes conceptual mixing of test to distinguish aldehyde and ketone and structural determination of compound from molecular formula.

This problem can be solved by undergoing following sequential step

- Identify the functional group using information provided in the question. Determine degree of unsaturation.
- Then approach towards correct structure of molecule.

2, 4 DNP test Since compound gives positive 2, 4 DNP test means the compound contain carbonyl group.

Iodoform test Compound show iodoform test means it must contain keto group (not aldehydic) $-\overset{\overset{O}{\|}}{C}-CH_3$.

This is also confirmed by negative tollens test and Fehling test.

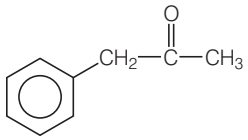
Baeyers test It does not decolorises bromine water mean there is no any free double bond or tripple bond.

Molecular formula = $C_9H_{10}O$

$$u = (9+1) - \frac{10}{2} = 5$$

Degree of unsaturation is 5 and no any double or tripple bond thus there must be a benzene ring and one C═O group.

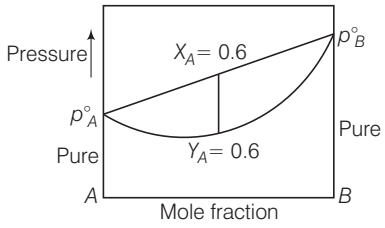
Out of given four choices only option (c) is satisfying all criterion observed by student.



Hence, (c) is the correct choice.

TEST Edge JEE Main asked these type of question in order to judge the analytical approach of student towards identification of structure of organic compound so students must go through infomation related to identification and separation of organic compounds such as lassiagne test, functional group identification test etc.

43. (*c*) $p_A^0 = 1000$ torr $p_B^0 = 1600$ torr



$X_A = 0.6 \quad Y_A = 0.6$ (from graph)

$$Y_A = \frac{p_A^0 X_A}{p_A^0 X_A + p_B^0 (1 - X_A)}$$

$$0.6 = \frac{1000\, X_A}{1000\, X_A + 1600\,(1 - X_A)}$$

$$X_A = 0.70 \quad X_B = 1 - X_A = 0.30$$

44. (*b*) **Idea** This problem is based on preparation of inorganic benzene and structural characteristics of diborane (·) student must be familiar with concept of bonding in diborane to solve this problem. Identify the compounds and complete the chemical reaction using information supplied in the question.

$$\underset{(x)}{B_2H_6} + 2NH_3 \xrightarrow{\text{Low temperature}} B_2H_6 \cdot 2NH_3$$

($3C$– $2e^-$, $2e^-$—$2e^-$ bonds)

$$\downarrow 200^\circ C$$

$$\underset{(Y)}{B_3N_3H_6}$$

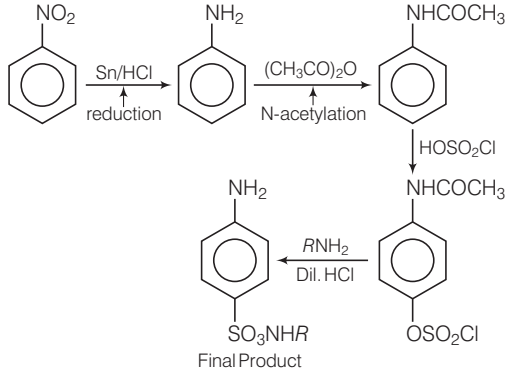
Y has structure similar to benzene (inorganic benzene)

$$B_2H_6 + NH_3 \xrightarrow[\text{temperature}]{\text{High}} \underset{(Z)}{(BN)_X}$$

TEST Edge JEE Main include this type of problem to judge the in depth knowledge of student in nature of bonding in inorganic compound. So student are advised to understand the concept of chemical bonding in inorganic molecular such as P_2O_5, P_4O_{10}, borax, etc.

45. (*a*) **Idea** This problem includes conceptual mixing of chemical properties of aromatic amines and their preparation. Students are advised to solve the question by undergoing a sequence of reaction according to information supplied in the question.

The reaction sequence involves various processes are shown as



TEST Edge Trend related problems are generally asked in JEE Main to judge the knowledge of studetn in the field of organic chemistry including knowledge of reagent, rearrangement etc.

46. (*c*) After addition of Cd it oxidises into Cd^{2+}

$$\underset{0.1-x}{NO_3^-(aq)} + \underset{0.4-4X}{4H^+(aq)} + 3e^- \longrightarrow NO(g) + 2H_2O(l)$$

$$X = 0.06$$

$[NO_3^-]$ remaining = 0.1 – 0.06 ≈ 0.4 M

$[H^+]$ remaining = 0.4 – 4 × 0.06 = 0.4 – 0.24

= 0.16 M

$$E_{NO_3^-/NO} = E_{NO_3^-/NO} - \frac{0.591}{3} \log \frac{1}{[NO_3^-][H^+]^4}$$

$$= 0.95 - \frac{0.591}{3} \log \frac{1}{(0.04)\,(0.16)^4}$$

$$= 0.95 - \frac{0.591}{3} \log \frac{1}{4 \times 10^{-2} \times 10^{-8} \times 2^{+16}}$$

$$= 0.95 - \frac{0.591}{3} \log \frac{10^{10}}{2^{18}} \approx 0.86\,V$$

47. (*c*) **Idea** This problem contains conceptual mixing of degree of unsaturation of organic compound and structure of given compound. This problem can be solved by following these steps

- Determine degree of unsaturation of given comopund.
- Write all possible structures and choose the correct one according to given information.

Degree of unsaturation (u) Degree of unsaturation is calculated as

$$u = (C+1) - \frac{H}{2} + \frac{N}{2}$$

where,

C = number of carbons
H = number of hydrogens
N = number of nitrogens.

Degree of unsaturation for C_6H_7N

$$u = (6+1) - \frac{7}{2} + \frac{1}{2}$$

$$= 7 - 3 = 4$$

$u = 4$ denotes various possibility

3 double bond + 1 ring

3 ring + 1 double bond

2 ring + 2 double bond

2 ring + 1 tripple bond.

Among the given option only (i) and (ii) are true structure of C_6H_7N.

TEST Edge This type of problem is asked in JEE Main to judge the analytical knowledge of chemistry to choose the correct option. Student must be smart during solving these problem by using the required important information only without being puzzled.

48. (*d*) **Idea** This problem includes concept of hybridisation in determination of structure of molecule using following formula.

$$H = \frac{V + M - C + A}{2}$$

where, V = number of valence electron on central metal
M = number of monoatomic members
C = number of catiom
A = number of aniom

$$Al + 6F^- \longrightarrow AlF_6^{3-}$$

$$H = \frac{3 + 6 + 3}{2} = 6$$

Hybridination = sp^3d^2

structure = octahedral

$[Be(C_2O_4)_2]^{2-}$

$$H = \frac{2 + 4 + 2}{2} = 4$$

Hybridisation = sp^3

structure = tetrahedral

TEST Edge In JEE Main these kind of questions are asked frequently, so the students are advised to keep understanding to molecular structure determination using VSEPR theory. Effect of lone pair or π bond pair on structure of molecule may also be asked. Students are required to study these topics.

49. (*a*) Moles of NO_2 required $= \left(\frac{1075}{63}\right) \times \frac{3}{2} \times \frac{1}{0.7} = 35.56$ g

Moles of NO required $= \frac{36.56}{0.60}$

Moles of NH_3 required $= \frac{36.56}{0.6} \times \frac{1}{0.40} = 152.325$

Volume of NH_3 at STP required $= 152.325 \times 22.4$

$\simeq 3413$ L

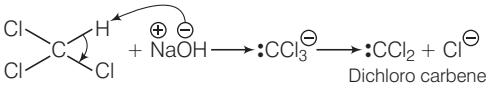
50. (*a*) $H_2O_2 + KI + Strach \longrightarrow$ Paper blue

Liberated I_2 makes starch paper blue while H_2O will not.

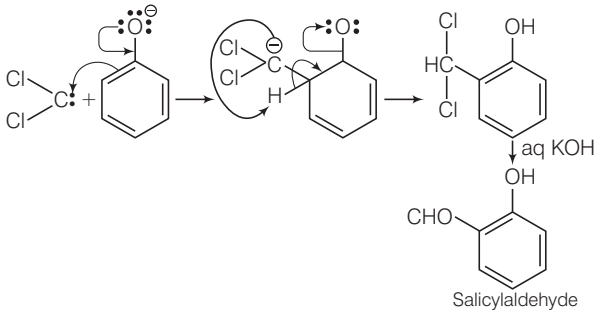
51. (*c*) **Idea** This problem includes conceptual mixing of reaction intermediate and Reimer tieman reaction. To solve this problem student must have the knowledge of generation of carbene by undergoing α elimination.

Riemer tieman reaction When phenol is treated with a mixture of $HCCl_3$ and NaOH it produces salicylaldehyde. The intermediate involved in the above reaction is carbene.

Step I



Step II



TEST Edge Student are advised to go through understanding of generation of reactive intermediates and their stability as well as chemical behaviour these question are asked very frequently.

52. (*a*) **Idea** To solve this problem complete the all reaction then choose the correct option.

Number of protons in $_6C^{14} = 6$

Number of neutrons in $_6C^{14} = 8$

New atomic mass of $_6C^{14} = \frac{8}{2} \times 6 \times 2$

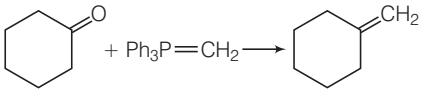
$= 4 + 16 = 16$

% increase in mass $= \frac{16-14}{14} \times 100 = 14.3\%$

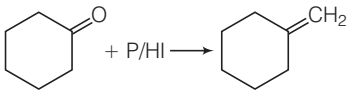
TEST Edge In JEE Main these type of questions are asked frequently, so students are advised to go through broad and clear understanding of preparation of various hydrocarbon and their derivatives such as alkenes, alkyl halide, aldehyde amines etc.

53. (*c*) **Idea** This problem includes various methods of preparation of alkene *i.e.*, Wittig reaction, elimination reaction etc.

I. Wittig reaction



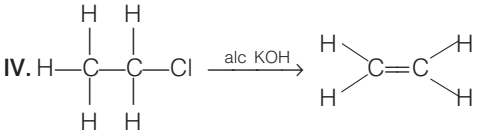
II. Reduction



III. Wurtz reaction

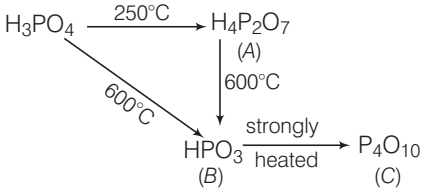
$$2\,CH_3I + 2\,Na \longrightarrow CH_3{-}CH_3$$

IV.



Hence, the correct choice is (c).

54. (*c*) **Idea** This problem can be solved by knowing the properties of phosphoric acid, so students are advised to learn the concept of effect of heat on H_3PO_4 at various temperatue.



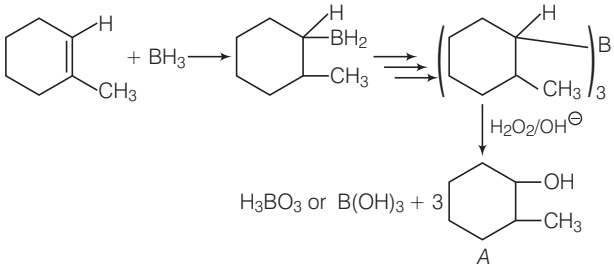
Oxidation state of P in H_3PO_4, $H_4P_2O_7$ and HPO_3 are +5, +5 and +5 respectively which here remain preserved during heating

TEST Edge JEE Main may include this type of question to judge the knowledge of student regarding properties of compound. So students are advised to study the concept of interconversion of red P, black P_1, white P also.

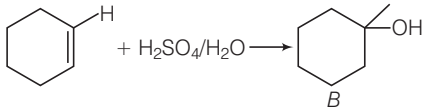
55. (*d*) **Idea** This problem includes conceptual mixing of hydroboration, oxidation and stereochemistry.

This problem can be solved by using the skill of electrophilic addition reaction in hydroboration oxidation reactor including isomerism. Follow the steps to solve this problem, complete reaction acid, then identify the isomerism in the out come products.

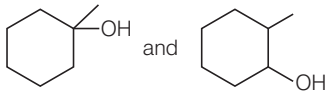
(a) **Hydroboration-oxidation** When alkene or substituted alkene is treated with B_2H_6 it gives alkyl borane which on treatment with $H_2O_2/OH^{\ominus}$ causes oxidation of alkyl borone to give alcohol. The above reaction is believed to proceed as



(b) When 2-methyl cyclohexane is treated with H_2O/H_2SO_4

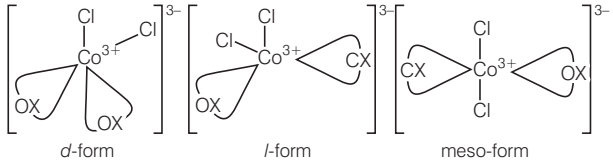


A and *B* have difference in position of OH only so *A* and *B* are position isomers.

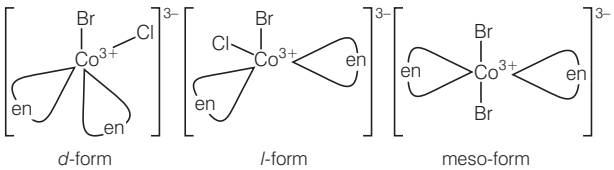


56. (*b*) **Idea** Students attempting such type of problem are advised to go through in depth study of isomerism such as geometrical and optical isomerism.

Isomers of $K_3[CoCl_2(C_2O_4)_2]$ are



Isomers of $[CoBrCl(en)_2]$ are



TEST Edge JEE Main includes such problems to judge the knowledge of student in isomerism in organic compound. Students must go through concept of isomerism along with study of magnetic character of complex. Which may also be asked.

57. (*a*) This problem includes conceptual mixing of lattice energy, hydration energy and solubility of dihydroxides of dipositive metals of group II element.

Solubility of hydroxides of Group II elements Lattice energy decreases much more than the hydration energy with increase in size. As we know when we move top to bottom along a group the size of ion increases causes decreases in value of lattice energy to a greater extent than hydration energy.

$Be(OH)_2$	Size of cation increases
$Mg(OH)_2$	
$Ca(OH)_2$	Solubility decreases
$Sr(OH)_2$	
$Ba(OH)_2$	

58. (*b*) **Idea** This question can be solved by knowing the structure of oxyacids of sulphur and determination of their oxidation state. Students are advised to write the molecular structure first, then to calculate the oxidation state of sulphur.

Marshell's acid = $H_2S_2O_8$ = peroxy disulphuric acid

Caro's acid = H_2SO_5 = peroxo-mono-sulphuric acid

The oxidation state of sulphur in both the acid is +6.

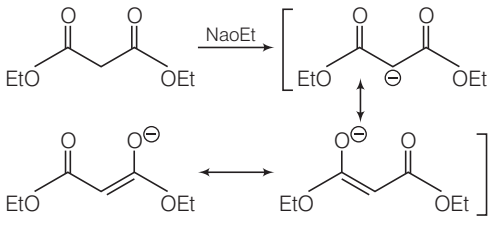
There are S—S linkages.

TEST Edge These type of question are asked in JEE Main very frequently, so students are advised to study the information relating structure of oxyacids of nitrogen, phosphorous and chlorine. Which may also be asked.

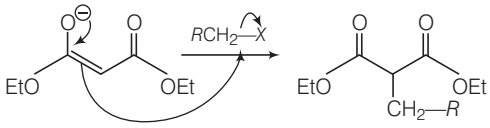
59. (*a*) **Idea** This problem involves conceptual mixing of formation of enolate anion and nucleophilic substitution reaction.

Students are advised to form the intermediate first followed by their chemical reaction.

Formation of enolate anion



Nucleophilic substitution reaction takes place on reaction with alkyl halide as follows



TEST Edge JEE Main may also include question with conceptual mixing of generation of reactive intermediate such as carbocation, carbanion, free radical etc and then their chemical reaction along with stereochemistry of product.

60. (*a*) Statement II is the correct explanation of Statement I.

e.g., $\underset{\text{Acid}}{H_3A} \rightleftharpoons \underset{\substack{\text{Conjugate base}\\(\text{weak acid})}}{H_2A^{\ominus}} + H^{\oplus}$

$\underset{\text{Acid}}{H_2A^{\ominus}} \rightleftharpoons \underset{\substack{\text{Conjugate base}\\(\text{weak acid})}}{H_2A^{2-}} + H^{\oplus}$

Mathematics

61. (*d*) **Idea** A differential equation of form

$$f(ax + by + c)\frac{dy}{dx} = c$$

is solved by writing $ax + by + c = t$.

Then, convert it into variable separable form.

Given that $(x + y)^2 \frac{dy}{dx} = a^2$ …(i)

Now, let $x + y = z$

Differentiation with respect to x, we get

$$\frac{dz}{dx} - 1 = \frac{a^2}{z^2} \Rightarrow \frac{dz}{dx} = \frac{a^2 + z^2}{z^2}$$

Using variable separation

$$\int \frac{z^2}{a^2 + z^2}\, dz = \int dx$$

Integrating on both sides, we get

$$x + c = z - a\tan^{-1}\left(\frac{z}{a}\right)$$

$$\Rightarrow \quad a\tan^{-1}\left(\frac{x + y}{a}\right) = y - c$$

$x = 0, y = 0 \Rightarrow c = 0$

$$\Rightarrow \quad \frac{y}{a} = \tan^{-1}\left(\frac{x + y}{a}\right)$$

$$\Rightarrow \quad y = a$$

$$\Rightarrow \quad 1 = \tan^{-1}\left(\frac{x}{a} + 1\right)$$

$$\Rightarrow \quad \frac{x}{a} = \tan 1 - 1$$

TEST Edge In JEE Main equation in which the variables are separable related questions are asked from this concept. *i.e.,* the equation $\frac{dy}{dx} = f(x, y)$ express it in the form $f(x)dx = g(y)dy$. Now, by integrating this, solution is

$$\int f(x)dx = \int g(y) + c.$$

62. (*a*) Here, direction ratios of the line *AB* are 1, 2, 2.

So, direction cosine are

$$\frac{1}{\sqrt{1^2+2^2+2^2}}, \frac{2}{\sqrt{1^2+2^2+2^2}}, \frac{2}{\sqrt{1^2+2^2+2^2}}$$

i.e., $\frac{1}{3}, \frac{2}{3}, \frac{2}{3}$

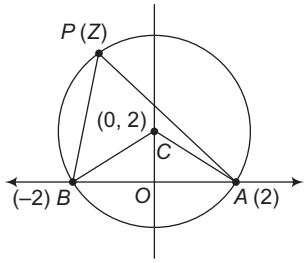
Projection of *AP* on the line *AB*, *i.e.*,

$$AN = l(x-a) + m(y-b) + n(z-c)$$
$$= 1/3(0-1) + 2/3(-1+3) + 2/3(3-2)$$
$$= 5/3$$

63. (*b*) **Idea** Circle whose centre is (x_0, y_0) *i.e.,* z_0 and radius $r, (x-x_0)^2 + (y-y_0)^2 = r^2$ or $|z - z_0| = r$

$\therefore$ If $z = a + ib$ then $\arg z = \theta = \tan^{-1}\frac{b}{a}$

Since, $|z - 2i| = 2\sqrt{2}$ represent the circle.



$$CA = CB = 2\sqrt{2}, OC = 2$$
$$OA = OB = 2$$

$\therefore$ $BC^2 + AC^2 = AB^2$

$\therefore$ $\angle BCA = 90°$

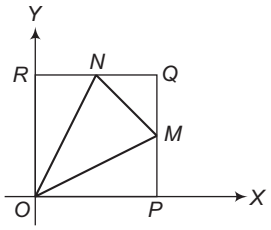
$$\angle BPA = 45°$$

$\therefore$ $\arg\left(\frac{z-2}{z+2}\right) = \tan 45°$

$$= \frac{\pi}{4}$$

TEST Edge From this concept the equation in complex variable of circles, number of solution of the given equations and a complex number lies on the circle related question are asked in JEE Mains to cracked such types of problem. Students are suggested that understand the concept of complex number related with circle.

64. (*d*) Assume that *OP* and *OR* are *x*-axis and *y*-axis respectively.



Let, $OP = a$, then

$$\text{ar}(OPQR) = a^2$$

Also, coordinates of *M* and *N* are $\left(a, \frac{a}{2}\right)$ and $\left(\frac{a}{2}, a\right)$ respectively.

$\therefore$ $$(\text{ar}\,\Delta OMN) = \frac{1}{2}\begin{vmatrix} a & a/2 \\ a/2 & a \end{vmatrix}$$

$$= \frac{3a^2}{8}$$

$\therefore$ $$a^2 : \frac{3a^2}{8} = \lambda : 6$$

$\Rightarrow$ $$\lambda = 16$$

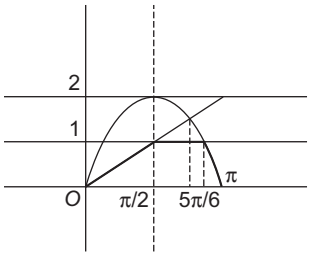
65. (*d*) **Idea** Here draw the graph of $\sin x$ and $\cos x$.

$\therefore$ The solution part below the both curves.

Then apply the formula of integration to get the value of $\int_0^{\pi} f(x)dx$ also use $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^d g(x)dx + \int_d^b h(x)dx$

We have given that

$\because$ $f(x) = \min\{2\sin x, 1 - \cos x, 1\}$



$$\int_0^{\pi} f(x)\,dx = \int_0^{\pi/2} (1-\cos x)\,dx + \int_{\pi/2}^{5\pi/6} dx + \int_{5\pi/6}^{\pi} 2\sin x\,dx$$

$$= [x - \sin x]_0^{\pi/2} + [x]_{\pi/2}^{5\pi/6} - 2[\cos x]_{5\pi/6}^{\pi}$$

$$= \frac{\pi}{2} - 1 + \frac{5\pi}{6} - \frac{\pi}{2} - 2\left[-1 + \frac{\sqrt{3}}{2}\right]$$

$$= 1 - \sqrt{3} + \frac{5\pi}{6}$$

TEST Edge This concept play a very important role in JEE Main the area between two curve, maximum or minimum area and maximum or minimum values of given curves related question are asked. To solve such type of questions students are advised to acquainted yourself how to draw the graph of trigonometric function and proper use of properties of definite integral.

66. (*a*) **Idea**

$\because$ $-1 \le \cos x \le 1$ we know as x increases from 0 to $\frac{\pi}{2}$ cos x decrease from 1 to 0

$\therefore$ When function is increasing the input and output inequality remain same but for decreasing function input or output inequality get reversed.

We have given that $f(x) = \cos x$

$\because$ $\frac{-\pi}{3} < x \le \frac{\pi}{6}$

$\because$ cos x is decreasing function.

$$\frac{-\pi}{3} < x < 0 \text{ and } 0 \le x \le \frac{\pi}{6}$$

$$\cos\frac{\pi}{3} \le \cos x < \cos 0 \quad \text{or} \quad \cos\frac{\pi}{6} \le \cos x \le \cos 0$$

$$\frac{\sqrt{3}}{2} < \cos x \le 1 \text{ or } \frac{1}{2} \le \cos x \le 1$$

$\therefore$ $$\frac{1}{2} < \cos x \le 1$$

$$a = \frac{1}{2}, b = 1$$

$$a + b = \frac{1}{2} + 1 = \frac{3}{2}$$

TEST Edge The interval in which the given function is increasing or decreasing related question are asked. To solve such type of questions student are advised to understand the concept of monotonic function.

67. (*a*) Cummulative frequency distribution for the data is given by

x_i	f_i	Cummulative frequency
2^0	1	1
2^1	2	3
2^2	3	6
2^3	4	10
2^4	5	15
2^5	6	21
2^6	7	28
2^7	8	36
2^8	9	45
2^9	10	55
2^{10}	11	66

$\therefore$ $$\frac{n}{2} = \frac{66}{2} = 33$$

Here, cummulative frequency is in 8th row,

So, median $= 2^7 = 128$

68. (*a*) **Idea**

$\because$ Here $AA^{-1} = I$, where I is a unit matrix *e.g.*, $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is a unit matrix of order 2.

Given, matrix is $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

Consider the equation $A^2 - 4A + I = 0$

$\Rightarrow$ $A^{-1}A^2 - 4AA^{-1} + A^{-1}I = 0$

$\Rightarrow$ $A - 4I + A^{-1} = 0$

$\Rightarrow$ $A^{-1} = 4I - A$

$$= 4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$$

Also,

$$A^{-1} = \begin{bmatrix} 2 & -3 \\ -1 & 1 \end{bmatrix}$$

{A^{-1} can be evaluated by elementary transformation}

Thus, the matrix A satisfies the equation $A^2 - 4A + I = 0$

TEST Edge Addition and subtraction of matrix properties of unit matrix related questions are asked from this concept. To solve these type of questions, students are advised to learn the properties of unit matrix and understand the addition and subtraction of two matrix.

69. (*c*) Here, it is given $E(X) = 3$ and $E(X^2) = 11$

Variance of $X = E(X^2) - (E(X))^2$

$= 11 - 3^2$

$= 11 - 9$

$= 2$

70. (*d*) **Idea**

$\because$ If $f(x)$ is continuous at $x = a$, then LHL = RHL = $f(a)$ and if $f(x)$ is differentiable at $x = a$ then $\lim\limits_{x \to a} \frac{f(x) \cdot f(a)}{x - a}$ exist finitely.

Consider the given function

$$f(x) = \begin{cases} x \sin(\log x^2), & x \ne 0 \\ 0, & x = 0 \end{cases}$$

Test of continuity

$$\text{LHL} = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} -h\sin\log(-h)^2$$

$$= -\lim_{h\to 0} h\sin\log h^2$$

Hence, $\sin\log h^2$

As $h \to 0, \log h^2 \to -\infty$

Hence, $\sin\log h^2$ oscillates between -1 and $+1$

So, $$\text{LHL} = -\lim_{h\to 0} h \times \lim_{h\to 0} (\sin\log h^2)$$

$$= 0 \times (\text{a number between } -1 \text{ and } +1)$$

$$= 0$$

$$\text{RHL} = \lim_{h\to 0} f(0+h) = \lim_{h\to 0} h\sin\log h^2$$

$$= \lim_{h\to 0} h \cdot \lim_{h\to 0} \sin\log h^2$$

$$= 0 \times (\text{oscillating between } -1 \text{ and } +1)$$

$$= 0$$

Also $f(0) = 0$ (given)

$\Rightarrow$ LHL = RHL = $f(0)$

$f(x)$ is continuous at $x = 0$

Test of differentiability

$$Lf'(0) = \lim_{h\to 0} \frac{f(0-h)-f(0)}{-h}$$

$$= \lim_{h\to 0} \frac{-h\sin\log(-h)^2 - 0}{-h}$$

$$= \lim_{h\to 0} \sin\log h^2$$

Since, $\sin\log h^2$ oscillates between -1 and $+1$,

Therefore, the limit does not exists.

LHD is not defined.

$f(x)$ is not differentiable at $x = 0$

TEST Edge Generally in JEE Main continuity at a point and differentiability of a function at a point related questions are asked. To solve these types of questions students are advised to understand the concept of continuity and differentiability at a point and also acquainted your self with properties of limit.

71. (b) Factor of the number 3630 are

$$3630 = 2\times 3\times 5\times 11^2$$

Odd divisor of 3630 are term of the product

$$= (3^0 + 3^1) + (5^0 + 5^1)(11^0 + 11^1 + 11^2)$$

$$= (1+3)(1+5)(1+11+11^2)$$

$$= (1+5+3+15)(1+11+121)$$

$$= (1+5+3+15+11+55+33+165 + 121+605+363+1815)$$

$\therefore$ Required divisor which left 1 as remainder when divided by 4 are 1, 5, 33, 165, 121, 605.

Hence, total divisor = 6

72. (a) **Idea**

$\therefore$ Let $\mathbf{OA} = \mathbf{a}, \mathbf{OB} = \mathbf{b}, \mathbf{OC} = \mathbf{c}$ and $\mathbf{OD} = \mathbf{d}$, then $\mathbf{AB} = \mathbf{b}-\mathbf{a}, \mathbf{CD} = \mathbf{d}-\mathbf{e}$,

$\mathbf{AB}\times\mathbf{CD} = (\mathbf{b}-\mathbf{a})\times(\mathbf{d}-\mathbf{e})$

use above condition to solve the problem.

Consider the four points A, B, C, D with position vectors

$\mathbf{OA} = \mathbf{a}, \mathbf{OB} = \mathbf{b}, \mathbf{OC} = \mathbf{c}$ and $\mathbf{OD} = \mathbf{d}$

It is given that

$$|\mathbf{AB}\times\mathbf{CD} + \mathbf{BC}\times\mathbf{AD} + \mathbf{CA}\times\mathbf{BD}|$$

$$= k \cdot \text{area of } \Delta ABC$$

$\Rightarrow$ $2|\mathbf{a}\times\mathbf{b} + \mathbf{b}\times\mathbf{c} + \mathbf{c}\times\mathbf{a}| = k \cdot \text{area of } \Delta ABC$

$\Rightarrow$ $4\cdot\frac{1}{2}|\mathbf{a}\times\mathbf{b} + \mathbf{b}\times\mathbf{c} + \mathbf{c}\times\mathbf{a}| = k \cdot \text{area of } \Delta ABC$

$\Rightarrow$ $4\times$ area of $\Delta ABC = k$ area of ΔABC

Then, $k = 4$

TEST Edge Generally in JEE Main area of triangle, area of parallelogram related questions are asked. To solve these types of questions students are advised to clear the formulae and understand the basic concept of cross product.

73. (d) If a, b and c are in AP, then

$$2b = a + c$$

$$8b^3 = (a+c)^3$$

$$8b^3 = a^3 + c^3 + 3ac(a+c)$$

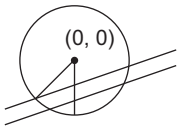
$$8b^3 = a^3 + c^3 + 3ac(2b) \quad [\because a + c = 2b]$$

$$a^3 + c^3 - 8b^3 = -6abc$$

74. (a) **Idea** If the line $x + y = a$ intersect the circle $x^2 + y^2 = r^2$ then $x^2 + (a-x)^2 = r^2$

$\because$ We know that for $ax^2 + bx + c = 0$, the roots are real then $b^2 - 4ac > 0$.

For the intersection of the line $x + y = n$ and circle $x^2 + y^2 = 4$. We must have



$$x^2 + (n-x)^2 = 4$$

$\Rightarrow$ $2x^2 + 2nx + n^2 - 4 = 0$

For $D > 0$,

$\Rightarrow$ $4n^2 - 8(n^2 - 4) > 0$

$\Rightarrow$ $n^2 - 8 < 0$

$\Rightarrow$ $-2\sqrt{2} < n < 2\sqrt{2}$

$\Rightarrow$ $n = 1$ or 2

Hence, lines are

$x + y = 1$ or $x + y = 2$

∴ Length of chord

$$l_1 = 2\left(\sqrt{4-\frac{1}{2}}\right) = \sqrt{14}$$

and $l_2 = 2\sqrt{4-2} = \sqrt{8}$

According to the given condition

$$11k = l_1^2 + l_2^2 = 22$$

$$\Rightarrow \quad k = 2$$

TEST Edge Length of chords when a line intersect the circle and distance from the centre of circle to the given line related questions are asked. Students are advised to understand the basic concept of circle and line.

75. (*d*) For the function

$f(x) = x^3 + ax^2 + bx + c$ has no extremum then

$f'(x) = 3x^2 + 2ax + b \neq 0 \ \ \forall\ x \in R$

i.e., $f'(x) = 3x^2 + 2ax + b = 0$ has no real roots.

$\Rightarrow$ Discriminant D

$$(2a)^2 - 4(3)(b) < 0$$

$$4a^2 - 12b < 0$$

$$a^2 < 3b$$

76. (*c*) Rewriting the given equations as

$$k(6x^2 + 3) + rx + 2x^2 - 1 = 0$$

$$\Rightarrow \quad (6K + 2)x^2 + rx + 3k - 1 = 0 \qquad \ldots(i)$$

and $6k(2x^2 + 1) + px + 4x^2 - 2 = 0$

$$\Rightarrow \quad (12k + 4)x^2 + px + 6k - 2 = 0 \qquad \ldots(ii)$$

$\because$ We know that $a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$ are two quadratic equations with equal roots, then

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\therefore \quad \frac{6k+2}{12k+4} = \frac{r}{p} = \frac{3k-1}{6k-2}$$

$$\frac{r}{p} = \frac{1}{2}$$

$$2r - p = 0$$

Hence, $2r - p = 0$

77. (*a*) **Idea**

$\because$ The equation of plane through the line of intersection of plane P and Q is $P + \lambda Q = 0$

$\therefore$ If plane $a_1x + b_2y + c_2z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ are perpendicular then $a_1a_2 + b_1b_2 + c_1c_2 = 0$

We know that the equation of any plane through the line of intersection of the planes.

$$ax + by + cz + d = 0$$

and $\alpha x + \beta y + \gamma z + e = 0$ is

$$(ax + by + cz + d) + k(\alpha x + \beta y + \gamma z + e) = 0 \qquad \ldots(i)$$

or $x(a + \alpha k) + y(b + \beta k) + z(c + \gamma k)$

$$+ (d + ek) = 0 \qquad \ldots(ii)$$

Equation of xy-plane is $z = 0$ ...(iii)

If the planes (ii) and (iii) are perpendicular to each other, then

$$0(a + \alpha k) +) + 0(b + \beta k) + 1(c + \gamma k) = 0$$

$$\Rightarrow \quad k = -c/\gamma$$

Putting the value of $k = \dfrac{-c}{\gamma}$ in Eq. (i)

$$(ax + by + cz + d) - c/\gamma(\alpha x + \beta y + \gamma z + e) = 0$$

$$\Rightarrow \quad (a\gamma - c\alpha)x + (b\gamma - c\beta)y + (d\gamma - ce) = 0$$

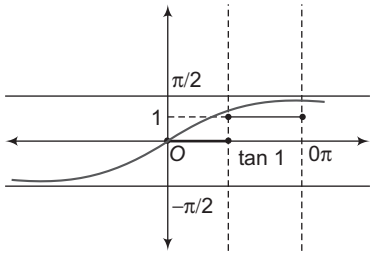
TEST Edge Equation of plane passing through the given points the plane are parallel or perpendicular to each other related questions are asked. Students are advised to learn the different equation of plane and condition.

78. (*a*) **Idea** We know that

$\because\ y = \tan^{-1}x, -\dfrac{\pi}{2} < y < \dfrac{\pi}{2}$ and $\forall\ -\infty < x < \infty$

$\because$ Draw the graph of $\tan^{-1}x$.

Let $f(x) = \tan^{-1}x$, then graph of $\tan^{-1}x$ is



$$\therefore \quad \int_0^{10\pi}[\tan^{-1}x]dx = \int_0^{\tan 1}[\tan^{-1}x]\,dx + \int_{\tan 1}^{10\pi}[\tan^{-1}x]dx$$

$$= \int_0^{\tan 1} 0\ dx + \int_{\tan 1}^{10\pi} dx$$

$$= [0 + x]_{\tan 1}^{10\pi} = 10\pi - \tan 1$$

TEST Edge Integration of greatest integer function and area bounded by curve related questions are asked. To solve these type of question students are advised to understand the graph transformation of inverse trigonometric function and also know about properties of greatest integer function.

79. (*c*) **Idea** $\because\ (1+x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + \ldots + {}^nC_rx^r + \ldots + {}^nC_nx^n$

coefficient of x^r is ${}^nC_r$

It is given that coefficient. of x^{101} in

$$1 + (1+x) + (1+x)^2 + \ldots (1+x)^{n+1}$$

(If $n \geq 100$) is ${}^{201}C_{101}$.

Let $S = 1 + (1+x) + (1+x)^2 + \ldots + (1+x)^n$

$(1+x)S = (1+x) + (1+x)^2 + (1+x)^3 + \ldots + (1+x)^n + (1+x)^{n+1}$

$\Rightarrow \quad S(1+x-1) = (1+x)^{n+1} - 1$

$\Rightarrow \quad S = \dfrac{(1+x)^{n+1} - 1}{x}$

$= [\text{coefficient of } x^{101} \text{ in } [(1+x)^{n+1} - 1]$

$= [\text{coefficient of } x^{101} \text{ in } (1+x)^{n+1}]$

$= {}^{n+1}C_{101} = {}^{201}C_{101}$

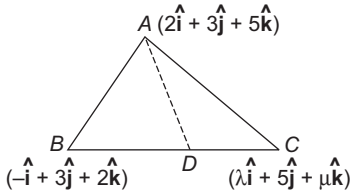
$\{\because \text{coefficient of } x^r \text{ in } (1+x)^n \text{ is } {}^nC_r\}$

$\therefore \quad n + 1 = 201$

$n = 200$

TEST Edge Number of terms sum of odd and even terms and find the larger or smaller number related questions are asked. To understand binomial expansions in different cases.

80. (*b*) Given that A, B and C are the vertices of ΔABC.



So, position of vector of D

$$= \frac{\lambda - 1}{2}\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + \frac{\mu + 2}{2}\hat{\mathbf{k}}$$

Direction of ratio of $AD = \dfrac{\lambda - 5}{2}, 1, \dfrac{\mu - 8}{2}$

But direction cosine of AD should be

$$\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \Rightarrow \frac{\lambda - 5}{2} = 1 = \frac{\mu - 8}{2}$$

$$\lambda = 7, \mu = 10 \quad \Rightarrow \quad 2\lambda - \mu = 4$$

81. (*b*) Given that $a > b > c > 0$

Now, $\tan^{-1}\left(\dfrac{1}{x}\right) = \begin{cases} \cot^{-1} x, & \text{for } x > 0 \\ -\pi + \cot^{-1} x, & \text{for } x < 0 \end{cases}$

$$\cot^{-1} x = \begin{cases} \tan^{-1}\dfrac{1}{x}, & \text{for } x > 0 \\ \pi + \tan^{-1}\dfrac{1}{x}, & \text{for } x < 0 \end{cases}$$

$$\therefore \cot^{-1}\left(\frac{ab+1}{a-b}\right) + \cot^{-1}\left(\frac{bc+1}{b-c}\right) + \cot^{-1}\left(\frac{ca+1}{c-a}\right)$$

$$= \tan^{-1}\left(\frac{a-b}{1+ab}\right) + \tan^{-1}\left(\frac{b-c}{1+bc}\right) + \pi + \tan^{-1}\left(\frac{c-a}{1+ca}\right)$$

$$= \tan^{-1} a - \tan^{-1} b + \tan^{-1} b - \tan^{-1} c + \pi + \tan^{-1} c - \tan^{-1} a$$

$$= \pi$$

82. (*d*) **Idea** For cube root unity

$\because \quad x^3 = 1 \Rightarrow x^3 - 1 = 0$

$(x-1)(x^2 + x - 1) = 0$

$x = 1 \text{ or } x^2 + x - 1 = 0$

$$x = \frac{-1 \pm \sqrt{1-4}}{2}$$

$$\Rightarrow \quad x = \frac{-1 \pm i\sqrt{3}}{2}, \frac{-1 - i\sqrt{3}}{2} \text{ and } 1$$

It is given that $A = \begin{bmatrix} \dfrac{-1+i\sqrt{3}}{2i} & \dfrac{-1-i\sqrt{3}}{2i} \\ \dfrac{1+i\sqrt{3}}{2i} & \dfrac{1-i\sqrt{3}}{2i} \end{bmatrix}, i = \sqrt{-i}$

and $f(x) = x^2 + 2$

Since, $\omega^3 = 1$

$$\omega = \frac{-1+i\sqrt{3}}{2} \text{ and } \omega^2 = \frac{-1-i\sqrt{3}}{2} \text{ and } \omega^2 + \omega = -1$$

Then,

$$A = \begin{bmatrix} \dfrac{\omega}{i} & \dfrac{\omega^2}{i} \\ \dfrac{-\omega^2}{i} & -\dfrac{\omega}{i} \end{bmatrix}$$

$$= \frac{\omega}{i}\begin{bmatrix} 1 & \omega \\ -\omega & -1 \end{bmatrix}$$

$$A^2 = \frac{\omega^2}{i^2}\begin{bmatrix} 1 & \omega \\ -\omega & -1 \end{bmatrix}\begin{bmatrix} 1 & \omega \\ -\omega & -1 \end{bmatrix}$$

$$= -\omega^2\begin{bmatrix} 1-\omega^2 & 0 \\ 0 & 1-\omega^2 \end{bmatrix}$$

$$= \begin{bmatrix} -\omega^2 + \omega^4 & 0 \\ 0 & -\omega^2 + \omega^4 \end{bmatrix}$$

$$= \begin{bmatrix} -\omega^2 + \omega & 0 \\ 0 & -\omega^2 + \omega \end{bmatrix}$$

$\therefore \quad f(x) = x^2 + 2$

$\Rightarrow \quad f(A) = A^2 + 2I$

$$= \begin{bmatrix} -\omega^2 + \omega & 0 \\ 0 & -\omega^2 + \omega \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -\omega^2 + \omega + 2 & 0 \\ 0 & -\omega^2 + \omega + 2 \end{bmatrix}$$

$$= (-\omega^2 + \omega + 2)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= 3 + 2\omega\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \{\because 1 + \omega + \omega^2 = 0\}$$

$$= \left[3 + 2\left(\frac{-1 + i\sqrt{3}}{2}\right)\right]\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$= (2 + i\sqrt{3})\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

TEST Edge Cube root unity in complex number and properties of matrix related questions are asked in JEE Main, to solve these type of question students are advised to understand the concept of addition multiplication of matrix and also acquainted yourself the concept of cube root unity.

83. (*a*) In 2×2 matrices total positions are 4 in each position there are 2 choices.

$\therefore \quad n(S) = 2^4 = 16$

Matrices which are not singular are

$$\begin{pmatrix}1 & 1\\ 0 & 1\end{pmatrix}, \begin{pmatrix}0 & 1\\ 1 & 1\end{pmatrix}, \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$$

$$\begin{pmatrix}1 & 0\\ 1 & 1\end{pmatrix}\begin{pmatrix}1 & 1\\ 1 & 0\end{pmatrix}$$

$\therefore$ Total matrices which are singular $= 16 - 6 = 10$

$\therefore$ Required probability $= \frac{10}{16} = \frac{5}{8}$

84. (*d*) **Idea** Find the equation of tangent and normal to the given parabolas.

$\therefore ax^2 + bx + c = 0$ for two distinct roots $b^2 - 4ac > 0$

Here, we know that a tangent to $y^2 = 4ax$ is

$ty = x + at^2$ ...(i)

A normal to $x^2 = 4by$ is

$Ty + x = 2bT + bT^3$ ...(ii)

If Eqs. (i) and (ii) are the same lines, then

$t = -T$ and $at^2 = -2bT - bT^3$

$\Rightarrow \quad bT^3 + aT^2 + 2bT = 0$

$T \neq 0$ and $bT^2 + aT + 2b = 0$

For two distinct such lines

$a^2 - 8b^2 > 0$

$a^2 > 8b^2$

TEST Edge Equation of tangents, normal and tangents, normal lying or not lying in the axis to the parabola related questions are asked. To understand the concept of parabola with tangent and normal.

85. (*c*) The truth table for the used operations is given below

p	q	$p \to q$	$p \leftrightarrow q$
T	T	T	T
T	F	F	F
F	T	T	F
F	F	T	T

So, it is clear that only option (c) is false.

86. (*a*) **Idea** Express the given function as polynomial equation and differentiate with respect to x.

It is given that $y = \cos^{-1}\left[\frac{\cos x + 4\sin x}{\sqrt{17}}\right]$

Put $\quad 1 = r\cos\theta$ and $4 = r\sin\theta$

$\Rightarrow \quad r = \sqrt{17}$ and $\theta = \tan^{-1} 4$

So, we have

$$\cos x + 4\sin x = \sqrt{17}\left[\frac{1}{\sqrt{17}}\cos x + \frac{4}{\sqrt{17}}\sin x\right]$$

$$= \sqrt{17}\cos(x - \tan^{-1} 4)$$

[Here $\cos\theta = \frac{1}{\sqrt{17}} \quad \therefore \quad \sin\theta = \frac{4}{\sqrt{17}}$

using $\cos(x - y) = \cos x \cos y \sin x \sin y$]

Thus, $\quad y = \cos^{-1}\left(\frac{\sqrt{17}\cos(x - \tan^{-1} 4)}{\sqrt{17}}\right)$

$\Rightarrow \quad y = x - \tan^{-1} 4$

$\Rightarrow \quad \frac{dy}{dx} = 1$

TEST Edge Find the derivative of different types of trigonometric and inverse trigonometric function. To understand the concept of differentiation and learn the trigonometric identities and formulae.

87. (*c*) The given equations are

$4^{\sin x} + 3^{\sec y} = 1$ ...(i)

and $\quad 5(16)^{\sin x} - 2(3)^{\sec y} = 2$ ...(ii)

Let $\lambda = 4^{\sin x}$

$\mu = 3^{\sec y}$

Then, Eqs. (i) and (ii) reduce to

$\lambda + \mu = 11$

$5\lambda^2 - 2\mu = 2 \Rightarrow 5\lambda^2 - 2(11 - \lambda) = 2$

$\Rightarrow \quad 5\lambda^2 + 2\lambda - 24 = 0$

$\Rightarrow \quad \lambda = \frac{-2 \pm \sqrt{484}}{10} \Rightarrow \lambda = 2, \frac{-12}{5}$

For $\quad \lambda = 2, \mu = 9$

$\Rightarrow \quad 4^{\sin x} = 2$ and $3^{\sec y} = 9$

$\Rightarrow \quad 2\sin x = 1$ and $\sec y = 2$

$\Rightarrow \quad \sin x = \frac{1}{2}$ and $\cos y = \frac{1}{2}$

$x = n\pi + (-1)^n \frac{\pi}{6}$ and $y = 2m\pi \pm \frac{\pi}{3}, \quad m, n \in z$

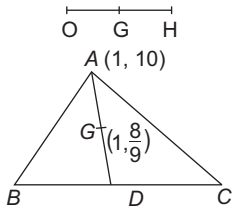
For $\quad \lambda = -\frac{12}{5}$, we have

$\Rightarrow (4)^{\sin x} = -\frac{12}{5}$, which is not possible as $4^{\sin x} > 0$

Thus, the solution of the given equations is given by

$x = n\pi + (-1)^n \frac{\pi}{6}, \; y = 2m\pi \pm \frac{\pi}{3}, \; m, n, \in z$

88. (*a*) It is given that, in ΔABC,



Circumcentre $O = \left(\frac{-1}{3}, \frac{2}{3}\right)$

Orthocentre $H = \left(\frac{11}{3}, \frac{4}{3}\right)$

Coordinates of G are $\left(1, \frac{8}{9}\right)$

$$A = (1, 0),\ G = \left(1, \frac{8}{9}\right)$$

$$\therefore \quad AG : GD = 2 : 1$$

$$\therefore \quad D = \left(1, \frac{-11}{3}\right)$$

$\therefore$ Coordinates of the mid-point of BC are $\left(1, \frac{-11}{3}\right)$

So, correct option is (a).

89. (*b*) **Statement I** Consider the expression

$$X \cap Y' \cap X' = X \cap X' \cap Y' = \phi \cap Y' = \phi$$

$\Rightarrow$ Statement I is true.

Statement II It is given that $n(X \cup Y) = m$

and $n(X \cap Y) = n$

Now, $X \Delta Y = (X - Y) \cup (Y - X) = (X \cup Y) - (X \cap Y)$

$\Rightarrow \quad n(X \Delta Y) = n[(X \cup Y) - (X \cap Y)]$

$\Rightarrow n(X \Delta Y) = n(X \cup Y) - n(X \cap Y) = m - n$

$\Rightarrow$ Statement II is true

But Statement II is not the correct explanation of Statement I.

90. (*a*) **Idea** Express the given equation into linear differential equation as $\frac{dy}{dx} + PY = Q$

$\because$ IF $= e^{\int P dx}$ and solution is $y \times \text{IF} = \int Q \times \text{IF}$

$$\frac{dy}{dx} + \left(\frac{1}{\sin x} + \cot x + \frac{1}{x}\right) y = \frac{1}{x}$$

$$\text{IF} = e^{\int \left(\frac{1}{\sin x} + \cot x + \frac{1}{x}\right) dx} = e^{\log\left(x \tan \frac{x}{2} \sin x\right)}$$

$$= x \tan \frac{x}{2} \times 2 \sin \frac{x}{2} \cos \frac{x}{2} = x(1 - \cos x)$$

$$yx(1 - \cos x) = \int \frac{1}{x} \cdot x(1 - \cos x)\, dx$$

$$= x - \sin x + c$$

$$y(\pi/2) = 1 - \frac{2}{\pi} \Rightarrow c = 0$$

$$y(x) = \frac{x - \sin x}{x(1 - \cos x)}$$

$$= \frac{x - \left(x - \frac{x^3}{6} \ldots\right)}{x\left[1 - \left(1 - \frac{x^2}{2} \ldots\right)\right]}$$

$$= \frac{x^2}{6} \frac{1}{\frac{x^2}{2}} = \frac{1}{3} \text{ as } x \to 0$$

So, it is clear that Statement I and Statement II both are true and Statement II is a correct explanation for Statement I.

TEST Edge Generally in JEE Main linear differential equation in x or y related questions are asked from this concept students are advised to understand the basic concept of these equation and also acquainted yourself with formulae of integrations.