JEE Main

Joint Entrance Examination

Question Booklet Code **P**

Practice Set 5

Duration: 3 Hours *Max. Marks: 360*

Read the Following Instructions Carefully

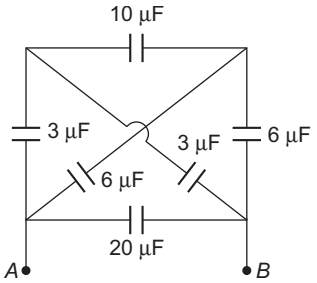
1. *Immediately fill the particulars on this page of the test booklet with blue / black ball point pen. Use of pencil is strictly prohibited.*
2. *The test is of 3 hours duration.*
3. *The test booklet consists of 90 questions. The maximum marks are 360.*
4. *There are three parts in the question paper A, B, C consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage. Each question is allotted 4 (four) marks for correct response.*
5. *Candidates will be awarded marks as stated in above instructions for correct response of each question. ¼ (one fourth) marks will be deducted for indicating incorrect response of each question. There is no negative marking for unattampted questions.*
6. *There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted accordingly as per above instructions.*
7. *No candidate is allowed to carry any textual material, printed or written, bits of papers, paper, mobile phone, any electronic device, etc., except the Admit Card inside the examination hall/room.*
8. *Rough work is to be done on the space provided for this purpose in the test booklet only. This space is given at the bottom of pages.*

Name of the Candidate (in Capital Letters)_____

Roll Number (in Figures) | | | | | | | | | in Words _____

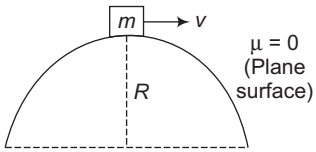
PART A Physics

1. The equivalent capacitance between A and B is



(a) 24.5 μF (b) 22 μF (c) 20 μF (d) 30 μF

2. A box is placed on a semicircular convex surface. It is given a horizontal speed v. Choose the correct option.



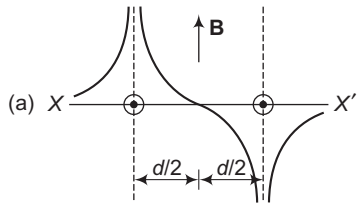
(a) if $v = \sqrt{Rg}$, the block will leave the surface at the same instant

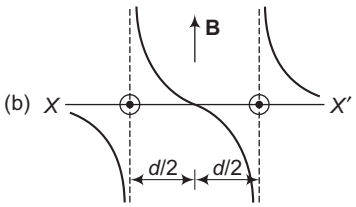
(b) if $v > \sqrt{Rg}$, the block will leave the surface at very next instant

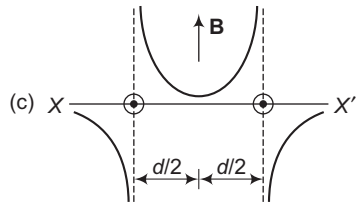
(c) if $v < \sqrt{Rg}$, the block will leave the surface after moving some distance along the surface

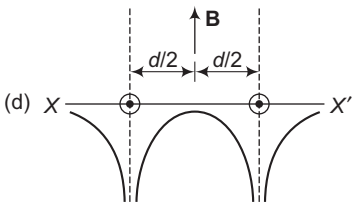
(d) None of the above

3. Two long parallel wires are at a distance d apart. They carry steady equal currents flowing out of the plane of paper. The variation of magnetic field along the line XX′ is given by [+ve direction of B is indicated in figure]









4. An L-C-R series circuit with a resistance of 100 Ω is connected to an AC source of 200 V (rms) and angular frequency 300 rad/s when only capacitor is removed. The current lags behind the voltage by 60°, when only inductor is removed, the current leads the voltage by 60°. Then, the power dissipiated in L-C-R circuit is

(a) zero (b) 200 (c) 400 (d) 800

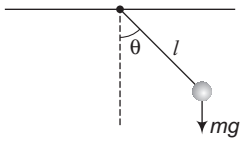
5. For a simple pendulum work done by

(a) $mg \cos\theta$ is positive

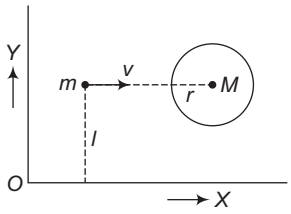
(b) tension is negative

(c) $mg \sin\theta$ could be zero at an instant

(d) None of the above



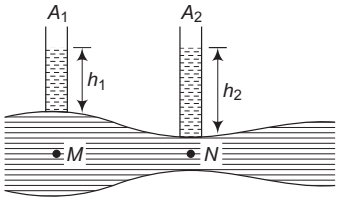
6. **A ring is placed on a horizontal plane (frictionless) and a point mass is moving towards the ring as shown in the figure. Point mass hit the ring and attached with it. The angular momentum of the system of ring and point mass about O after the collision**



(a) $(M + m)\, v^2 l$ (b) mlv

(c) $mv^2 l$ (d) $\dfrac{mvl}{(M + m)}$

7. **A liquid is in steady flow inside a tube of non-uniform variation. Two capillaries are joined with the tube at two places, then**



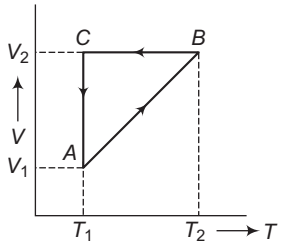
(a) $h_1 = h_2$

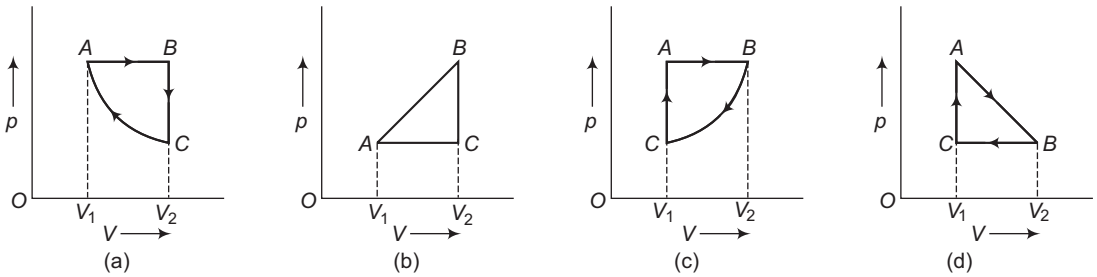
(b) h_1 must be greater than h_2

(c) h_2 may be greater than h_1

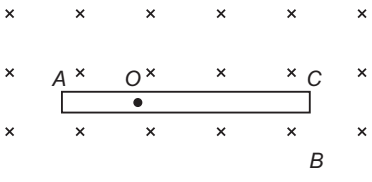
(d) h_2 must be greater than h_1

8. **A cyclic process is performed with the constant mass of an ideal gas corresponding to p - V graph**





9. **A conducting rod AC of length 4l and mass m is rotating about point O with angular momentum J. A uniform magnetic field B is directed into the paper. Given, AO = l and OC = 3l. Then, $V_A - V_C$ is**



(a) $\dfrac{5}{12}\dfrac{BJ}{m}$ (b) $\dfrac{7}{12}\dfrac{BJ}{ml^2}$

(c) $\dfrac{12}{7}\dfrac{BJ}{m}$ (d) $\dfrac{12}{7}\cdot\dfrac{BJ}{ml^2}$

10. **Pick up the incorrect statement.**

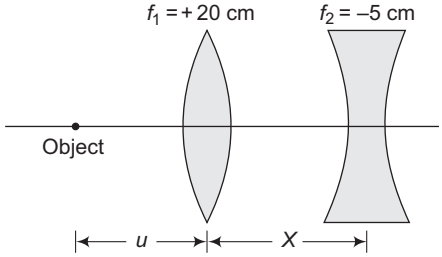
(a) Parallel *L-C-R* circuit at resonance acts as a rejector circuit

(b) Series *L-C-R* circuit at resonance acts as a acceptor circuit

(c) Resonance frequency of both the circuits is always given by $\nu = \dfrac{1}{2\pi\sqrt{LC}}$

(d) Acceptor circuit is used for voltage magnification while rejector circuit is used for current magnification

11. If the final image formed at 5 cm from the concave lens, then



(a) $u = -20$ cm, $X = -5$ cm
(b) $u = -10$ cm, $X = -10$ cm
(c) $u = -40$ cm, $X = -20$ cm
(d) $u = -20$ cm, X = could have any value

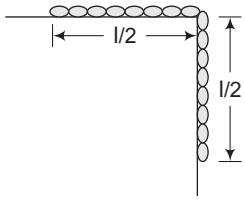
12. In a revolving satellite,

(a) there is no force acting on the people present in the satellite
(b) the acceleration of satellite is zero (as seen from the Earth) as in the satellite the condition is of free fall
(c) Both (a) and (b) are correct
(d) Both (a) and (b) are wrong

13. Centrifugal force

(a) must be applied if the motion is circular
(b) could be towards the centre of the circle
(c) may be applied if the motion is circular
(d) is a real force

14. A chain of uniform mass density is hanging as shown in the figure. The total mass is M and length is l. When the chain completely falls from the table, then its speed is

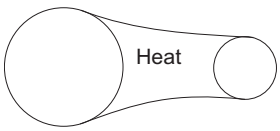


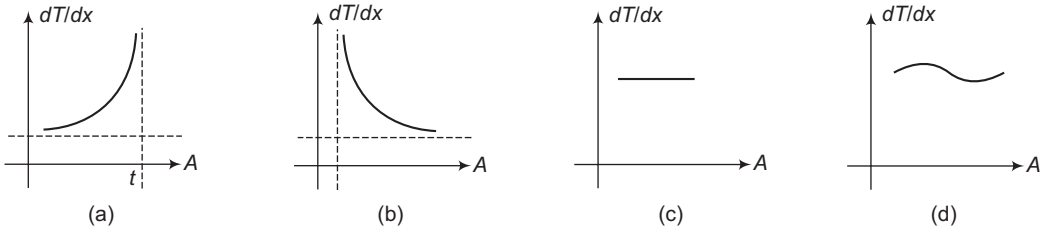
(a) $\frac{\sqrt{3gl}}{2}$
(b) $\sqrt{\frac{3gl}{8}}$
(c) $\sqrt{\frac{3gl}{2}}$
(d) $\frac{\sqrt{3gl}}{4}$

15. A uniform rod of length l and mass M is suspended on two vertical inextensible string as shown in the figure. Calculate tension T in left string at the instant when right string snaps.

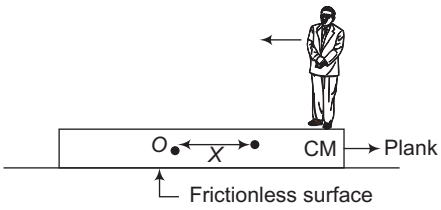
(a) $mg/2$ (b) mg (c) $mg/4$ (d) $mg/8$

16. An irregular rod of same uniform material as shown in figure, is conducting heat at a steady rate. The temperature gradient at various sections versus area of cross-section graph will be





17. **A boy is standing at the end of a plank, where the CM of the system lies at a distance X from the centre of plank. Boy starts running on the plank towards the left, when the boy reach at the centre of the plank then, [The length of plank = l]**



(a) The CM of the system is displaced by a distance X towards left

(b) The boy had covered a distance of $\left(\frac{l}{2} - X\right)$ with respect to plank

(c) The boy had moved a distance $l/2$ with respect to the plank

(d) None of the above

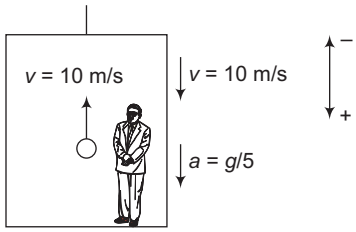
18. **A train is moving towards a platform with the speed of 54 km/h and the wind is blowing in the direction opposite to the direction of train's motion, what is observed frequency of the horn of train by the passenger who is standing on the platform? Original frequency of horn f_o = 300 Hz and ν_{air} = 108 km / h, ν_{sound} in still air = 330 m/s.**

(a) 335 Hz (b) 300 Hz (c) 285 Hz (d) 315 Hz

19. **An electric sphere of charge 10 μC is placed at origin (0, 0) of x-y coordinate system. Two points A and B are situated at $(\sqrt{2}, \sqrt{2})$ and (2, 0) respectively. The potential difference between the points A and B will be**

(a) zero (b) 9 V (c) 2 V (d) 4.5 V

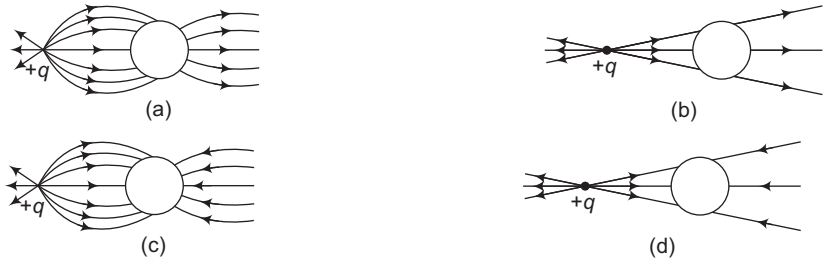
20. **A boy is in a lift which is moving downwards with an acceleration g/4. At an instant when the velocity of the lift is 10 m/s, then boy throws a ball upwards with a velocity 10 m/s. The time after which ball will return into the hand of boy is**



(a) 3 s (b) 4 s

(c) 5 s (d) Not possible

21. **A point positive charge is brought near an isolated conducting sphere as shown below. The electric field is best given by**



22. **If the error in the radius of a sphere is 5%. What will be error in its volume?**

(a) 5% (b) 25% (c) 30% (d) 15%

23. **Two wires A and B are made of same material and having their lengths in the ratio 6 : 1 are connected in series. The potential difference across the wires are 3 V and 2 V respectively. If r_A and r_B are the radii of A and B, then r_B / r_A is**

(a) 1/2 (b) 1/4 (c) 1/6 (d) 1/3

24. **A square frame of side 1m carries a current I, produces a magnetic field B at its centre. The same current is passed through circular coil having same perimeter as the square. The magnetic field at the centre of the circular coil is B′. The ratio B/B′ is**

(a) $8/\pi^2$ (b) $8\sqrt{2}/\pi^2$ (c) $16/\pi^2$ (d) $\frac{8}{\sqrt{2}\,\pi^2}$

25. **Two identical charged spheres are suspended from a common point by two massless strings of length l are initially at distance d (d << l) because of mutual repulsion. Charge begins to leak from both the spheres at a constant rate. As a result charges approach each other with velocity v. Then, as a function of distance x between them,**

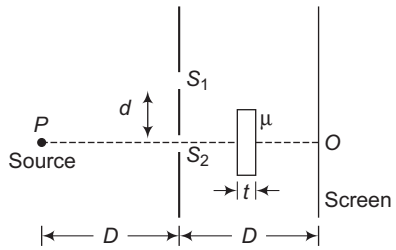
(a) $x \propto x^{-1}$ (b) $v \propto x^{1/2}$

(c) $v \propto x$ (d) $v \propto x^{-1/2}$

26. **Half-life of a radioactive substance is 2 h. The time between 20% and 90% decay will be**

(a) 4 h (b) 6 h (c) 5 h (d) 2 h

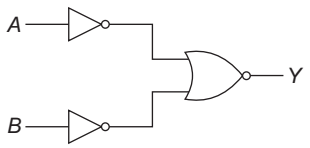
27. **Consider a double slit experiment set up. What shall be the value of t to produce central maxima at O ? Given, $\mu = 1.5$ (refractive index)**



(a) d^2/D (b) $2\,d/D$

(c) $2 \cdot \frac{d^2}{D}$ (d) It is not possible for any value of t

28. **Which logic gate is represented by the following combination of logic gates?**



(a) OR (b) AND (c) NAND (d) NOR

29. **Statement I In Young's double slit experiment, the ratio $\frac{I_{max}}{I_{min}}$ is infinite.**

Statement II If width of any one of the slits is slightly increased, then this ratio will decrease.

(a) Statement I is true, Statement II is also true and Statement II is the correct explanation of the Statement I

(b) Statement I is true, Statement II is also true and Statement II is not the correct explanation of the Statement I

(c) Statement I is true, Statement II is false

(d) Statement I is false, Statement II is false

30. **Statement I The Moon revolves around the earth due to a centripetal force.**

Statement II The net force on the moon is $F_{Gravitational} + F_{Centripetal}$.

(a) Statement I is true, Statement II is also true and Statement II is the correct explanation of the Statement I

(b) Statement I is true, Statement II is also true and Statement II is not the correct explanation of the Statement I

(c) Statement I is true, Statement II is false

(d) Statement I is false, Statement II is false

PART B Chemistry

31. **Which of the following statement is correct regarding following process?**

(i) $Cl \xrightarrow{EA} Cl^-$ (ii) $Cl^- \xrightarrow{IE} Cl$ (iii) $Cl \xrightarrow{IE} Cl^+$ (iv) $Cl \xrightarrow{IE} Cl^{2+}$

(a) |IE of Process (iv)| = |IE of Process (iii)|

(b) |IE of Process (iii)| = |IE of Process (ii)|

(c) |IE of Process (ii)| = |EA of Process (i)|

(d) |IE of Process (iv)| = |EA of Process (i)|

32. **A flask containing 0.5 atm pressure of $A_2(g)$, some solid AB added into flask which undergoes dissociation according to $2AB(s) \rightleftharpoons A_2(g) + B_2(g)$, $K_p = 0.06\ atm^2$. The total pressure (atm) at equilibrium will be**

(a) 0.5 atm (b) 0.2 atm (c) 0.7 atm (d) 1.4 atm

33. **Determine the value of percentage of nitrogen in the compound, When in Duma's method for estimation of nitrogen 0.30 g of organic compound gave 50 cm^3 of nitrogen gas collected at 300 K and 715 nm pressure (Hint vapour pressure of water at 300 K is 15 mm Hg)**

(a) 17.46 (b) 28.56 (c) 28 (d) 15.68

34. **Which of the following is a cyclic oxoacid?**

(a) $H_4P_2O_7$ (b) $H_4P_2O_6$ (c) $H_3P_3O_9$ (d) $H_5P_5O_{15}$

35. **Given reaction cis Z $\underset{k_b}{\overset{k_f}{\rightleftharpoons}}$ trans Z is first order in both directions. At 28°C the equilibrium constant is 10^{-2} and the rate constant $k_f = 3 \times 10^{-5}\ s^{-1}$. In an experiment starting with the pure cis-form, how long would it take for half of the equilibrium amount of trans-isomer to be formed?**

(a) 5.6×10^4 s (b) 9.8×10^2 s (c) 2.3×10^4 s (d) 7.5×10^3 s

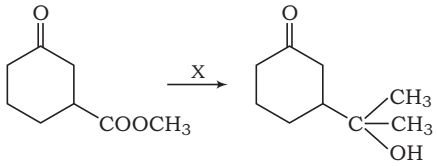
36. **$FeCr_2O_4 \xrightarrow{X} Na_2CrO_4 \xrightarrow{Y} Cr_2O_3 \xrightarrow{Z} Cr$, then X, Y and Z are**

	X	Y	Z
(a)	NaOH/air	carbon	carbon
(b)	Na_2CO_3/air	carbon	carbon
(c)	Na_2CO_3/air	NH_4Cl	Al
(d)	NaOH/air	carbon	Al

37. **The oxidation number of nitrogen atoms in ammonium nitrate are**

(a) – 3, + 5 (b) – 3, – 3 (c) + 5, + 5 (d) + 2, – 3

38. **X in the below reaction will be**



(a) $HOCH_2—CH_2OH | H^+, LiAlH_4$ | ether, $2CH_3MgBr, H_3O^+$

(b) $2CH_3MgBr$ and H_3O^+

(c) $HOCH_2—CH_2OH | H^+, 2CH_3MgBr, H_3O^+$

(d) $HOCH_2—CH_2OH — H^+, H_2$ Pt, CH_3OH, H^+

39. Choose the one molecule or ion from each set having smallest bond angle.

(i) NH_3, PH_3 or AsH_3 (ii) O_3^+, O_3 (iii) NO_2^- or O_3 (iv) X — S — X angle in $SOCl_2$ or SOF_2

(a) NH_3, O_3^+, O_3, $SOCl_2$ (b) AsH_3, O_3, NO_2^-, SOF_2 (c) PH_3, O_3^+, NO_2^- (d) AsH_3, O_3^+, O_3, SOF_2

40. What percentage of oxygen is present in the compound $CaCO_3$. 3 $Ca_3(PO_4)_2$?

(a) 40.54 % (b) 35.56 % (c) 15.40 % (d) 41.95%

41. The polymer obtained by addition polymerisation of [x]. Which can be obtained by reaction between 1-chloro-2-phenyl ethane and potassium tertiary butoxide?

(a) Polystyrene (b) Urea formaldehyde resin

(c) Bakelite (d) Glyptal

42. A detergent ($C_{12}H_{25}$ $SO_4^-Na^+$) solution becomes a colloidal solution at a concentration of 10^{-4} M. On an average 10^{13} colloidal particles are present in 1 mm^3. What is the average number of ions contained in one colloidal particle (micelle)? {Given, $N_A = 6 \times 10^{23}$}

(a) 10^{12} (b) 10^{13} (c) 12 (d) 6

43. $[PdCl_2(PMe_3)_2]$ is a diamagnetic complex of Pd (II). How many total isomers are possible for $[NiCl_2(PMe_3)_2]$ having oxidation state of Ni is (II)?

(a) 3 (b) 2 (c) 1 (d) Zero

44. Arrange the following in increasing order of basic strength in aqueous medium.

$(CH_3)_3$ N, $(CH_3)_2$ NH, $CH_3CH_2NH_2$

(a) $(CH_3)_3$ N > $(CH_3)_2$ NH > $CH_3CH_2NH_2$ (b) $(CH_3)_2$ NH > $(CH_3)_3$ N > $CH_3CH_2NH_2$

(c) $(CH_3)_3$ N < $(CH_3)_2$ NH < $CH_3CH_2NH_2$ (d) $(CH_3)_3$ N < $CH_3CH_2NH_2$ < $(CH_3)_2$NH

45. The angular velocity (ω) of an electron occupying second orbit of Li^{2+} ion.

(a) $\frac{9\pi^2K^2me^4}{h^3}$ (b) $\frac{27\,\pi^3\,mK^2e^4}{h^2}$ (c) $\frac{9\pi^3\,K^2m^2e^4}{h^3}$ (d) $\frac{27\pi^2m^3\,K^3e^2}{h^2}$

46. Which of the following is the correct order of stability of conformational isomers of 2-amino ethan-l-ol?

(a) gauche > eclipsed > anti (b) gauche > anti > eclipsed

(c) eclipsed > gauche > anti (d) anti > eclipsed > gauche

47. A red coloured metal oxide A on treatment with conc. HNO_3 gives a compound B. B with HCl produces a chloride C which is insoluble in cold water but soluble in hot water. C can also be formed by treating A with conc. HCl (·) compounds A, B and C are

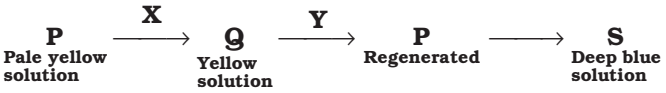
(a) $Fe_2O_3, FeO, FeCl_3$ (b) $ZnO_2, PbO_2, ZnCl_2$

(c) $Mn_2O_3, MnO, MnCl_2$ (d) $Pb_3O_4, PbO_2, PbCl_2$

48. 10 mL of 0.2 M solution of compounds $Na_2CO_3 \cdot NaHCO_3 \cdot 2H_2O$ is titrated against 0.05 M HCl, y mL of HCl is used when phenolphthalein is used as an indicator and x mL of HCl is used (·) when methyl orange is the indicator in two separate titrations (·) hence (x – y) will be

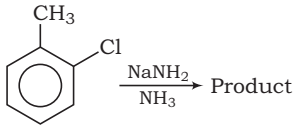
(a) 120 mL (b) 80 mL (c) 100 mL (d) 40 mL

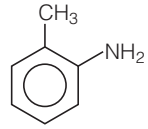
49. If P is $[Fe(CN)_6]^{4-}$, Q is $[Fe(CN)_6]^{3-}$, S is $Fe_4[Fe(CN)_6]_3$, then X, Y and Z respectively are

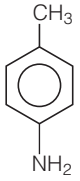


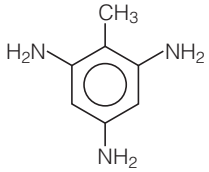
(a) H_2O_2 / OH^-, H_2O_2 neutral medium, Co^{2+} solution
(b) H_2O_2 / H^+, H_2O_2 neutral medium, Cu^{2+} salt solution
(c) H_2O_2 / H^+, H_2O_2 / OH^-, Fe^{3+} salt solution
(d) H_2O_2 / OH^-, H_2O_2 / H^+, Fe^{3+} salt solution

50. Identify the correct product that will obtained after the reaction.



(a) 

(b) 

(c) 

(d) None of these

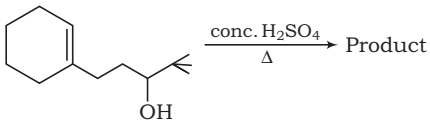
51. Calculate the potential of the following cell.

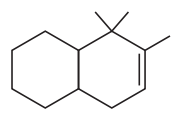
$$Pt\,|H_2\,(g, 0.1\,atm)\,|H^+\,(aq, 10^{-2}M)||MNO_4^-\,(aq, 0.1\,M)$$
$$Mn^{2+}\,(aq),\ 0.1\,M),\ H^+\,(aq,\ 0.01\,M)\ Pt$$

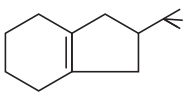
Given, $E^\circ_{MnO_4^- / Mn^{2+}} = 1.50$ V

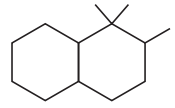
(a) 1.21 V (b) 1.41 V (c) 1.82 V (d) 1.91 V

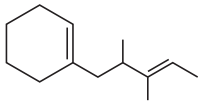
52. The major product and number of asymmetric carbon are



(a)  , 2

(b)  , 1

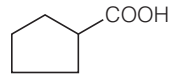
(c)  , 3

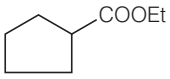
(d)  , 1

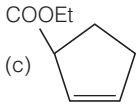
53. **Which of the following compounds is consumed during the preparation of Na_2CO_3 by Solvay's process?**

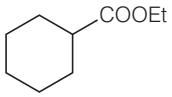
(a) $NH_3 + CaCO_3 + NaCl$ (b) $NaCl + NH_4HCO_3$ (c) $NH_4Cl + CaO + NaCl$ (d) $CaCO_3 + NaCl$

54. **What will be the product when α-bromo cyclohexanone is treated with NaOEt?**

(a) 

(b) 

(c) 

(d) 

55. **What is the reactant [X] which produces cyclohexanone and formaldehyde on reaction with HIO_4?**

(a) Cyclohexan, 1, 2-diol (b) Cyclohexan, 1, 2, 3-triol

(c) 1-hydroxy cyclohexyl methanol (d) 2-hydroxy cyclohexyl methanol

56. **Which of the following order is correct regarding covalent bond character?**

(a) $LiCl > BeCl_2 > BCl_3 > CCl_4$ (b) $LiCl < BeCl_2 < BCl_3 < CCl_4$

(c) $LiCl > BeCl_2 > CCl_4 > BCl_3$ (d) $LiCl < BeCl_2 , BCl_3 > CCl_4$

57. **The forward rate constant for the reversible gaseous reaction $C_2H_6 \rightleftharpoons 2CH_3$ is $3.14 \times 10^{+2}\ s^{-1}$ at 200 K. What is the rate constant for the backward reaction at this temperature, if 10^{-5} moles of CH_3 and 100 mol of C_2H_6 are present in 10 L vessel at equilibrium?**

(a) $1.57 \times 10^{14}\ Lmol^{-1}s^{-1}$ (b) $3.14 \times 10^{15}\ L\ mol^{-1}\ s^{-1}$

(c) $1.57 \times 10^{7}\ Lmol^{-1}\ s^{-1}$ (d) $3.14 \times 10^{10}\ Lmol^{-1}\ s^{-1}$

58. **Germanium (II) compounds are powerful reducing agents whereas lead (IV) compounds are strong oxidising agents even they belongs to same group (14), this is due to**

(a) more powerful inert pair effect in Pb than Ge

(b) the ionisation energy of Pb < IE of Ge

(c) Pb is more electronegative than Ge

(d) the ionic radius of Ge^{2+} and Ge^{4+} are greater than Pb^{2+} and Pb^{4+}

59. **Statement I The vapour pressure of 0.1 M sugar solution is more that of 0.1 M KCl solution.**

Statement II Lowering of vapour pressure is directly proportional to the number of species present in the solution.

(a) Statement I is true, Statement II is also true and Statement II is the correct explanation of Statement I

(b) Statement I is true, Statement II is also true and Statement II is not the correct explanation of Statement I

(c) Statement I is true, Statement II is false

(d) Statement I is false, Statement II is true

60. **Statement I m-RNA and DNA has sequence of base are CATTAAACC and TGAACCATT respectively.**

Statement II In DNA, nitrogeneous base has no hydrogen bond.

Read these two statements written above and choose the correct option.

(a) Statement I is true, Statement II is also true and Statement II is the correct explanation of Statement I

(b) Statement I is true, Statement II is also true and Statement II is not the correct explanation of Statement I

(c) Statement I is true, Statement II is false

(d) Statement I is false, Statement II is true

PART C Mathematics

61. **The equation of tangent at the vertex of parabola $x^2 + 8x + 4y = 0$ is**

(a) $x = -4$ (b) $y = 8$
(c) $y = 4$ (d) $y = -8$

62. **Let z_1 and z_2 are two complex numbers such that $|z_1| = |z_2|$ and $\arg(z_1) + \arg(z_2) = \pi$, then z_1 equals to**

(a) z_2 (b) $-z_2$ (c) $\bar{z}_2$ (d) $-\bar{z}_2$

63. **A curve pass through (2, 0) and slope of the tangent at the point (x, y) is $\frac{(x+1)^2 + y - 3}{x+1}$. Then, the area bounded by the curve and the x-axis is**

(a) 2 (b) 1/3 (c) 4/3 (d) 4

64. **If $\lim_{x \to 0} \frac{a \cos x + bx \sin x - 5}{x^4}$ exists and is finite, then b equals**

(a) – 2.5 (b) 2.5 (c) 5 (d) – 5

65. **Consider the following statements,**

1. Identity relation in a finite set A is the greatest relation in A.
2. The universal relation in a set containing at least 2 elements is not anti-symmetric.
3. The union and intersection of two symmetric relations are also symmetric relations.

Which of these is/are correct?

(a) Only 1 (b) Only 2, 3 (c) Only 1, 3 (d) All of these

66. **The equation of the plane bisecting the acute angle between the planes $2x - y + 2z + 3 = 0$ and $3x - 2y + 6z + 8 = 0$ is**

(a) $23x - 13y + 32z + 45 = 0$ (b) $11x - 11y + 20z + 20 = 0$
(c) $5x - y + z + 40 = 0$ (d) $23x + 13y + 32z + 45 = 0$

67. $\int \frac{f(x)\,\phi'(x) - f'(x)\,\phi(x)}{f(x)\,\phi(x)} \log \frac{\phi(x)}{f(x)}\,dx$

(a) $\log \frac{\phi(x)}{f(x)} + C$ (b) $\frac{1}{2}\left\{\log \frac{\phi(x)}{f(x)}\right\}^2 + C$ (c) $\frac{\phi(x)}{f(x)} \log \frac{\phi(x)}{f(x)} + C$ (d) None of these

68. **If the median AD of a triangle ABC makes an angle θ with side AB. Then, $\sin(A - \theta)$ is**

(a) $\frac{b}{c} \sin\theta$ (b) $\frac{c}{b} \sin\theta$ (c) $\frac{a}{b} \sin\theta$ (d) $\frac{b}{a} \sin\theta$

69. **A die is thrown three times and the sum of the three numbers thrown is 15, then the probability that the first throw was a four is**

(a) $\frac{1}{5}$ (b) $\frac{1}{4}$ (c) $\frac{1}{6}$ (d) $\frac{2}{15}$

70. **The point ([P + 1], [P]), (where, [·] denotes the greatest integer function inside the region bounded by the circle $x^2 + y^2 - 2x - 15 = 0$ and $x^2 + y^2 - 2x - 7 = 0$, then**

(a) $P \in [-1, 0) \cup [0, 1) \cup [1, 2)$ (b) $P \in [-1, 2) - \{0, 1\}$
(c) $P \in (-1, 2)$ (d) None of these

71. **If three points whose position vectors are $\mathbf{A} = a\hat{\mathbf{j}} + b\hat{\mathbf{j}} + c\hat{\mathbf{k}}$, $\mathbf{B} = \hat{\mathbf{i}} + c\hat{\mathbf{j}}$ and $\mathbf{C} = -\hat{\mathbf{i}} - \hat{\mathbf{j}}$ are collinear, then**

(a) $a - 2b = 1$ (b) $a - 2b = 2$ (c) $a - 2b = 3$ (d) $a - 2b = 0$

72. **The two consecutive terms in the expansion of $(3 + 2x)^{74}$ whose coefficients are equal, is/are**

(a) 30^{th} and 31^{st} terms (b) 29^{th} and 30^{th} terms
(c) 31^{st} and 32^{nd} terms (d) 28^{th} and 29^{th} terms

73. **If the adjoint of a 3×3 matrix P is $\begin{bmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{bmatrix}$, then the possible values of the determinant of P are**

(a) ± 2 (b) ± 1 (c) ± 3 (d) ± 4

74. **If p is true and q is false then which of the following is having its truth value as true?**

(a) $p \rightarrow \sim q$ (b) $p \leftrightarrow q$ (c) $q \rightarrow \sim p$ (d) Both (a) and (c)

75. **The roots of the quadratic equation $ax^2 + bx + c = 0$ be α, β $(\alpha, \beta \notin \{-1\})$. Roots of quadratic equation $a(x + 1)^2 - b(x + 1)x + cx^2 = 0$ are**

(a) $-(\alpha + 1), -(\beta + 1)$ (b) $\frac{-\alpha}{\alpha + 1}, \frac{-\beta}{\beta + 1}$ (c) $\frac{-1}{\alpha + 1}, \frac{-1}{\beta + 1}$ (d) $\frac{1}{\alpha + 1}, \frac{1}{\beta + 1}$

76. **Consider that $f'(x) > g'(x)$ for all real x, and $f(0) = g(0)$, then $f(x) < g(x)$ for all x belonging to**

(a) $(-\infty, 2)$ (b) $(-\infty, 0)$ (c) $(-\infty, 3)$ (d) $(-\infty, 1)$

77. **Number of positive unequal solution of the equation $x + y + z = 6$ is**

(a) 4^1 (b) 6^1 (c) 5^1 (d) 2×4^1

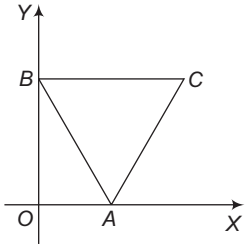
78. $\int \frac{e^x}{x + 2} \{1 + (x + 2) \log (x + 2)\} dx$

(a) $e^x \log(x + 2) + C$ (b) $\frac{e^x}{x + 2} + C$ (c) $e^x (x + 2) + C$ (d) $e^x (x - 2) + C$

79. **If two sets A and B are having 99 elements in common, then the number of elements common to each of the sets $A \times B$ and $B \times A$ are**

(a) 2^{99} (b) 99^2 (c) 100 (d) 18

80. **Adjacent figure represents a equilateral triangle ABC of side length 2 units. Locus of vertex C as the side AB slides along the coordinate axis is**



(a) $x^2 + y^2 - xy + 1 = 0$ (b) $x^2 + y^2 + xy\sqrt{3} = 1$
(c) $x^2 + y^2 = 1 + xy\sqrt{3}$ (d) $x^2 + y^2 - xy\sqrt{3} + 1 = 0$

81. **If the median of 40 observations is 160 and the observations greater than the median are increased by 10, then new median will be**

(a) 170 (b) 40 (c) 150 (d) None of these

82. **If, $y^{\frac{1}{m}} + y^{-\frac{1}{m}} = 2x$, then $(x^2 - 1)\, y_2$ is equal to**

(a) $xy_1 + m^2y$ (b) $xy_1 - m^2y$

(c) $-xy_1 + m^2y$ (d) $-xy_1 - m^2y$

83. **If** $I = \begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}\begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, **then**

(a) $a = 1, b = 1$ (b) $a = \cos 2\theta, b = \sin 2\theta$

(c) $a = \sin 2\theta, b = \cos 2\theta$ (d) None of these

84. **The coordinates of the foot of the perpendicular drawn from the origin to the plane $3x + 4y - 6z + 1 = 0$ is**

(a) $\left(\frac{3}{61}, \frac{4}{61}, \frac{6}{61}\right)$ (b) $\left(\frac{-3}{61}, \frac{6}{61}, \frac{6}{61}\right)$ (c) $\left(\frac{-3}{61}, \frac{6}{61}, \frac{-6}{61}\right)$ (d) $\left(\frac{-3}{61}, \frac{-4}{61}, \frac{6}{61}\right)$

85. **Out of 15 tokens consecutively numbered from 1 to 15, 3 tokens are drawn at random. The probability that the numbers on them in AP is**

(a) 7/65 (b) 6/65 (c) 7/55 (d) 5/56

86. **a, b, c, d, e are positive real numbers such that $a + b + c + d + e = 15$ and $ab^2c^3d^4e^5 = (120)^3\,(50)$, then the value of $a^2 + b^2 + c^2 + d^2 + e^2$ is**

(a) 40 (b) 50 (c) 45 (d) 55

87. **The orthogonal trajectory of the family of unit circles with centres on x-axis which passes through the point (0, 1) is**

(a) $x = \sqrt{1-y^2} - \log\left(\frac{1+\sqrt{1-y^2}}{y}\right)$ (b) $x = -\sqrt{1-y^2} + \log\left(\frac{1+\sqrt{1-y^2}}{y}\right)$

(c) $x = 2\sqrt{1-y^2} + \log\left(\frac{2+\sqrt{1-y^2}}{3y}\right)$ (d) None of these

88. **Consider $\hat{a}$ and $\hat{b}$ be two unit vectors such that $\hat{a} + \hat{b}$ is also a unit vector. Then, the angle between $\hat{a}$ and $\hat{b}$ is**

(a) acute angle (b) right angle (c) obtuse angle (d) straight angle

89. **Statement I** Consider the identity, $\dfrac{\sin\frac{\theta}{2} - \sin\frac{\phi}{2}}{\cos\frac{\theta}{2} + \cos\frac{\phi}{2}} = \tan\dfrac{\theta - \phi}{4}$

$$\left(\frac{\cos A + \cos B}{\sin A - \sin B}\right)^n + \left(\frac{\sin A + \sin B}{\cos A - \cos B}\right)^n = \begin{cases} 2\cot^n\dfrac{A-B}{2} & , n \text{ is odd} \\ 0 & , n \text{ is even} \end{cases}$$

Statement II $\dfrac{\cos A + \cos B}{\sin A - \sin B} = \cot\dfrac{A - B}{2}$

(a) Statement I is true, Statement II is also true and Statement II is the correct explanation of the Statement I

(b) Statement I is true, Statement II is also true and Statement II is not the correct explanation of the Statement I

(c) Statement I is true, Statement II is false

(d) Statement I is false, Statement II is true

90. Statement I If the vertices of a triangle are having rational coordinates, then its centroid, circumcentre and orthocentre are rational.

Statement II In any triangle orthocentre, centroid and circumcentre are collinear and centroid divide the line joining orthocentre and circumcentre in the ratio 2 : 1.

(a) Statement I is true, Statement II is also true and Statement II is the correct explanation of Statement I

(b) Statement I is true, Statement II is also true and Statement II is not the correct explanation of Statement I

(c) Statement I is true, Statement II is false

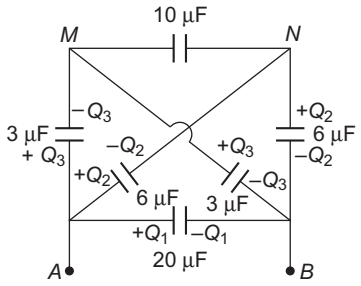
(d) Statement I is false, Statement II is true

Analytical Explanations

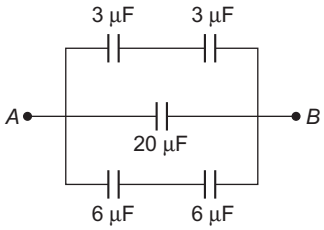
Physics

1. (*a*) **Idea** This question is based on the distribution of charges through capacitors. Here one more thing should be noted that the net charge of the isolated part must be zero.

From the charge distribution,



Now, we redraw the circuit.

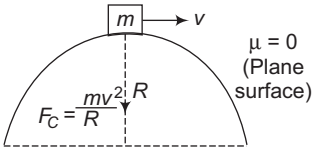


$\Rightarrow \quad C_{eq} = 24.5\ \mu F$

TEST Edge The question based on combination of capacitors could be solved by distribution of charges, series and parallel combination and Wheat stone bridge.

2. (*c*) **Idea** This question is of circular motion and one thing should be noticed that the centripetal force will be provided by the net external force acting on that body (there could be more than one force).

Always remember that centripetal force is not a real force. Here in FBD, it is provided by *mg*



If $(F_C)_{required} < mg$

Then, $mg - N = \frac{mv^2}{R} \quad \Rightarrow \quad N = W$

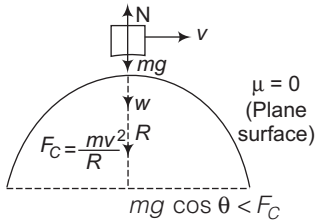
For v_{max}, the box will not press the surface below and the entire *m* will use as the centripetal force.

$\Rightarrow \quad N = W = 0$

$\Rightarrow \quad mg = \frac{mv^2_{max}}{R} \quad \Rightarrow \quad v_{max} = \sqrt{Rg}$

As the block moves downward, its speed starts increasing (increasing the F_C) and the component of force ($mg \cos\theta$) towards centre will start decreasing.

The block will leave the surface when



or $mg < F_C$, highest point

Now, you can check why option (c) is correct.

TEST Edge In almost every question of circular motion first one must see the forced acting on that body and then one should find net external force acting towards the centre. This net force (towards the centre) will act as a centripetal force.

3. (*b*) As direction of current is out of plane. So by right hand thumb rule, direction of field to the left of wire is downwards and to the right is upwards. By using this and superimposing the fields due to two wires. We get the net field as shown by option (b).

Note Strength of field $\propto \frac{1}{\text{distance}}$

So, near the wire strength of field is more and approaching to infinity and it declines gradually as the distance increases.

4. (*c*) $\tan\phi = \frac{|X_L - X_C|}{R}$

***Case* I** When capacitor is removed, then

$$\tan 60° = \frac{X_L}{R} \quad \Rightarrow \quad X_L = \sqrt{3}\,R$$

***Case* II** When inductor is removed, then

$X_C = \sqrt{3}\,R$

So, $Z = \sqrt{R^2 + (\sqrt{3}\,R - \sqrt{3}\,R)^2} = R$

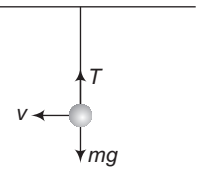
So, *L-C-R* would acts as pure resistor and thus power factor $\cos\phi = 1$

Power dissipated is $P = \frac{V^2}{R} = \frac{200 \times 200}{100}$

$= 400\ W$

5. (*d*) **Idea** At the lowest point $mg \sin\theta = 0$.

So work done by it will be zero at the lowest point.



At the lowest point,

$$mg \sin\theta = 0$$

It seems, $W_{mg\sin\theta} = 0$ at the lowest point, but there is no such thing "work done at an instant".

TEST Edge Questions on vertical circular loop could also be asked on this topic. In these questions, one must know the **conservation of mechanical energy** and how to find **centripetal force expression**.

6. (*b*) **Idea** In this question, one can apply conservation of linear momentum and conservation of angular momentum.

From conservation of angular momentum theorem,

Initial, $L_i = mv\,l$

After collision, the common velocity of the system v'.

From linear momentum conservation,

$$mv = (M + m)\,v'$$

$$v' = \frac{m}{(M + m)}\,v$$

So, the angular momentum of the system after collision

$$L_f = (M + m)\,v'\,l$$

$$= (M + m)\,\frac{mv}{(M + m)}\,l$$

$$= mlv$$

Note Here, important thing to note is that the angular speed of ring will be zero.

TEST Edge Different types of questions could be asked on the above concepts. A question is usually asked where a point mass hits a stationary rod and stick to it the angular speed of the rod after collision could be asked. Here just apply conservation of linear momentum and conservation of angular momentum.

7. (*c*) $p_M > p_N$

[pressure at M is greater than pressure at N]

So, velocity

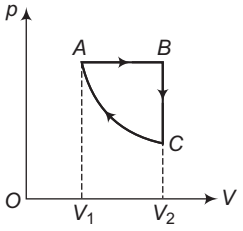
$$v_M < v_N$$

From this observation, it seems that $h_1 > h_2$ but there is no relation given between A_1 and A_2. So, there could be any relation possible between h_1 and h_2.

8. (*a*)

Name of the process	Information obtained from the graph	Pressure	Volume	Nature of p-V graph
$A \to B$	$V \propto T$ $\therefore p =$ constant V and T both are increasing	Constant	Increasing	Straight line parallel to V-axis as $p =$ constant
$B \to C$	$V =$ constant $\therefore \quad p \propto T$ T is decreasing. $\therefore \quad p$ will decrease	Decreasing	Constant	Straight line parallel to p-axis, as $V =$ constant
$C \to A$	$T =$ constant $\therefore \quad p \propto \frac{1}{V}$ V is decreasing $\therefore p$ should increase	Increasing	Decreasing	Rectangular hyperbola as $p \propto \frac{1}{V}$

The corresponding graph is shown in figure given below



9. (*c*) Moment of inertia of rod about point O is

$I =$ Moment of inertia of OA + Moment of inertia of OC

$$= \frac{1}{3}\left(\frac{M}{4}\right)\cdot(l)^2 + \frac{1}{3} \times \frac{3M}{4} \times (3l)^2$$

$$= \frac{Ml^2}{12} + \frac{27\,Ml^2}{12}$$

$$= \frac{7}{3}\,Ml^2$$

As $L = I\omega = J$

$$\Rightarrow \quad \omega = \frac{L}{I} = \frac{3J}{7Ml^2}$$

Now, for rotating rod induced emf is

$$e = \frac{1}{2}\,Bl^2\omega$$

For part AO, $e_{OA} = e_O - e_A = \frac{1}{2}\,Bl^2\omega$

For part OC, $e_{OC} = e_O - e_C = \frac{1}{2} B (3l)^2 \omega$

$\therefore \quad e_A - e_C = 4Bl^2\omega$

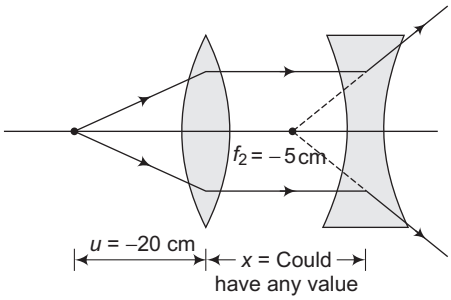
$= 4Bl^2 \times \frac{3J}{7Ml^2}$

$= \frac{12\,BJ}{7M}$

10. (c) Resonance frequency of series L-C-R circuit is $\frac{1}{2\pi\sqrt{LC}}$ only when $Z = 0$ of that circuit.

11. (d) **Idea** Above question is based on combination of lenses. The easier way to solve this question is to draw the ray diagram.

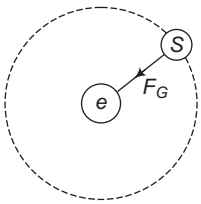
From figure, it is clear that the final (virtual) image will be at the focus of concave lens. So, the object has to be at $u = -20$ cm (at the focus of the convex lens)



TEST Edge The questions on combination of lenses could also be solved by the direct formulae but in this question it will be easier to draw ray diagram and solve this question.

12. (d) **Idea** For this question, one must understand the concept of centripetal force.

The gravitational force acts as a centripetal force, *i.e.,* people in the satellite feels weightlessness.

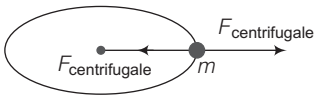


If we observe from the earth, the satellite needs a centripetal acceleration that is provided by the gravitational force.

TEST Edge Free fall does not mean that there is, no gravitational force is acting, it just means that the person is not getting normal reaction. In satellite, the gravitational force acts on the person but it acts like centripetal force to revolve it. So, the person will not get any normal reaction (free fall).

13. (c) **Idea** To solve this question, one must understand that centrifugal force is a pseudo force and it will be considered only when we observe from the accelerated frame.

Centrifugal force is a pseudo force (not a real force) and we apply it only when we observe from the accelerated frame. (*i.e.,* frame of the revolving body).

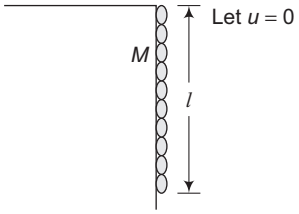


So, option (c) is correct.

TEST Edge Questions on pseudo force are generally asked in **Laws of motion**.

Just remember that we will apply pseudo force only if we are observing from accelerated frame.

14. (a) When chain falls completely out of the table.



Loss of PE $= U_i - U_f$

$$U_i = -\left[\frac{M}{2} \times g \times \frac{l}{4}\right]$$

[from centre of mass concept]

$$U_f = -M \times g \times \frac{l}{2}$$

[at table level let $U = 0$]

So, $\quad U_i - U_f = \frac{-Mgl}{8} + \frac{Mgl}{2}$

$$= \frac{-Mgl}{2}\left(\frac{1}{4} - 1\right)$$

$$\Delta U = \frac{3\,Mgl}{8}$$

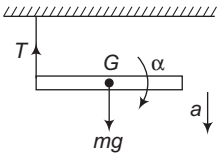
From mechanical energy conservation,

$$\Delta U = \Delta \text{ KE}$$

$$\frac{3\,Mgl}{8} = \frac{1}{2} Mv^2$$

$$v = \sqrt{\frac{3gl}{4}} = \frac{\sqrt{3gl}}{2}$$

15. (*c*) Let the tension in the string be T and the acceleration of the centre of mass of the rod downwards be a.



Then, $mg - T = ma$ ...(i)

Again, $\frac{mgl}{2} = I\alpha = \frac{ml^2}{3} \times \frac{a}{l/2}$

$a = \frac{3}{4}g$ ...(ii)

$\therefore$ $T = \frac{mg}{4}$

16. (*b*) $H = KA\frac{dT}{dx}$ is same in steady state condition.

$\therefore$ $A\frac{dT}{dx} =$ constant

$\left(\frac{dT}{dx} = \text{Temperature gradient}\right)$

So, graph will be hyperbolic graph with shifted origin.

17. (*c*) As the external force on the system is zero then the centre of mass of the system will not change its position. This condition is possible when boy and centre of plank reaches at centre of mass.

So, the boy had moved a distance $\frac{l}{2}$ with respect to plank.

18. (*d*) **Idea** This question is based on Doppeler's effect. Hence, one thing should be understood that air flow will only change the speed of sound, it will not effect relative motion between the observer and source.

The motion of air just changes the speed of sound in air.

v_{sound} in blowing air $= 330 - 108 \times \frac{5}{18}$

$= 300$ m/s

$\Rightarrow$ $f' = \frac{v_{sound}}{v_{sound} - v_s} f_0$

$= \frac{300}{300 - 15} \times 300$

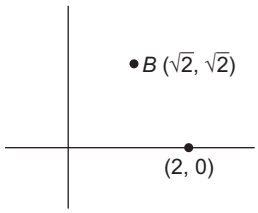
$= \frac{300 \times 300}{285}$

$= 315$ Hz

TEST Edge For these types of questions where the air flow is considered, then one can take the frame of reference of air to solve the question.

19. (*a*) $V_A = \frac{q}{4\pi\varepsilon_0 r_A}$ and $V_B = \frac{q}{4\pi\varepsilon_0 r_B}$

$\Rightarrow$ $V_A - V_B = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_A} - \frac{1}{r_B}\right]$



Now, r_A = distance of point A from origin = 2 units

and r_B = distance of point B from origin

$= [(\sqrt{2})^2 + (\sqrt{2})^2)^{1/2} = 2$ units

As $r_A = r_B$

$\therefore$ $V_A - V_B =$ zero

20. (*c*) **Idea** This question is based on one-dimensional motion.

Here, one thing should be noticed that the final displacement of boy and ball is same.

When the ball come back into the hands of the boy, the displacement of boy and ball will be same.

Boy	Ball
$u = +10$ m/s, $a = +2$ m/s^2	$u = -10$ m/s, $a = +10$ m/s^2

$\because$ $S_{Boy} = S_{Ball}$

$10t + \frac{1}{2} \times 2 \times t^2 = -10t + \frac{1}{2} \times 10 \times t^2$

$20t = 5t^2 - t^2$,

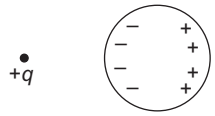
$20t = 4t^2$

$t = 5$ s

TEST Edge In these types of questions, we can solve the question from any frame of reference (whether it is lift or ground frame).

21. (*a*) **Idea** This question shows that the electric field lines are always perpendicular to a conducting surface.

Charge distribution would be like this,



Now, (i) field lines originate from positive charge and ends at negative charge.

(ii) Field lines are always perpendicular to surface of a conductor.

TEST Edge The electric field inside a conducting sphere will also be zero. The electric field inside a conducting shell will be zero if there is no charge is placed inside it.

22. (*d*) **Idea** This is a simple question based on errors.

If $x = A^n$ then $\frac{\Delta x}{x}\% = \frac{n\Delta A}{A}\%$

Δx is error in calculation of x.

$$\frac{\Delta r}{r}\% = 5\%$$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{\Delta V}{V} = 3\frac{\Delta r}{r}$$

$$\left(\frac{\Delta V}{V}\right)\% = 3\left(\frac{\Delta r}{r}\right)\%$$

$$\frac{\Delta V}{V}\% = 3 \times 5 = 15\,\%$$

TEST Edge On errors different questions could be asked. Some questions could be asked on absolute error, error in subtraction, addition, multiplication and division.

23. (*a*) In series, potential difference $\propto$ resistance

Hence, $\frac{r_A}{r_B} = \frac{3}{2}$

Now, $R \propto L/A$ and A = area $= \pi r^2$

$$\Rightarrow \quad \frac{L_A}{L_B} \times \left(\frac{r_B}{r_A}\right)^2 = \frac{3}{2}$$

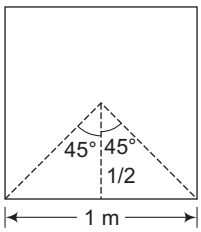
$$\Rightarrow \quad \frac{6}{1} \times \left(\frac{r_B}{r_A}\right)^2 = \frac{3}{2}$$

$$\Rightarrow \quad \frac{r_B}{r_A} = \frac{1}{2}$$

24. (*b*) **Idea** This question is based on magnetic field due to a current carrying configurations.

Radius of circular loop is

$$R = \frac{4}{2\pi} \qquad [\because 2\pi R = 4]$$



So, field at centre of circle is

$$B' = \frac{\mu_0 i}{2R} = \frac{\pi \mu_0 i}{4}$$

Field due to square is

$$B = 4\left[\frac{\mu_0}{4\pi} \cdot \frac{i}{1/2}(\sin 45° + \sin 45°)\right]$$

$$\Rightarrow \quad B = \frac{2\sqrt{2}\,\mu_0 i}{\pi}$$

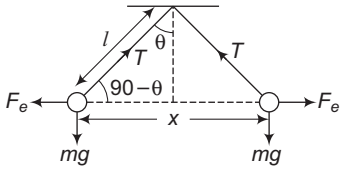
So, $B/B' = 8\sqrt{2}/\pi^2$

TEST Edge Different types of current configurations like straight wire, circular wire, solenoid and toroid are also exists. So, questions based on these configurations could also be asked.

25. (*d*) T is tension in the strong, then

$$T \sin\theta = F_e = \frac{kq^2}{x^2} \qquad \ldots(i)$$

and $T\cos\theta = mg$ …(ii)



From Eqs. (i) and (ii), we get

$$\Rightarrow \quad \tan\theta \times mg = \frac{kq^2}{x^2}$$

$$\Rightarrow \quad q^2 = \frac{mg}{k}\tan\theta \cdot x^2$$

Now, $\tan\theta \simeq \theta = \frac{x}{2l}$ $\qquad [\because d << l]$

$$\Rightarrow \quad q^2 = \frac{mg}{2kl}x^3$$

or $q = C \cdot x^{3/2}$ $\qquad \left[\text{Here, } C = \sqrt{\frac{mg}{2kl}}\right]$

$$\Rightarrow \quad \frac{dq}{dt} = C \cdot \frac{3}{2}x^{1/2}\frac{dx}{dt}$$

$\frac{dx}{dt}$ = velocity

$$\Rightarrow \quad \frac{dq}{dt} = \frac{3}{2}C \cdot x^{1/2}\,v$$

or $v \propto x^{-1/2}$

$[\because \frac{dq}{dt}$ is also constant as given in question]

26. (*b*) **Idea** This question is based on radioactivity. Just apply the formula of radioactive decay and solve the question.

$T_{\text{half}} = 20$ min

Now, $N = N_0(1/2)^{t/T}$, for given conditions we have,

$$0.8\,N_0 = N_0(1/2)^{t_1/T}$$

and $0.1\,N_0 = N_0(1/2)^{t_2/T}$

Taking their ratio, we get

$$\frac{0.1}{0.8} = \left(\frac{1}{2}\right)^{\frac{t_2 - t_1}{T}}$$

$$\left(\frac{1}{2}\right)^3 = (1/2)^{\frac{t_2 - t_1}{T}}$$

$$\frac{t_2 - t_1}{2} = 3 \quad \Rightarrow \quad t_2 - t_1 = 6 \text{ h}$$

TEST Edge From radioactivity some question could be asked (from concept of) α and β decay. Some questions could be asked from half-life.

27. (*c*) Path difference of light reaching *O* is

$$\Delta x = (PS_1 + S_1O) - [PS_2 + S_2O + (\mu - 1)\, t]$$

Note $(\mu - 1)t$ is the net increase of path length due to introduction of glass plate.

Now, $PS_1 = S_1O = \sqrt{d^2 + D^2}$

$$= D\left[1 + \frac{d^2}{2D^2}\right] \text{(using binomial expansion).}$$

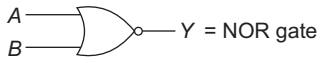
and $PS_2 = S_2O = D$,

also given is $\mu = 1.5$ and $\Delta x = 0$ for central maxima.

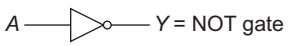
Hence, $0 = 2\left[D + \frac{d^2}{2D}\right] - [2(D) + 0.5\, t]$

$$\Rightarrow \qquad 0 = \frac{d^2}{D} - 0.5\, t \quad \Rightarrow \quad t = \frac{2d^2}{D}$$

28. (*b*)



Here, $Y = \overline{A + B}$



Here, $Y = \overline{A}$

So, in the given problem

$$Y = \overline{\overline{A} + \overline{B}}$$

$$= \overline{\overline{A}}\,.\,\overline{\overline{B}} \quad \text{(using De-Morgan's law)}$$

$$= AB$$

∴ AND gate.

29. (*b*) **Idea** In YDSE $I_{max} = (a + a)^2 = 4a^2$

$$= 4I_0$$

and $I_{min} = (a - a) = 0$

In YDSE, $I_{max} = 4I_0$ [I_0 is the intensity of single beam] and $I_{min} = 0$ *i.e.,* minima is perfectly dark.

$$\therefore \qquad \frac{I_{max}}{I_{min}} \to \infty$$

But when width of any one of the slit is slightly increased then intensity of two waves would become different and thus minimas will not be perfectly dark and thus $I_{min} \neq 0$

$\therefore \frac{I_{max}}{I_{min}}$ would become finite.

TEST Edge In YDSE when we use white light we get central white fringe and red and blue dominant fringes on both the sides of central fringe.

30. (*d*) **Idea** The centripetal force needed for the moon is provided by the gravitational force. Here, one should notice that the only real force here is gravitational force.

The moon revolve around the earth due to a real gravitational force. As that force acts towards the centre also called centripetal force.

The net force on the moon = $\mathbf{F}_{Grav}$

TEST Edge In the case of a car moving on a circular banked road, the centripetal force is provided by the friction and the component of the normal reaction.

Chemistry

31. (*c*) **Idea** This problem includes conceptual mixing of IE and EA. This problem can be solved by knowing the exact concept involved in IE and EA.

Amount of energy involved in loosing electron from outermost shell is known as IE while amount of energy involved in accepting one extra electron is known as EA and we know that

$$\text{IE} = \text{EA}$$

In process (ii), IE is involved

$$Cl^- \longrightarrow Cl + e^-$$

While in process (i), EA is involved.

$$Cl + e^- \longrightarrow Cl^-$$

So, same amount of energy is released when electron is absorbed.

So, |IE of process (ii)| = |EA of process (i)|

TEST Edge JEE Main includes this type of problems in examination to judge the understanding of the student in basic inorganic periodic properties. So students are advised to go through better understanding of periodic trends of various atomic properties, such as electronegativity, metallic character, size of atom etc.

32. (*c*) **Idea** This problem includes concept of determination of equilibrium constant. This problem can be solved by using the skill of partial pressure and K_p.

$$2AB\,(s) \rightleftharpoons A_2\,(g) + B_2\,(g)$$

$$(0.5 + p_0)\; p_0$$

$$K_p = p_{A_2} \times p_{B_2}$$

$$0.06 = (0.5 + p_0)\; p_0$$

$$p_0^2 + 0.5\, p_0 - 0.06 = 0$$

$$p_0 = 0.1$$

$$p_{total} = p_{A_2} + p_{B_2} = (0.5 + p_0) + p_0$$

$$= 0.6 + 0.1 = 0.7 \text{ atm}$$

TEST Edge Similar questions including conceptual mixing of K_p, K_c can be asked in JEE Main which is a general trend of JEE Main and can be solved by using relation

$$K_p = K_c RT^{\Delta n}$$

33. (*a*) **Idea** This problem includes conceptual mixing of various calculation involved in Duma's method *viz,* determination of volume of nitrogen and determination of % age of nitrogen as follows

- Determine the % of N_2 at STP using $\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$
- Determine the mass of N_2 using gram molar volume then calculate % of N_2.

Determination of volume of nitrogen

Given, mass of substance = 0.30 g

V_{N_2} (volume of N_2) = 50 cm^3

Atmospheric pressure = 715 mm Hg

Room temperature = 300 K

Vapour pressure of water at 300 K = 15 mm

Actual vapour pressure of dry gas

= 715 – 15 = 700 mm Hg

Now, the volume at experimental condition is standarize by converting it into STP.

$p_1 = 700$ mm $\quad p_2 = 760$ mm

$V_1 = 50\ cm^3$, $N_2 = ?$

$T_1 = 300$ K, $\quad T_2 = 273$ K

As we know $\quad \frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$

$$\frac{700 \times 50}{300} = \frac{760 \times V_2}{273}$$

$$V_2 = \frac{273 \times 700 \times 50}{300 \times 760} = 41.9\ cm^3$$

The mass of nitrogen in 41.9 cm^3 gas is calculated by using concept of gram molar volume.

22400 cm^3 of N_2 at STP weigh = 28 g

41.9 cm^3 of N_2 at STP weigh

$$= \frac{28 \times 41.9}{22400}\ g$$

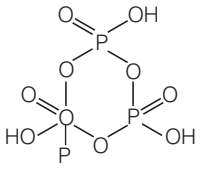
Percentage of N_2

$$= \frac{\text{Mass of } N_2}{\text{Total mass of organic compound}} \times 100$$

$$= \frac{28 \times 41.9}{22400} \times \frac{100}{0.3} = 17.46\ \%$$

TEST Edge Similar questions having % of P or % of S may also be asked in JEE Main so students are advised to go through quantitative calculation of % of P or % of S.

34. (*c*) $H_3P_3O_9$ is a cyclic trimetaphosphoric acid.



Structure of $H_3P_3O_9$

35. (*c*) **Idea** This problem can be solved by using the concept of determination of rate of reversible reaction as follows

- Write the chemical transformation at $t = 0$, $t = t$ and $t = t_{eq}$ and calculate the value of k_b
- using relation $k_{(aq)} = \frac{k_f}{k_b}$
- Now, calculate the value of t using relation

$$k = \frac{2.303}{t} \log \frac{x}{x - a}$$

$$cis\text{-}Z \underset{k_b}{\overset{k_f}{\rightleftharpoons}} trans\text{-}Z$$

Initial	a	0	$k_{(aq)} = \frac{k_f}{k_b}$
at time t	$a - x$	x	$k_b = \frac{3 \times 10^{-5}}{10^{-2}}$
At equilibrium	$a - x_e$	x_e	$= 3 \times 10^{-3} = k_b$

According to relation

$$(k_f + k_b) = \frac{2.303}{t} \log \frac{x_e}{x_e - x}$$

Given, $\quad x = \frac{x_e}{2}$

$$(k_f + k_b) = \frac{2.303}{t} \log 2$$

$$(3 \times 10^{-5} + 3 \times 10^{-3}) = \frac{0.693}{t}$$

$$t = \frac{0.693}{3 \times 10^{-3}(1 + .01)} = \frac{0.693}{3 \times 10^{-3} \times 0.01}$$

$$= 2.3 \times 10^4\ s$$

TEST Edge In JEE Main, these questions are asked frequently to know the in depth knowledge of student in rate equation of different types of questions. So students are advised to go through in depth study of consecutive reaction and parallel reaction also.

36. (*c*) $4FeCr_2O_4 + \underset{(X)}{8Na_2CO_3} + 7O_2 \xrightarrow[\text{air}]{\text{Heat}} 8Na_2CrO_4 + 2Fe_2O_3 + 8CO_2$

$$Na_2CrO_4 + 2NH_4Cl \longrightarrow (NH_4)_2Cr_2O_7 + 2NaCl$$

$$\downarrow \text{Heat}$$

$$Cr_2O_3 \xrightarrow[(Z)]{Al} Cr$$

37. (*a*) **Idea** This problem is based on concept of oxidation number of elements in various compound. This problem can be solved by considering the oxidation state of element is equal to x. (which has to determine).

NH_4NO_3 exists as $NH_4^+ NO_3^-$

In NH_4^+, N have oxidation state

$$x + 4 = +1 \Rightarrow x = -3$$

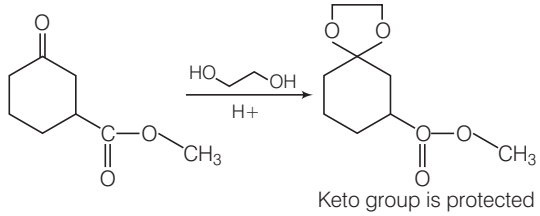
In NO_3^-, N have oxidation state

$$x - 6 = -1 \Rightarrow x + 5$$

TEST Edge This problem is asked in JEE Main to judge the basic understanding of students in calculation of oxidation state. Students are advised to go through calculation of oxidation state of elements oxoacid of chlorine and sulphur.

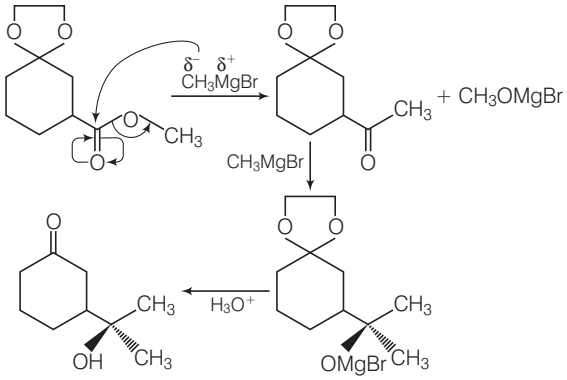
38. (*c*) **Idea** This problem includes conceptual mixing of protecting of carbonyl group and addition reaction to the carbonyl group. This problem is solved by protecting the keto group first followed by nucleophilic addition reaction to carbonyl centre and subsequent hydrolysis to get the desired product.

Protection of carbonyl group Aldehyde and ketone are protected by formation of acetal.



Keto group is protected

Addition of Grignard reagent to ester group leading to formation of ketone and then alcohol after hydrolysis.



Hence, the correct choice is (c).

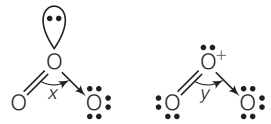
TEST Edge Similar problems taking the keto group or another group under protection and reaction of ther group are also be asked in JEE Main very frequently.

39. (*b*) This problem includes conceptual mixing of bond angle, Drago rule and Bent rule.

(i) **Drago rule** According to Drago rule bond angle of NH_3, PH_3 and AsH_3 decreases top to bottom

$$NH_3 > PH_3 > AsH_3$$

Here, AsH_3 has smallest bond angle.

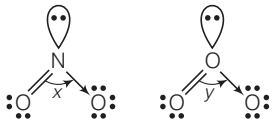


$x < y$

Lone pair-double bond repulsion is O_3 is greater than single electron-double bond repulsion in O_3^+

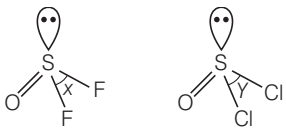
The repulsion between lone-pair single bond in O_3 is greater than repulsion between single electron single bond in O_3.

(ii) According to Bent's rule bond order of N-O bond in NO_2^- has more *p* character than O—O bond in O_3.



O_3 has sp^2 hybridisation central oxygen atom.

(iii) According to Bent rule *p*-character of S-Cl bond in $SOCl_2$ is lower than *p*-character of S - F bond in SOF_2.



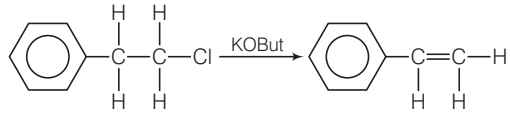
Hence, the correct choice regarding smallest bond angle is (b).

40. (*d*) % of $O = \dfrac{16 \times 27}{100 + 3 \times 310} \times 100 = 41.95\ \%$

41. (*a*) **Idea** This problem involves conceptual mixing of preparation of styrene and addition polymerisation of styrene. Students are advised to follow these steps.

- Complete the reaction first using information provided in question. (use elimination reaction)
- Then, using the product of above reaction as starting material identify the correct product.

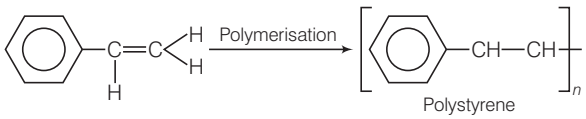
Preparation of styrene



1,chloro 2-phenyl ethane Styrene

Styrene is obtained due to elimination reaction of 1-chloro-2 phenyl ethane in presence of strong base KO But

Polymerisation of styrene



TEST Edge Problems related to preparation of bakelite, nylon 6, nylon 6,6 and other polymers using different modes of reaction to prepare the monomer are asked in JEE Main frequently, students are advised to go through these topics.

42. (*d*) Number of sodium lauryl sulphate

$[C_{12}H_{25}SO_4^-Na^+]$ in 1 L solution

$$= 10^{-4} \times 6 \times 10^{23} = 6 \times 10^{19}$$

Number of sodium lauryl sulphate per mm^3

$$= 6 \times 10^{13}$$

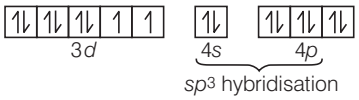
Number of colloidal particles per $mm^3 = 10^{13}$

Number of molecules per colloidal particle

$$= \frac{6 \times 10^{13}}{10^{13}} = 6$$

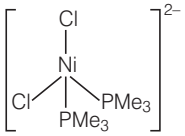
43. (*d*) $[NiCl_2(PMe_3)_2]$

Ni^{2+} in $[NiCl_2(PMe_3)_2]$



sp^3 hybridisation

Structure: tetrahedral



So, no isomer is possible, so answer is zero.

44. (*b*) **Idea** This problem is a conceptual mixing of basic character, inductive effect, steric effect, solvation effect.

Students are advised to go through study of effect of each factor and then come to a final conclusion using the effect of all factors comparatively.

Inductive effect CH_3 group has $+I$ effect as the number of CH_3 group around N-atom increases the basic strength increases hence according to this concept basic strength order is

$$(CH_3)_3N > (CH_3)_2NH > CH_3CH_2NH_2$$

Steric effect Greater the CH_3 group around N-atom greater is the steric hindrance which causes decrease in basic strength.

Solvation effect As the number of H-atom directly attached to N-atom increases solvation increases hence basic strength increases on this basics strengths basic are as follows $(CH_3)_3NH < (CH_3)_2N < CH_3CH_2NH_2$.

The basic strength of 1, 2 and 3 amine are the weighted result of these above three factors which is as follows.

$$(CH_3)_2NH > (CH_3)_3N > CH_2NH_2$$

TEST Edge Problems related to basicity of aliphatic and substituted aromatic amines as well as heterocyclic compound are generally asked in JEE Main. Students are advised to study these concepts using inductive effect in depth.

45. (*a*) $v = r\omega, r_n = \dfrac{n^2h^2}{4\pi^2 KZme^2}$ and $v_n = \dfrac{2\pi KZe^2}{nh}$

$$\frac{2\pi KZe^2}{nh} = \frac{n^2h^2}{4\pi^2 KmZ \cdot e^2}\omega$$

$$\omega = \frac{8\pi^3 me^4K^2Z^2}{n^3h^3}$$

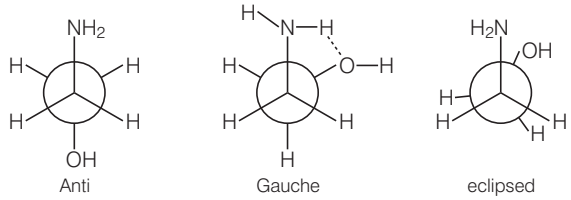
Given, $n = 2, Z = 3$

$$\omega = \frac{9\pi^3K^2me^4}{h^3}$$

46. (*b*) **Idea** This problem includes conceptual mixing of conformational structure of molecule and H-bonding. Students are advised to draw the conformational structure of organic molecule first followed by analysing the effect of H-bonding on molecule.

Gauche > anti > eclipsed

Hydrogen bonding The presence of hydrogen bonding stabilises, the structure. The conformations of 2-amino ethane 1-ol are



Out of above gauche form have more stability due to hydrogen bonding.

TEST Edge The problem related to preparation of such molecule and then relating the concept of conformational isomerism an H-bonding as well as steric hindrance are asked very frequently in JEE Main. Students are recommended to study the effect of these concepts in conformational isomerism.

47. (*d*) Pb_3O_4 (*X*) is red coloured mixed oxide (PbO and PbO_2)

$$\underset{(A)}{Pb_3O_4} + 4HNO_3 \longrightarrow 2Pb(NO_3)_2 + \underset{(B)}{PbO_2} + 2H_2O$$

$$\underset{(B)}{PbO_2} + 4HCl \longrightarrow \underset{(C)}{PbCl_2} + 2H_2O + Cl_2$$

$PbCl_2$ is insoluble in cold water but soluble in hot water.

$$\underset{(A)}{Pb_3O_4} + 8HCl \longrightarrow 3PbCl_2 + 4H_2O + Cl_2$$

48. (*b*) When phenolphthalein is used as an indicator

$0.05\ y = 10 \times 0.2 \times 1$

$$y = 40\ mL$$

When methyl orange is used as an indicator

$$\therefore \quad 0.05\ x = 10 \times 0.2 \times 3$$

$$x = 120\ mL$$

Hence, $x - y = 80\ mL$

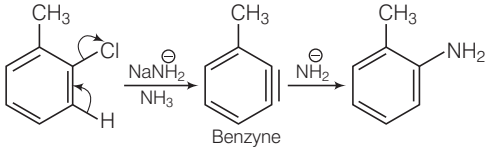
49. (*c*) $\underset{(P)}{2[Fe(CN)_6]^{4-}} + \underset{(X)}{H_2O_2} + 2H^+ \longrightarrow \underset{\text{Yellow (Q)}}{2[Fe(CN)_6]^{3-}} + 2H_2O$

$$\underset{(Y)}{2[Fe(CN)_6]^{3-}} + 2OH^- + H_2O_2 \longrightarrow \underset{(P)}{2[Fe(CN)_6]^{4-}} + 2H_2O + O_2$$

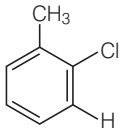
$$3[Fe(CN)_6]^{4-} + \underset{(Z)}{4Fe^{3+}} \longrightarrow \underset{\text{(Deep blue)}}{Fe_4[Fe(CN)_6]_3}$$

50. (*a*) **Idea** This reaction includes the nucleophilic substitution reaction on haloarene.

- Form the benzyne intermediate first.
- Now, complete the reaction using the NH_2 as a nucleophile keeping in mind the stability of product.



Benzyne formation Benzyne formation can be detected if and only if there is presence of hydrogen atom at ortho position to chlorine/halogen in haloarene.



TEST Edge In JEE Main, questions containing the substrate having substituent different directive influence are also be asked in JEE Main frequently.

51. (*b*) **Idea** This problem includes concept of Nernst equation along with its application to determine cell potential.

Anode $H_2(g) \longrightarrow 2H^+(aq) + 2e^-, E° = 0\ V$

Cathode $MnO_4^-(aq) + 8H^+(aq) + 5e^- \longrightarrow Mn^{2+}(aq) + 4H_2O(e), \quad E° = 1.50\ V$

$$E°_{cell} = E_{RP\ (RHS)} - E_{RP\ (LHS)}$$

$$= \left(1.50 - \frac{0.06}{5}\log\frac{[Mn^{2+}]}{[MnO_4^-][H^+]^8}\right) - \left[0 - \frac{0.06}{2}\log\frac{P_{H_2}}{[H^+]^2}\right] \quad \{0.0591 \approx 0.06\}$$

$$= \left(1.50 - \frac{0.06}{5}\log\frac{10^{-2}}{0.1 \times (10^{-2})^8}\right) + \frac{0.06}{2}\log\frac{0.1}{(10^{-2})^2}$$

$$= 1.50 - \frac{0.06}{5}\log 10^{15} + 0.03 \times \log 10^3$$

$$= 1.50 - 0.012 \times 15 + 0.03 \times 3$$

$$= 1.50 - 0.18 + 0.09$$

$$= 1.41\ V$$

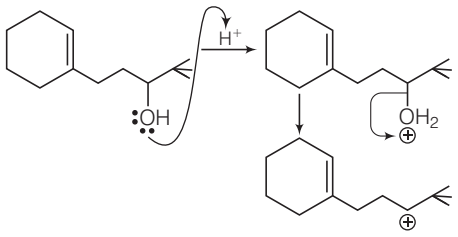
TEST Edge Problem related calculation of equilibrium constant and pH of cell are also asked frequently in JEE Main so students are advised to build up the skill in determination of these parameters using Nernst equation.

52. (*c*) **Idea** This problem includes conceptual mixing of rearrangement of carbocation ring cyclisation and number of assymmetric carbon.

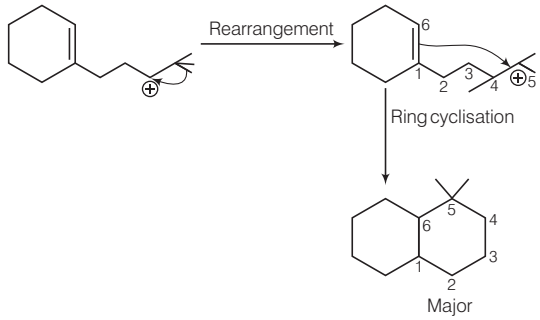
While solving this question, students are advised to follow this route:

- Generate the stablest carbocation (intermediate) using rearrangement.
- Complete the reaction using process of interamolecular nucleophilic substitution reaction (double bond acts as a nucleophile here.)

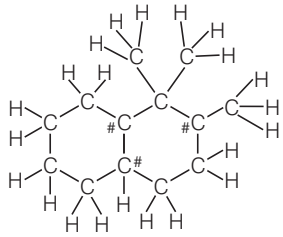
Rearrangement and ring cyclisation The reaction proceed through rearrangement of carbocation and ring cyclisation leading to the formation of 6 membered ring as follows.



Rearrangement reaction is the characteristic property of carbocation where carbocation undergoes rearrangement to stabilise the carbocation to more extent.



Assymmetric carbon The carbon attached to the four different substituents are known as assymetric carbon. In the given product, we may count the total number of assymetric carbon which is equal to 3 (denoted by #).



TEST Edge Problems involving concept of various rearrangement reactions such as pinacol-pinacolone rearrangement, nucleophilic addition reaction to carbonyl compound and identification of isomerism in product are also asked frequently.

53. (*b*) Solvay soda ammonia process is

$$NH_3 + H_2O + CO_2 \longrightarrow NH_4HCO_3$$

$$NaCl + NH_4HCO_3 \longrightarrow NaHCO_3 + NH_4Cl$$

$$NH_3 + H_2O + CO_2 + NaCl \longrightarrow NaHCO_3 + NH_2Cl$$

$$2NaHCO_3 \xrightarrow{\Delta} Na_2CO_3 + H_2O + CO_2$$

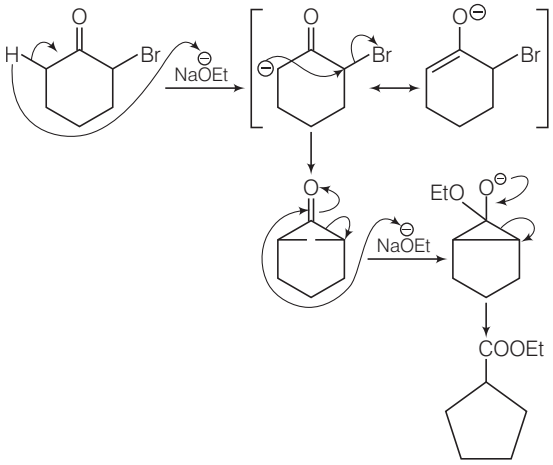
54. (*b*) **Idea** This problem includes conceptual mixing of favorskii rearrangement and migrating aptitude.

While solving such problems, students are advised to follow these steps

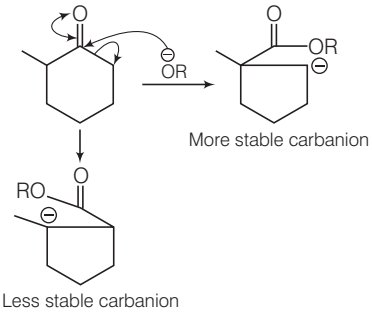
Formation of carbanion takes place first by α-H abstraction then formation of cyclopropane by removal to leaving group.

Reattack of nucleophilic $\overset{\ominus}{O}Et$ to formation of rearranged product.

Favorskii rearrangement Conversion of α-bromo ketone to ester in presence of base is known as favorskii rearrangement. This reaction proceeds by intramolecular nucleophilic substitution reaction through cyclopropane intermediate.



Note In case of α substituted α-bromo α-alkyl ketone undergo rearrangement then the rearrangement occurs in such a way to stabilise carbanion.

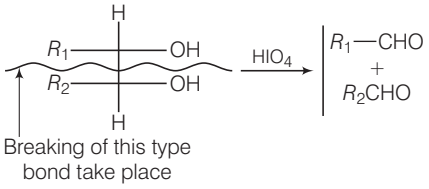


TEST Edge Problem related to Beckmann rearrangement, Hoffmann bromaamide reaction are generally asked in JEE Main. Students are advised to study these topics in depth.

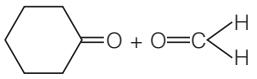
55. (*c*) **Idea** This problem includes conceptual mixing of malaprade oxidation and nomenclature.

While solving this problem, students are advised to write the structure of both product of the reaction and given choices then choose the correct starting material keeping the basic concept of Malaprade oxidation in mind.

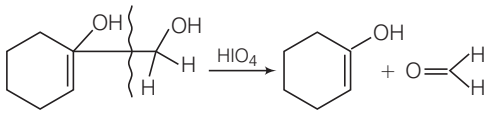
Malaprade oxidation This oxidation is done by treating vicinal diol with HIO_4.



The product of reaction in above given question is



means they must be made by Malaprade oxidation of (1-hydroxy cyclohexyl) methanol as follows



TEST Edge Similar questions having Malaprade oxidation of α-amino alcohol are also be asked in JEE Main, so students are advised to sutdy in depth these and preparation of their starting material also.

56. (*b*) **Idea** This problem includes conceptual mixing of covalent character and Fazan's rule. While solving the problems student is advised to choose the common ion in all and then arrange the trend of polarisibility of cation and anion on the basis of these two trend. One can easily find the answer as polarisation ∝ covalent character.

Fazan's rule According to Fazan's rule, the polarisibility of smaller cation and large anion are larger.

Here out of LiCl, $BeCl_2$, BCl_3, CCl_4 all the anion are same *i.e.,* Cl. While cations are different hence polarising power will depend upon size of cation. Polarisation of cation of $Li^+, Be^{2+}, B^{3+}, C^{4+}$ are as follows

$$Li^+ < Be^{2+} < B^{3+} < C^{4+} \longrightarrow$$

Size of cation decreases.

Polarisation increases.

Covalent nature increases.

As we know polarisation is directly proportional to covalent charactor. Hence, correct answer is (b).

TEST Edge Question based on Fazan's rule in determination of covalent character, ionic character, acidic character are also asked very frequently, students are advised to go through these concepts.

57. (*b*) $k_{eq} = \dfrac{k_f}{k_b} = \dfrac{[CH_3]^2}{[C_2H_6]}$

$$[CH_3] = \frac{10^{-5}}{10}\text{mol L}^{-1} = 10^{-6}\text{ mol L}^{-1}$$

$$[C_2H_6] = \frac{100}{10} = 10\,\text{mol L}^{-1} = 10\,\text{M}$$

$$k_{eq} = \frac{k_f}{k_b}$$

$$\frac{[10^{-6}]^2}{10} = \frac{3.14\times10^2\,\text{s}^{-1}}{k_b}$$

$$k_b = 3.14\times10^{3+12}\,\text{L mol}^{-1}\text{s}^{-1}$$

$$= 3.14\times10^{15}\,\text{L mol}^{-1}\text{s}^{-1}$$

58. (*a*) Inert pair effect increases in 14th group as the atomic number increases.

On moving top to bottom on group 14

	(*Z*)	
C	26	• Atomic number increases
Si	14	
Ge	32	
Sn	50	• Inert pair effect increase.
pb	82	

59. (*b*) Relative lowering in vapour pressure is a colligative properties *i.e.,* depends upon the number of particles. More the number of particles lower is the vapour pressure. Thus, 0.1M sugar solution has higher vapour pressure than the KCl solution of same concentration.

Relative lowering in vapour pressure is directly proportional to the mole fraction of the solute (x_B).

$$\frac{p^0 - p}{p^0} \propto x_B$$

60. (*d*) This problem includes conceptual mixing of sequence of amino acid in nucleic acids and bonding in them.

Sequence of amino acid in nucleic acid The presence of different sequence of amino acid is responsible for the different types of nucleic acid.

Sequence of base in DNA and RNA are as follows

Types of nucleic acid	Sequence of base
DNA	TGAACCCTT
mRNA	ACUUGGGAA

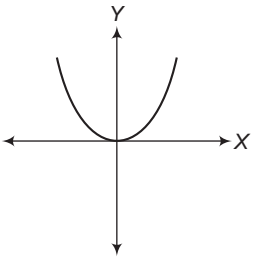
According to base pairing principle
T in DNA faces A in mRNA
G in DNA faces C in mRNA
A in DNA face U in mRNA.
Bonding in nucleic acid is hydrogen bonding which is responsible for passing of different bases to each other.

TEST Edge Similar questions relating type of linkage between nitrogeneous based and number of hydrogen bonding between the two nucleic acids are also asked generally in JEE Main, so students are recommended to study these topic in depth.

Mathematics

61. (*c*) **Idea** Convert the given equation of tangent as $x^2 = 4ay$, find the vertex.

Given, the equation of tangent $x^2 + 8x + 4y = 0$



$\Rightarrow \quad (x+4)^2 = -4(y-4)$

$\Rightarrow \quad X^2 = -4Y$ (by shifting the origin)

$X = x + 4, Y = y - 4$

$\therefore \quad$ Vertex $= -4$

Tangent at vertex is $Y = 0$

$y - 4 = 0 \quad$ or $\quad y = 4$

TEST Edge Equation of parabola of tangent and in other form related questions are asked. To solve these types of questions, students are advised to understand the basic concept of parabola.

62. (*d*) We have given that

$|z_1| = |z_2|$ and $\arg(z_1) = \pi - \arg(z_2)$

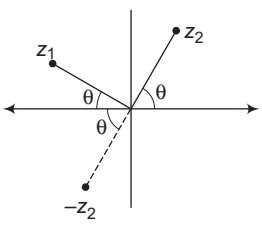
$z_2 = re^{i\theta}, \bar{z}_2 = re^{-i\theta}$

$z_1 = re^{i(\pi-\theta)} = re^{i\pi} \cdot e^{-i\theta}$

$= r(\cos\pi + i\sin\pi) \cdot e^{-i\theta}$

$= -re^{-i\theta} \Rightarrow -\bar{z}_2$

Alternate Solution



From figure, it is clear that $z_1 = -\bar{z}_2$

63. (*c*) **Idea** Convert the given equation as linear differential equation such as $\frac{dy}{dx} + Py = Q$

$\text{IF} = e^{\int P\cdot dx}$ and solution is

$y \times \text{IF} = \int (\text{IF} \times Q)\, dx$

Given that slope of the tangent,

$$\frac{dy}{dx} = x + 1 + \frac{y-3}{x+1}$$

Let $x + 1 = X, y - 3 = Y$

$$\frac{dY}{dX} - \frac{Y}{X} = X, \text{IF} = \frac{1}{X}$$

[This is linear differential equation]

$$\frac{Y}{X} = X + c \Rightarrow \frac{y-3}{x+1} = x + 1 + c$$

$x = 2, y = 0 \Rightarrow c = -4$

$y - 3 = (x+1)^2 - 4(x+1) = x^2 - 2x - 3$

$y = x^2 - 2x$ meets x-axis are 3 at $(0, 2)$

$\text{Area} = \int_0^2 (2x - x^2)dx = 4 - \frac{8}{3} = 4/3$

Area = 4/3 sq units

TEST Edge Area bounded by the ellipse, parabola, circle and axis related questions are asked, students are advised to understand the concept of area bounded by the curve and also acquainted yourself with properties of definite integral.

64. (*b*) **Idea** $\because \sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} \ldots$

and $\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \ldots$

Apply these formulae and go with concept of limit.

It is given that $\lim_{x\to 0} \frac{a\cos x + bx\sin x - 5}{x^4}$ exists and is finite.

Now, $\lim_{x\to 0} \frac{a\cos x + bx\sin x - 5}{x^4}$

$$= \lim_{x\to 0} \frac{a\left(1 - \frac{x^2}{2} + \frac{x^4}{24} + \ldots\right) + bx\left(x - \frac{x^3}{6} + \ldots\right) - 5}{x^4}$$

$$= \lim_{x\to 0} \frac{(a-5) + \left(\frac{-a}{2} + b\right)x^2 + \left(\frac{a}{24} - \frac{b}{6}\right)x^4 + \ldots}{x^4}$$

Since limit is finite.

$\therefore\ a - 5$ and $-\frac{a}{2} + b$ must be equal to zero.

$\Rightarrow \quad a - 5 = 0 \Rightarrow a = 5$

and $\quad -\frac{a}{2} + b = 0$

$\Rightarrow \quad b = \frac{5}{2} = 2.5$

TEST Edge Algebraic limit, exponential limit and logrithmic limits related questions are asked. To solve these types of questions students are advised to understand the direct substitution method, factorisation method, rationalisation method.

65. (*b*) 1. Universal relation is the largest relation and identity relation in a finite set A is a subset of universal relation. Therefore, identity relation on a finite set A is not the greatest relation.

2. Consider the set $A = \{1, 2\}$

$\Rightarrow$ $R\ \{(1,1),(1,2),(2,1),(2,2)\}$

$\Rightarrow$ R is a symmetric relation

$\Rightarrow$ R is not anti symmetric

3. By definition, union and intersection of 2 symmetric relations are also symmetric relation.

So, only 2 and 3 are correct.

66. (*a*) **Idea** The equation of planes bisecting the angles between planes

$$a_1x + b_1y + c_1z + d_1 = 0$$

and $a_2x + b_2y + c_2z + d_2 = 0$ are

$$\frac{a_1x + b_1y + c_1z + d_1}{\sqrt{a_1^2 + b_1^2 + c_1^2}} = \pm\frac{a_2x + b_2y + c_2z + d_2}{\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

The given planes are

$$2x - y + 2z + 3 = 0$$

and $$3x - 2y + 6z + 8 = 0$$

$$a_1a_2 + b_1b_2 + c_1c_2 = 6 + 2 + 12 = 24 > 0$$

So that, the acute angle bisectors is

$$\frac{2x - y + 2z + 3}{3} = \frac{-3x - 2y + 6z + 8}{7}$$

Solving we get,

$\Rightarrow$ $23x - 13y + 32z + 45 = 0$

TEST Edge Generally in JEE Main, equation of parallel planes, distance of a point from a plane related questions are asked. To solve these types of questions, students are advised to understand the basic concept of the different forms of equation of plane.

67. (*b*) We have given that

$$\int \frac{f(x)\,\phi'(x) - f'(x)\,\phi(x)}{f(x)\,\phi(x)} \log\frac{\phi(x)}{f(x)}\,dx$$

Let, $$\log\frac{\phi(x)}{f(x)} = t$$

$$\frac{f(x)}{\phi(x)}\frac{d}{dx}\left(\frac{\phi(x)}{f(x)}\right) = dt$$

$$\frac{f(x)}{\phi(x)} \cdot \frac{f(x)\,\phi'(x) - \phi(x)\,f'(x)}{[f(x)]^2}\,dx = dt$$

$$\frac{f(x)\,\phi'(x) - \phi(x)\,f'(x)}{f(x)\,\phi(x)}\,dx = dt$$

$$\int t\,dt = \frac{t^2}{2} + C = \frac{1}{2}\left(\log\frac{\phi(x)}{f(x)}\right)^2 + C$$

68. (*b*) **Idea** Here, for a triangle ABC

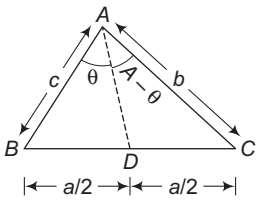
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Given that in ΔABC, median AD makes an angle θ with side AB.

In ΔABD,

$$\frac{BD}{\sin\theta} = \frac{AD}{\sin B}$$

$\Rightarrow$ $$AD = BD\frac{\sin B}{\sin\theta} \quad \ldots(i)$$



In ΔACD,

$$\frac{CD}{\sin(A-\theta)} = \frac{AD}{\sin C}$$

$\Rightarrow$ $$AD = CD\frac{\sin C}{\sin(A-\theta)} \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$BD\frac{\sin B}{\sin\theta} = \frac{\sin C}{\sin(A-\theta)} \cdot CD$$

$\Rightarrow$ $$\frac{\sin B}{\sin C} = \frac{\sin\theta}{\sin(A-\theta)}$$

$\Rightarrow$ $$\frac{b}{c} = \frac{\sin\theta}{\sin(A-\theta)}$$

$\Rightarrow$ $$\sin(A-\theta) = \frac{c}{b}\sin\theta$$

TEST Edge Generally in JEE Main, properties of triangle based questions are asked. To solve these types of questions, students are advised to understand the basic concept of properties of triangle.

69. (*a*) **Idea** Probability of occurence of A given that B has already happened is $P(A/B)$

$$P(A/B) = \frac{n(A \cap B)}{n(B)}$$

Let A be the event that sum of the all three dies is 15.

B be the event that the first throw was four.

$$A = \{6, 6, 3\} = \frac{3!}{2!} = 3$$

$= \{6, 5, 4\} = 3! = 6$

$\{5, 5, 5\} = 1! = 1$

$n(A) = 10$

$n(B) = 36$ but $n(A \cap B) = 2$

Required probability

$$P(B/A) = \frac{n(A \cap B)}{n(A)} = \frac{2}{10} = \frac{1}{5}$$

TEST Edge Multiplication theorem on probability, conditional probability related questions are asked. To solve these types of questions, students are advised to understand the basic concept of probability.

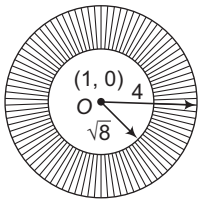
70. (*d*) Centre and radius of first and second circles are

$$C_1(1, 0), r_1 = 4$$

and $C_2(1, 0), r_2 = \sqrt{8}$

∵ The region $([P + 1], [P])$ lines inside the circle $x^2 + y^2 - 2x - 15 = 0$, then

$$[P + 1]^2 + [P]^2 - 2[P + 1] - 15 < 0$$



$\Rightarrow ([P] + 1)^2 + [P]^2 - 2([P] + 1) - 15 < 0$

$\Rightarrow 2[P]^2 - 16 < 0 \Rightarrow [P]^2 < 8$ ...(i)

∵ Circles are concentric.

∴ Point $([P + 1], [P])$ outside the circle

$$x^2 + y^2 - 2x - 7 = 0$$

$\therefore [P + 1]^2 + [P]^2 - 2[P + 1] - 7 > 0$

$\Rightarrow ([P] + 1)^2 + [P]^2 - 2([P] + 1) - 7 > 0$

$\Rightarrow 2[P]^2 - 8 > 0$

$\Rightarrow [P]^2 > 4$ ...(ii)

From Eqs. (i) and (ii), we get

$4 < [P]^2 < 8$, which is impossible.

∴ For no value of P the point will be within the region.

71. (*a*) If A, B and C are collinear, then $\overline{AB} = \lambda\, \overline{BC}$

Comparing, we get

$c = 0, 1 - a = -2\lambda$ and $b = \lambda$

$\Rightarrow a - 2b = 1$

72. (*a*) **Idea** Here, $(a + x)^n = a^n\left(1 + \frac{x}{a}\right)^n$

coefficient of T_r^{th} in $\left(1 + \frac{x}{a}\right)^n = {}^nC_{r-1}\left(\frac{x}{a}\right)^{r-1}$

and $T_{r+1} = {}^nC_r\left(\frac{x}{a}\right)^r$

Let, the consecutive terms T_r and T_{r+1} having equal coefficient.

∴ Coefficient of T_r^{th} term = coefficient of T_{r+1}^{th} term

$$\therefore \quad {}^{74}C_{r-1}\, 3^{74}\left(\frac{2}{3}\right)^{n-1} = {}^{74}C_r\, 3^{74}\left(\frac{2}{3}\right)^r$$

$$\Rightarrow \quad {}^{74}C_{r-1} \times 3 = {}^{74}C_r \times 2$$

$$\Rightarrow \quad \frac{3 \times 74!}{(r-1)!\,(75-r)!} = \frac{2 \times 74!}{r!\,(74-r)!}$$

$$\Rightarrow \quad \frac{3}{75-r} = \frac{2}{r}$$

$$\Rightarrow \quad 150 - 2r = 3r \quad \Rightarrow \quad r = 30$$

∴ T_{30} and T_{31} are two consecutive terms whose coefficients are same.

73. (*a*) **Idea** Here, if A is non-singular square matrix then,

$\therefore$ adj (adj A) $= |A|^{n-2} A$

It is given that $\text{adj}(P) = \begin{bmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{bmatrix}$

$\therefore |\text{adj}(P)| = 1(3 - 7) - 4(6 - 7) + 4(2 - 1)$

$= -4 + 4 + 4$

$= 4$

$\because \quad |\text{adj}(P)| = |P|^2$ {$\because |\text{adj } A| = |A|^{(n-1)}$}

$\Rightarrow \quad |P|^2 = 4$

$\Rightarrow \quad |P| = \pm 2$

TEST Edge Properties of matrix related questions are asked in JEE Main. To solve these types of questions, students are advised to understand the basic concept of matrix.

74. (*d*) Truth table for the mathematical conditions is given as

p	q	$\sim p$	$\sim q$	$p \to \sim q$	$q \to \sim p$	$p \leftrightarrow q$
T	F	F	T	T	T	F

As per the above truth table, it is clear that $P \to \sim q$ is true (T) and $q \to \sim P$ is true (T).

75. (*c*) ∵ α and β are roots of the equation

$$ax^2 + bx + c = 0 \quad \ldots(i)$$

$$a(x + 1)^2 - b(x + 1)x + cx^2 = 0$$

$$a\left(\frac{x+1}{x}\right)^2 - b\left(\frac{x+1}{x}\right) + c = 0$$

$$a\left(-\frac{x+1}{x}\right)^2 + b\left(-\frac{x+1}{x}\right) + c = 0 \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$\alpha = -\left(\frac{x+1}{x}\right) \Rightarrow \alpha x = -x - 1$$

$$\Rightarrow \quad x(\alpha+1)=-1 \Rightarrow x=-\frac{1}{1+\alpha}$$

$\therefore$ Required roots are $=\frac{-1}{1+\alpha}$ and $\frac{-1}{1+\beta}$

76. (*b*) Here, suppose that $h(x)=f(x)-g(x)$

$$h'(x)=f'(x)-g'(x)>0 \ \forall\ x\in R$$

$h(x)$ is an increasing function and

$$h(0)=f(0)-g(0)=0$$

$\therefore \quad h(x)>0 \ \forall\ x\in(0,\infty)$

and $\quad h(x)<0 \ \forall\ x\in(-\infty,0)$

77. (*b*) **Idea** Number of permutation of n different things taken all at a time is $n!$

We have given that, $x+y+z=6$ is true for

$$x=1, y=2, z=3$$

These values of x, y and z can be exchanged by x, y and z, hence total number of ways

$$=3!=3\times2=6$$

TEST Edge Generally in JEE Main permutation under certain condition such as repetition of items are allowed or not related questions are asked. To solve these types of questions students are advised to understand the basic concept of permutation under certain condition.

78. (*a*) We have given that

$$\int \frac{e^x}{x+2}\{1+(x+2)\log(x+2)\}\,dx$$

$$=\int e^x\left\{\frac{1}{x+2}+\log(x+2)\right\}dx$$

Here, $f(x)=\log(x+2)$

$$f'(x)=\frac{1}{x+2}$$

$\because \quad \int e^x\,[f(x)+f'(x)]\,dx=e^x f(x)+C$

$\therefore$ Required solution is $e^x\log(x+2)+C$

79. (*b*) Here, given that $n(A\cap B)=99$

Now,

$$(A\times B)\cap(B\times A)=n(A\cap B)\times(B\cap A)$$
$$=n(A\cap B)\cdot n(B\cap A)$$
$$=n(A\cap B)\cdot n(A\cap B)$$
$$=99\cdot99$$
$$=99^2$$

So, correct option is (b).

80. (*c*) **Idea** $\sin(A+B)=\sin A\cos B+\cos A\sin B$, and $\cos(A-B)=\cos A\cos B+\sin A\sin B$

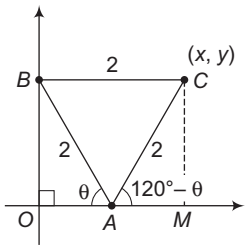
From the figure,

$$OA=AB\cos\theta$$

$$OA=2\cos\theta$$

$$x=OA+AM$$

$$=2\cos\theta+2\cos(120°-\theta)$$



$$x=2\cos\theta+2\cos(120°-\theta)$$
$$=2\cos\theta+2[\cos120°\cos\theta+\sin120°\sin\theta]$$
$$=2\cos\theta+2\left[\left(-\frac{1}{2}\right)\cos\theta+\left(\frac{\sqrt3}{2}\right)\sin\theta\right]$$
$$=2\cos\theta-\cos\theta+\sqrt3\sin\theta$$
$$x=\cos\theta+\sqrt3\sin\theta \qquad ...(i)$$
$$y=2\sin(120°-\theta)$$
$$=2(\sin120°\cos\theta-\cos120°\sin\theta)$$
$$=2\left(\frac{\sqrt3}{2}\cos\theta+\frac{1}{2}\sin\theta\right)$$
$$y=\sqrt3\cos\theta+\sin\theta$$
$$x^2+y^2=4+4\sqrt3\cos\theta\sin\theta$$
$$xy=\sqrt3+4\sin\theta\cos\theta$$
$$x^2+y^2=4+\sqrt3(xy-\sqrt3)$$
$$x^2+y^2=4+\sqrt3\,xy-3$$
$$x^2+y^2=\sqrt3\,xy+1$$

TEST Edge Generally in JEE Main, coordinate geometry in two dimensional and properties of triangle based questions are asked. To solve these type of questions, students are advised to understand the basic concept of coordinate geometry.

81. (*d*) If the observations greater than the median is increased by 10, then there will be no change in median.

82. (*c*) The given equation is

$$y^{\frac{1}{m}}+y^{-\frac{1}{m}}=2x$$

Differentiating on both sides, we get

$$\frac{1}{m}\left[y^{\frac{1}{m}-1}-y^{-\frac{1}{m}-1}\right]y_1=2$$

$$\Rightarrow \quad 2my=\left[y^{\frac{1}{m}}-y^{-\frac{1}{m}}\right]y_1$$

$$\Rightarrow \quad 4m^2y^2=\left[y^{\frac{1}{m}}-y^{-\frac{1}{m}}\right]^2y_1^2$$

$$= \left[\left(y^{\frac{1}{m}} + y^{-\frac{1}{m}}\right)^2 - 4\right] y_1^2$$

$$\Rightarrow \quad 4(x^2-1)\,y_1^2 = 4m^2\,y^2$$

$$\Rightarrow \quad (x^2-1)\,y_1^2 = m^2y^2$$

Again differentiating, we get

$$2xy_1^2 + 2y_1(x^2-1)\,y_2 = 2m^2yy_1$$

$$\Rightarrow \quad xy_1 + (x^2-1)\,y_2 = m^2y$$

$$\Rightarrow \quad (x^2-1)y_2 = m^2y - xy_1$$

83. (*b*) **Idea** Here, for a square matrix A

$$A^{-1} = \frac{1}{|A|}.\text{adj}\,A \quad [\because A \text{ is non-singular matrix}]$$

If two matrices are equal then their corresponding elements are also equal.

Here, it is given that

$$I = \begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}\begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix} \times \frac{1}{1+\tan^2\theta}\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}$$

$$= \frac{1}{1+\tan^2\theta}\begin{bmatrix} 1-\tan^2\theta & -2\tan\theta \\ 2\tan\theta & 1-\tan^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1-\tan^2\theta}{1+\tan^2\theta} & \frac{-2\tan\theta}{1+\tan^2\theta} \\ \frac{2\tan\theta}{1+\tan^2\theta} & \frac{1-\tan^2\theta}{1+\tan^2\theta} \end{bmatrix} = \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$$

Since, $$I = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\Rightarrow \quad a = \cos 2\theta,\ b = \sin 2\theta$$

TEST Edge Generally in JEE Main, adj of a matrix, inverse of square matrix related questions are asked to solve these types of questions. Students are advised to understand the basic concept of non-singular matrix.

84. (*d*) **Idea** Write the equation of line passing through $P(x_1, y_1, z_1)$ and normal to the given plane as

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

It is given that the equation of plane

$$3x + 4y - 6z + 1 = 0 \qquad \ldots(i)$$

The direction ratios of the normal to the plane (i) are 3, 4, – 6

Now, equation of line through (0, 0, 0) and perpendicular to the plane (i) are

$$\frac{x}{3} = \frac{y}{4} = \frac{z}{-6} = k \qquad \ldots(ii)$$

The coordinates of any point P on (ii) are $(3k, 4k, -6k)$. If this point lies on the plane 1 then

$$3(3k) + 4(4k) - 6(-6k) + 1 = 0$$

i.e., $$k = -\frac{1}{61}$$

Put $$k = -1/61$$

We get, coordinates of the foot of the perpendicular P are $\left(-\frac{3}{61}, \frac{-4}{61}, \frac{6}{61}\right)$

TEST Edge Image of a point in plane, intersection of line and plane related questions are asked. To solve such types of questions, students are advised to understand the basic concept of plane and line.

85. (*a*) If a, b, c are in AP, then $\frac{a+c}{2} = b \Rightarrow a + c = 2b$

i.e., sum of a and c must be even.

$\therefore$ Both a and c are either even or both are odd. Here even number is 7 and odd number is 8

$\therefore$ Favourable case to choose a and $c = {}^7C_2 + {}^8C_2$

Total possible cases to choose a, b and $c = {}^{15}C_3$

Hence, required probability

$$= \frac{{}^7C_2 + {}^8C_2}{{}^{15}C_3} = \frac{\frac{7!}{5!\,2!} + \frac{8!}{6!\,2!}}{\frac{15!}{12!\cdot 3!}} = \frac{7}{65}$$

86. (*d*) We have given that $a + b + c + d + e = 15$

$$\text{AM} = \frac{a + \frac{b}{2} + \frac{b}{2} + \frac{c}{3} + \frac{c}{3} + \frac{c}{3} + \frac{d}{4} + \frac{d}{4} + \frac{d}{4} + \frac{d}{4} + \frac{e}{5} + \frac{e}{5} + \frac{e}{5} + \frac{e}{5} + \frac{e}{5}}{15}$$

$$\text{AM} = 1$$

$$\text{GM} = \left(a \cdot \frac{b^2}{2^2} \cdot \frac{c^3}{3^3} \cdot \frac{d^4}{4^4} \cdot \frac{e^5}{5^5}\right)^{\frac{1}{15}}$$

$$\text{GM} = \frac{(120)^3 \cdot 50}{2^2 \cdot 3^3 \cdot 4^4 \cdot 5^5} = 1$$

$$\text{GM} = 1$$

$\therefore$ $$\text{AM} = \text{GM}$$

Hence, $$a = \frac{b}{2} = \frac{c}{3} = \frac{d}{4} = \frac{e}{5}$$

$$\Rightarrow \quad a = 1, b = 2, c = 3, d = 4, e = 5$$

$$\therefore \ a^2 + b^2 + c^2 + d^2 + e^2$$

$$= 1^2 + 2^2 + 3^2 + 4^2 + 5^2$$

$$= \frac{5 \times 6 \times 11}{6} = 55$$

87. (*b*) **Idea** Equation of circle with (a,b) as radius r
$(x-a)^2+(y-b)^2=r^2$
Now, use differentiation convert the equation as polynomial in derivative and apply the concept of orthogonal trajectories.

The circles are $(x-h)^2+y^2=1$ ...(i)

Differentiating w.r.t. x, we get

$$(x-h)+y\frac{dy}{dx}=0 \Rightarrow \frac{dy}{dx}=\frac{-(x-h)}{y} \quad ...(ii)$$

To eliminating $(x-h)$, put $(x-h)^2=1-y^2$ from

From Eqs. (i) and (ii), we get

$$\left(\frac{dy}{dx}\right)^2=\frac{1-y^2}{y^2}\Rightarrow\frac{dy}{dx}=\frac{\sqrt{1-y^2}}{y}$$

For orthogonal trajectories replacing $\frac{dy}{dx}$ by $-\frac{1}{(dy/dx)}$

$$\frac{dy}{dx}=-\frac{y}{\sqrt{1-y^2}}\Rightarrow\frac{\sqrt{1-y^2}}{y}dy+dx=0$$

$$\int\frac{\sqrt{1-y^2}}{y}dy+\int dx=0$$

$$x+c=-\int\frac{\sqrt{1-y^2}}{y}dy, \text{ let } y=\sin\theta$$

$$x+c=-\int\frac{\cos^2\theta}{\sin\theta}d\theta=\int(\sin\theta-\operatorname{cosec}\theta)\,d\theta$$

$$=-\cos\theta+\log(\operatorname{cosec}\theta+\cot\theta)$$

$$x=0,\ y=1,\theta=\pi/2\Rightarrow c=0$$

$$x=-\cos\theta+\log\left(\frac{1+\cos\theta}{\sin\theta}\right)$$

$$=-\sqrt{1-y^2}+\log\left(\frac{1+\sqrt{1-y^2}}{y}\right)$$

TEST Edge Different types of differential equations based questions are asked. To solve these types of questions students are advised to understand the basic concept of differential equation.

88. (*c*) **Idea** Here $\mathbf{a}\cdot\mathbf{b}=|\mathbf{a}||\mathbf{b}|\cos\theta$ for a unit vector such as $\mathbf{a}\Rightarrow|\mathbf{a}|=1$

Given that $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ be unit vectors, then

$$(\hat{\mathbf{a}}+\hat{\mathbf{b}})\cdot(\hat{\mathbf{a}}+\hat{\mathbf{b}})=1$$

$$\Rightarrow \quad \hat{\mathbf{a}}\cdot\hat{\mathbf{a}}+\hat{\mathbf{b}}\cdot\hat{\mathbf{b}}+2\hat{\mathbf{a}}\cdot\hat{\mathbf{b}}=1$$

$$\Rightarrow \quad 1+1+2\hat{\mathbf{a}}\cdot\hat{\mathbf{b}}=1$$

$$2\hat{\mathbf{a}}\cdot\hat{\mathbf{b}}=-1$$

$$\hat{\mathbf{a}}\cdot\hat{\mathbf{b}}=-1/2$$

$$\cos\theta=-1/2$$

$$\cos\theta=\cos 120^\circ$$

$$\theta=120^\circ$$

So, θ is a obtuse angle.

TEST Edge Angle between two vectors, vectors are perpendicular or parallel related questions are asked. To solve these types of questions students are advised to understand to basic concept of vectors.

89. (*d*) Consider the given expression,

$$\left(\frac{\cos A+\cos B}{\sin A-\sin B}\right)^n+\left(\frac{\sin A+\sin B}{\cos A-\cos B}\right)^n$$

$$=\left(\frac{2\cos\frac{A+B}{2}\cos\frac{A-B}{2}}{2\cos\frac{A+B}{2}\sin\frac{A-B}{2}}\right)^n+\left(\frac{2\sin\frac{A+B}{2}.\cos\frac{A-B}{2}}{2\sin\frac{B-A}{2}.\sin\frac{A+B}{2}}\right)^n$$

$$=\cot^n\frac{A-B}{2}+\cot^n\frac{B-A}{2}$$

$$=\begin{cases}0, & \text{if } n \text{ is odd}\\ 2\cot^n\frac{A-B}{2}, & \text{if } n \text{ is even}\end{cases}$$

Statement I is false. Statement II is true by using given identity.

So, correct option is (d).

90. (*c*) Statement I is true and Statement II is false.

Centroid $\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$

is rational point. Orthocentre is intersection point of two altitudes which will have rational coefficient when expressed as a straight line. So, orthocentre is also rational. Similarly, circumcentre will also possesses rational point.

But Statement II is false as for equilateral triangle all four points coincide at one.