Electrostatic Potential and Capacitance

APPROACH TO CHAPTER: ELECTROSTATIC POTENTIAL AND CAPACITANCE

The numerical problems based on electric potential are relatively easy. Numerical problems based on capacitors, especially capacitor combinations, need cautious approach. Derivations are simple and learning the same can help you perform better in the examinations.

(i) Units and Symbols for numericals

Quantity	Symbol	SI Unit
Electric potential/Electric potential difference	V	Volt (V)
Capacitance	C	Farad (F)
Dielectric constant	K	No unit
Polarisation	P	Cm^{-2}

(ii) Key Formulae

Electric Potential & Potential difference

(i) General formulae:

(a) Potential, $V = \frac{W}{q}$

(b) Potential difference, $V = \frac{W_B - W_A}{q}$

Formula Tips

Based on the idea that work per unit charge gives potential.

Helps define the unit – Volt.

(ii) Electric potential due to a point charge (in vacuum):

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

Helps to think of the variation of electric potential

(i) with distance

(ii) if the charge is surrounded by a medium.

(iii) Electric potential at a point due to a system of charges

$$V = \sum_{i=1}^{n} \frac{1}{4\pi\varepsilon_0}\frac{q_i}{r_i}$$

(iv) Electric potential due to a dipole:

(i) along the axial line, $V = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2}$

Remember: It's r^2, not r, in the denominator.

(ii) equatorial line, V = 0

(v) Relation between electric potential and electric field intensity:

$$E = -\frac{dV}{dr}$$

Helps to find out field strength when potential difference between two points and the distance between them are given.

Electrostatic potential energy

(i) Potential energy of a system of charges:

(a) A two – charge system (in vacuum) $U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$

Remember: potential energy relates to the work done on the system that gets stored.

(b) An n–charge system (in vacuum) $U = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\sum_{\substack{j=1 \\ j\neq i}}^{n}\frac{q_i q_j}{r_{ij}}$

(ii) Potential energy of a dipole in an electric field

$$U = -\vec{p}.\vec{E}$$

Formula Tips

CAPACITANCE

(a) General form $C = \frac{q}{V}$

Capacitance is the charge required per unit rise in electric potential.

(b) Parallel plate capacitor

$C = \frac{\varepsilon_0 A}{d}$ (with vacuum between the plates)

Helps to think of how a di–electric medium affect the capacitance of a system.

$C = \frac{\varepsilon_0 K A}{d}$ (with a medium of dielectric constant K between the plates)

(c) Effective capacitance:

(i) Series combination $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$

(ii) Parallel combination $C = C_1 + C_2 + \ldots$

In series combination, the effective capacitance is less than the least individual capacitance.

In parallel combination, the effective capacitance is more than the largest individual capacitance.

(d) Energy stored by a capacitor: $U = \frac{1}{2}CV^2$

Helps you to express energy stored in terms of q & V as well as q & C.

B. DERIVATIONS

(i) Potential energy of a dipole in an electric field

Derivation Tips

Start with the torque experienced by a dipole in an electric field.

Find the work done in twisting the dipole against the torque

$$dW = \tau\, d\theta = pE\sin\theta\, d\theta$$

$$W = \int_{\theta_1}^{\theta_2} pE\sin\theta\, d\theta = pE\left(\cos\theta_2 - \cos\theta_1\right)$$

$$W = -pE\cos\theta = \vec{p}.\vec{E}$$

(ii) Capacitance of a parallel plate capacitor

Derivation Tips

Get expressions for the charge q and the potential difference (V) between the plates.

Apply $C = \frac{q}{V}$

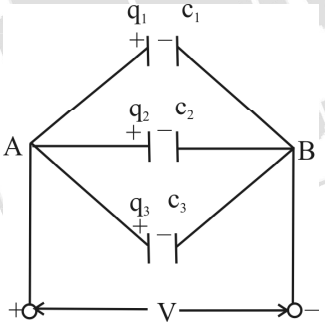
$$E = \frac{\sigma}{\varepsilon_0}$$

$$V = Ed$$

$$q = A\sigma$$

(iii) Effective capacitance:

(a) Parallel Combination



Derivation Tips

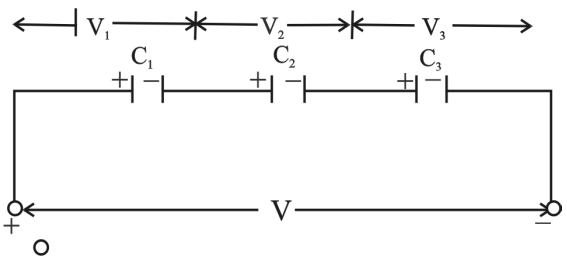
Combination	Same	Different	Approach
Parallel	Voltage	Charge	Find the total charge
Series	Charge	Voltage	Find the total voltage

$q = q_1 + q_2 + q_3 + \ldots$

$CV = C_1V + C_2V + C_3V + \ldots$

$C = C_1 + C_2 + \ldots$

(b) Series combination



$$V = V_1 + V_2 + V_3 + \ldots$$

$$\frac{q}{C} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3} + \ldots$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$$

(iv) Energy stored by a capacitor:

$q = CV$

$dq = C\,dV$

$dW = V\,dq$

$$U = \frac{1}{2}CV^2$$

Derivation Tips

Start with q = CV

Then differentiate

Write the work done to move charge dq against a pd of V

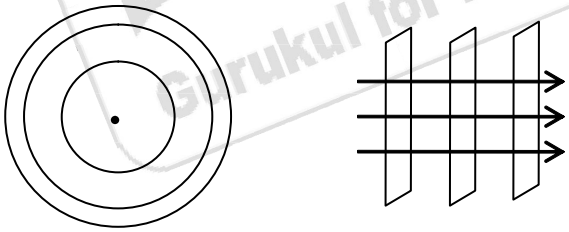
Substitute for dq Integrate it to get the total work done to build up charge from 0 to q

Work done is stored as potential energy

C. Properties / Definitions / Descriptions

Equipotential surface:

Any surface which has the same electrostatic potential at every point is called an equipotential surface.



Equipotential surfaces are :

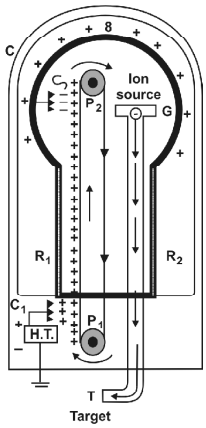
1. Concentric spheres for an isolated point charge
2. Parallel planes for a uniform electric field

Van de Graaff's generator

Use: To generate high electric potential of the order of millions of volts.

Principle: Charge given to a hollow conductor gets transferred to the outer surface.

Diagram:



Description Tips

Write down the use of the device and the principle based on which it functions

Parts:

Hollow sphere

Conveyer belt

Two combs

Function of each part:

Bottom (first) comb: To spray charge on to the conveyer belt.

Conveyer belt: Carries charge to the top (second) comb.

Top (second) comb: Carries charge to the hollow sphere.

Hollow sphere gets charged to a high potential.

2004

1. The graph shows the variation of voltage, 'V' across the plates of two capacitors A and B versus increase of charge, 'Q' stored on them. Which of the two capacitors has higher capacitance. Give reason for your answer. **(2 Marks)**

2. An electric dipole of length 4 cm, when placed with its axis making an angle of 60° with a uniform electric field experiences a torque of $4\sqrt{3}$ Nm. Calculate the (*i*) magnitude of the electric field. (*ii*) potential energy of the dipole, if the dipole has charges of ± 8 nC. **(2 Marks)**

[Ans.: $(i)\ \mathrm{E} = 2.5 \times 10^{10}\ \frac{\mathrm{N}}{\mathrm{C}}$, $(ii)\ -1$ J**]**

3. Find the total energy stored in the capacitors in the given network. **(3 Marks)**
[Ans.: 4.5 μJ**]**

4. A 10 μF capacitor is charged by a 30 V dc supply and then connected across an uncharged 50 μFcapacitor. Calculate: **(3 Marks)**

(*i*) the final potential difference across the combination. **[Ans.:** 5V**]**

(*ii*) the initial and final energies. How will you account for the difference in energy.

[Ans.: $\mathrm{U}_i = 4.5\ m\mathrm{J}$, $\mathrm{U}_f = 0.35\ m\mathrm{J}$**]**

2005

1. A parallel plate capacitor with air between the plates has a capacitance of 8 pF. What will be the capacitance if the distance between the plates be reduced by half and the space between them is filled with a substance of dielectric constant $k = 6$. **[Ans.:** 96 pF**]**

2. Two dielectric slabs of dielectric constants K_1 and K_2 are filled in between the two plates, each of area A, of the parallel plate capacitor as shown in the figure. Find the net capacitance of the capacitor. **(2 Marks)**

[Ans.: $\frac{\mathrm{A}\varepsilon_0}{2d}[\mathrm{K}_1 + \mathrm{K}_2]$**]**

3. A 4 μFcapacitor is charged by a 200 V supply. The supply is then disconnected and the charged capacitor is connected to another uncharged 2 μF capacitor. How much electrostatic energy of the first capacitor is lost in the process of attaining the steady situation. **[Ans.:** $\frac{8}{3} \times 10^{-2}$ J**]**

4. A 5 μFcapacitor is charged by a 100 V supply. The supply is then disconnected and the charged capacitor is connected to another uncharged 3 μFcapacitor. How much electrostatic energy of the first capacitor is lost in the process of attaining the steady situation. **[Ans.:** 0.037 J**]**

5. Deduce an expression for the electric potential due to an electric dipole at any point on its axis. Mention one contrasting feature of electric potential of a dipole at a point as compared to that due to a single charge.

2006

1. Define the term 'dielectric constant' of a medium in terms of capacitance of a capacitor. **(1 Mark)**

2. The electric field and electric potential at any point due to a point charge kept in air is 20 NC^{-1} and 10 JC^{-1} respectively. Compute the magnitude of this charge. **[Ans.:** 0.55×10^{-9} C**]** **(2 Marks)**

3. The given graph shows the variation of charge *q* versus potential difference V for two capacitors. The two capacitors C_1 and C_2 have same plate separation but the plate area of C_2 is double than that of C_1. Which of the lines in the graph correspond to C_1 and C_2 and why. **[Ans.:** C_2 Corresponds to A, C_1 Corresponds to B**]**

4. Two capacitors of capacitance 6 μF and 12 μFare connected in series with a battery. The voltage across the 6 μFcapacitor is 2 V. Compute the total battery voltage. **[Ans.:** V = 3 Volt**]**

5. A parallel plate capacitor with air between the plates has a capacitance of 8 pF. The separation between the pates is now reduced by half and the space between them is filled with a medium of dielectric constant 5. Calculate the value of capacitance of the capacitor in the second case.

[Ans.: C = 80 μF**]**

2007

1. A parallel plate capacitor, each with plate area A and separation *d*, is charged to a potential difference V. The battery used to charge it is then disconnected. A dielectric slab of thickness d and dielectric constant K is now placed between the plates. What change, if any, will take place in

(*i*) charge on the plates (*ii*) electric field intensity between the plates

(*iii*) capacitance of the capacitor.

Justify your answer in each case.

2. Derive the expression for the energy stored in a parallel plate capacitor of capacitance C with air as medium between its plates having charges Q and –Q. Show that this energy can be expressed in terms of electric field as $\frac{1}{2}\varepsilon_0 E^2 A d$ where **A** is the area of each plate and *d* is the separation between the plates.

4. How will the energy stored in a fully charged capacitor change when the separation between the plates is doubled and a dielectric medium of dielectric constant 4 is introduced between the plates.

2008

1. Derive an expression for the potential energy of an electric dipole of the dipole moment in an electric field.

2. (*a*) Using Gauss' law, derive an expression for the electric field intensity at any point outside a uniformly charged thin spherical shell of radius R and surface charge density σ C /m. Draw the field lines when the charge density of the sphere. (*i*) positive, (*ii*) negative.

(*b*) A uniformly charged conducting sphere of 2.5 m in diameter has a surface charge density of 100 C/m^2. Calculate the

(*i*) charge on the sphere (*ii*)total electric flux passing through the sphere

[Ans. (*i*) 156.25 π, (*ii*) $\cong 2\pi \times 10^{13}$ (V – m)**]**

3. (*a*) Derive an expression for the energy stored in a parallel plate capacitor C, charged to a potential difference V.

(*b*) Obtain the equivalent capacitance of the network given below. For a supply of 300 V, determine the charge and voltage across C_4.

[Ans. C = 500 pF, *q* = 150 *n*C, V = 300 V**]**

4. Explain the principle on which Van De Graaff generator operates. Draw a schematic sketch and write briefly its working. A Van De Graaff type generator is capable of building up potential difference of 15×10^6 V. The dielectric strength of the gas surrounding the electrode is 5×10^7 Vm^{-1}. What is the minimum radius of the spherical shell required. [**Ans.** 0.3m]

(5 Marks)

2009

1. Two parallel plate capacitor, X and Y, have the same area of plates and same separation between them, connected by battery 12 volt battery in series. X has air between the plates while Y contains a dielectric of medium $\varepsilon_r = 4$.

(*i*) Calculate capacitance of each capacitor if equivalent capacitance of the combination is 4 miro Farad.

[Ans. $C_x = 5$ μF $C_y = 20$ μF**]**

(*ii*) Calculate the potential difference between the plates of X and Y. **[Ans.** 12 V**]**

(*iii*) What is the ratio of electrostatic energy stored in X and Y. **[Ans.** 4**]**

2. The equivalent capacitance of the combination between A and B in the given figure is 4 micro Farad.

(*i*) Calculate the capacitance of the capacitor C. **[Ans.** 5 μF**]**

(*ii*) Calculate charge on each capacitor if a 12 volt battery is connected across terminals A and B.

[Ans. 48 μC**]**

(*iii*) What will be the potential drop across each capacitor.

[Ans.: $\frac{48.5}{5}$V, $\frac{12}{5}$V **]**

2010

1. In which orientation, a dipole placed in a uniform electric field is in (*i*) stable, (*ii*) unstable equilibrium.

2. (*a*) Depict the equi-potential surfaces for a system of two identical positive point charges placed a distance '*d*' apart.

(*b*) Deduce the expression for the potential energy of a system of two point charges q_1 and q_2 brought from infinity to the points $\vec{r}_1$ and $\vec{r}_2$ respectively in the presence of external electric field $\vec{E}$.

3. A parallel-plate capacitor is charged to a potential difference V by a d.c. source. The capacitor is then disconnected from the source. If the distance between the plates is doubled, state with reason how the following change:

(*i*) electric field between the plates (*ii*) capacitance, and (*iii*) energy stored in the capacitor.

4. A parallel plate capacitor each with plate area A, and sepration '*d*' is charges to a potential difference V, if battery remains connected and dielectric slab of thickness *d* and constant K is now placed between the plates what is the change if any will take place in :-

(*i*) charge on plates, (*ii*) electric field intensity (*iii*) capacitance of capacitor

Justify your answer in each case.

2011

1. A point charge Q is placed at point O as shown in the figure. Is the potential difference $V_A - V_B$ positive, negative or zero, if Q is (i) positive (ii) negative.

2. Figure shows two identical capacitors C_1 and C_2 each of 1.5 μF capacitance, connected to a battery of 2 V. Initially switch 'S' is closed. After sometime 'S' is left open and dielectric slabs of dielectric constant K = 2 are inserted to fill completely the space between the plates of the two capacitors. How will the (i) charge and (ii) potential difference between the plates of the capacitors be affected after the slabs are inserted.

2012

1. When electrons drift in a metal from lower to higher potential, does it mean that all the free electrons of the metal are moving in the same direction?

2. Explain the principle of a device that can build up high voltages of the order of a few million volts. Draw a schematic diagram and explain the working of this device.

Is there any restriction on the upper limit of the high voltage set up in this machine? Explain.

Or

(a) Define electric flux. Write its S.I. units.

(b) Using Gauss's law, prove that the electric field at a point due to a uniformly charged infinite plane sheet is independent of the distance from it.

(c) How is the field directed if (i) the sheet is positively charged, (ii) negatively charged?

