Resonance
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PHYSICS

Target : JEE(IITs)

Gravitation

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Contents

JEE (IITs) Syllabus 2012

Law of gravitation; Gravitational potential and field; Acceleration due to gravity; Motion of planets and satellites in circular orbits only ; Escape velocity.

GRAVITATION

1. INTRODUCTION

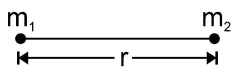
The motion of celestial bodies such as the sun, the moon, the earth and the planets etc. has been a subject of fascination since time immemorial. Indian astronomers of the ancient times have done brilliant work in this field, the most notable among them being Arya Bhatt the first person to assert that all planets including the earth revolve round the sun.

A millennium later the Danish astronomer Tycobrahe (1546-1601) conducted a detailed study of planetary motion which was interpreted by his pupil Johnaase Kepler (1571-1630), ironically after the master himself had passed away. Kepler formulated his important findings in three laws of planetary motion. The basis of astronomy is gravitation.

2. UNIVERSAL LAW OF GRAVITATION : NEWTON'S LAW

According to this law "Each particle attracts every other particle. The force of attraction between them is directly proportional to the product of their masses and inversely proportional to square of the distance between them".

$F \propto \frac{m_1 m_2}{r^2}$ or $F = G\frac{m_1 m_2}{r^2}$



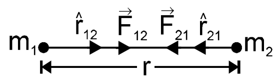
where $G = 6.67 \times 10^{-11}$ $Nm^2\ kg^{-2}$ is the universal gravitational constant. This law holds good irrespective of the nature of two objects (size, shape, mass etc.) at all places and all times. That is why it is known as universal law of gravitation.

Dimensional formula of G :

$$F = \frac{Fr^2}{m_1 m_2} = \frac{[MLT^{-2}]\,[L^2]}{[M^2]} = [M^{-1}\,L^3\,T^{-2}]$$

Newton's Law of gravitation in vector form :

$$\vec{F}_{12} = \frac{Gm_1m_2}{r^2}\,\hat{r}_{12} \qquad \& \qquad \vec{F}_{21} = \frac{Gm_1m_2}{r^2}\,\hat{r}_{21}$$



Where $\vec{F}_{12}$ is the force on mass m_1 exerted by mass m_2 and vice-versa.

Now $\hat{r}_{12} = -\hat{r}_{21}$, Thus $\vec{F}_{21} = \frac{-G\,m_1\,m_2}{r^2}\,\hat{r}_{12}$. Comparing above, we get $\vec{F}_{12} = -\vec{F}_{21}$

Important characteristics of gravitational force

(i) Gravitational force between two bodies form an action and reaction pair i.e. the forces are equal in magnitude but opposite in direction.

(ii) Gravitational force is a central force i.e. it acts along the line joining the centers of the two interacting bodies.

(iii) Gravitational force between two bodies is independent of the nature of the medium, in which they lie.

(iv) Gravitational force between two bodies does not depend upon the presence of other bodies.

(v) Gravitational force is negligible in case of light bodies but becomes appreciable in case of massive bodies like stars and planets.

(vi) Gravitational force is long range-force i.e., gravitational force between two bodies is effective even if their separation is very large. For example, gravitational force between the sun and the earth is of the order of 10^{27} N although distance between them is 1.5×10^7 km

Solved Example

Example 1.

The centres of two identical spheres are at a distance 1.0 m apart. If the gravitational force between them is 1.0 N, then find the mass of each sphere. (G = 6.67 × 10^{-11} m^3 kg^{-1} sec^{-1})

Solution Gravitational force $F = \frac{Gm.m}{r^2}$

on substituting F = 1.0 N , r = 1.0 m and G = 6.67 × 10^{-11} m^3 kg^{-1} sec^{-1}

we get m = 1.225 × 10^5 kg

Example 2.

Two particles of masses m_1 and m_2, initially at rest at infinite distance from each other, move under the action of mutual gravitational pull. Show that at any instant their relative velocity of approach is $\sqrt{2G(m_1+m_2)/R}$, where R is their separation at that instant.

Solution The gravitational force of attraction on m_1 due to m_2 at a separation r is

$$F_1 = \frac{Gm_1m_2}{r^2}$$

Therefore, the acceleration of m_1 is $a_1 = \frac{F_1}{m_1} = \frac{Gm_2}{r^2}$

Similarly, the acceleration of m_2 due to m_1 is $a_2 = -\frac{Gm_1}{r^2}$

the negative sign being put as a_2 is directed opposite to a_1. The relative acceleration of approach is

$$a = a_1 - a_2 = \frac{G(m_1+m_2)}{r^2} \quad .... (1)$$

If v is the relative velocity, then $a = \frac{dv}{dt} = \frac{dv}{dr}\frac{dr}{dt}$.

But $-\frac{dr}{dt}$ = v (negative sign shows that r decreases with increasing t).

$$\therefore a = -\frac{dv}{dr} v. \quad .... (2)$$

From (1) and (2), we have $v\,dv = -\frac{G(m_1+m_2)}{r^2} dr$

Integrating, we get $\frac{v^2}{2} = \frac{G(m_1+m_2)}{r} + C$

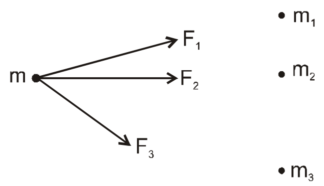
At r = ∞, v = 0 (given), and so C = 0.

$$\therefore \quad v^2 = \frac{2G(m_1+m_2)}{r}$$

Let v = v_R when r = R. Then $v_R = \sqrt{\left(\frac{2G(m_1+m_2)}{R}\right)}$

Principle of superposition

The force exerted by a particle on other particle remains unaffected by the presence of other nearby particles in space.

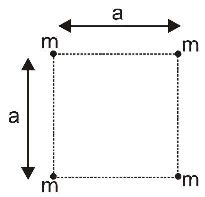


Total force acting on a particle is the vector sum of all the forces acted upon by the individual masses when they are taken alone.

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ...$$

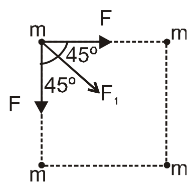
Solved Example

Example 3.



Four point masses each of mass 'm' are placed on the corner of square of side 'a' . Calculate magnitude of gravitational force experienced by each particle.

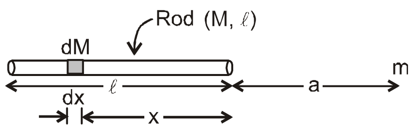
Solution :



F_r = resultant force on each particle = 2F cos 45° + F_1

$$= \frac{2G.m^2}{a^2} \cdot \frac{1}{\sqrt{2}} + \frac{Gm^2}{(\sqrt{2}a)^2} \qquad = \frac{G.m^2}{2a^2}\ (2\sqrt{2}+1)$$

Example 4.



Find gravitational force exerted by point mass 'm' on uniform rod (mass 'M' and length 'ℓ')

Solution : dF = force on element in horizontal direction = $\dfrac{G \cdot dM \cdot m}{(x+a)^2}$

where dM = $\dfrac{M}{\ell}$ dx.

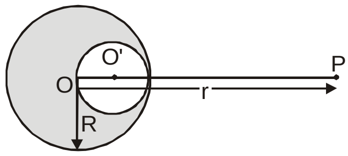
$$\therefore \qquad F = \int dF = \int_0^{\ell} \frac{G.Mm\,dx}{\ell(x+a)^2} = \frac{G.Mm}{\ell} \int_0^{\ell} \frac{dx}{(x+a)^2}$$

$$= \frac{G.Mm}{\ell}\left[-\frac{1}{(\ell+a)} + \frac{1}{a}\right] = \frac{GMm}{(\ell+a)a}$$

Example 5.

A solid sphere of lead has mass M and radius R. A spherical hollow is dug out from it (see figure). Its boundary passing through the centre and also touching the boundary of the solid sphere. Deduce the gravitational force on a mass m placed at P, which is distant r from O along the line of centres.

Solution : Let O be the centre of the sphere and O' that of the hollow (figure).



For an external point the sphere behaves as if its entire mass is concentrated at its centre. Therefore, the gravitational force on a mass `m` at P due to the
the
original sphere (of mass M) is

$$F = G\frac{Mm}{r^2}, \text{ along PO.}$$

The diameter of the smaller sphere (which would be cut off) is R, so that its radius OO' is R/2. The force on m at P due to this sphere of mass M' (say) would be

$$F' = G\frac{M'm}{(r-\frac{R}{2})^2} \text{ along PO}'. \qquad [\because \text{ distance PO}' = r - \frac{R}{2}]$$

As the radius of this sphere is half of that of the original sphere, we have

$$M' = \frac{M}{8}.$$

$$\therefore \qquad F' = G\frac{Mm}{8(r-\frac{R}{2})^2} \text{ along PO}'.$$

As both F and F′ point along the same direction, the force due to the hollowed sphere is

$$F - F' = \frac{GMm}{r^2} - \frac{GMm}{8r^2(1-\frac{R}{2r})^2} = \frac{GMm}{r^2}\left\{1 - \frac{1}{8(1-\frac{R}{2r})^2}\right\}.$$

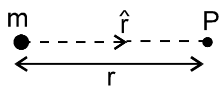
3. GRAVITATIONAL FIELD

The space surrounding the body within which its gravitational force of attraction is experienced by other bodies is called gravitational field. Gravitational field is very similar to electric field in electrostatics where charge 'q' is replaced by mass 'm' and electric constant 'K' is replaced by gravitational constant 'G'. The intensity of gravitational field at a point is defined as the force experienced by a unit mass placed at that point.

$$\vec{E} = \frac{\vec{F}}{m}$$

The unit of the intensity of gravitational field is N kg^{-1}.

Intensity of gravitational field due to point mass :



The force due to mass m on test mass m_0 placed at point P is given by :

$$F = \frac{GMm_0}{r^2}$$

Hence $\qquad E = \frac{F}{m_0} \quad \Rightarrow \quad E = \frac{GM}{r^2}$

In vector form $\qquad \vec{E} = -\frac{GM}{r^2}\hat{r}$

Dimensional formula of intensity of gravitational field = $\frac{F}{m} = \frac{[MLT^{-2}]}{[M]} = [M^0LT^{-2}]$

Solved Example

Example 6.

Find the distance of a point from the earth's centre where the resultant gravitational field due to the earth and the moon is zero. The mass of the earth is 6.0×10^{24} kg and that of the moon is 7.4×10^{22} kg. The distance between the earth and the moon is 4.0×10^5 km.

Solution : The point must be on the line joining the centres of the earth and the moon and in between them. If the distance of the point from the earth is x, the distance from the moon is (4.0×10^5 km-x). The magnitude of the gravitational field due to the earth is

$$E_1 = \frac{GM_e}{x^2} = \frac{G \times 6 \times 10^{24}\text{ kg}}{x^2}$$

and magnitude of the gravitational field due to the moon is

$$E_2 = \frac{GM_m}{(4.0 \times 10^5\text{ km} - x)^2} = \frac{G \times 7.4 \times 10^{22}\text{ kg}}{(4.0 \times 10^5\text{ km} - x)^2}$$

These fields are in opposite directions. For the resultant field to be zero $E_1 = E_2$.

or, $$\frac{6 \times 10^{24}\text{ kg}}{x^2} = \frac{7.4 \times 10^{22}\text{ kg}}{(4.0 \times 10^5\text{km} - x)^2}$$

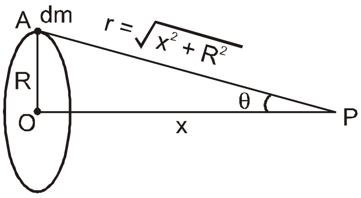
or, $$\frac{x}{4.0 \times 10^5\text{km} - x} = \sqrt{\frac{6 \times 10^{24}}{7.4 \times 10^{22}}} = 9$$

or, $x = 3.6 \times 10^5$ km.

Example 7.

Calculate gravitational field intensity due to a uniform ring of mass M and radius R at a distance x on the axis from center of ring.

Solution :



Consider any particle of mass dm. Gravitational field at point P due to dm

$$dE = \frac{G\,dm}{r^2} \text{ along PA}$$

Component along PO is

$$dE \cos\theta = \frac{G\,dm}{r^2} \cos\theta$$

Net gravitational field at point P is

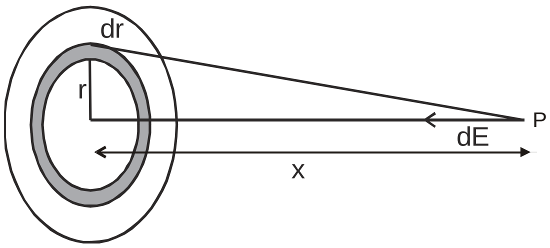
$$E = \int \frac{G\,dm}{r^2} \cos\theta = \frac{G\cos\theta}{r^2} \int dm$$

$$= \frac{GMr}{\left(R^2 + x^2\right)^{3/2}} \text{ towards the center of ring}$$

Example 8.

Calculate gravitational field intensity at a distance x on the axis from centre of a uniform disc of mass M and radius R.

Solution : Consider a elemental ring of radius r and thickness dr on surface of disc as shown in figure



Gravitational field due to elemental ring

$$dE = \frac{GdMx}{(x^2 + r^2)^{3/2}} \qquad \text{Here} \quad dM = \frac{M}{\pi R^2} \cdot 2\pi r dr = \frac{2M}{R^2} r\,dr$$

$$\therefore\ dE = \frac{G.2Mxrdx}{R^2(x^2 + r^2)^{3/2}}$$

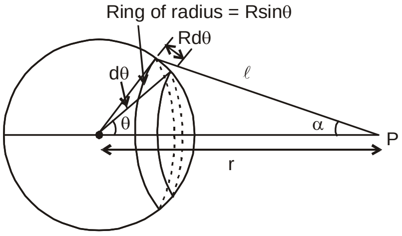
$$\therefore\ E = \int_0^R \left(\frac{2GMx}{R^2}\right) \frac{rdr}{(x^2 + r^2)^{3/2}}$$

$$\therefore\ E = \frac{2GMx}{R^2}\left[\frac{1}{x} - \frac{1}{\sqrt{x^2 + R^2}}\right]$$

Example 9.

For a given uniform spherical shell of mass M and radius R, find gravitational field at a distance r from centre in following two cases (a) $r \geq R$ (b) $r < R$

Solution :



$$dE = \frac{GdM}{\ell^2} \cdot \cos\alpha \qquad r \geq R$$

$$dM = \frac{M}{4\pi R^2} \times 2\pi R \sin\theta\, Rd\theta$$

$$dM = \frac{M}{2} \sin\theta\, d\theta$$

$$\therefore \quad dE = \frac{GM \sin\theta \cos\alpha\, d\theta}{2\ell^2}$$

Now $\ell^2 = R^2 + r^2 - 2rR\cos\theta$ ....(1)

$R^2 = \ell^2 + r^2 - 2\ell r\cos\alpha$ ....(2)

$$\therefore \quad \cos\alpha = \frac{\ell^2 + r^2 - R^2}{2\ell r}$$

$$\cos\theta = \frac{R^2 + r^2 - \ell^2}{2rR}$$

differentiating (1)

$$\therefore \quad 2\,\ell\, d\,\ell = 2rR\sin\theta\, d\theta$$

$$\therefore \quad dE = \frac{GM}{2\ell^2}\cdot\frac{\ell d\ell}{Rr}\cdot\frac{\ell^2 + r^2 - R^2}{(2\ell r)} \quad \Rightarrow \quad dE = \frac{GM}{4Rr^2}\left[1 + \frac{r^2 - R^2}{\ell^2}\right] d\ell$$

$$\therefore \quad E = \int dE = \frac{GM}{4Rr^2}\left[\int_{r-R}^{r+R} d\ell + (r^2 - R^2)\int_{r-R}^{r+R}\frac{d\ell}{\ell^2}\right].$$

$$\Rightarrow \quad E = \frac{GM}{r^2}, \qquad r \ge R$$

If point is inside the shell limit changes to [(R–r) to R + r]

$E = 0$ when $r < R$.

Example 10.

Find the relation between the gravitational field on the surface of two planets A & B of masses m_A, m_B & radius R_A & R_B respectively if

(i) they have equal mass

(ii) they have equal (uniform) density

Solution : Let E_A & E_B be the gravitational field intensities on the surface of planets A & B.

then, $$E_A = \frac{Gm_A}{R_A^2} = \frac{G\frac{4}{3}\pi R_A^3 \rho_A}{R_A^2} = \frac{4G\pi}{3}\rho_A R_A$$

Similarly, $$E_B = \frac{Gm_B}{R_B^2} = \frac{4G}{3}\pi\ \rho_B R_B$$

(i) for $m_A = m_B$ $$\frac{E_A}{E_B} = \frac{R_B^2}{R_A^2}$$

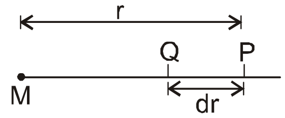
(ii) For & $\rho_A = \rho_B$ $$\frac{E_A}{E_B} = \frac{R_A}{R_B}$$

4. GRAVITATIONAL POTENTIAL

The gravitational potential at a point in the gravitational field of a body is defined as the amount of work done by an external agent in bringing a body of unit mass from infinity to that point, slowly (no change in kinetic energy). Gravitational potential is very similar to electric potential in electrostatics.

Gravitational potential due to a point mass :

Let the unit mass be displaced through a distance dr towards mass M, then work done is given by



$$dW = F\,dr = \frac{GM}{r^2}dr$$

Total work done in displacing the particle from infinity to point P is

$$W = \int dW = \int_{\infty}^{r}\frac{GM}{r^2}dr = \frac{-GM}{r}.$$

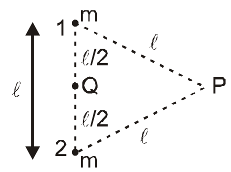
Thus gravitational potential, $\boxed{V = -\frac{GM}{r}}$.

The unit of gravitational potential is J kg^{-1}. Dimensional Formula of gravitational potential

$$= \frac{\text{Work}}{\text{mass}} = \frac{[ML^2T^{-2}]}{[M]} = [M^\circ L^2 T^{-2}].$$

Solved Example

Example 11.



Find out potential at P and Q due to the two point mass system. Find out work done by external agent in bringing unit mass from P to Q. Also find work done by gravitational force.

Solution : (i) V_{P1} = potential at P due to mass 'm' at '1' = $-\frac{Gm}{\ell}$

$V_{P2} = -\frac{Gm}{\ell}$

$\therefore$ $V_P = V_{P1} + V_{P2} = -\frac{2Gm}{\ell}$

(ii) $V_{Q1} = -\frac{GM}{\ell/2}$ $\Rightarrow$ $V_{Q2} = -\frac{Gm}{\ell/2}$

$\therefore$ $V_Q = V_{Q1} + V_{Q2} = -\frac{Gm}{\ell/2} - \frac{Gm}{\ell/2} = -\frac{4Gm}{\ell}$

Force at point Q = 0

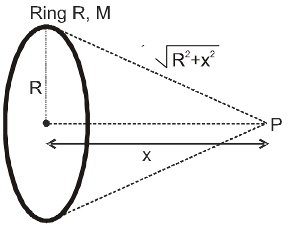
(iii) work done by external agent = $(V_Q - V_P) \times 1 = -\frac{2GM}{\ell}$

(iv) work done by gravitational force = $V_P - V_Q = \frac{2GM}{\ell}$

Example 12.

Find potential at a point 'P' at a distance 'x' on the axis away from centre of a uniform ring of mass M and radius R.

Solution :



Ring can be considered to be made of large number of point masses (m_1 , m_2 ....etc)

$$V_P = -\frac{Gm_1}{\sqrt{R^2+x^2}} - \frac{Gm_2}{\sqrt{R^2+x^2}} - ....$$

$$= -\frac{G}{\sqrt{R^2+x^2}}(m_1 + m_2.....) = -\frac{GM}{\sqrt{R^2+x^2}}$$, where $M = m_1 + m_2 + m_3 + ...$

Potential at centre of ring $= -\frac{G.M}{R}$

5. RELATION BETWEEN GRAVITATIONAL FIELD AND POTENTIAL

The work done by an external agent to move unit mass from a point to another point in the direction of the field E, slowly through an infinitesimal distance dr = Force by external agent × distance moved = – Edr.

Thus $dV = -Edr \Rightarrow E = -\frac{dV}{dr}$.

Therefore, gravitational field at any point is equal to the negative gradient at that point.

Solved Example

Example 13.

The gravitational field in a region is given by $\vec{E} = -(20N/kg)\ (\hat{i} + \hat{j})$. Find the gravitational potential at the origin (0, 0) – (in J/kg)

(A) zero (B) $20\sqrt{2}$ (C) $-20\sqrt{2}$ (D) can not be defined

Solution : $V = -\int E.dr = \left[\int Ex.dx + \int Ey.dy\right] = 20x + 20y$

at origin V = 0

Ans. (A)

Example 14.

In above problem, find the gravitational potential at a point whose co-ordinates are (5, 4): (in J/kg)

(A) – 180 (B) 180 (C) – 90 (D) zero

Solution : $V = 20 \times 5 + 20 \times 4 = 180$ J/kg

Ans. (B)

Example 15.

In the above problem, find the work done in shifting a particle of mass 1 kg from origin (0, 0) to a point (5, 4): (In J)

(A) – 180 (B) 180 (C) – 90 (D) zero

Solution : $W = m\,(V_f - V_i) = 1\,(180 - 0) = 180$ J

Ans. (B)

Example 16

$v = 2x^2 + 3y^2 + zx$,

Find gravitational field at a point (x, y, z).

Solution : $E_x = \frac{-\partial v}{\partial x} = -4x - z$

$E_y = -by$

$E_z = -x$

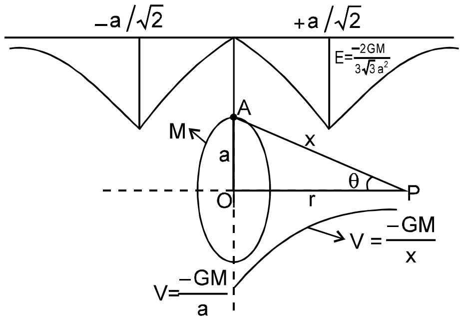
$\therefore$ Field $= \vec{E} = -[\,(4x + z)\ \hat{i} + by\ \hat{j} + x\hat{k}\,]$

$\vec{E} = -\nabla V$.

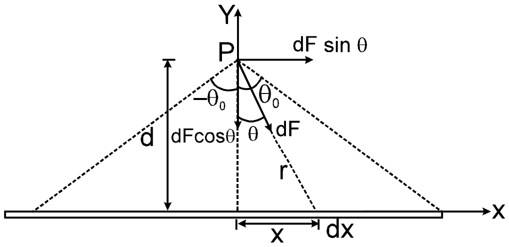
6. GRAVITATIONAL POTENTIAL & FIELD FOR DIFFERENT OBJECTS

I. **Ring.** $V = \frac{-GM}{x}$ or $\frac{-GM}{(a^2+r^2)^{1/2}}$ & $E = \frac{-GMr}{(a^2+r^2)^{3/2}}\hat{r}$ or $E = -\frac{GM\cos\theta}{x^2}$

Gravitational field is maximum at a distance, $r = \pm\ a/\sqrt{2}$ and it is $-\ 2GM/3\sqrt{3}\,a^2$



II. **A linear mass of finite length on its axis :**



(a) Potential :

$$\Rightarrow \quad V = -\frac{GM}{L}\ell n\,(\sec\theta_0 + \tan\theta_0) = -\frac{GM}{L}\ln\left\{\frac{L+\sqrt{L^2+d^2}}{d}\right\}$$

(b) Field intensity :

$$\Rightarrow \quad E = -\frac{GM}{Ld}\sin\theta_0 = \frac{GM}{d\sqrt{L^2+d^2}}$$

III. **An infinite uniform linear mass distribution of linear mass density λ, Here $\theta_0 = \frac{\pi}{2}$.**

And noting that $\lambda = \frac{M}{2L}$ in case of a finite rod

we get, for field intensity $E = \frac{2G\lambda}{d}$

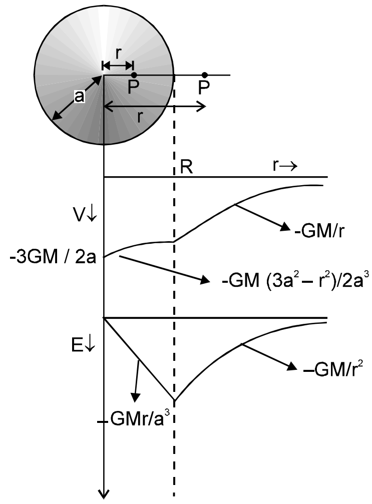
Potential for a mass-distribution extending to infinity is not defined. However even for such mass distributions potential-difference is defined. Here potential difference between points P_1 and P_2 respectively at distances d_1 and d_2 from the infinite rod, $v_{12} = 2G\lambda\ \ell n\frac{d_2}{d_1}$

IV. **Uniform Solid Sphere**

(a) Point P inside the shell. $r \le a$, then

$$V = -\frac{GM}{2a^3}(3a^2 - r^2)\ \&\ E = -\frac{GMr}{a^3}\text{, and at the centre } V = -\frac{3GM}{2a} \text{ and } E = 0$$

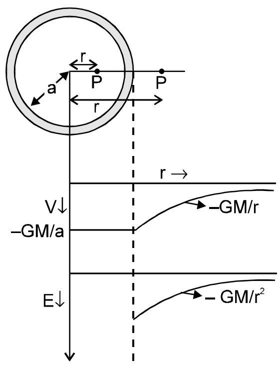
(b) Point P outside the shell. $r \geq a$, then $V = -\frac{GM}{r}$ & $E = -\frac{GM}{r^2}$



V. Uniform Thin Spherical Shell

(a) Point P Inside the shell. $r \leq a$, then $V = \frac{-GM}{a}$ & $E = 0$

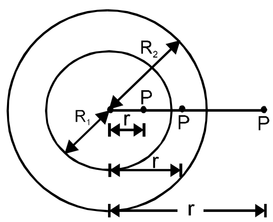
(b) Point P outside shell. $r \geq a$, then $V = \frac{-GM}{r}$ & $E = -\frac{GM}{r^2}$



VI. Uniform Thick Spherical Shell

(a) Point outside the shell

$$V = -G\frac{M}{r} \quad ; \quad E = -G\frac{M}{r^2}$$



(b) Point inside the Shell

$$V = -\frac{3}{2}GM\left(\frac{R_2 + R_1}{R_2^2 + R_1R_2 + R_1^2}\right)$$

$E = 0$

(c) Point between the two surface

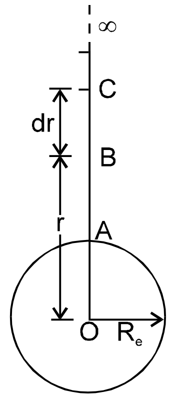
$$V = -\frac{GM}{2r}\left(\frac{3rR_2^2 - r^3 - 2R_1^3}{R_2^3 - R_1^3}\right) \; ;$$

$$E = -\frac{GM}{r^2}\;\frac{r^3 - R_1^3}{R_2^3 - R_1^3}$$

7. GRAVITATIONAL POTENTIAL ENERGY

Gravitational potential energy of two mass system is equal to the work done by an external agent in assembling them, while their initial separation was infinity. Consider a body of mass m placed at a distance r from another body of mass M. The gravitational force of attraction between them is given by,

$$F = \frac{GMm}{r^2}.$$

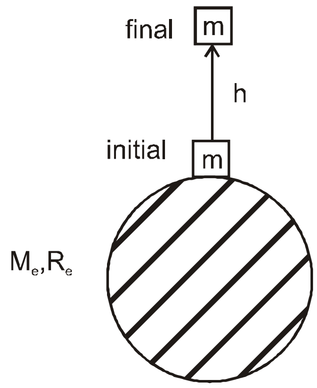


Now, Let the body of mass m is displaced from point. C to B through a distance 'dr' towards the mass M, then work done by internal conservative force (gravitational) is given by,

$$dW = F\,dr = \frac{GMm}{r^2}\,dr \qquad \Rightarrow \qquad \int dW = \int_{\infty}^{r} \frac{GMm}{r^2}\,dr$$

$\therefore$ Gravitational potential energy, $\boxed{U = -\frac{GMm}{r}}$

Increase in gravitational potential energy :



Suppose a block of mass m on the surface of the earth. We want to lift this block by ‘h’ height.
Work required in this process = increase in P.E. = $U_f - U_i = m(V_f - V_i)$

$$W_{ext} = \Delta U = (m)\left[-\left(\frac{GM_e}{R_e + h}\right) - \left(-\frac{GM_e}{R_e}\right)\right]$$

$$W_{ext} = \Delta U = GM_e m \left(\frac{1}{R_e} - \frac{1}{R_e + h}\right) = \frac{GM_e m}{R_e}\left(1 - \left(1 + \frac{h}{R_e}\right)^{-1}\right)$$

(as $h << R_e$, we can apply Bionomical theorem)

$$W_{ext} = \Delta U = \frac{GM_e m}{R_e}\left(1 - \left(1 - \frac{h}{R_e}\right)\right) = (m)\left(\frac{GM_e}{R_e^2}\right)h$$

$$W_{ext} = \Delta U = mgh$$

* This formula is valid only when $h << R_e$

Example 17.

A body of mass m is placed on the surface of earth. Find work required to lift this body by a height

(i) $h = \frac{R_e}{1000}$ (ii) $h = R_e$

Solution :

(i) $h = \frac{R_e}{1000}$, as $h << R_e$, so

we can apply

$W_{ext} = U\uparrow = mgh$

$$W_{ext} = (m)\left(\frac{GM_e}{R_e^{\,2}}\right)\left(\frac{R_e}{1000}\right) = \frac{GM_e m}{1000\,R_e}$$

(ii) $h = R_e$, in this case h is not very less than R_e, so we cannot apply $\Delta U = mgh$

so we cannot apply $\Delta U = mgh$

$W_{ext} = U\uparrow = U_f - U_i = m(V_f - V_i)$

$$W_{ext} = m\left[\left(-\frac{GM_e}{R_e + R_e}\right) - \left(-\frac{GM_e}{R_e}\right)\right]$$

$$W_{ext} = -\frac{GM_e m}{2R_e}$$

Example 18.

Calculate the velocity with which a body must be thrown vertically upward from the surface of the earth so that it may reach a height of 10 R, where R is the radius of the earth and is equal to 6.4×10^8 m. (Earth's mass = 6×10^{24} kg, Gravitational constant G = 6.7×10^{-11} N-m²/kg²)

Solution : The gravitational potential energy of a body of mass m on earth's surface is

$$U(R) = -\frac{GMm}{R}$$

where M is the mass of the earth (supposed to be concentrated at its centre) and R is the radius of the earth (distance of the particle from the centre of the earth). The gravitational energy of the same body at a height 10 R from earth's surface, i.e. at a distance 11R from earth's centre is

$$U(11R) = -\frac{GMm}{R}$$

$\therefore$ change in potential energy $\quad U(11R) - U(R) = -\frac{GMm}{11R} - \left(-\frac{GMm}{R}\right) = \frac{10}{11}\frac{GMm}{R}$

This difference must come from the initial kinetic energy given to the body in sending it to that height. Now, suppose the body is thrown up with a vertical speed v, so that its initial kinetic energy is $\frac{1}{2}mv^2$. Then

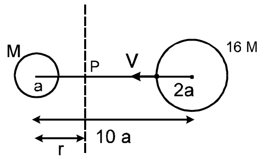
$$\frac{1}{2}mv^2 = \frac{10}{11}\frac{GMm}{R} \quad \text{or} \quad v = \sqrt{\left(\frac{20}{11}\frac{GMm}{R}\right)}.$$

Putting the given values : $v = \sqrt{\left(\frac{20 \times (6.7 \times 10^{-11} N - m^2/kg^2) \times (6 \times 10^{24} kg)}{11(6.4 \times 10^6 m)}\right)} = 1.07 \times 10^4$ m/s.

Example 19.

Distance between centres of two stars is 10 a. The masses of these stars are M and 16 M and their radii are a & 2a respectively. A body is fired straight from the surface of the larger star towards the smaller star. What should be its minimum initial speed to reach the surface of the smaller star?

Solution : Let P be the point on the line joining the centres of the two planets s.t. the net field at it is zero



Then, $\frac{GM}{r^2} - \frac{G.16M}{(10a-r)^2} = 0 \Rightarrow (10\,a-r)^2 = 16\,r^2$

$\Rightarrow 10a - r = 4r \Rightarrow r = 2a$

Potential at point P, $v_P = \frac{-GM}{r} - \frac{G.16M}{(10a-r)} = \frac{-GM}{2a} - \frac{2GM}{a} = \frac{-5GM}{2a}$.

Now if the particle projected from the larger planet has enough energy to cross this point, it will reach the smaller planet.

For this, the K.E. imparted to the body must be just enough to raise its total mechanical energy to a value which is equal to P.E. at point P.

i.e. $\frac{1}{2}mv^2 - \frac{G(16M)m}{2a} \frac{-GMm}{8a} = mv_P$

or, $\frac{v^2}{2} \frac{-8GM}{a} \frac{-GM}{8a} = \frac{-5GMm}{2a}$

or, $v^2 = \frac{45GM}{4a}$ or, $v_{min} = \frac{3}{2}\sqrt{\frac{5GM}{a}}$

8. GRAVITATIONAL SELF-ENERGY

The gravitational self-energy of a body (or a system of particles) is defined as the work done by an external agent in assembling the body (or system of particles) from infinitesimal elements (or particles) that are initially an infinite distance apart.

Gravitational self energy of a system of n particles

Potential energy of n particles at an average distance 'r' due to their mutual gravitational attraction is equal to the sum of the potential energy of all pairs of particle, i.e.,

$$U_s = -G \sum_{\substack{\text{all pairs} \\ j \neq i}} \frac{m_i m_j}{r_{ij}}$$

This expression can be written as $U_s = -\frac{1}{2}G \sum_{i=1}^{i=n} \sum_{\substack{j=1 \\ j\neq i}}^{j=n} \frac{m_i m_j}{r_{ij}}$

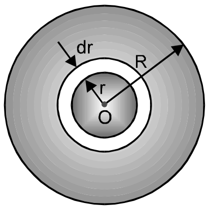
If consider a system of 'n' particles, each of same mass 'm' and separated from each other by the same average distance 'r', then self energy

or $U_s = -\frac{1}{2}G \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j\neq i}}^{n} \left(\frac{m^2}{r}\right)_{ij}$

Thus on the right hand side 'i' comes 'n' times while 'j' comes (n – 1) times. Thus

$$U_s = -\frac{1}{2}Gn\,(n-1)\,\frac{m^2}{r}$$

Gravitational Self energy of a Uniform Sphere (star)



$$U_{sphere} = -G\,\frac{\left(\frac{4}{3}\pi r^3\rho\right)\left(4\pi r^2 dr\,\rho\right)}{r} \quad \text{where } \rho = \frac{M}{\left(\frac{4}{3}\right)\pi R^3}$$

$$= -\frac{1}{3}\,G\,(4\,\pi\rho)^2\, r^4\, dr,$$

$$U_{star} = -\frac{1}{3}G\,(4\,\pi\rho)^2 \int_0^R r^4\,dr = -\frac{1}{3}G\,(4\pi\rho)^2 \left[\frac{r^5}{5}\right]_0^R = -\frac{3}{5}G\left(\frac{4\pi}{3}R^3\rho\right)^2 \frac{1}{R}.$$

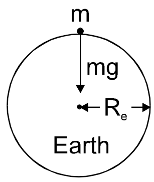
$$\therefore\; U_{star} = -\frac{3}{5}\,\frac{GM^2}{R}$$

9. ACCELERATION DUE TO GRAVITY :

It is the acceleration, a freely falling body near the earth's surface acquires due to the earth's gravitational pull. The property by virtue of which a body experiences or exerts a gravitational pull on another body is called **gravitational mass m_G**, and the property by virtue of which a body opposes any change in its state of rest or uniform motion is called its **inertial mass m_I** thus if $\vec{E}$ is the gravitational field intensity due to the earth at a point P, and $\vec{g}$ is acceleration due to gravity at the same point, then $m_I\,\vec{g} = m_G\,\vec{E}$.

Now the value of inertial & gravitational mass happen to be exactly same to a great degree of accuracy for all bodies. Hence, $\vec{g} = \vec{E}$

The gravitational field intensity on the surface of earth is therefore numerically equal to the acceleration due to gravity (g), there. Thus we get,



$$g = \frac{GM_e}{{R_e}^2}$$

where , M_e = Mass of earth
R_e = Radius of earth

Note :

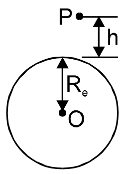
- Here the distribution of mass in the earth is taken to be spherical symmetrical so that its entire mass can be assumed to be concentrated at its center for the purpose of calculation of g.

10. VARIATION OF ACCELERATION DUE TO GRAVITY

(a) Effect of Altitude

Acceleration due to gravity on the surface of the earth is given by, $g = \frac{GM_e}{R_e^2}$

Now, consider the body at a height 'h' above the surface of the earth, then the acceleration due to gravity at height 'h' given by



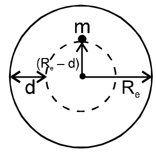
$$g_h = \frac{GM_e}{(R_e + h)^2} = g\left(1 + \frac{h}{R_e}\right)^{-2} \simeq g\left(1 - \frac{2h}{R_e}\right) \text{ when } h << R.$$

The decrease in the value of 'g' with height h = $g - g_h = \frac{2gh}{R_e}$. Then percentage decrease in the value of

$$\text{'g'} = \frac{g - g_h}{g}\times 100 = \frac{2h}{R_e}\times 100\%$$

(b) Effect of depth

The gravitational pull on the surface is equal to its weight i.e. mg = $\frac{GM_e m}{R_e^2}$



$\therefore \quad mg = \frac{G \times \frac{4}{3}\pi R_e^3 \rho m}{R_e^2}$ or $g = \frac{4}{3}\pi G R_e \rho$ ....(1)

When the body is taken to a depth d, the mass of the sphere of radius (R_e – d) will only be effective for the gravitational pull and the outward shall will have no resultant effect on the mass. If the acceleration due to gravity on the surface of the solid sphere is g_d, then

$g_d = \frac{4}{3}\pi G (R_e - d)\rho$ ....(2)

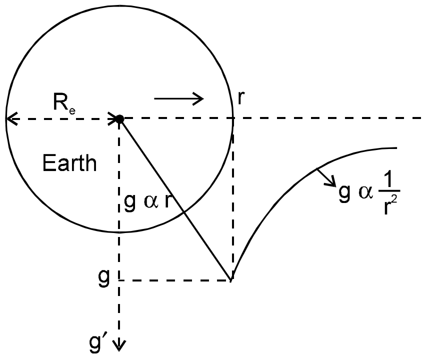
By dividing equation (2) by equation (1)

$\Rightarrow \quad g_d = g\left(1 - \frac{d}{R_e}\right)$

IMPORTANT POINTS

(i) At the center of the earth, d = R_e, so $g_{centre} = g\left(1 - \frac{R_e}{R_e}\right) = 0$.

Thus weight (mg) of the body at the centre of the earth is zero.



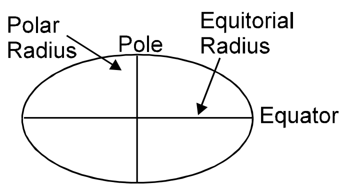
(ii) Percentage decrease in the value of 'g' with the depth

$= \left(\frac{g - g_d}{g}\right) \times 100 = \frac{d}{R_e} \times 100$.

(c) Effect of the surface of Earth

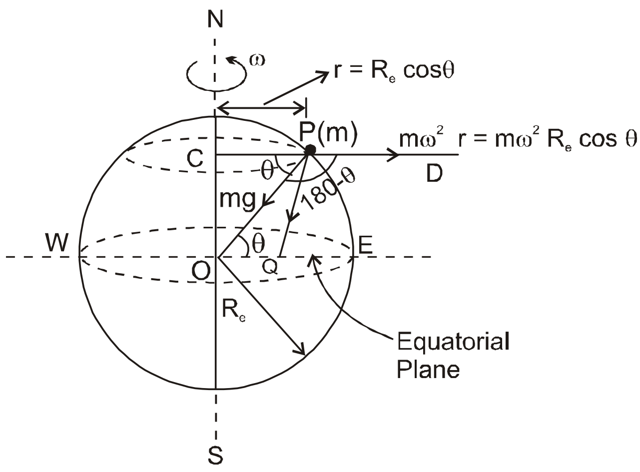
The equatorial radius is about 21 km longer than its polar radius.

We know, $g = \frac{GM_e}{R_e^2}$ Hence $g_{pole} > g_{equator}$. The weight of the body increase as the body taken from the equator to the pole.



(d) Effect of rotation of the Earth

The earth rotates around its axis with angular velocity ω. Consider a particle of mass m at latitude θ. The angular velocity of the particle is also ω.



According to parallelogram law of vector addition, the resultant force acting on mass m along PQ is

$$F = [(mg)^2 + (m\omega^2 R_e \cos\theta)^2 + \{2mg \times m\omega^2 R_e \cos\theta\} \cos(180 - \theta)]^{1/2}$$
$$= [(mg)^2 + (m\omega^2 R_e \cos\theta)^2 - (2m^2 g\omega^2 R_e \cos\theta) \cos\theta]^{1/2}$$

$$= mg\left[1 + \left(\frac{R_e\,\omega^2}{g}\right)^2 \cos^2\theta - 2\frac{R_e\,\omega^2}{g}\cos^2\theta\right]^{1/2}$$

At pole $\theta = 90° \Rightarrow g_{pole} = g$, At equator $\theta = 0 \Rightarrow g_{equator} = g\left[1 - \frac{R_e\,\omega^2}{g}\right]$.

Hence $g_{pole} > g_{equator}$

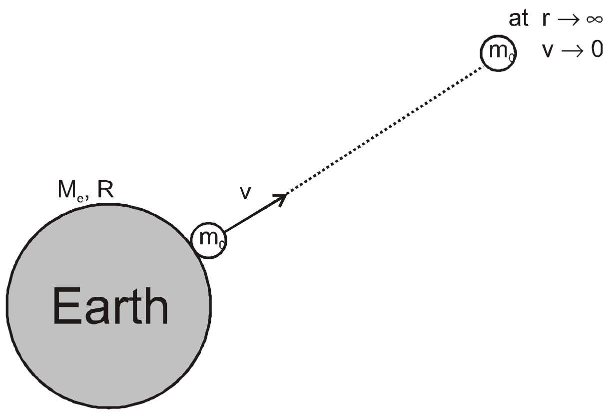
If the body is taken from pole to the equator, then $g' = g\left(1 - \frac{R_e\omega^2}{g}\right)$.

Hence % change in weight = $\dfrac{mg - mg\left(1 - \frac{R_e\omega^2}{g}\right)}{mg} \times 100 = \dfrac{mR_e\,\omega^2}{mg} \times 100 = \dfrac{R_e\,\omega^2}{g} \times 100$

11. ESCAPE SPEED

The minimum speed required to send a body out of the gravity field of a planet (send it to $r \to \infty$)

11.1 Escape speed at earth's surface :



Suppose a particle of mass m is on earth's surface
We project it with a velocity V from the earth's surface, so that it just reaches $r \to \infty$ (at $r \to \infty$, its velocity become zero)
Applying energy conservation between initial position (when the particle was at earth's surface) and find positions (when the particle just reaches to $r \to \infty$)

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}mv^2 + m_0\left(-\frac{GM_e}{R}\right) = 0 + m_0\left(-\frac{GM_e}{(r \to \infty)}\right)$$

$$\Rightarrow \qquad v = \sqrt{\frac{2GM_0}{R}}$$

Escape speed from earth is surface $v_e = \sqrt{\dfrac{2GM_e}{R}}$

If we put the values of G, Me, R the we get

$V_e = 11.2$ km/s.

11.2 Escape speed depends on :

(i) Mass (M_e) and size (R) of the planet
(ii) Position from where the particle is projected.

11.3 Escape speed does not depend on :

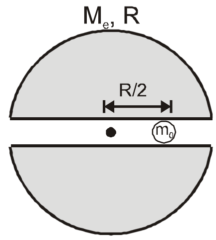
(i) Mass of the body which is projected (m_0)
(ii) Angle of projection.

If a body is thrown from Earth's surface with escape speed, it goes out of earth's gravitational field and never returns to the earth's surface. But it starts revolving around the sun.

Solved Example

Example 20.

A very small groove is made in the earth, and a particle of mass m_0 is placed at R/2 distance from the centre. Find the escape speed of the par ticle from that place.



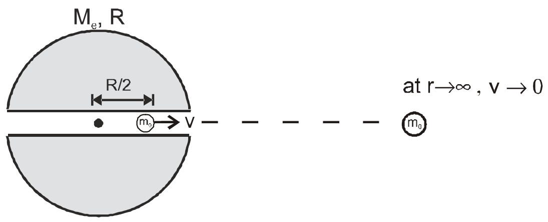
Solution :

Suppose we project the particle with speed v, so that it just reaches at ($r \to \infty$).

Applying energy conservation

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}m_0v^2 + m_0\left(-\frac{GM_e}{2R^3}\left(3R^2 - \left(\frac{R}{2}\right)^2\right)\right) = 0 + 0$$



$$v = \sqrt{\frac{11GM_e}{4R}} = V_e \text{ at that position.}$$

Example 21.

Find radius of such planet on which the man escapes through jumping. The capacity of jumping of person on earth is 1.5 m. Density of planet is same as that of earth.

Solution : For a planet : $\frac{1}{2}mv^2 - \frac{GM_Pm}{R_p} = 0 \quad \Rightarrow \quad \frac{1}{2}mv^2 = \frac{GM_Pm}{R_p}$

On earth $\to$ $\frac{1}{2}mv^2 = m\left(\frac{GM_E}{R_E^2}\right)h$

$\therefore \quad \frac{GM_Pm}{R_p} = \frac{GM_E.m}{R_E^2} . h \quad \Rightarrow \quad \frac{M_p}{R_p} = \frac{M_Eh}{R_E^2}$

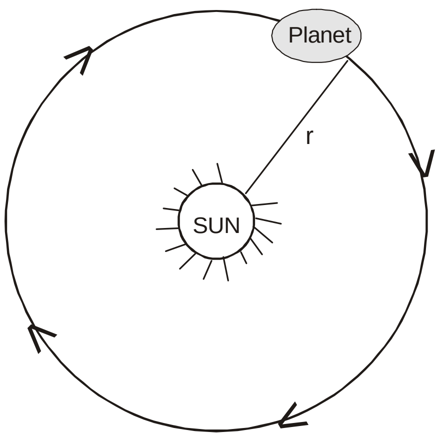
$\therefore$ Density (ρ) is same $\quad \Rightarrow \quad \frac{4/3\,\pi R_p^3\,\rho}{R_p} = \frac{4/3\,\pi R_E^2\,h\,\rho}{R_E^2} \quad \Rightarrow \quad R_P = \sqrt{R_Eh}$

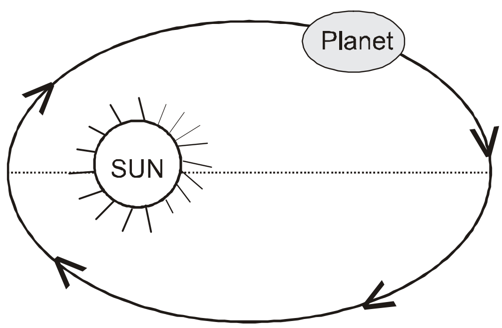
12. KEPLER'S LAW FOR PLANETARY MOTION

Suppose a planet is revolving around the sun, or a satellite is revolving around the earth, then the planetary motion can be studied with help of Kepler's three laws.

12.1 Kepler's Law of orbit

Each planet moves around the sun in a circular path or elliptical path with the sun at its focus. (In fact circular path is a subset of elliptical path)





12.2 Law of areal velocity :

To understand this law, lets understand the angular momentum conservation for the planet.

If a planet moves in an elliptical orbit, the gravitation force acting on it always passes through the centre of the sun. So torque of this gravitation force about the centre of the sun will be zero. Hence we can say that angular momentum of the planet about the centre of the sun will remain conserved (constant)

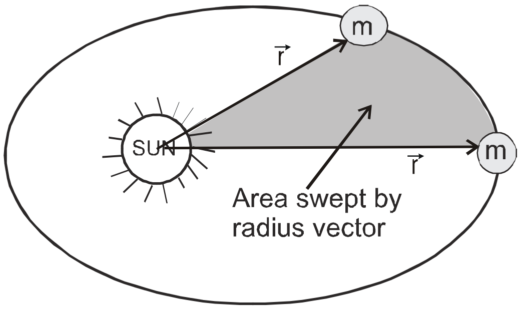
τ about the sun = 0

$\Rightarrow \quad \frac{dJ}{dt} = 0 \quad \Rightarrow \quad J_{planet}$ / sun = constant $\quad \Rightarrow \quad$ mvr sinθ = constant

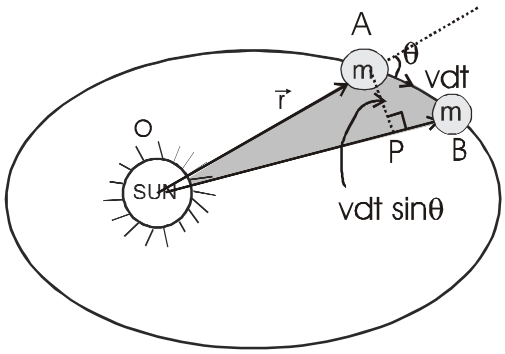
Now we can easily study the Kepler's law of aerial velocity.

If a planet moves around the sun, the radius vector ($\vec{r}$) also rotates are sweeps area as shown in figure.

Now lets find rate of area swept by the radius vector ($\vec{r}$).



Suppose a planet is revolving around the sun and at any instant its velocity is v, and angle between radius vector ($\vec{r}$) and velocity ($\vec{v}$). In dt time, it moves by a distance vdt, during this dt time, area swept by the radius vector will be OAB which can be assumed to be a triangle



dA = 1/2 (Base) (Perpendicular height)

dA = 1/2 (r) (vdtsinθ)

so rate of area swept $\frac{dA}{dt} = \frac{1}{2}$ vr sinθ

we can write $\frac{dA}{dt} = \frac{1}{2}\frac{mvr\sin\theta}{m}$

where mvr sinθ = angular momentum of the planet about the sun, which remains conserved (constant)

$\Rightarrow \quad \frac{dA}{dt} = \frac{L_{planet/sun}}{2m}$ = constant

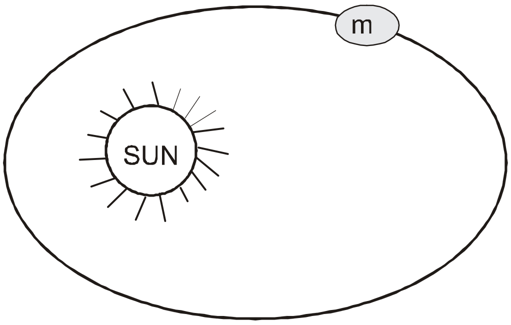
so ***Rate of area swept by the radius vector is constant***

Solved Example

Example 22.

Suppose a planet is revolving around the sun in an elliptical path given by $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Find time period of revolution. Angular momentum of the planet about the sun is L.



Solution :

Rate of area swept $\frac{dA}{dt} = \frac{L}{2m}$ = constant

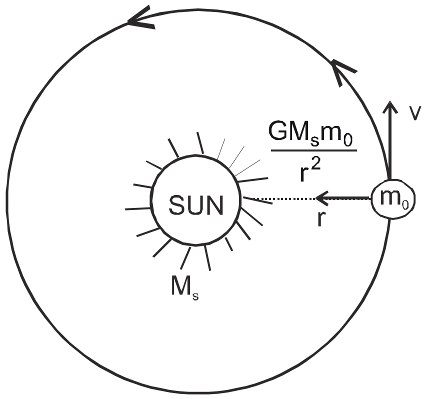
$$\Rightarrow \qquad dA = \frac{L}{2m}dt$$

$$\int_{A=0}^{A=\pi ab} dA = \int_{t=0}^{t=T} \frac{L}{2m}dt$$

$$\Rightarrow \qquad \pi ab = \frac{L}{2m}T \qquad \Rightarrow T = \frac{2\pi mab}{L}$$

12.3 Kepler's law of time period :

Suppose a planet is revolving around the sun in circular orbit



then $\frac{m_0 v^2}{r} = \frac{GM_s m_0}{r^2}$

$$v = \sqrt{\frac{GM_s}{r}}$$

Time period of revolution is

$$T = \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{r}{GM_s}}$$

$$T^2 = \left(\frac{4\pi^2}{GM_s}\right) r^3$$

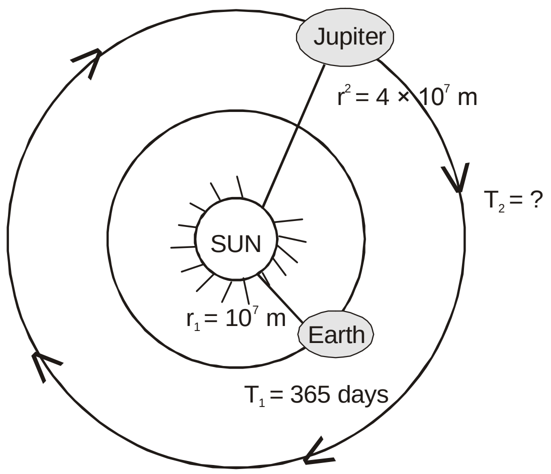
$\Rightarrow \qquad T^2 \alpha\, r^3$

For all the planet of a sun , $T^2 \propto r^3$

Solved Example

Example 23.

The Earth and Jupiter are two planets of the sun. The orbital radius of the earth is 10^7 m and that of Jupiter is 4×10^7 m. If the time period of revolution of earth is T = 365 days, find time period of revolution of the Jupiter.



Solution :

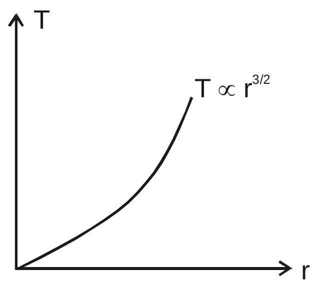
For both the planets

$T^2 \propto r^3$

$$\left(\frac{T_{jupiter}}{T_{earth}}\right)^2 = \left(\frac{T_{jupiter}}{r_{earth}}\right)^3 \quad \Rightarrow \quad \left(\frac{T_{jupiter}}{365\,\text{days}}\right)^2 = \left(\frac{4\times 10^7}{10^7}\right)^3$$

$T_{jupiter} = 8 \times 365$ days

Graph of T vs r :-

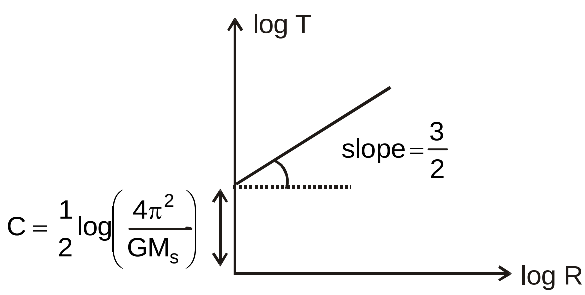


Graph of log T v/s log R :-

$$T^2 = \left(\frac{4\pi^2}{GM_s}\right)R^3$$

$$\Rightarrow \qquad 2\log T = \log\left(\frac{4\pi^2}{GM_s}\right) + 3\log R$$

$$\log T = \frac{1}{2}\log\left(\frac{4\pi^2}{GM_s}\right) + \frac{3}{2}\log R$$



* If planets are moving in elliptical orbit, then $T^2 \propto a^3$ where a = semi major axis of the elliptical path.

Solved Example

Example 24.

A satellite is launched into a circular orbit 1600 km above the surface of the earth. Find the period of revolution if the radius of the earth is R = 6400 km and the acceleration due to gravity is 9.8 m/sec^2. At what height from the ground should it be launched so that it may appear stationary over a point on the earth's equator?

Solution: The orbiting period of a satellite at a height h from earth's surface is $T = \dfrac{2\pi r^{3/2}}{gR^2}$ where r = R + h

then, $T = \dfrac{2\pi(R+h)}{R}\sqrt{\left(\dfrac{R+h}{g}\right)}$

Here, R = 6400 km, h = 1600 km = R/4.

Then $T = \dfrac{2\pi\left(R+\dfrac{R}{4}\right)}{R}\sqrt{\left(\dfrac{R+\dfrac{R}{4}}{g}\right)} = 2\pi(5/4)^{3/2}\sqrt{\dfrac{R}{g}}$

Putting the given values : $T = 2 \times 3.14 \times \sqrt{\left(\dfrac{6.4\times10^6\,m}{9.8\,m/s^2}\right)}\,(1.25)^{3/2}$ = 7092 sec = 1.97 hours

Now, a satellite will appear stationary in the sky over a point on the earth's equator if its period of revolution round the earth is equal to the period of revolution of the earth round its own axis which is 24 hours. Let us find the height h of such a satellite above the earth's surface in terms of the earth's radius. Let it be nR. then

$T = \dfrac{2\pi(R+nR)}{R}\sqrt{\left(\dfrac{R+nR}{g}\right)} = 2\pi\sqrt{\left(\dfrac{R}{g}\right)}\,(1+n)^{3/2} = 2 \times 3.14\sqrt{\left(\dfrac{6.4\times10^6\text{ meter/sec}}{9.8\text{ meter/sec}^2}\right)}\,(1+n)^{3/2}$

= (5075 sec) $(1 + n)^{3/2}$ = (1.41hours) $(1 + n)^{3/2}$

For T = 24 hours, we have

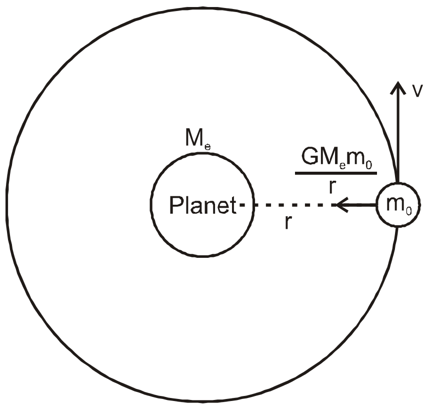
(24 hours) = (1.41) hours) $(1 + n)^{3/2}$

or $(1+n)^{3/2} = \dfrac{24}{1.41} = 17$

or $1 + n = (17)^{2/3} = 6.61$ or $n = 5.61$

The height of the geo-stationary satellite above the earth's surface is nR = 5.61 × 6400 km = 3.59 × 10^4 km.

13. CIRCULAR MOTION OF A SATELLITE AROUND A PLANET



Suppose at satellite of mass m_0 is at a distance r from a planet. If the satellite does not revolve, then due to the gravitational attraction, it may collide to the planet.

To avoid the collision, the satellite revolve around the planet, for circular motion of satellite.

$$\Rightarrow \qquad \frac{GM_e m_0}{r^2} = \frac{m_0 v^2}{r} \qquad ....(1)$$

$$\Rightarrow \qquad v = \sqrt{\frac{GM_e}{r}} \text{ this velocity is called orbital velocity } (v_0)$$

$$v_0 = \sqrt{\frac{GM_e}{r}}$$

13.1 Total energy of the satellite moving in circular orbit :

(i) $\quad KE = \frac{1}{2} m_0 v^2$ and from equation (1)

$$\frac{m_0 v^2}{r} = \frac{GM_e m_0}{r^2} \Rightarrow m_0 v^2 = \frac{GM_e m_0}{r} \quad \Rightarrow \quad KE = \frac{1}{2} m_0 v^2 = \frac{GM_e m_0}{2r}$$

(ii) Potential energy

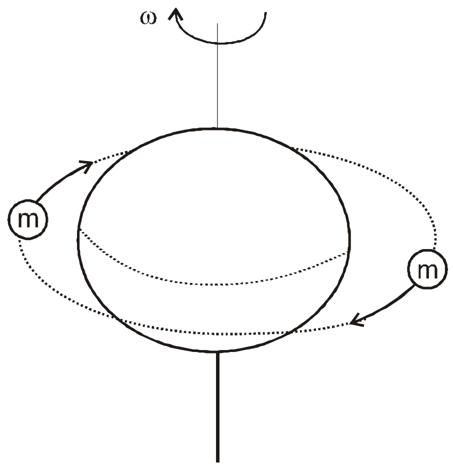
$$U = -\frac{GM_e m_0}{r}$$

$$\text{Total energy} = KE + PE = \left(\frac{GM_e m_0}{2r}\right) + \left(\frac{-GM_e m_0}{r}\right)$$

$$TE = -\frac{GM_e m_0}{2r}$$

Total energy is –ve. It shows that the satellite is still bounded with the planet.

14. GEO - STATIONARY SATELLITE :



We know that the earth rotates about its axis with angular velocity ω_{earth} and time period T_{earth} = 24 hours. Suppose a satellite is set in an orbit which is in the plane of the equator, whose ω is equal to ω_{earth}, (or its T is equal to T_{earth} = 24 hours) and direction is also same as that of earth. Then as seen from earth, it will appear to be stationery. This type of satellite is called geo- stationary satellite. For a geo-stationary satellite,

$$w_{satellite} = w_{earth}$$

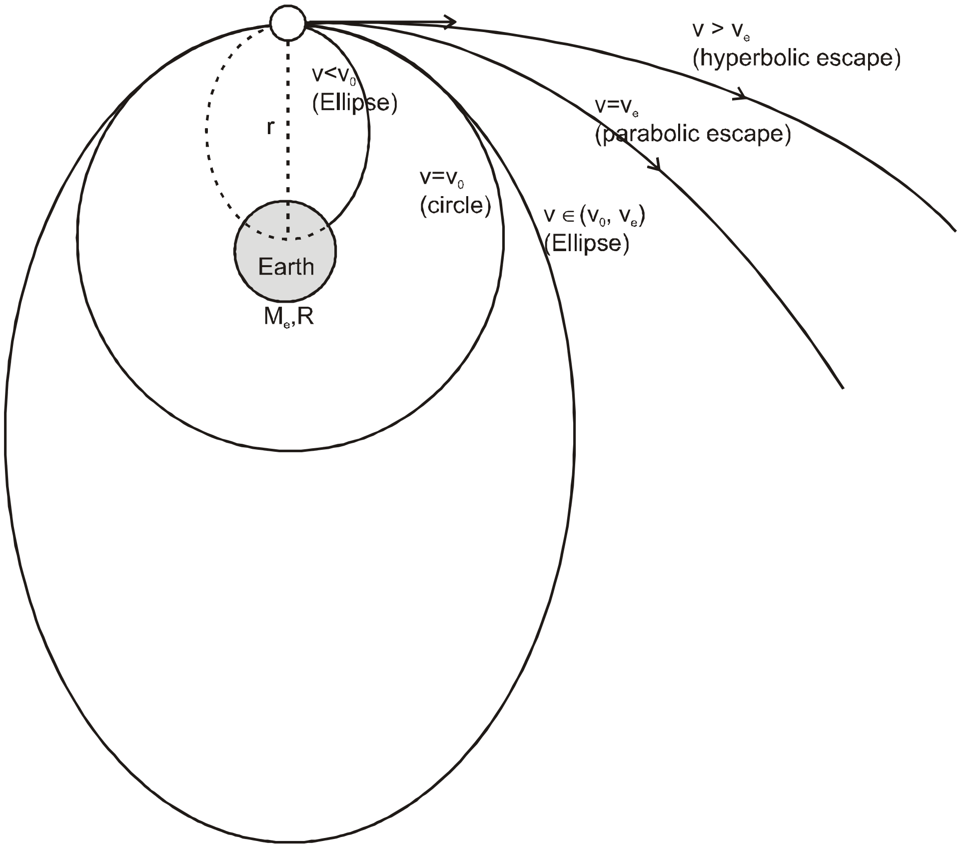
$$\Rightarrow \qquad T_{satellite} = T_{earth} = 24 \text{ hr.}$$

So time period of a geo-stationery satellite must be 24 hours. To achieve T = 24 hour, the orbital radius geo-stationary satellite :

$$T^2 = \left(\frac{4\pi^2}{GM_e}\right) r^3$$

Putting the values, we get orbital radius of geo stationary satellite $r = 6.6\ R_e$ (here Re = radius of the earth) height from the surface $h = 5.6\ R_e$.

15. PATH OF A SATELLITE ACCORDING TO DIFFERENT SPEED OF PROJECTION



Suppose a satellite is at a distance r from the centre of the earth. If we give different velocities (v) to the satellite, its path will be different

(i) If $v < v_0$ $\left(\text{or } v < \sqrt{\frac{GM_e}{r}}\right)$ then the satellite will move is an elliptical path and strike the earth's surface.

But if size of earth were small, the satellite would complete the elliptical orbit, and the centre of the earth will be at is farther focus.

(ii) If $v = v_0$ $\left(\text{or } v = \sqrt{\frac{GM_e}{r}}\right)$, then the satellite will revolve in a circular orbit.

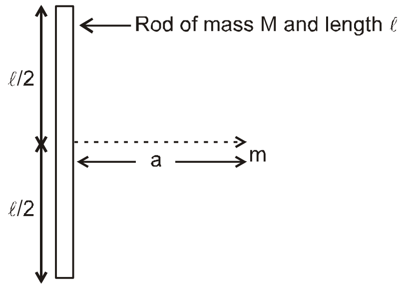
(iii) If $v_0 > v > v_0$ $\left(\text{or } \sqrt{\frac{2GM_e}{r}} > v > \sqrt{\frac{GM_e}{r}}\right)$, then the satellite will revolve in a elliptical orbital, and the centre of the earth will be at its nearer focus.

(iv) If $v = v_e$ $\left(\text{or } v = \sqrt{\frac{2GM_e}{r}}\right)$, then the satellite will just escape with parabolic path.

Solved Miscellaneous Problems

Problem 1.

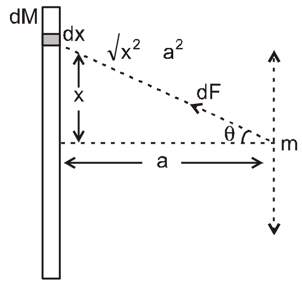
Calculate the force exerted by point mass m on rod of uniformly distributed mass M and length ℓ (Placed as shown in figure).



Solution :

$\because$ Direction of force is changing at every element. We have to make components of force and then integrate.

Net vertical force = 0.



dF = force on element = $\dfrac{G.dM.m}{(x^2+a^2)}$

dF_h = dF cos θ = force on element in horizontal direction = $\dfrac{G.dM.m}{(x^2+a^2)}\cos\theta$

$$\therefore \qquad F_h = \int \frac{G.M.m\cos\theta\, dx}{\ell(x^2+a^2)} = \frac{G.M.m}{\ell}\int_{-\ell/2}^{\ell/2}\frac{\cos\theta\cdot dx}{(x^2+a^2)} = \frac{GMm}{\ell a^2}\int_{-\ell/2}^{\ell/2}\frac{\cos\theta\cdot dx}{\sec^2\theta}$$

where x = a tan θ then dx = a sec^2 θ . dθ

$$= \frac{GMm}{\ell a}[\sin\theta]_{-\ell/2}^{\ell/2} \qquad \tan\theta = \frac{x}{a}\text{, then } \sin\theta = \frac{x}{\sqrt{x^2+a^2}}$$

$$= \frac{GMm}{\ell a}\left[\frac{x}{\sqrt{x^2+a^2}}\right]_{-\ell/2}^{\ell/2} \qquad = \frac{GMm\ell}{\ell a\sqrt{\frac{\ell^2}{4}+a^2}} = \frac{GMm}{a\sqrt{\frac{\ell^2}{4}+a^2}}$$

Problem 2.

Three identical bodies of mass M are located at the vertices of an equilateral triangle with side L. At what speed must they move if they all revolve under the influence of one another's gravity in a circular orbit circumscribing the triangle while still preserving the equilateral triangle ?

Solution :

Let A, B and C be the three masses and O the centre of the circumscribing circle. The radius of this circle is

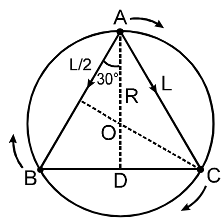
$$R = \frac{L}{2}\sec 30° = \frac{L}{2}\times\frac{2}{\sqrt{3}} = \frac{L}{\sqrt{3}}.$$

Let v be the speed of each mass M along the circle. Let us consider the motion of the mass at A. The force of gravitational attraction on it due to the masses at B and C are

$$\frac{GM^2}{L^2}\text{ along AB} \qquad \text{and} \qquad \frac{GM^2}{L^2}\text{ along AC}$$

The resultant force is therefore

$$2\frac{GM^2}{L^2}\cos 30° = \frac{\sqrt{3}\,GM^2}{L^2}\text{ along AD.}$$



This, for preserving the triangle, must be equal to the necessary centripetal force. That is ,

$$\frac{\sqrt{3}\,GM^2}{L^2} = \frac{Mv^2}{R} = \frac{\sqrt{3}\,Mv^2}{L} \qquad [\because R = L/\sqrt{3}\,] \quad \text{or} \quad v = \sqrt{\frac{GM}{L}}$$

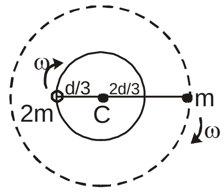
Problem 3.

In a double star, two stars (one of mass m and the other of 2m) distant d apart rotate about their common centre of mass. Deduce an expression of the period of revolution. Show that the ratio of their angular momentum about the centre of mass is the same as the ratio of their kinetic energies.

Solution :

The centre of mass C will be at distances d/3 and 2d/3 from the masses 2m and m respectively. Both the stars rotate round C in their respective orbits with the same angular velocity ω. The gravitational force acting on each star due to the other supplies the necessary centripetal force.

The gravitational force on either star is $\frac{G(2m)m}{d^2}$. If we consider the rotation of the smaller star, the centripetal force $(m r \omega^2)$ is $\left[m\left(\frac{2d}{3}\right)\omega^2\right]$ and for bigger star $\left[\frac{2md\omega^2}{3}\right]$ i.e. same



$$\therefore \quad \frac{G(2m)m}{d^2} = m\left(\frac{2d}{3}\right)\omega^2 \qquad \text{or} \qquad \omega = \sqrt{\left(\frac{3Gm}{d^3}\right)}$$

Therefore, the period of revolution is given by $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\left(\frac{d^3}{3Gm}\right)}$

The ratio of the angular momentum is $\frac{(I\omega)_{big}}{(I\omega)_{small}} = \frac{I_{big}}{I_{small}} = \frac{(2m)\left(\frac{d}{3}\right)^2}{m\left(\frac{2d}{3}\right)^2} = \frac{1}{2}$,

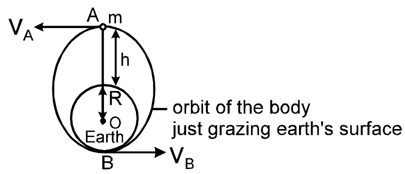
since ω is same for both. The ratio of their kinetic energies is $\frac{(\frac{1}{2}I\omega^2)_{big}}{(\frac{1}{2}I\omega^2)_{small}} = \frac{I_{big}}{I_{small}} = \frac{1}{2}$,

which is the same as the ratio of their angular momentum.

Problem 4.

For a particle projected in a transverse direction from a height h above Earth's surface, find the minimum initial velocity so that it just grazes the surface of earth path of this particle would be an ellipse with center of earth as the farther focus, point of projection as the apogee and a diametrically opposite point on earth's surface as perigee.

Sol. Suppose velocity of projection at point A is v_A & at point B, the velocity of the particle is v_B.
then applying Newton's 2nd law at point A & B, we get,



$$\frac{mv_A^2}{r_A} = \frac{GM_e m}{(R+n)^2} \quad \& \quad \frac{mv_B^2}{r_B} = \frac{GM_e m}{R^2}$$

Where r_A & r_B are radius of curvature of the orbit at points A & B of the ellipse,
but $r_A = r_B = r$(say).
Now applying conservation of energy at points A & B

$$\frac{-GM_e m}{R+h} + \frac{1}{2}mv_A^2 = \frac{-GM_e m}{R} + \frac{1}{2}mv_B{}^2$$

$$\Rightarrow \quad GM_e m\left(\frac{1}{R} - \frac{1}{(R+h)}\right) = \frac{1}{2}(mv_B{}^2 - mv_A{}^2) = \left(\frac{1}{2}\rho GM_e m\left(\frac{1}{R^2} - \frac{1}{(R+h)^2}\right)\right)$$

$$\text{or,} \quad r = \frac{2R(R+h)}{2R+h} = \frac{2Rr}{R+r} \quad \therefore \quad V_A{}^2 = \frac{rGM_e}{(R+h)^2} = 2GM_e\frac{R}{r(r+R)}$$

where r = distance of point of projection from earth's centre = R + h.

Problem 5.

A rocket starts vertically upward with speed v_0. Shown that its speed v at height h is given by

$$v_0^2 - v^2 = \frac{2hg}{1+\frac{h}{R}},$$

where R is the radius of the earth and g is acceleration due to gravity at earth's surface. Hence deduce an expression for maximum height reached by a rocket fired with speed 0.9 times the escape velocity.

Sol. The gravitational potential energy of a mass m on earth's surface and that a height h is given by

$$U(R) = -\frac{GMm}{R} \text{ and } U(R+h) = -\frac{GMm}{R+h}$$

$$\therefore \quad U(R+h) - U(R) = -GMm\left(\frac{1}{R+h} - \frac{1}{R}\right) = \frac{GMmh}{(R+h)R} = \frac{mhg}{1+\frac{h}{R}} \quad [\because \ GM = gR^2]$$

This increase in potential energy occurs at the cost of kinetic energy which correspondingly decreases. If v is the velocity of the rocket at height h, then the decrease in kinetic energy is $\frac{1}{2}mv_0^2 - \frac{1}{2}mv^2$.

Thus, $\frac{1}{2}mv_0^2 - \frac{1}{2}mv^2 = \frac{mhg}{1+\frac{h}{R}}$, or $v_0^2 - v^2 = \frac{2gh}{1+\frac{h}{R}}$

Let h_{max} be the maximum height reached by the rocket, at which its velocity has been reduced to zero. Thus, substituting v = 0 and h = h_{max} in the last expression, we have

$$v_0^2 = \frac{2gh_{max}}{1+\frac{h_{max}}{R}} \quad \text{or} \quad v_0^2\left(1+\frac{h_{max}}{R}\right) = 2\,gh_{max}$$

or $v_0^2 = h_{max}\left(2g - \frac{v_0^2}{R}\right)$ or $h_{max} = \frac{v_0^2}{2g - \frac{v_0^2}{R}}$

Now, it is given that $v_0 = 0.9 \times$ escape velocity $= 0.9 \times \sqrt{(2gR)}$

$$\therefore h_{max} = \frac{(09 \times 0.9)2gR}{2g - \frac{(09 \times 0.9)2gR}{R}}$$

$$= \frac{1.62gR}{2g - 1.62R} = \frac{1.62R}{0.38} = 4.26\ R$$

Exercise # 1

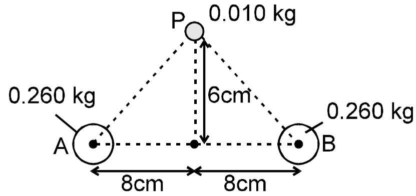
PART - I : SUBJECTIVE QUESTIONS

SECTION (A) : UNIVERSAL LAW OF GRAVITATION

A 1. The typical adult human brain has a mass of about 1.4 kg. What force does a full moon exert on such a brain when it is directly above with its centre 378000 km away ? (Mass of the moon = 7.34×10^{22} kg)

A 2. Two uniform solid spheres of same material and same radius 'r' are touching each other. If the density is 'ρ' then find out gravitational force between them.

A 3. Two uniform spheres, each of mass 0.260 kg are fixed at points 'A' and 'B' as shown in the figure. Find the magnitude and direction of the initial acceleration of a sphere with mass 0.010 kg if it is released from rest at point 'P' and acted only by forces of gravitational attraction of sphere at 'A' and 'B'(give your answer in terms of G)



SECTION (B) : GRAVITATIONAL FIELD AND POTENTIAL

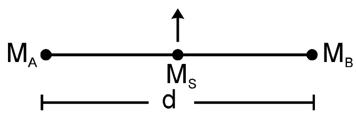
B 1. The gravitational potential in a region is given by V = (20x + 40y) J/kg. Find out the gravitational field (in newton / kg) at a point having co-ordinates (2, 4). Also find out the magnitude of the gravitational force on a particle of 0.250 kg placed at the point (2, 4).

B 2. Radius of the earth is 6.4×10^6 m and the mean density is 5.5×10^3 kg/m^3. Find out the gravitational potential at the earth's surface.

SECTION (C) : GRAVITATIONAL POTENTIAL ENERGY AND SELF ENERGY

C 1. A body which is initially at rest at a height R above the surface of the earth of radius R, falls freely towards the earth. Find out its velocity on reaching the surface of earth. Take g = acceleration due to gravity on the surface of the Earth.

C 2. Two planets A and B are fixed at a distance d from each other as shown in the figure. If the mass of A is M_A and that of B is M_B, then find out the minimum velocity of a satellite of mass M_S projected from the mid point of two planets to infinity.



SECTION (D) : KEPLER'S LAW FOR SATELLITES, ORBITAL SPEED AND ESCAPE SPEED

D 1. If the radius of earth is R and height of a geostationary satellite above earth's surface is h then find the minimum co-latitude which can directly receive a signal from geostationary satellite.

D 2. A satellite is established in a circular orbit of radius r and another in a circular orbit of radius 1.01 r. How much nearly percentage the time period of second-satellite will be larger than the first satellite .

D 3. Two identical stars of mass M, orbit around their centre of mass. Each orbit is circular and has radius R, so that the two stars are always on opposite sides on a diameter.

(a) Find the gravitational force of one star on the other.

(b) Find the orbital speed of each star and the period of the orbit.

(c) Find their common angular speed.

(d) Find the minimum energy that would be required to separate the two stars to infinity.

(e) If a meteorite passes through this centre of mass perpendicular to the orbital plane of the stars. What value must its speed exceed at that point if it escapes to infinity from the star system.

D 4. Two earth satellites A and B each of equal mass are to be launched into circular orbits about earth's centre. Satellite 'A' is to orbit at an altiude of 6400 km and B at 19200 km. The radius of the earth is 6400 km. Determine-

(a) the ratio of the potential energy

(b) the ratio of kinetic energy

(c) which one has the greater total energy

D 5. The Saturn is about six times farther from the Sun than The Mars. Which planet has :
(a) the greater period of revolution ?
(b) the greater orbital speed and
(c) the greater angular speed ?

SECTION (E) : THE EARTH AND OTHER PLANETS GRAVITY

E 1. The acceleration due to gravity at a height (1/20)th the radius of the earth above earth's surface is $9\ m/s^2$. Find out its approximate value at a point at an equal distance below the surface of the earth.

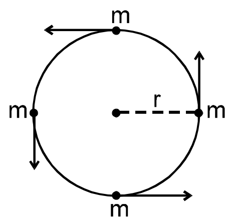
E 2. If a pendulum has a period of exactly 1.00 sec. at the equator, what would be its period at the south pole ? Assume the earth to be spherical and rotational effect of the Earth is to be taken.

PART - II : OBJECTIVE QUESTIONS

***Marked questions are More than one choice type**

SECTION (A) : UNIVERSAL LAW OF GRAVITATION

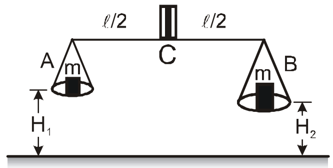
A 1. Four similar particles of mass m are orbiting in a circle of radius r in the same direction and same speed because of their mutual gravitational attractive force as shown in the figure. Speed of a particle is given by



(A) $\left[\dfrac{Gm}{r}\left(\dfrac{1+2\sqrt{2}}{4}\right)\right]^{\frac{1}{2}}$ (B) $\sqrt[3]{\dfrac{Gm}{r}}$

(C) $\sqrt{\dfrac{Gm}{r}\left(1+2\sqrt{2}\right)}$ (D) zero

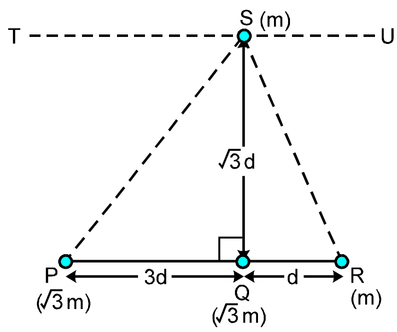
A 2. Two blocks of masses m each are hung from a balance as shown in the figure. The scale pan A is at height H_1 whereas scale pan B is at height H_2. Net torque of weights acting on the system about point 'C', will be (length of the rod is ℓ and H_1 & H_2 are << R) ($H_1 > H_2$)



(A) $mg\left(\dfrac{1-2H_1}{R}\right)\ell$ (B) $\dfrac{mg}{R}(H_1 - H_2)\,\ell$

(C) $\dfrac{2mg}{R}(H_1 + H_2)\,\ell$ (D) $2mg\dfrac{H_2 H_1}{H_1 + H_2}\,\ell$

A 3. Three particles P, Q and R are placed as per given figure. Masses of P, Q and R are $\sqrt{3}$ m, $\sqrt{3}$ m and m respectively. The gravitational force on a fourth particle 'S' of mass m is equal to



(A) $\dfrac{\sqrt{3}\,GM^2}{2d^2}$ in ST direction only

(B) $\dfrac{\sqrt{3}\,Gm^2}{2d^2}$ in SQ direction and $\dfrac{\sqrt{3}\,Gm^2}{2d^2}$ in SU direction

(C) $\dfrac{\sqrt{3}\,Gm^2}{2d^2}$ in SQ direction only

(D) $\dfrac{\sqrt{3}\,Gm^2}{2d^2}$ in SQ direction and $\dfrac{\sqrt{3}\,Gm^2}{2d^2}$ in ST direction

A-4. Three identical stars of mass M are located at the vertices of an equilateral triangle with side L. The speed at which they will move if they all revolve under the influence of one another's gravitational force in a circular orbit circumscribing the triangle while still preserving the equilateral triangle :

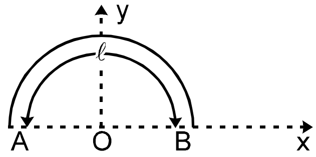
(A) $\sqrt{\dfrac{2GM}{L}}$ (B) $\sqrt{\dfrac{GM}{L}}$ (C) $2\sqrt{\dfrac{GM}{L}}$ (D) not possible at all

SECTION (B) : GRAVITATIONAL FIELD AND POTENTIAL

B-1. Let gravitation field in a space be given as E = – (k/r). If the reference point is at distance d_i where potential is V_i then relation for potential is :

(A) $V = k\,\ell n\dfrac{1}{V_i} + 0$ (B) $V = k\,\ell n\dfrac{r}{d_i} + V_i$ (C) $V = \ell n\dfrac{r}{d_i} + kV_i$ (D) $V = \ell n\dfrac{r}{d_i} + \dfrac{V_i}{k}$

B 2. Gravitational field at the centre of a semicircle formed by a thin wire AB of mass m and length ℓ as shown in the figure. is :



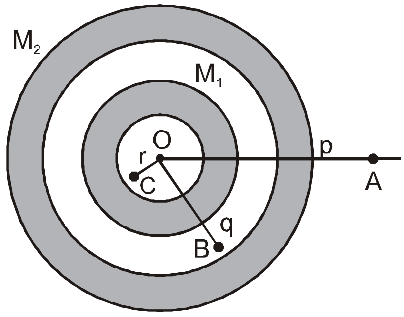
(A) $\dfrac{Gm}{\ell^2}$ along +x axis (B) $\dfrac{Gm}{\pi\ell^2}$ along +y axis

(C) $\dfrac{2\pi Gm}{\ell^2}$ along + x axis (D) $\dfrac{2\pi Gm}{\ell^2}$ along + y axis

B-3. A very large number of particles of same mass m are kept at horizontal distances of 1m, 2m, 4m, 8m and so on from (0,0) point. The total gravitational potential at this point is :

(A) – 8G m (B) – 3G m (C) – 4G m (D) – 2G m

B 4. Two concentric shells of uniform density of mass M_1 and M_2 are situated as shown in the figure. The forces experienced by a particle of mass m when placed at positions A, B and C respectively are (given OA = p, OB = q and OC = r).



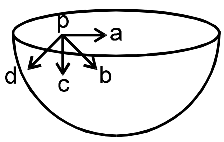
(A) zero, $G\dfrac{M_1 m}{q^2}$ and $G\dfrac{(M_1+m_2)m}{p^2}$

(B) $G\dfrac{(M_1+M_2)m}{p^2}$, $G\dfrac{(M_1+M_2)m}{q^2}$ and $G\dfrac{M_1 m}{r^2}$

(C) $G\dfrac{M_1 m}{q^2}$, $\dfrac{G(M_1+M_2)m}{p^2}$, $G\dfrac{M_1 m}{q^2}$ and zero

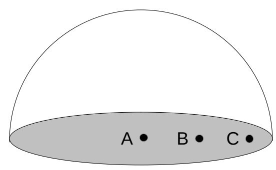
(D) $\dfrac{G(M_1+M_2)m}{p^2}$, $G\dfrac{M_1 m}{q^2}$ and zero

B-5. Figure show a hemispherical shell having uniform mass density. The direction of gravitational field intensity at point P will be along:



(A) a (B) b (C) c (D) d

B-6. Mass M is uniformly distributed only on curved surface of a thin hemispherical shell. *A*, *B* and *C* are three points on the circular base of hemisphere, such that *A* is the centre. Let the gravitational potential at points A, B and C be V_A, V_B, V_C respectively. Then



(A) $V_A > V_B > V_C$ (B) $V_C > V_B > V_A$

(C) $V_B > V_A$ and $V_B > V_C$ (D) $V_A = V_B = V_C$

SECTION (C) : GRAVITATIONAL POTENTIAL ENERGY AND SELF ENERGY

C 1. A body starts from rest at a point, distance R_0 from the centre of the earth of mass M, radius R. The velocity acquired by the body when it reaches the surface of the earth will be

(A) $GM\left(\dfrac{1}{R}-\dfrac{1}{R_0}\right)$ (B) $2\,GM\left(\dfrac{1}{R}-\dfrac{1}{R_0}\right)$ (C) $\sqrt{2GM\left(\dfrac{1}{R}-\dfrac{1}{R_0}\right)}$ (D) $2GM\sqrt{\left(\dfrac{1}{R}-\dfrac{1}{R_0}\right)}$

C 2. Three equal masses each of mass 'm' are placed at the three-corners of an equilateral triangle of side 'a'.

(a) If a fourth particle of equal mass is placed at the centre of triangle, then net force acting on it, is equal to :

(A) $\frac{Gm^2}{a^2}$ (B) $\frac{4Gm^2}{3a^2}$ (C) $\frac{3Gm^2}{a^2}$ (D) zero

(b) In above problem, if fourth particle is at the mid-point of a side, then net force acting on it, is equal to:

(A) $\frac{Gm^2}{a^2}$ (B) $\frac{4Gm^2}{3a^2}$ (C) $\frac{3Gm^2}{a^2}$ (D) zero

(c) If above given three particles system of equilateral triangle side a is to be changed to side of 2a, then work done on the system is equal to :

(A) $\frac{3Gm^2}{a}$ (B) $\frac{3Gm^2}{2a}$ (C) $\frac{4Gm^2}{3a}$ (D) $\frac{Gm^2}{a}$

(d) In the above given three particle system, if two particles are kept fixed and third particle is released. Then speed of the particle when it reaches to the mid-point of the side connecting other two masses:

(A) $\sqrt{\frac{2Gm}{a}}$ (B) $2\sqrt{\frac{Gm}{a}}$ (C) $\sqrt{\frac{Gm}{a}}$ (D) $\sqrt{\frac{Gm}{2a}}$

SECTION : (D) KEPLER'S LAW FOR SATELLITES, ORBITAL VELOCITY AND ESCAPE VELOCITY

D-1. Periodic-time of satellite revolving around the earth is - (ρ is density of earth)

(A) Proportional to $\frac{1}{\rho}$ (B) Proportional to $\frac{1}{\sqrt{\rho}}$

(C) Proportional ρ (D) does not depend on ρ.

D-2. An artificial satellite of the earth releases a package. If air resistance is neglected the point where the package will hit (with respect to the position at the time of release) will be

(A) ahead (B) exactly below

(C) behind (D) it will never reach the earth

D-3*. An orbiting satellite will escape if :

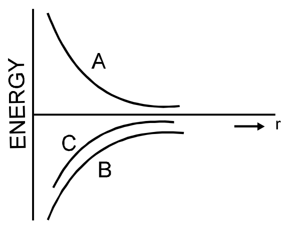
(A) its speed is increased by $(\sqrt{2}-1)100\%$

(B) its speed in the orbit is made $\sqrt{1.5}$ times of its initial value

(C) its KE is doubled

(D) it stops moving in the orbit

D-4. The figure shows the variation of energy with the orbit radius of a body in circular planetary motion. Find the correct statement about the curves A, B and C



(A) A shows the kinetic energy, B the total energy and C the potential energy of the system

(B) C shows the total energy, B the kinetic energy and A the potential energy of the system

(C) C and A are kinetic and potential energies respectively and B is the total energy of the system

(D) A and B are the kinetic and potential energies and C is the total energy of the system.

D-5*. In case of an orbiting satellite if the radius of orbit is decreased :

(A) its Kinetic Energy decreases

(B) its Potential Energy decreases

(C) its Mechanical Energy decreases

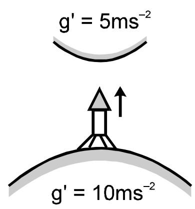
(D) its speed decreases

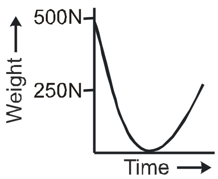
D-6. A planet of mass m revolves around the sun of mass M in an elliptical orbit. The minimum and maximum distance of the planet from the sun are r_1 & r_2 respectively. If the minimum velocity of the planet is $\sqrt{\dfrac{2GMr_1}{(r_1+r_2)r_2}}$ then it's maximum velocity will be :

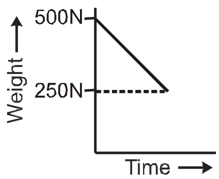
(A) $\sqrt{\dfrac{2GMr_2}{(r_1+r_2)r_1}}$ (B) $g\sqrt{\dfrac{2GMr_1}{(r_1+r_2)r_2}}$ (C) $\sqrt{\dfrac{2Gmr_2}{(r_1+r_2)r_1}}$ (D) $\sqrt{\dfrac{2GM}{r_1+r_2}}$

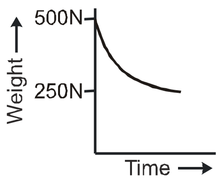
SECTION (E) : EARTH AND OTHER PLANETS GRAVITY

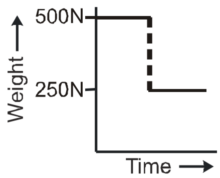
E 1. If acceleration due to gravity on the surface of earth is 10 ms^{-2} then let acceleration due to gravitational acceleration at surface of another planet of our solar system be 5 ms^{-2}. An astronaut weighing 50 kg on earth goes to this planet in a spaceship with a constant velocity. The weight of the astronaut with time of flight is roughly given by



(A) 

(B) 

(C) 

(D) 

E 2*. In case of earth :
(A) gravitational field is zero, both at centre and infinity
(B) gravitational potential is zero, both at centre and infinity
(C) gravitational potential is same, both at centre and infinity but not zero
(D) gravitational potential is minimum at the centre

Exercise # 2

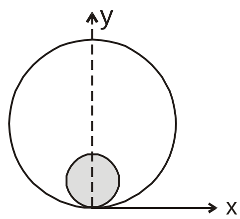
PART - I : SUBJECTIVE QUESTIONS

1. The gravitational field in a region is given by $\vec{E} = (3\hat{i} - 4\hat{j})$ N/kg. Find out the work done (in joule) in displacing a particle of mass 1 kg by 1 m along the line 4y = 3x + 9.

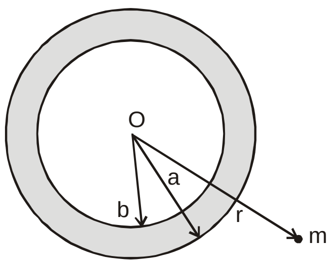
2. A solid sphere of mass m and radius r is placed inside a hollow spherical shell of mass 4 m and radius 4r find gravitational field intensity at :

(a) r < y < 2r (b) 2r < y < 8r (c) y > 8r

here y coordinate is measured from the point of contact of the sphere and the shell.



3. A sphere of density ρ and radius a has a concentric cavity of radius b, as shown in the figure.



(a) Sketch the gravitational force F exerted by the sphere on the particle of mass m, located at a distance r from the centre of the sphere as a function of r in the range $0 \le r \le \infty$.
(b) Sketch the corresponding curve for the potential energy u (r) of the system.

4. Two stars of mass M_1 & M_2 are in circular orbits around their centre of mass. The star of mass M_1 has an orbit of radius R_1, the star of mass M_2 has an orbit of radius R_2. (assume that their centre of mass is not accelerating and distance between stars is fixed)

(a) Show that the ratio of the orbital radii of the two stars equals the reciprocal of the ratio of their masses, that is $R_1/R_2 = M_2/M_1$.

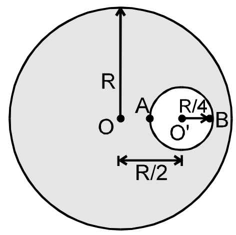
$R_1/R_2 = M_2/M_1$

(b) Explain why the two stars have the same orbital period and show that the period,

$$T = 2\pi \frac{(R_1+R_2)^{3/2}}{\sqrt{G(M_1+M_2)}}.$$

(c) The two stars in a certain binary star system move in circular orbits. The first star, α moves in an orbit of radius 1.00×10^9 km. The other star, β moves in an orbit of radius 5.00×10^8 km. The orbital period is 44.5 year. What are the masses of each of the two stars ?

5. In a solid sphere of radius 'R' and density 'ρ' there is a spherical cavity of radius R/4 as shown in figure. A particle of mass 'm' is released from rest from point 'B' (inside the cavity). Find out -



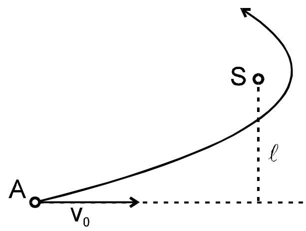
(a) The position where this particle strikes the cavity.

(b) Velocity of the particle at this instant.

6. **(a)** What is the escape speed for an object in the same orbit as that of Earth around sun (Take orbital radius R) but far from the earth ? (mass of the sun = M_s)

(b) If an object already has a speed equal to the earth's orbital speed, what minimum additional speed must it be given to escape as in (a) ?

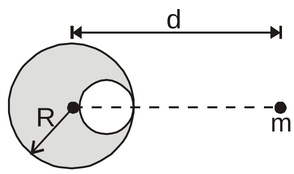
7. A cosmic body A moves towards the Sun with velocity v_0 (when far from the Sun) and aiming parameter ℓ, the direction of the vector v_0 relative to the centre of the Sun as shown in the figure. Find the minimum distance by which this body will get to the Sun. (Mass of Sun = M_S)



PART - II : OBJECTIVE QUESTIONS

Single Choice type :

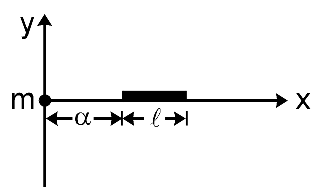
1. A spherical hollow cavity is made in a lead sphere of radius R, such that its surface touches the outside surface of the lead sphere and passes through its centre. The mass of the sphere before hollowing was M. With what gravitational force will the hollowed-out lead sphere attract a small sphere of mass 'm', which lies at a distance d from the centre of the lead sphere on the straight line connecting the centres of the spheres and that of the hollow, if d = 2R :



(A) $\frac{7GMm}{18R^2}$ (B) $\frac{7GMm}{36R^2}$

(C) $\frac{7GMm}{9R^2}$ (D) $\frac{7GMm}{72R^2}$

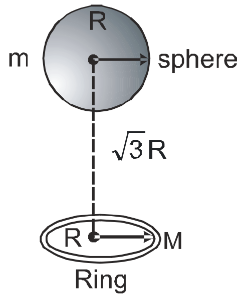
2. A straight rod of length ℓ extends from x = α to x = $\ell + \alpha$. as shown in the figure. If the mass per unit length is (a + bx²). The gravitational force it exerts on a point mass m placed at x = 0 is given by



(A) $Gm\left(a\left(\frac{1}{\alpha}-\frac{1}{\alpha+\ell}\right)+b\ell\right)$ (B) $\frac{Gm(a+bx^2)}{\ell^2}$

(C) $Gm\left(\alpha\left(\frac{1}{a}-\frac{1}{a+\ell}\right)+b\ell\right)$ (D) $Gm\left(a\left(\frac{1}{\alpha+\ell}-\frac{1}{\alpha}\right)+b\ell\right)$

3. A uniform ring of mass M is lying at a distance $\sqrt{3}$ R from the centre of a uniform sphere of mass m just below the sphere as shown in the figure where R is the radius of the ring as well as that of the sphere. Then gravitational force exerted by the ring on the sphere is :



(A) $\frac{GMm}{8R^2}$ (B) $\frac{GMm}{3R^2}$

(C) $\sqrt{3}\frac{GMm}{R^2}$ (D) $\sqrt{3}\frac{GMm}{8R^2}$

4. The gravitational potential of two homogeneous spherical shells A and B (separated by large distance) of same surface mass density at their respective centres are in the ratio 3 : 4. If the two shells coalesce into single one such that surface mass density remains same, then the ratio of potential at an internal point of the new shell to shell A is equal to :

(A) 3 : 2 (B) 4 : 3 (C) 5 : 3 (D) 3 : 5

5. A projectile is fired from the surface of earth of radius R with a speed $k\nu_e$ in radially outward direction (where ν_e is the escape velocity and k < 1). Neglecting air resistance, the maximum hight from centre of earth is

(A) $\frac{R}{k^2+1}$ (B) k^2 R (C) $\frac{R}{1-k^2}$ (D) kR

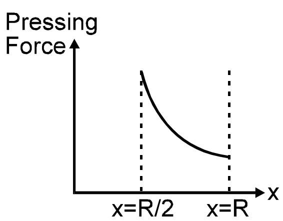
More than one choice type

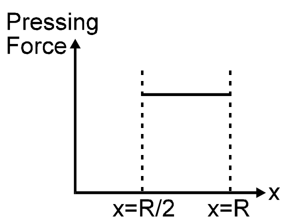
6. For a satellite to appear stationary to an observer on earth
(A) It must be rotating about the earth's axis.
(B) It must be rotating in the equatorial plane.
(C) Its angular velocity must be from west to east.
(D) Its time period must be 24 hours.

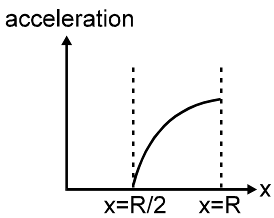
7. Inside an isolated uniform spherical shell :
(A) The gravitation potential is not zero
(B) The gravitational field is not zero
(C) The gravitational potential is same everywhere
(D) The gravitational field is same everywhere.

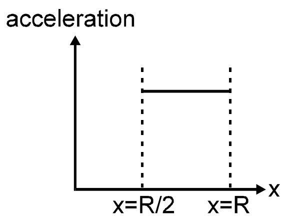
8. Which of the following statements are correct about a planet rotating around the sun in an elliptic orbit:
(A) its mechanical energy is constant
(B) its angular momentum about the sun is constant
(C) its areal velocity about the sun is constant
(D) its time period is proportional to r^3

9. A tunnel is dug along a chord of the earth at a perpendicular distance R/2 from the earth's centre. The wall of the tunnel may be assumed to be frictionless. A particle is released from one end of the tunnel. The pressing force by the particle on the wall and the acceleration of the particle varies with x (distance of the particle from the centre) according to :

(A)


(B)


(C)


(D)


10. A planet revolving around sun in an elliptical orbit has a constant

(A) kinetic energy (B) angular momentum bout the sun

(C) potential energy (D) Total energy

11. A satellite close to the earth is in orbit above the equator with a period of revolution of 1.5 hours. If it is above a point P on the equator at some time, it will be above P again after time

(A) 1.5 hours

(B) 1.6 hours if it is rotating from west to east

(C) 24/17 hours if it is rotating from east to west

(D) 24/17 hours if it is rotating from west to east

Exercise # 3

PART - I : MATCH THE COLUMN

1. A particle is taken to a distance r (> R) from centre of the earth. R is radius of the earth. It is given velocity V which is perpendicular to $\vec{r}$. With the given values of V in column I you have to match the values of total energy of particle in column II and the resultant path of particle in column III. Here 'G' is the universal gravitational constant and 'M' is the mass of the earth.

Column I (Velocity)	Column II (Total energy)	Column III (Path)
(A) $V = \sqrt{GM/r}$	(p) Negative	(t) Elliptical
(B) $V = \sqrt{2GM/r}$	(q) Positive	(u) Parabolic
(C) $V > \sqrt{2GM/r}$	(r) Zero	(v) Hyperbolic
(D) $\sqrt{GM/r} < V < \sqrt{2GM/r}$	(s) Infinite	(w) Circular

2. Let V and E denote the gravitational potential and gravitational field respectively at a point due to certain uniform mass distribution described in four different situations of column-I. Assume the gravitational potential at infinity to be zero.The value of E and V are given in column-II. Match the statement in column-I with results in column-II.

Column-I	Column-II
(A) At centre of thin spherical shell	(p) $E = 0$
(B) At centre of solid sphere	(q) $E \neq 0$
(C)A solid sphere has a non-concentric spherical cavity. At the centre of the spherical cavity	(r) $V \neq 0$
(D) At centre of line joining two point masses of equal magnitude	(s) $V = 0$

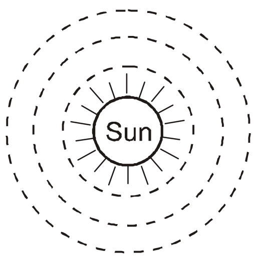
PART - II : COMPREHENSION

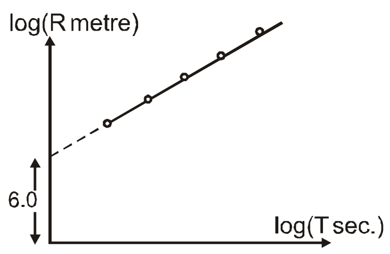
Comprehensiion # 1

Many planets are revolving around the fixed sun, in circular orbits of different radius (R) and different time period (T). To estimate the mass of the sun, the orbital radius (R) and time period (T) of planets were noted. Then $\log_{10}$ T v/s $\log_{10}$ R curve was plotted.

The curve was found to be approximately straight line (as shown in figure) having y intercept = 6.0

(Neglect the gravitational interaction among the planets [Take G = $\frac{20}{3} \times 10^{-11}$ in MKS, $\pi^2 = 10$]





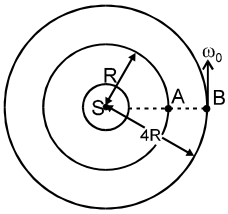
3. The slope of the line should be :

(A) 1 (B) $\frac{3}{2}$ (C) $\frac{2}{3}$ (D) $\frac{19}{4}$

4. Estimate the mass of the sun :

(A) 6×10^{29} kg (B) 5×10^{20} kg (C) 8×10^{25} kg (D) 3×10^{35} kg

5. Two planets A and B, having orbital radius R and 4R are initially at the closest position and rotating in the same direction. If angular velocity of planet B is ω_0, then after how much time will both the planets be again in the closest position?(Neglect the interaction between planets).



(A) $\frac{2\pi}{7\omega_0}$ (B) $\frac{2\pi}{9\omega_0}$ (C) $\frac{2\pi}{\omega_0}$ (D) $\frac{2\pi}{5\omega_0}$

COMPREHENSION - 2

An artificial satellite is moving in a circular orbit around the earth with a speed equal to half the magnitude of escape velocity from the surface of earth. R is the radius of earth and g is acceleration due to gravity at the surface of earth. (R = 6400 km)

6. Then the distance of satellite from the surface of earth is

(A) 3200 km (B) 6400 km (C) 12800 km (D) 4800 km

7. The time period of revolution of satellite in the given orbit is

(A) $2\pi\sqrt{\frac{2R}{g}}$ (B) $2\pi\sqrt{\frac{4R}{g}}$ (C) $2\pi\sqrt{\frac{8R}{g}}$ (D) $2\pi\sqrt{\frac{6R}{g}}$

8. If the satellite is stopped suddenly in its orbit and allowed to fall freely onto the earth, the speed with which it hits the surface of the earth.

(A) $\sqrt{gR}$ (B) $\sqrt{1.5gR}$ (C) $\sqrt{\frac{gR}{2}}$ (D) $\sqrt{\frac{gR}{\sqrt{2}}}$

COMPREHENSION # 3

A pair of stars rotates about their center of mass. One of the stars has a mass M and the other has mass m such that M = 2m. The distance between the centres of the stars is d (d being large compared to the size of either star).

9. The period of rotation of the stars about their common centre of mass (in terms of d, m, G.) is

(A) $\sqrt{\frac{4\pi^2}{Gm}d^3}$ (B) $\sqrt{\frac{8\pi^2}{Gm}d^3}$ (C) $\sqrt{\frac{2\pi^2}{3Gm}d^3}$ (D) $\sqrt{\frac{4\pi^2}{3Gm}d^3}$

10. The ratio of the angular momentum of the two stars about their common centre of mass (L_m/L_M) is

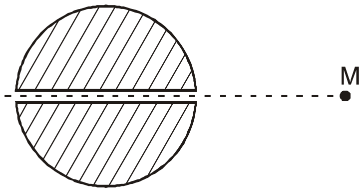
(A) 1 (B) 2 (C) 4 (D) 9

11. The ratio of kinetic energies of the two stars (K_m/K_M.) is

(A) 1 (B) 2 (C) 4 (D) 9

PART - III : ASSERTION / REASON

12. **STATEMENT-1 :** In free space a uniform spherical planet of mass M has a smooth narrow tunnel along its diameter as shown in the figure. This planet and another superdense small particle of mass M start approaching towards each other from rest under action of their gravitational forces . When the particle passes through the centre of the planet, sum of kinetic energies of both the bodies is maximum.



STATEMENT-2 : When the resultant of all forces acting on a particle or a particle like object (initially at rest) is constant in direction, the kinetic energy of the particle keeps on increasing.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

PART - IV : TRUE / FALSE

13._ (i) The gravitation potential due to a uniform solid sphere is maximum at its centre.

(ii) A communication satellite must have orbit in the equatorial plane of the earth and the angular velocity and direction of rotation must be the same as those of the rotating earth.

(iii) Select True/False
For an elliptical path of a planet around the sun, the total energy of the system is positive. False

PART - V : FILL IN THE BLANKS

14. **Fill in the blanks :**

(i) The numerical value of the angular velocity of rotation of the earth about its own axis should be..............rad/s in order to make the apparent acceleration due to gravity at equator equal to zero. **(1984 ; 2M)**

(ii) According to Kepler's second law, the radius vector to a planet from the sun sweeps out equal areas in equal intervals of time.This law is a consequence of the conservation of................ **.**

(iii) A geostationary satellite is orbiting the earth at a height of 6 R above the surface of the earth where R is the radius of earth. The time period of another satellite at a height of 2.5 R from the surface of the earth is ....hours. **(1987 ; 2M)**

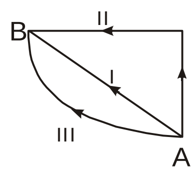
Exercise # 4

PART - I : IIT–JEE PROBLEMS (LAST 10 YEARS)

1. A geostationary satellite orbits around the earth in a circular orbit of radius 36000 km. Then, the time period of a spy satellite orbiting a few hundred kilometers above the earth's surface (R_{Earth} = 6400 km) will approximately be : **[JEE(Scr) - 2002, 3/84]**

(A) 1/2 hr (B) 1 hr (C) 2 hr (D) 4 hr

2. A particle of mass m is taken through the gravitational field produced by a source S, from A to B, along the three paths as shown in figure. If the work done along the paths I, II and III is W_I, W_{II} and W_{III} respectively, then : **[JEE (Scr.) - 2003, 3/84]**



(A) $W_I = W_{II} = W_{III}$ (B) $W_{II} > W_{III} = W_{II}$
(C) $W_{III} = W_{II} > W_I$ (D) $W_I > W_{II} > W_{III}$

3. A projectile is fired vertically up from the bottom of a crater (big hole) on the moon. The depth of the crater is R/100, where R is the radius of the moon. If the initial velocity of the projectile is the same as the escape velocity from the moon surface, determine in terms of R, the maximum aproximate height attained by the projectile above the lunar (moon) surface. **[JEE 2003(Main), 4/60]**

4. A double star system consists of two stars A and B which have time period T_A and T_B. Radius R_A and R_B and mass M_A and M_B. Choose the correct option. **[JEE 2006, +3, –1/184]**

(A) If $T_A > T_B$ then $R_A > R_B$ (B) If $T_A > T_B$ then $M_A > M_B$

(C) $\left(\frac{T_A}{T_B}\right)^2 = \left(\frac{R_A}{R_B}\right)^3$ (D) $T_A = T_B$

5. **Column I** describes some situations in which a small object moves. **Column II** describes some characteristics of these motions. Match the situations in **Column I** with the characteristics in **Column II**

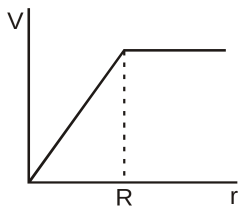
[IIT-JEE 2007, 6/162]

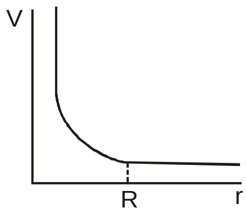
Column I		**Column II**	
(A)	The object moves on the x–axis under a conservative force in such a way that its "speed" and "position" satisfy $v = c_1\sqrt{c_2 - x^2}$, where c_1 and c_2 are positive constants.	(p)	The object executes a simple harmonic motion.
(B)	The object moves on the x–axis in such a way that its velocity and its displacement from the origin satisfy $v = -kx$, where k is a positive constant.	(q)	The object does not change its direction,
(C)	The object is attached to one end of a massless spring of a given spring constant. The other end of the spring is attached to the ceiling of an elevator. Initially everything is at rest. The elevator starts going upwards with a constant acceleration a. The motion of the object is observed from the elevator during the period it maintains this acceleration.	(r)	The kinetic energy of the object keeps on decreasing.
(D)	The object is projected from the earth's surface vertically upwards with a speed $2\sqrt{GM_e/R_e}$, where M_e is the mass of the earth and R_e is the radius of the earth. Neglect forces from objects other than the earth.	(s)	The object can change its direction only once.

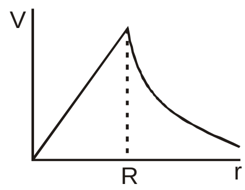
6. A spherically symmetric gravitational system of particles has a mass density **[JEE 2008, +3, –1/82]**

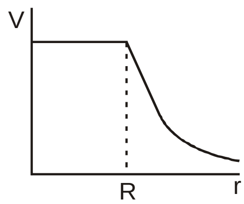
$$\rho = \begin{cases} \rho_0 & \text{for} \quad r \le R \\ 0 & \text{for} \quad r > R \end{cases}$$

where ρ_0 is a constant. A test mass can undergo circular motion under the influence of the gravitational field of particles. Its speed V as a function of distance r ($0 < r < \infty$) from the centre of the system is represented by

(A) 

(B) 

(C) 

(D) 

7. **STATEMENT -1** **[JEE 2008,+3, –1/82]**

An astronaut in an orbiting space station above the Earth experiences weightlessness.

and

STATEMENT -2

An object moving around the Earth under the influence of Earth's gravitational force is in a state of 'free-fall.

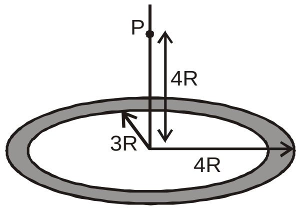
(A) STATEMENT -1 is True, STATEMENT -2 is True; STATEMENT -2 is a correct explanation for STATEMENT -1

(B) STATEMENT -1 is True, STATEMENT -2 is True; STATEMENT -2 is NOT a correct explanation for STATEMENT -1

(C) STATEMENT -1 is True, STATEMENT -2 is False

(D) STATEMENT -1 is False, STATEMENT -2 is True.

8. A thin uniform annular disc (see figure) of mass M has outer radius 4R and inner radius 3R. The work required to take a unit mass from point P on its axis to infinity is : **[JEE 2010, 3/163, –1]**



(A) $\frac{2GM}{7R}\left(4\sqrt{2}-5\right)$ (B) $-\frac{2GM}{7R}\left(4\sqrt{2}-5\right)$

(C) $\frac{GM}{4R}$ (D) $\frac{2GM}{5R}\left(\sqrt{2}-1\right)$

9. A binary star consists of two stars A (mass 2.2 M_S) and B (mass 11 M_S) where M_s is the mass of the sun. They are separated by distance d and are rotating about their centre of mass, which is stationary. The ratio of the total angular momentum of the binary star to the angular momentum of star B about the centre of mass is : **[JEE 2010, 3/163]**

10. Gravitational acceleration on the surface of a planet is $\frac{\sqrt{6}}{11}g$, where g is the gravitational acceleration on the surface of the earth. The average mass density of the planet is $\frac{2}{3}$ times that of the earth. If the escape speed on the surface of the earth is taken to be 11 kms^{-1}, the escape speed on the surface of the planet in kms^{-1} will be : **[JEE 2010, 3/163]**

11. A satellite is moving with a constant speed 'V' in a circular orbit about the earth. An object of mass *'m'* is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of its ejection, the kinetic energy of the object is **[JEE 2011, 3/160, –1]**

(A) $\frac{1}{2}mV^2$ (B) mV^2 (C) $\frac{3}{2}mV^2$ (D) $2mV^2$

12. Two spherical planets P and Q have the same unfirom density ρ, masses M_P and M_Q, an surface areas A and 4A, respectively. A spherical planet R also has unfirom density ρ and its mass is $(M_P + M_Q)$. The escape velocities from the planets P, Q and R, are V_P, V_Q and V respectivley. Then: **[IIT-JEE-2012, Paper-2; 4/66]**

(A) $V_Q > V_R > V_P$ (B) $V_R > V_Q > V_P$ (C) $V_R/V_P = 3$ (D) $V_P/V_Q = \frac{1}{2}$

PART - II : AIEEE PROBLEMS (PREVIOUS YEARS)

1. If g_E and g_m are the accelerations due to gravity on the surfaces of the earth and the moon respectively and if Millikan's oil drop expriment could be performed on the two surfaces, one will find the ratio

$\frac{\text{Electronic charge on the moon}}{\text{electronic charge on the earth}}$ to be **[AIEEE-2007, 3/120]**

(1) 1 (2) 0 (3) g_E/g_M (4) g_M/g_E

2. A planet in a distant solar system is 10 times more massive than the earth and its radius is 10 times smaller. Given that the escape velocity from the earth is 11 km s^{-1}, the escape velocity from the surface of the planet would be **[AIEEE-2008, 3/105]**

(1) 11 km s^{-1} (2) 110 km s^{-1} (3) 0.11 km s^{-1} (4) 1.1 km s^{-1}

3. The height at which the acceleration due to gravity becomes $\frac{g}{9}$ (where g = the acceleration due to gravity on the surface of the earth) in terms of R, the radius of the earth, is **[AIEEE-2009, 4/144]**

(1) $\frac{R}{\sqrt{2}}$ (2) $\frac{R}{2}$ (3) $\sqrt{2}$ R (4) 2R

4. Two bodies of masses m and 4 m are placed at a distance r. The gravitational potential at a point on the line joining them where the gravitational field is zero is : **[AIEEE - 2011, 4/120, –1]**

(1) zero (2) $-\frac{4Gm}{r}$ (3) $-\frac{6Gm}{r}$ (4) $-\frac{9Gm}{r}$

5. Two particles of equal mass 'm' go around a circle of radius R under the action of their mutual gravitational attraction. The speed of each partial with respect to their centre of mass is :**[AIEEE 2011, 11 May; 4/120 -1]**

(1) $\sqrt{\frac{Gm}{4R}}$ (2) $\sqrt{\frac{Gm}{3R}}$ (3) $\sqrt{\frac{Gm}{2R}}$ (4) $\sqrt{\frac{Gm}{R}}$

6. The mass of a spaceship is 1000 kg. It is to be launched from the earth's surface out into free space. The value of 'g' and 'R' (radius of earth) are 10 m/s^2 and 6400 km respectively. The required energy for this work will be : **[AIEEE 2012 ; 4/120, –1]**

(1) 6.4×10^{11} Joules (2) 6.4×10^{8} Joules

(3) 6.4×10^{9} Joules (4) 6.4×10^{10} Joules

Answers

EXERCISE - 1
PART - I

SECTION (A)

A 1. $F = G\dfrac{m_1m_2}{r^2} = 6.67 \times 10^{-11} \times \dfrac{1.4\times7.34\times10^{22}}{(378\times10^6)^2}$

$= 4.8 \times 10^{-5}$ N

A 2. $\dfrac{4}{9}\pi^2\rho^2Gr^4$

A 3. 31.2 G m/sec^2 = 2.1×10^{-9} m/s^2, down

SECTION (B) :

B 1. $-20\hat{i} - 40\hat{j}$, $|\vec{F}| = 5\sqrt{5}$ N, $\vec{F} = -5\hat{i} - 10\hat{j}$

B 2. $-\dfrac{4}{3}\pi G$ $5.5 \times 10^3 \times (6.4 \times 10^6)^2$ J/Kg = 6.3×10^7 J/Kg

SECTION (C) :

C 1. $\sqrt{gR}$ **C 2.** $2\sqrt{\dfrac{G(M_A + M_B)}{d}}$

SECTION (D) :

D 1. $\sin^{-1}\left(\dfrac{R}{R+h}\right)$ **D 2.** 1.5%

D 3. **(a)** $F = \dfrac{GM^2}{4R^2}$ **(b)** $\sqrt{\dfrac{GM}{4R}}$; $T = 4\pi\sqrt{\dfrac{R^3}{GM}}$

(c) $\sqrt{\dfrac{GM}{4R^3}}$ **(d)** $\dfrac{GM^2}{4R}$ **(e)** $\sqrt{\dfrac{4GM}{R}}$

D 4. (a) $\dfrac{U_A}{U_B} = \dfrac{25600}{12800} = 2$ (b) $\dfrac{K_A}{K_B} = \dfrac{m_A}{m_B}\,\dfrac{r_B}{r_A} = 2$

(c) B is having more energy. B

D 5. **(a)** The Saturn **(b)** The Mars **(c)** The Mars

SECTION (E) :

E 1. $\dfrac{19}{2}$ m/s^2 = 9.5 m/s^2

E 2. $T = 1 - \dfrac{1}{2}\dfrac{4\pi^2}{(86400)^2} \times 6400 \times \dfrac{10^3}{9.8}$ = 0.998 s

PART - II

SECTION (A) :

A 1. (A) **A 2.** (B) **A 3.** (C) **A-4.** (B)

SECTION (B) :

B-1. (B) **B 2.** (D) **B-3.** (D) **B 4.** (D)
B-5. (C) **B-6.** (D)

SECTION (C) :

C 1. (C) **C 2.** **(a)** (D) **(b)** (B) **(c)** (B) **(d)** (B)

SECTION (D) :

D-1. (B) **D-2.** (D) **D-3.** (A)(C)
D-4. (D) **D-5.** (B)(C) **D-6.** (A)

SECTION (E) :

E 1. (A) **E 2.** (A)(D)

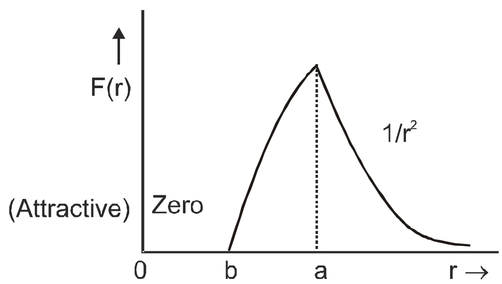
EXERCISE - 2
PART - I

1. zero

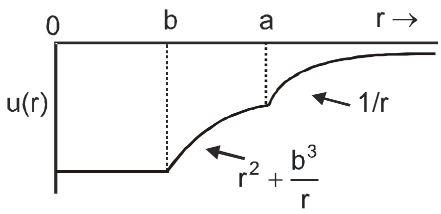
2. (a) $\left(\dfrac{Gm(y-r)}{r^3}(-\hat{j})\right)$ (b) $\left(\dfrac{Gm}{(y-r)^2}(-\hat{j})\right)$

(c) $\left(\dfrac{4Gm}{(y-4r)^2} + \dfrac{Gm}{(y-r)^2}\right)(-\hat{j})$

3. **(a)**



(b)



4. $M_\alpha = \dfrac{4\pi^2[1.5\times10^{12}]^3}{3G[44.5\times365\times86400]^2} = 3.376 \times 10^{29}$ kg,

$M_\beta = 2M_\alpha = 6.75 \times 10^{29}$ kg

5. (a) Since force is always acting towards centre of solid sphere. Hence it will strike at 'A'.

(b) $v = \sqrt{\dfrac{2\pi G\rho R^2}{3}}$

6. **(a)** $\sqrt{\dfrac{2GM_S}{R}}$ **(b)** $\left(\sqrt{2}-1\right)\sqrt{\dfrac{GM_S}{R}}$

7. $r_{min} = (GM_S / v_0^2)\ [\sqrt{1+(\ell v_0^2/GM_S)^2} - 1]$

PART - II

1.	(B)	**2.**	(A)	**3.**	(D)	**4.**	(C)
5.	(A)	**6.**	(A)(B)(C)(D)			**7.**	(A) (C) (D)
8.	(A) (B) (C)	**9.**	(B)(C)			**10.**	(B)(D)
11.	(B)(C)						

EXERCISE - 3

1.

I	II	III
A	p	w
B	r	u
C	q	v
D	p	t

2. (A) p,r (B) p,r (C) q,r (D) p,r

3.	(C)	**4.**	(A)	**5.**	(A)	**6.**	(B)
7.	(C)	**8.**	(A)	**9.**	(D)	**10.**	(B)
11.	(B)	**12.**	(A)				

13. (i) False (ii) False (iii) False

14. **(i)** 1.24×10^{-3} rad/s
(ii) angular momentum
(iii) 8.48 h

EXERCISE - 4

PART - I

1.	(C)	**2.**	(A)	**3.**	99R	**4.**	(D)
5.	(A) $\rightarrow$ (p) ; (B) $\rightarrow$ (q, r) ; (C) $\rightarrow$ (p) ; (D) $\rightarrow$ (q, r)						
6.	(C)	**7.**	(A)	**8.**	(A)	**9.**	6
10.	3	**11.**	(B)	**12.**	(B,D)		

PART - II

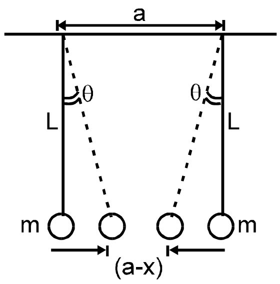
1.	(1)	**2.**	(2)	**3.**	(4)	**4.**	(4)
5.	(1)	**6.**	(4)				

Advanced Level Problems

PART - I : OBJECTIVE QUESTIONS

Single Choice type :

1. Two small balls of mass m each are suspended side by side by two equal threads of length L as shown in the figure. If the distance between the upper ends of the threads be a, the angle θ that the threads will make with the vertical due to attraction between the balls is



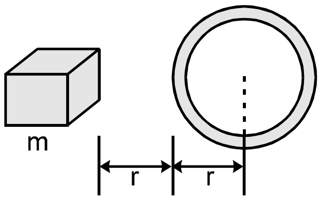
(A) $\tan^{-1}\dfrac{(a-X)g}{mG}$

(B) $\tan^{-1}\dfrac{mG}{(a-X)^2 g}$

(C) $\tan^{-1}\dfrac{(a-X)^2 g}{mG}$

(D) $\tan^{-1}\dfrac{(a^2-X^2)g}{mG}$

2. A block of mass m is lying at a distance r from a spherical shell of mass m and radius r as shown in the figure. Then



(A) only gravitational field inside the shell is zero
(B) gravitational field and gravitational potential both are zero inside the shell
(C) gravitational potential as well as gravitational field inside the shell are not zero
(D) can't be ascertained.

3. In a spherical region, the density varies inversely with the distance from the centre. Gravitational field at a distance r from the centre is :

(A) proportional to r (B) proportional to $\dfrac{1}{r}$ (C) proportional to r^2 (D) same everywhere

4. In above problem, the gravitational potential is -

(A) linearly dependent on r (B) proportional to $\dfrac{1}{r}$

(C) proportional to r^2 (D) same every where.

5. A point P lies on the axis of a fixed ring of mass M and radius R, at a distance 2R from its centre O. A small particle starts from P and reaches O under gravitational attraction only. Its speed at O will be

(A) zero (B) $\sqrt{\dfrac{2GM}{R}}$ (C) $\sqrt{\dfrac{2GM}{R}(\sqrt{5}-1)}$ (D) $\sqrt{\dfrac{2GM}{R}\left(1-\dfrac{1}{\sqrt{5}}\right)}$

6. A body of mass m is lifted up from the surface of earth to a height three times the radius of the earth. The change in potential energy of the body is (g = gravity field at the surface of the earth)

(A) mgR (B) $\dfrac{3}{4}$mgR (C) $\dfrac{1}{3}$mgR (D) $\dfrac{2}{3}$mgR

7. Assuming that the moon is a sphere of the same mean density as that of the earth and one quarter of its radius, the length of a seconds pendulum on the moon (its length on the earth's surface is 99.2 cm) is:

(A) 24.8 cm (B) 49.6 cm (C) 99.2 (D) $\frac{99.2}{\sqrt{2}}$ cm

8. A satellite can be in a geostationary orbit around a planet at a distance r from the centre of the planet. If the angular velocity of the planet about its axis doubles, a satellite can now be in a geostationary orbit around the planet if its distance from the centre of the planet is

(A) $\frac{r}{2}$ (B) $\frac{r}{2\sqrt{2}}$ (C) $\frac{r}{(4)^{1/3}}$ (D) $\frac{r}{(2)^{1/3}}$

More than one choice type :

9. An object is weighed at the equator by a beam balance and a spring balance, giving readings W_b and W_s respectively. It is again weighed in the same manner at the north pole, giving readings of W_b' and W_s' respectively. Assume that the acceleration due to gravity is the same every where on the earth's surface and that the balances are quite sensitive.

(A) $W_b = W_b'$ (B) $W_b = W_s$ (C) $W_b' = W_s'$ (D) $W_s' > W_s$

10. If a body is projected with speed lesser than escape velocity :

(A) the body can reach a certain height and may fall down following a straight line path

(B) the body can reach a certain height and may fall down following a parabolic path

(C) the body may orbit the earth in a circular orbit

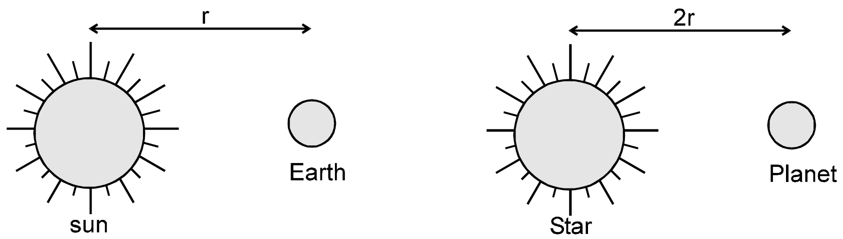
(D) the body may orbit the earth in an elliptic orbit

11. A double star is a system of two stars of masses m and 2m, rotating about their centre of mass only under their mutual gravitational attraction. If r is the separation between these two stars then their time period of rotation about their centre of mass will be proportional to

(A) $r^{3/2}$ (B) r (C) $m^{1/2}$ (D) $m^{-1/2}$

PART - II : SUBJECTIVE QUESTIONS

1. Let a star be much brighter than our sun but its mass is same as that of sun. If our earth has average life span of a man as 70 years. In the earth like planet of this star system having double the distance from our star find the average life span of a man on this planet in terms of our year.



2. A ring of radius R = 8m is made of a highly dense-material. Mass of the ring is $m_R = 2.7 \times 10^9$ kg distributed uniformly over its circumference. A particle of mass (dense) $m_p = 3 \times 10^8$ kg is placed on the axis of the ring at a distance $x_0 = 6$m from the centre. Neglect all other forces except gravitational interaction. Determine :

(a) closest distance of their approach (from centre).

(b) displacement of ring by this moment.

(c) speed of the particle at this instant.

3. Consider a spacecraft in an elliptical orbit around the earth. At the lowest point or perigee, of its orbit it is 300 km above the earth's surface at the highest point or apogee, it is 3000 km above the earth's surface.

(a) What is the period of the spacecraft's orbit ?

(b) Using conservation of angular momentum, find the ratio of the spacecraft's speed at perigee to its speed at apogee.

(c) Using conservation of energy, find the speed at perigee and the speed at apogee.

(d) It is derised to have the spacecraft escape from the earth completely. If the spacecraft's rockets are fired at perigee, by how much would the speed have to be increased to achieve this ? What if the rockets were fired at apogee ? Which point in the orbit is the most efficient to use ?
$T^2 = k a^3$, a = semi major axis. [$k = 1 \times 10^{-13}$ sec^2/m^3]

4. A planet A moves along an elliptical orbit around the Sun. At the moment when it was at the distance r_0 from the Sun its velocity was equal to v_0 and the angle between the radius vector r_0 and the velocity vector v_0 was equal to α. Find the maximum and minimum distance that will separate this planet from the Sun during its orbital motion. (Mass of Sun = M_S)

5. A satellite is put into a circular orbit with the intention that it hover over a certain spot on the earth's surface. However, the satellite's orbital radius is erroneously made 1.0 km too large for this to happen. At what rate and in what direction does the point directly below the satellite move across the earth's surface ?
R = Radius of earth = 6400 km
r = radius of orbit of geostationary satellite = 42000 km
T = Time period of earth about its axis = 24 hr.

6. What are : **(a)** the speed and **(b)** the period of a 220 kg satellite in an approximately circular orbit 640 km above the surface of the earth ? Suppose the satellite loses mechanical energy at the average rate of 1.4×10^5 J per orbital revolution. Adopting the reasonable approximation that due to atmospheric resistance force, the trajectory is a "circle of slowly diminishing radius". Determine the satellite's **(c)** altitude **(d)** speed & **(e)** period at the end of its 1500^{th} revolution.**(f)** Is angular momentum around the earth's centre conserved for the satellite or the satellite-earth system.

7. A planet of mass m moves along an ellipse around the Sun so that its maximum and minimum distance from the Sun are equal to r_1 and r_2 respectively. Find the angular momentum J of this planet relative to the centre of the Sun. (Mass of Sun = M_S)

8. Our sun, with mass 2×10^{30} kg revolves on the edge of our milky way galaxy, which can be assumed to be spherical, having radius 10^{20}m. Also assume that many stars, identical to our sun are uniformly distributed in the spherical milky way galaxy. If the time period of the sun is 10^{15} second and number of star in the galaxy are nearly $3 \times 10^{(x)}$, find value of 'x' (take $\pi^2 = 10$, G $\frac{20}{3} \times 10^{-11}$ in MKS)

9. What will be the corresponding expression for the energy needed to completely disassemble the planet earth against the gravitational pull amongst its constituent particles. Assume earth to be a sphere of uniform mass density. Calculate this energy, given the product of mass of earth and radius of earth to be 2.5×10^{31} kg-m and g = 10 m/s^2. **[JEE '92, 10 Part (b)]**

Answers

PART - I

1.	(B)	**2.**	(C)	**3.**	(D)	**4.**	(A)	**5.**	(D)
6.	(B)	**7.**	(A)	**8.**	(C)	**9.**	(A),(C), (D)		
10.	(A),(B),(C),(D)	**11.**	(A) (D)						

PART - II

1. $\frac{70}{(2)^{3/2}} \approx 25$ years **2.** (a) zero ; (b) 0.6 m; (c) 9cm/s

3. **(a)** 7.16×10^3 sec. **(b)** 1.4 **(c)** $V_p = 8.35 \times 10^3$ m/s, $V_a = 5.95 \times 10^3$ m/s

(d) $\Delta V = 14\times 10^2\sqrt{67} - V_P = 3.09 \times 10^3$ m/s , perigee

4. $r_m = \frac{r_0}{2-\eta}[1 \pm \sqrt{1-(2-\eta)\eta\sin^2\alpha}\,]$, where $\eta = r_0v_0^2/GM_S$.

5. $V_{rel} = \frac{3\Delta rR\pi}{rT} = \frac{\pi}{189}$ m/sec $\approx$ 1.66 cm/sec., to the east along equator

$V_{rel} = \frac{3\Delta rR\pi}{rT} = \frac{\pi}{189}$ m/sec $\approx$ 1.66 cm/sec.,

6. **(a)** $\frac{448}{\sqrt{3520}}$ km/s = 7.527 km/s **(b)** $\frac{220\pi}{7}\sqrt{3520}$ sec. $\approx$ 1.63 hour

(c) $\left[\frac{22\times14\times64^2\times7040}{22\times14\times64^2+7040\times6}-6400\right]$ km $\approx$ 411.92 km **(d)** $\frac{448}{\sqrt{3406}}$ km/sec. $\approx$ 7.67 km/s

(e) $\frac{1703\pi}{56}\sqrt{3406}$ sec. $\approx$ 1.55 hour **(f)** No

7. $J = m\sqrt{2GM_Sr_1r_2/(r_1+r_2)}$ **8.** 11 **9.** 15×10^{31} J