

**JEE-2015****JEE ADVANCED-3****Vidyamandir  
Classes**

Gurukul for IITJEE Preparation

**CODE**

ACEG

29/06/2014

**PAPER - 1**

MAX. MARKS : 180

10:00 AM - 01:00 PM

TIMING : 3.0 Hrs

NAME :

to h-15 h-0142 Yugank Singh

ROLL NO. :

10 h-15 h-0142

**Read the following Instructions very carefully before you proceed.**

1. The question paper consists of 3 parts (Part I : Chemistry, Part II : Physics, Part III : Mathematics). Each Part has 2 sections (Section I & Section II).
2. Section I contains **10 Multiple Choice Questions**. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY OR MORE CHOICES** may be correct.
3. Section II contains **10 Single Integer Value Type Questions**. The answer to each of the questions is a single-digit integer, ranging from 0 to 9 (both inclusive)
4. For answering a question, an ANSWER SHEET (OMR SHEET) is provided separately. Please fill your **Test Code, Roll No.** and **Group** properly in the space given in the ANSWER SHEET.
5. For each question in **Section I and Section II**, you will be given **3 Marks** if you have darkened only the bubble corresponding to the correct answer and zero mark if no bubble is darkened. There is **NO NEGATIVE MARKING**.
6. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc., except the Admit Card inside the examination hall/room.
7. On completion of the test, the candidate must hand over the Answer Sheet to the Invigilator on duty in the Room/Hall. However, the candidates are allowed to take away this Test Booklet with them.
8. **No one will be permitted to leave the test room before the end of the test, i.e. 01.00 PM**



## SECTION - I

## MULTIPLE CORRECT ANSWERS TYPE

This section contains 10 Multiple Choice Questions. Each Question has 4 choices A, B, C & D, out of which ONE or MORE Choices may be Correct:

1. For which of the following reaction enthalpy change of reaction is equal to standard enthalpy of formation of product?

- (A)  $S_{(\text{rhombic})}(\text{s}) + 3F_{2(\text{g})} \longrightarrow SF_{6(\text{g})}$   
 (B)  $P_{4(\text{white})}(\text{s}) + 5O_{2(\text{g})} \longrightarrow P_4O_{10}(\text{s})$   
 (C)  $C_{(\text{graphite})}(\text{s}) + 2S_{(\text{rhombic})}(\text{s}) \longrightarrow CS_{2(\text{l})}$   
 (D)  $C_{(\text{diamond})}(\text{s}) + O_{2(\text{g})} \longrightarrow CO_{2(\text{g})}$

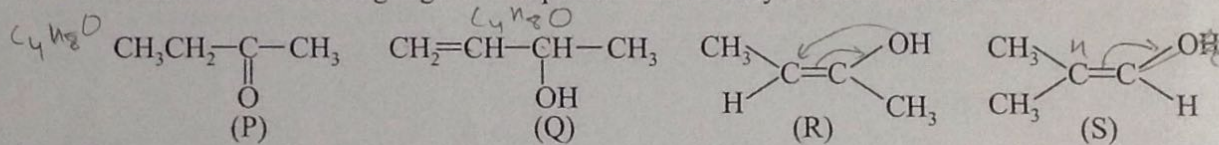
2. In which of the following phenomena hydrogen bonding plays a major role?

- (A) Solid form of water is less dense than its liquid form  
 (B) *o*-nitrophenol is more volatile than *p*-nitrophenol  
 (C) The boiling point of HF is lower than that of water  
 (D) orthophosphoric acid is more viscous than sulphuric acid.

3. Which of the following substance on heating yield a paramagnetic substance?

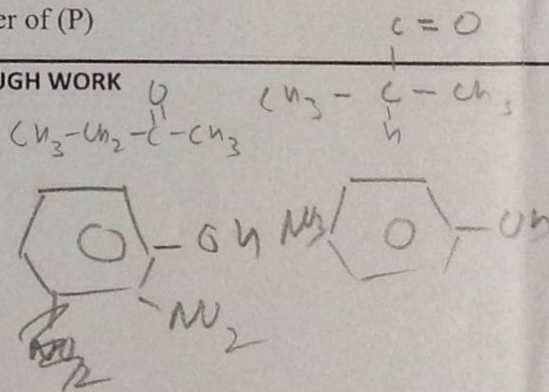
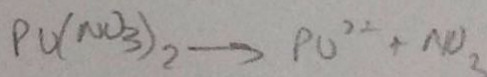
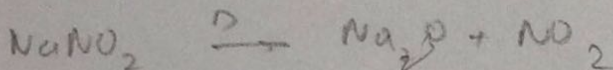
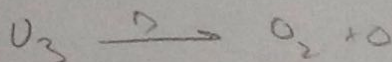
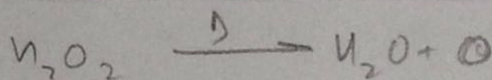
- (A)  $H_2O_2 \xrightarrow{\Delta}$  (B)  $O_3 \xrightarrow{\Delta}$  (C)  $NaNO_3 \xrightarrow{\Delta}$  (D)  $Pb(NO_3)_2 \xrightarrow{\Delta}$

4. Consider the following organic compounds and identify **correct** combinations.



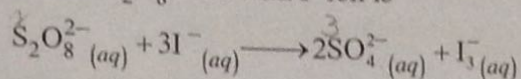
- (A) (P) and (Q) are functional isomers and both gives positive iodoform test.  
 (B) (P) and (R) are tautomers and (P) gives positive iodoform test.  
 (C) (Q) is an unsaturated alcohol while (S) is an enol  
 (D) Tautomeric form of (S) is positional isomer of (P)

SPACE FOR ROUGH WORK





5. - The reaction of  $S_2O_8^{2-}$  ion with  $I^-$  ion is



The following data are collected at a certain temperature,

| Experiment | $[S_2O_8^{2-}](M)$ | $[I^-](M)$ | Initial rate (M/s)   |
|------------|--------------------|------------|----------------------|
| 1          | 0.080              | 0.034      | $2.2 \times 10^{-4}$ |
| 2          | 0.080              | 0.017      | $1.1 \times 10^{-4}$ |
| 3          | 0.16               | 0.017      | $2.2 \times 10^{-4}$ |

$K[A]^a[B]^b$   
 $a=1 \quad b=1$

Identify the **correct** statement for above reaction.

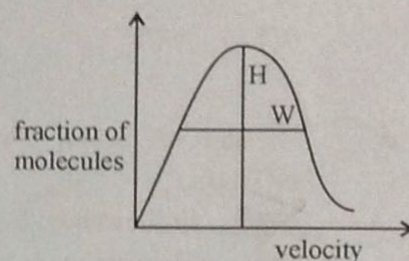
(A) It is a redox reaction

(B) Unit of its rate constant is  $L \text{ mol}^{-1} \text{ sec}^{-1}$

(C)  $I_3^-$  is linear species

(D) Oxidation state of S changes from +7 to +6

6. - The Maxwell-Boltzmann distribution for molecular speeds is shown in the following figure. In the figure, **H** is the height of the peak, **L** is the location of the maximum, **A** is area under the curve and **W** is the width at half height. As the temperature is increased identify the correct statement.



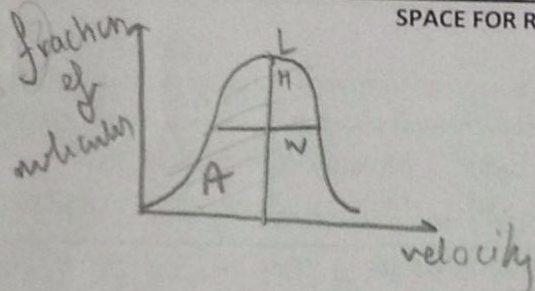
(A) H decreased

(B) L increases

(C) W increases

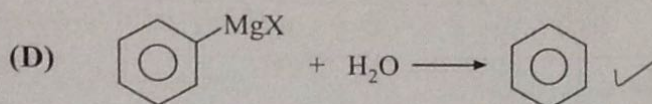
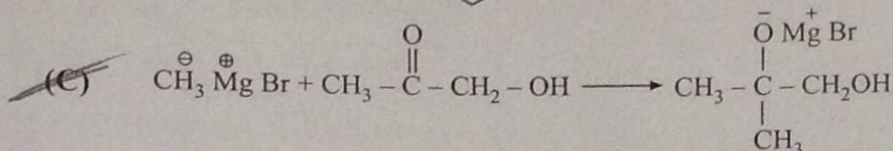
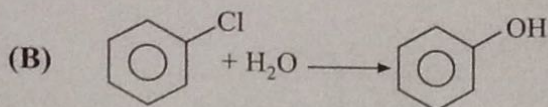
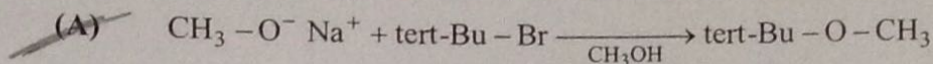
(D) A increases

SPACE FOR ROUGH WORK

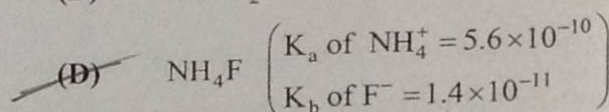
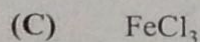
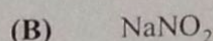
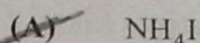




7. Which of the following reaction are **incorrect**?



8. Which of the following solution(s) will be acidic?



9. Be and Al exhibit diagonal relationship. Which of the following statements about them is(are) **NOT** true?

(A) Both react with HCl to liberate  $\text{H}_2$

(B) They are made passive by  $\text{HNO}_3$

(C) Their carbides gives acetylene on treatment with water

(D) Their oxides are amphoteric ✓

10. 40.0 g of a solute is dissolved in 500 ml of a solvent to give a solution with a volume of 515 ml. The solvent has a density of 1.00 g/ml. Which statement(s) about this solution is(are) correct?

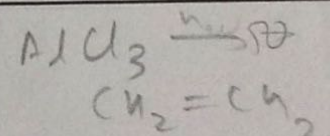
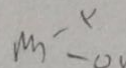
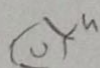
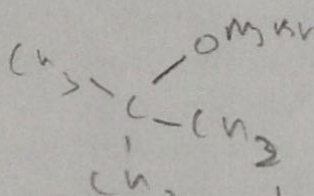
(A) Molarity of solution is 1.94 M

(B) Molality of solution is 2m

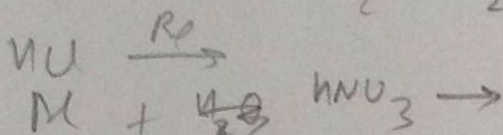
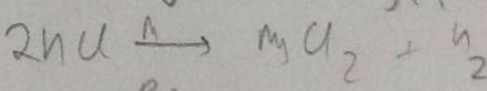
(C) Molarity is greater than the molality

(D) Molarity is lower than the molality

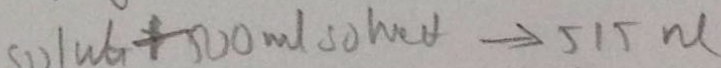
SPACE FOR ROUGH WORK



$M = \frac{\text{mles}}{V} = \frac{103}{515}$



$m = \frac{\text{mles}}{W}$



$M = \frac{\text{mles}}{V} \times 1000 = \frac{103}{515} \times 1000 = 199$

$d = 1\text{g/ml}$   
 $m = 2\text{m}$

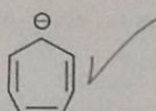
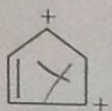
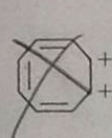
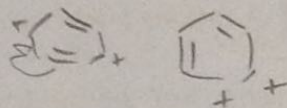
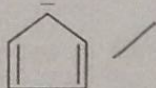
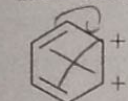


## SECTION - II

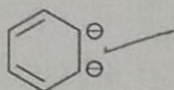
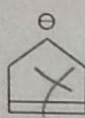
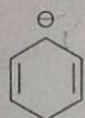
## SINGLE INTEGER VALUE CORRECT TYPE

This section contains 10 single Integer Value Correct type Questions. Each question has an integer answer between 0 and 9. Fill the answer bubbles in the OMR Sheet APPROPRIATELY and CAREFULLY.

11. How many of them are aromatic :



(3)



12. How many of the following will give positive iodoform test?

~~(1)~~ Formaldehyde

~~(2)~~ Acetaldehyde

~~(3)~~ Acetone

~~(4)~~ Benzaldehyde

~~(5)~~ Benzophenone

~~(6)~~ Acetophenone

~~(7)~~ 2-Methyl cyclohexanone

~~(8)~~ Ethanol

~~(9)~~ tert-Butyl alcohol

(5)

13. How many of the following combination can act as buffer.

(1)  $\text{HCl} + \text{NaOH}$

(2)  $\text{HCl} + \text{CH}_3\text{COO}^- \text{Na}^+$

(3)  $\text{H}_2\text{SO}_4 + \text{NaHSO}_4$

~~(4)~~  $\text{H}_2\text{CO}_3 + \text{NaOH}$

(5)  $\text{NaOH} + \text{Ph}-\text{CO}_2\text{H}$

~~(6)~~  $\text{HBr} + \text{NH}_4\text{OH}$

~~(7)~~  $\text{CH}_3\text{COOH} + \text{NH}_4\text{OH}$

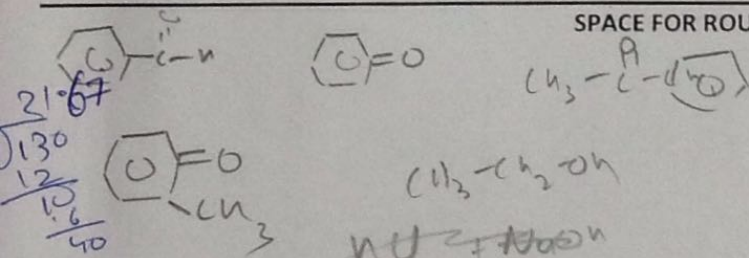
(8)  $\text{NaOH} + \text{NH}_4\text{OH}$

(9)  $\text{HCl} + \text{CH}_3\text{COOH}$

3

14. Teeth enamel is largely hydroxyapatite  $\text{Ca}_5(\text{PO}_4)_3\text{OH}$ , when dissolve in water it dissociates to give  $\text{Ca}^{2+}$ ,  $\text{PO}_4^{3-}$  and  $\text{OH}^-$  with a degree of dissociation is 0.5. The ratio of the observed colligative properties of the aqueous solution to the value of colligative properties in the absence of ionic dissociation is 5.

SPACE FOR ROUGH WORK



$$\alpha = \frac{i-1}{n-1}$$

$$0.5 = \frac{i-1}{8}$$

$$5 = i$$



15. At 25°C, the standard enthalpy of formation of HF(aq) is given by -320.1 kJ/mol; of OH<sup>-</sup>(aq) it is -229.6 kJ/mol; of F<sup>-</sup>(aq) it is -329.1 kJ/mol; and of H<sub>2</sub>O(l), it is -285.8 kJ/mol. If the standard enthalpy change for the reaction  $H^+_{(aq)} + OH^-_{(aq)} \longrightarrow H_2O(l)$  is -56.2 kJ then the standard enthalpy change for the reaction  $HF_{(aq)} \longrightarrow H^+_{(aq)} + F^-_{(aq)}$  will be -x kJ/mol. The numerical value of x is 9.
16. Photons of energy 7eV are incident on two metals A and B with work functions 6 eV and 3eV respectively. The minimum de-Broglie wavelengths of the emitted photoelectrons with maximum energies are  $\lambda_A$  and  $\lambda_B$ , respectively where  $\frac{\lambda_A}{\lambda_B}$  is nearly equal to 42.
17. The weight percent (W/V) of sucrose (Molecular weight = 342 g mol<sup>-1</sup>) in an aqueous solution is 34.2. The density of the solution is 1 g/ml, the concentration of sucrose in the solution in mol L<sup>-1</sup> is equal to 1.
18. How many of the following molecular species are regular tetrahedral in shape?  
 $NH_4^+$ ,  $BF_4^-$ ,  $XeF_4$ ,  $SiF_4$ ,  $SF_4$ ,  $POCl_3$ ,  $BrF_4^-$ ,  $PCl_4^+$ ,  $C(CN)_4$ . 5
19. In the structure of borax, the number of B-O-B units are 2.
20. If equilibrium concentration of both benzene and acetylene for reversible reaction  $3C_2H_{2(g)} \rightleftharpoons C_6H_{6(g)}$  is 0.5 mol lit<sup>-1</sup> then find the value of equilibrium constant  $K_c$  for the reaction. 4

SPACE FOR ROUGH WORK

$H^+$   $\Delta_f H = -320.1 \text{ kJ/mol}$   
 $OH^-$   $\Delta_f H = -229.6 \text{ kJ/mol}$   
 $F^-$   $\Delta_f H = -329.1 \text{ kJ/mol}$   
 $H_2O$   $\Delta_f H = -285.8 \text{ kJ/mol}$

$H^+ + OH^- \longrightarrow H_2O(l) \Delta_f H = -56.2$   
 $HF \longrightarrow H^+ + F^-$

$-285.8 + 229.6 - x = -56.2$   
 $x = 9$

$E = h\nu = 7 \text{ eV}$   
 $\lambda_A = \frac{hc}{E_A}$   
 $\lambda_B = \frac{hc}{E_B}$   
 $\frac{\lambda_A}{\lambda_B} = \frac{E_B}{E_A} = \frac{3}{6} = \frac{1}{2}$

$M_0 = 342 \text{ g/mol}$   
 $w = 34.2$   
 $d = 1 \text{ g/ml}$   
 $M = \frac{10w}{M_0} = \frac{10 \times 34.2}{342} = 1$

$K_c = \frac{0.5}{(0.5)^3} = \frac{1}{0.125} = 8$

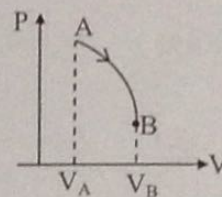


## SECTION - I

## MULTIPLE CORRECT ANSWERS TYPE

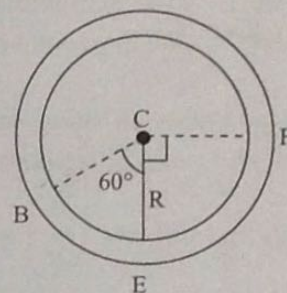
This section contains 10 Multiple Choice Questions. Each Question has 4 choices A, B, C & D, out of which ONE or MORE Choices may be Correct:

21. One mole of ideal monoatomic gas is taken through process  $AB$  given by  $P = \alpha - \beta V^2$  (where  $\alpha$  and  $\beta$  are positive constant) on  $P$ - $V$  diagram. Which of the following is the correct statement regarding the given process?



- (A) Temperature is maximum at  $V = \sqrt{\frac{\alpha}{3\beta}}$
- (B) Temperature is minimum at  $V = \sqrt{\frac{\alpha}{3\beta}}$
- (C) Rate of increase of temperature of gas with volume is maximum at A
- (D) Rate of increase of temperature of gas with volume is maximum at  $V = \sqrt{\frac{\alpha}{\beta}}$

22. As shown in figure  $BEF$  is a fixed vertical smooth circular narrow tube. A particle of mass  $m$  starts moving in the tube at point  $B$  with velocity  $V$  towards  $E$ . It is just able to complete the vertical circle, then



- (A) Velocity at  $B$  must be  $\sqrt{3Rg}$
- (B) Velocity at  $F$  must be  $\sqrt{2Rg}$
- (C) Normal reaction at point  $F$  is  $2mg$ .
- (D) The normal reaction at point  $E$  is  $6mg$ .

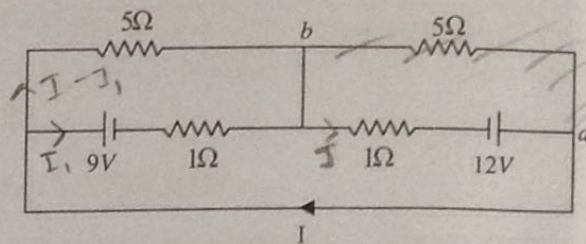
SPACE FOR ROUGH WORK

 $\sqrt{2Rg}$



23. In the circuit shown aside :

- (A) The P.d between  $a$  and  $b$  is 8.75 volts  
 (B) The P.d between  $a$  and  $b$  is 12.25 V  
 (C) The current  $I$  is 3A  
 (D) The current  $I$  is 1.5A



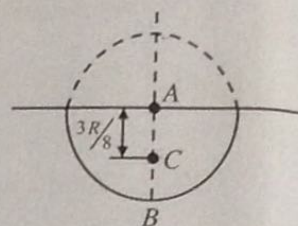
24. Consider a solid hemisphere of mass  $m$  and radius  $R$ .  $A$  is the centre of the complete sphere,  $C$  is the COM of the hemisphere and  $B$  is a point on the surface of the hemisphere collinear with  $AC$ . Consider the axes normal to the same plane of the paper passing through these points as  $AA'$ ,  $CC'$  and  $BB'$  and moment of inertia of the hemisphere about these axes as  $I_{AA'}$ ,  $I_{CC'}$  and  $I_{BB'}$  respectively. Then.

(A)  $I_{AA'} = \frac{2}{5} mR^2$

(B)  $I_{AA'} = \frac{mR^2}{5}$

(C)  $I_{CC'} = \frac{83}{320} mR^2$

(D)  $I_{BB'} = \frac{13}{20} mR^2$



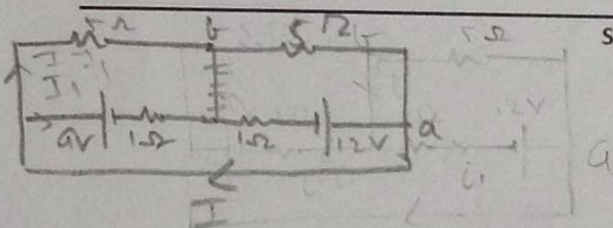
25. A particle of mass 2 kg and charge 1 mC is projected vertically with a velocity of  $10 \text{ ms}^{-1}$ . There is a uniform horizontal electric field of  $10^4 \text{ N/C}$

- (A) The horizontal range of the particle is 10 m  
 (B) The time of flight of the particle is 2 sec  
 (C) The maximum height reached is 5 m  
 (D) The horizontal range of the particle is 5 m

26. Two points on string are being observed as a traveling wave passes them. The points are at  $x_1 = 0$  and  $x_2 = 1 \text{ m}$ . The transverse motions of two points are found to be as follows  $y_1 = A \sin(3\pi t)$  and  $y_2 = A \sin\left(3\pi t + \frac{\pi}{8}\right)$  where  $t$  is in seconds and  $y$  in meters. Mark correct options.

- (A) Frequency of wave is 3 Hz  
 (B) Frequency of wave is 1.5 Hz  
 (C) Wavelength may be 16 m  
 (D) Wavelength may be  $\frac{16}{15} \text{ m}$

SPACE FOR ROUGH WORK



$$-6(i_1 - i_2) + 9 = 0 \Rightarrow -6i_1 + 12 + 9 = 0$$

$$12 - 6i_1 = -9 \Rightarrow 36i_1 = 21 \Rightarrow i_1 = \frac{7}{4}$$

$$12 - 6i_1 = -9 \Rightarrow 12 - 6\left(\frac{7}{4}\right) = -9$$

$$12 - \frac{42}{4} = -9 \Rightarrow 12 - 10.5 = -9 \Rightarrow 1.5 = -9$$

$$F = \frac{1}{2} \rho v^2$$

$$F = 10^4 \times 10^{-3} = 10 \text{ N}$$

$$12 - 9 - 2I = 0$$

$$I = 1.5 \text{ A}$$

$$-10(i_1 - i_2) + 2(i_1) + 3 = 0$$

$$-10I + 15 + 3 + 3 = 0$$

$$I = \frac{20}{10} = 2 \text{ A}$$



27. A uniform rod of mass  $M$  and length  $a$  lies on a smooth horizontal plane. A particle of mass  $m$  moving at a speed  $v$  perpendicular to the length of the rod and strikes it at a distance  $\frac{a}{4}$  from the centre and stops after the collision.

(A) Velocity of centre of mass of rod is  $\frac{mv}{M}$

(B) Angular velocity of rod about its centre just after the collision  $\omega = \frac{2mv}{Ma}$

(C)  $\omega = \frac{3mv}{Ma}$  [Just after the collision]

(D) Velocity of centre of mass of rod is  $\frac{Mv}{m}$

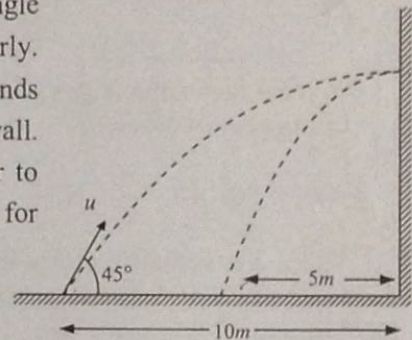
28. A ball of mass  $20 \text{ gm}$  is projected with velocity  $u \text{ m/s}$  at an angle of  $45^\circ$  with horizontal and it hits a vertical wall perpendicularly. The wall is  $10 \text{ m}$  away from the point of projection. It rebounds perpendicularly from the wall and falls  $5 \text{ m}$  away from the wall. The entire motion is in the same vertical plane perpendicular to ground and the wall. The ball remained in contact with the wall for  $0.01 \text{ seconds}$ .

(A) Average force exerted by wall on the ball is  $30 \text{ N}$

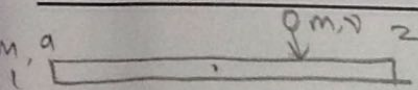
(B) Average force exerted by ball on the wall is  $70 \text{ N}$

(C) Time taken by particle from ground to wall is  $1 \text{ sec}$

(D) Time taken by particle from wall to ground is  $0.5 \text{ sec}$



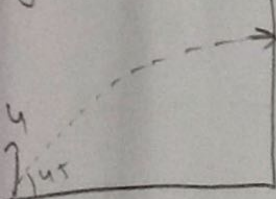
SPACE FOR ROUGH WORK



$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$mv = Mx$$

$$0.02 \times v = \frac{Mv}{M}$$



$$\omega = \frac{v}{r} = \frac{2mv}{Ma}$$

$$u = \frac{u^2 \sin 2\theta}{2g} = u^2 \quad \omega = \frac{u^2}{10} \times \frac{1}{2}$$

$$u^2 = 20 \quad \text{K.E.} \quad \frac{1}{2}mv^2 = \frac{1}{2} \times 0.02 \times 20 \times 20 = 0.2 \text{ J}$$

$$h = \frac{u \sin \theta}{g} = \frac{20 \times 1}{\sqrt{2} \times 10} = \sqrt{2} = 1.414$$

$$\frac{1}{2}mv^2 = mgh = \frac{1}{2}m(ev_2)^2$$

$$200 = 10h \quad h = 20$$

$$mgh = \frac{1}{2}m(ev_2)^2$$

$$g h = \frac{1}{2}(ev_2)^2 \quad \omega = ev_2$$

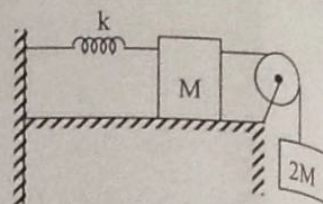
$$V = \omega r \quad 2ah = \frac{u^2 \sin 2\theta}{2g} \quad \omega = \frac{V}{r} \quad 2 \times 10 \times 10 = u^2$$

$$R = \frac{u^2 \sin 2\theta}{2g} \quad (u = 20 \text{ m/s})$$



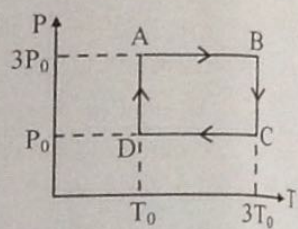
29.

Two blocks of masses  $M$  and  $2M$ , are connected to a light string of spring constant  $k$  that has one end fixed, as shown in figure. The horizontal surface and the pulley are frictionless. The blocks are released from rest when the spring is non deformed. (Assume  $M$  does not collide with pulley during the motion)



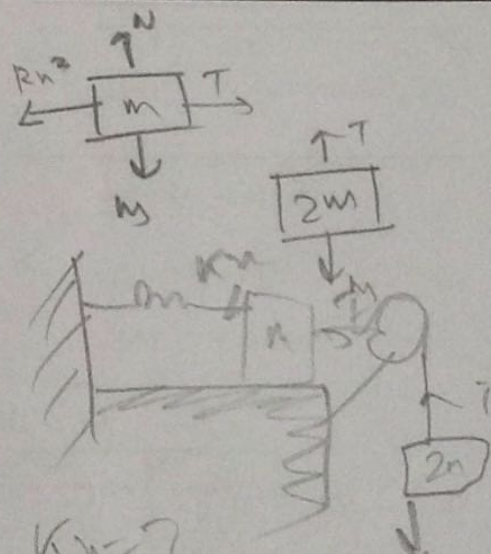
- (A) Maximum extension in the spring is  $\frac{4Mg}{k}$
- (B) Maximum kinetic energy of the system is  $\frac{2M^2g^2}{k}$
- (C) Maximum energy stored in the spring is four times that of maximum kinetic energy of the system
- (D) When kinetic energy of the system is maximum, energy stored in the spring is  $\frac{2M^2g^2}{k}$

30. An ideal monoatomic gas goes under cyclic process shown in following diagram.



- (A) For AB,  $\frac{\Delta U}{W}$  is  $\frac{2}{3}$
- (B) For BC,  $\Delta Q = W$
- (C) For complete cycles  $W > 0$
- (D) For complete cycle  $\Delta Q > 0$

SPACE FOR ROUGH WORK

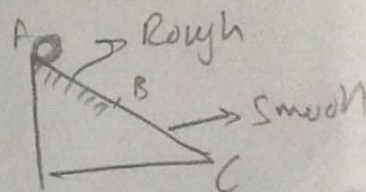


$$T = 2Mg$$

$$Kn = 2Mg$$

$$n = \frac{2Mg}{k}$$

$$KE = \frac{1}{2}mv^2$$



$$\frac{1}{2}mv^2 + \frac{1}{2}mv^2 = \frac{1}{2}mv^2$$

$$\frac{1}{2}mv^2 \left(1 + \frac{1}{v}\right)$$

$$Kn = 2Mg$$

$$n = \frac{2Mg}{k}$$

$$\frac{1}{2}Kn^2 = \frac{1}{2}mv^2$$

$$\frac{2M^2g^2}{2k} = \frac{1}{2}mv^2$$

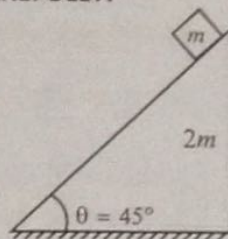


## SECTION - II

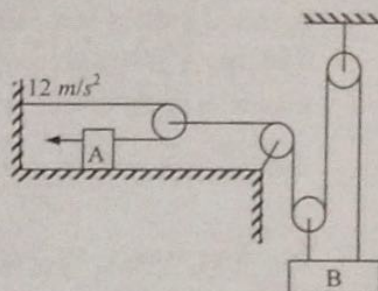
## SINGLE INTEGER VALUE CORRECT TYPE

This section contains 10 single Integer Value Correct type Questions. Each question has an integer answer between 0 and 9. Fill the answer bubbles in the OMR Sheet APPROPRIATELY and CAREFULLY.

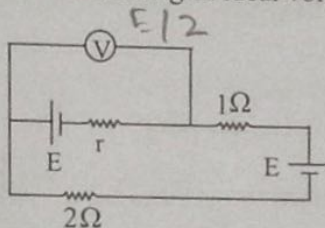
31. A wedge of mass  $2m$  and a cube of mass  $m$  are shown in figure. Between cube and wedge, there is no friction. The minimum coefficient of friction between wedge and ground so that wedge does not move is  $\mu$  then find the value of  $10\mu$ . **5**



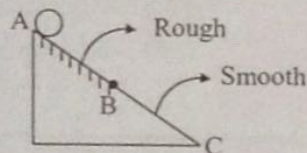
32. In the situation as shown in figure the acceleration of block A is  $12 \text{ m/s}^2$ . Assuming that the block B always remains horizontal, find acceleration of B in  $\text{m/s}^2$ . **3**



33. In the given circuit the reading of ideal voltmeter is  $E/2$ . Find the internal resistance of the battery in  $\Omega$ . **1**



34. In figure a solid cylinder rolls from A to B and then from B to C. Find the ratio of translational to rotational kinetic energy when cylinder reaches C. Given  $AB = BC$  (AB is rough and BC is smooth and there is rolling without slipping from A to B). **7**



SPACE FOR ROUGH WORK

Handwritten rough work for Question 31:

Handwritten rough work for Question 32:

Handwritten rough work for Question 33:

Handwritten rough work for Question 34:

$N = mg \cos 45^\circ = \frac{mg}{\sqrt{2}}$   
 $N \sin 45^\circ = 2mg = 2mg \mu$   
 $\frac{mg}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = 2mg \mu$   
 $\mu = \frac{1}{4} = 0.25$   
 $10\mu = 2.5$

Handwritten rough work for Question 31 (continued):

$2E - (3r + r)I = 0$   
 $2E - 4rI = 0$   
 $2E = 4rI$   
 $r = \frac{E}{2I}$

Handwritten rough work for Question 32 (continued):

$V = E - iV$   
 $\frac{E}{2} = E - iV$   
 $iV = \frac{E}{2}$   
 $V = \frac{E}{2}$

Handwritten rough work for Question 33 (continued):

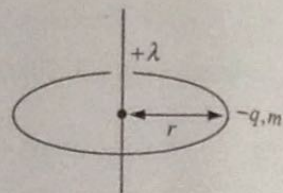
$V = \frac{E}{2}$   
 $\frac{E}{2} = E - iV$   
 $iV = \frac{E}{2}$   
 $V = \frac{E}{2}$



35. A particle of mass  $m$  kept on the top of a fixed smooth sphere is slightly pushed. If its tangential acceleration, when it breaks off the sphere is  $\sqrt{5}g/n$  then the value of  $n$  is 2.

36. A particle of charge  $-q$  and mass  $m$  moves in a circle of radius  $r$  around an infinitely long line charge of linear charge density  $+\lambda$ . Find the time period (in ms) to the nearest integer. (Neglect gravity)

Given  $m = 1g$ ,  $\lambda = 1.0 \times 10^{-4} C/m$ ,  $q = 0.5 \mu C$ ,  $r = \frac{9}{\pi} cm$



37. A cubical block of copper of side  $10 cm$  is floating in a vessel containing mercury. Water is poured into the vessel so that the copper block just gets submerged. The height of water column (in  $cm$ ) is 7.3.

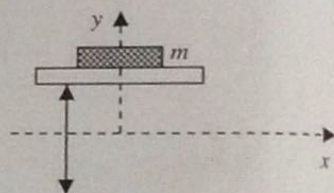
38. A nail is located at a certain distance vertically below the point of suspension of a simple pendulum. The length of simple pendulum is  $1 m$ . The pendulum bob is released from a position where the string makes an angle of  $60^\circ$  with the vertical. The distance of nail from the point of suspension such that the bob will just perform revolution with the nail as centre is  $\frac{a}{10} m$ . Find  $a$ .

39. Standing waves are set up in a string of length  $240 cm$  clamped horizontally at both ends. The separation between any two consecutive points where displacement amplitude is  $3\sqrt{2} cm$  is  $20 cm$ . The standing waves were set by two traveling waves of equal amplitude of  $3 cm$ . The overtone in which the string is vibrating will be 6.

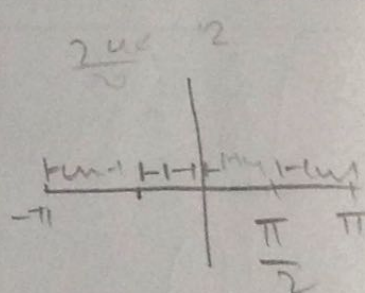
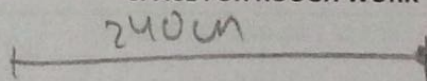
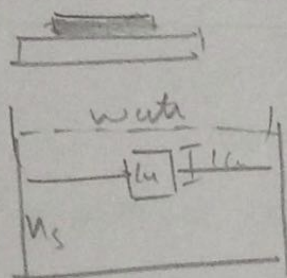
40. A horizontal platform with a mass  $m$  placed on it is executing

$SHM$  along  $y$ -axis. The time period of oscillation is  $\frac{\pi}{10} sec$ .

The maximum amplitude of  $SHM$  for the mass not to be detached from the platform is  $A$  (in  $mm$ ) then the value of  $A/5$  is 2. (Take  $g = 10 m/sec^2$ )



SPACE FOR ROUGH WORK





## SECTION - I

## MULTIPLE CORRECT ANSWERS TYPE

This section contains 10 Multiple Choice Questions. Each Question has 4 choices A, B, C & D, out of which ONE or MORE Choices may be Correct:

41. - Let  $f(x) = \begin{cases} x^2 & ; 0 < x < 2 \\ 2x-3 & ; 2 \leq x < 3 \\ x+2 & ; x \geq 3 \end{cases}$  then which of the following is (are) true?

~~(A)~~  $f\left(f\left(f\left(\frac{3}{2}\right)\right)\right) = f\left(\frac{3}{2}\right)$

~~(B)~~  $1 + f\left(f\left(f\left(\frac{5}{2}\right)\right)\right) = f\left(\frac{5}{2}\right)$

~~(C)~~  $f(f(f(2))) = f(1)$

~~(D)~~  $\underbrace{f(f(f(\dots f(4)\dots)))}_{1004 \text{ times}} = 2012$

42. - The equation of a line passing through the point of intersection of the lines,  $x - 2y = 3$  and  $x + 3y = 8$  having equal intercept on the co-ordinate axes is (are):

~~(A)~~  $x + y = 6$  (B)  $x - 5y = 0$  (C)  $5x - y = 0$  (D)  $x - y = 4$

43. If  $|z - 1 - i| = 1$ , then the locus of points represented by the complex number  $(5(z - i) - 6)$  is a circle such that:

(A) Centre of circle is (1, 0)

~~(B)~~ Centre is (-1, 0)

(C) Radius is 5 units

~~(D)~~ Radius is 3 units

44. If  $f(x) = \min(1, \cos x, 1 - \sin x)$ ,  $-\pi \leq x \leq \pi$ , then:

(A)  $f(x)$  is not differentiable at  $x = 0$

~~(B)~~  $f(x)$  has local maximum at  $x = 0$

(C)  $f(x)$  is differentiable at  $x = \frac{\pi}{4}$

~~(D)~~  $f(x)$  is continuous and bounded in  $x \in [-\pi, \pi]$

SPACE FOR ROUGH WORK

$$f\left(f\left(f\left(\frac{3}{2}\right)\right)\right)$$

$$f(f(2.25))$$

$$f(1.5) = f(2.25)$$

$$x - 2y = 3$$

$$x + 3y = 8$$

$$\frac{15y = 15}{y = 1}$$

$$y = 1$$

$$1 + f\left(f\left(f\left(\frac{5}{2}\right)\right)\right)$$

$$f(2)$$

$$1 + f(1) = 2$$

$$x - 2 = 3$$

$$x = 5$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$= \frac{1004}{2} [8 + (1003)2]$$

$$\frac{x}{a} + \frac{y}{a} = 1$$

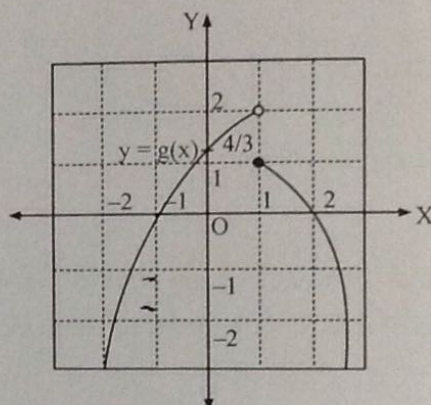
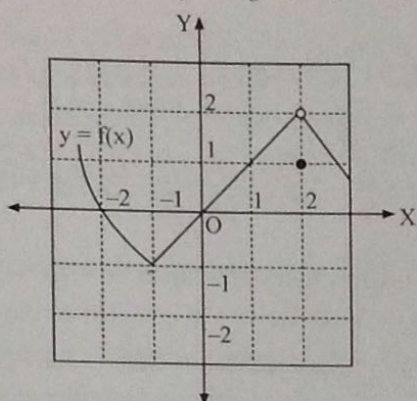
$$\frac{2}{a} = 1 \quad \frac{6}{a} =$$

$$2 = a$$

$$x + y = 2$$



45. If  $\tan \theta_i$ ;  $i = 1, 2, 3, 4$  are the roots of equation  $x^4 - x^3 \sin 2\beta + x^2 \cos 2\beta - x \cos \beta - \sin \beta = 0$ , then  $\tan(\theta_1 + \theta_2 + \theta_3 + \theta_4)$  is equal to :
- (A)  $\frac{1 - \cos 2\beta}{\sin 2\beta}$  (B)  $\frac{1 + \cos 2\beta}{\sin 2\beta}$  (C)  $\tan \beta$  (D)  $\cot \beta$
46. The co-ordinates of the vertices  $A, B, C$  of a triangle  $ABC$  whose orthocentre is  $H(-3, 10)$  and circumcentre  $O(-2, -3)$  and mid point of  $BC$  is  $D(1, 3)$  are :
- (A)  $(-9, -2)$  (B)  $(3, 2)$  (C)  $(11, 8)$  (D)  $(-1, 4)$
47. The graph of functions  $f$  and  $g$  are shown below.



[Note :  $[k]$  denotes the greatest integer less than or equal to  $k$ .]

Consider the following statements

I.  $\lim_{x \rightarrow 1^-} [f(x) + g(x)]$  exist and is equal to 2.

II.  $\lim_{x \rightarrow 2^+} [f(x) + g(x)]$  exist and is equal to 1.

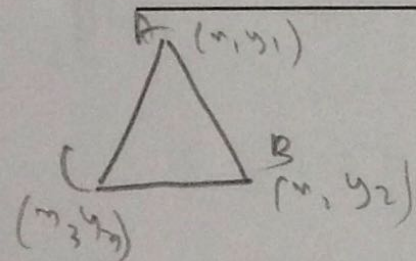
III.  $\lim_{x \rightarrow 0} [f(g(x))]$  exist and is equal to 1.

IV.  $\lim_{x \rightarrow 2} [g(f(x))]$  exist and is equal to -1.

Which of the statements I, II, III and IV given above are **CORRECT**?

- (A) I, II and III (B) I, II, III and IV (C) I, II and IV (D) II and III

SPACE FOR ROUGH WORK



$$\begin{aligned} \frac{x_2 + x_3}{2} &= 1 & y_2 + y_3 &= 6 \\ x_2 + x_3 &= 2 & 6 + y_1 &= -9 \\ \frac{2 + x_1}{3} &= -2 & y_1 &= -15 \\ 2 + x_1 &= -6 & & \\ x_1 &= -8 & & \end{aligned}$$



48. Let  $f(x) = x^3 + ax^2 + bx + c$  where  $a, b, c \in R$ , then which of the following statement(s) is (are) CORRECT?
- (A) If the equation  $f(x) = 0$  has exactly one real root then  $f(x)$  must be strictly increasing on  $R$
- (B) If  $f(x)$  has a negative point of local minimum, then both roots of equation  $f'(x) = 0$  must be negative and distinct
- (C) If  $f(x_1) \cdot f(x_2) < 0$ ,  $x_1 < x_2$ , then the equation  $f(x) = 0$  has at least one real root in  $(x_1, x_2)$ .
- (D)  $f(x)$  possesses exactly one point of inflexion
49. The function  $f(x) = \sqrt{ax^3 + bx^2 + cx + d}$  has its non-zero local minimum and local maximum values at  $-2$  and  $2$  respectively. Given  $a$  is root of the equation  $x^2 - x - 6 = 0$ . The value of  $(a + b + c)$  is a LCM of:
- (A) 16 (B) 2 (C) 11 (D) 22
50. If  $y = mx + 5$  is a tangent to the curve  $x^3 y^3 = ax^3 + by^3$  at  $P(1, 2)$ , then:
- (A)  $a + b = \frac{18}{5}$  (B)  $a > b$  (C)  $a < b$  (D)  $a + b = \frac{19}{5}$

SPACE FOR ROUGH WORK

$$f(x) = x^3 + ax^2 + bx + c$$

$$f'(x) = 3x^2 + 2ax + b$$

$$f''(x) = 6x + 2a$$

$$x^2 - 3x + 2x - 6 = 0$$

$$(x-3)(x+2) = 0$$

$$x = 3, -2$$

$$8 = a + 8b$$

$$2 = m + 5$$

$$m = -3$$

$$f(x) = \sqrt{ax^3 + bx^2 + cx + d}$$

$$f'(x) = \frac{3ax^2 + 2bx + c}{2\sqrt{ax^3 + bx^2 + cx + d}}$$

$$f'(x) = \frac{1}{2\sqrt{ax^3 + bx^2 + cx + d}}$$

$$\times 3ax^2 + 2bx + c$$

$$f'(x) < 0 \quad f'(x) > 0$$

$$\frac{3ax^2 + 2bx + c}{2\sqrt{ax^3 + bx^2 + cx + d}}$$

$$+ 12a - 4b + c < 0 \quad 12a + 4b + c > 0$$

$$-8a + 4b - 2c + d$$

$$24 - 4b + c < 0$$

$$24 + 4b + c > 0$$

$$-8b < 0$$

$$b =$$



## SECTION - II

## SINGLE INTEGER VALUE CORRECT TYPE

This section contains 10 single Integer Value Correct type Questions. Each question has an integer answer between 0 and 9. Fill the answer bubbles in the OMR Sheet APPROPRIATELY and CAREFULLY.

51. A differentiable function  $f(x)$  satisfying  $f(xy) = (f(x))^y \forall x, y \in \mathbb{R}$  and  $f(1) = \frac{1}{4}$   
 If  $\lim_{n \rightarrow \infty} (1 \cdot \sqrt{f(x+1)} + 2 \cdot \sqrt{f(x+2)} + \dots + n \cdot \sqrt{f(x+n)}) = k \sqrt{f(x)}$ , then  $k$  is equal to 6.
52. If  $\cos^{-1}\left(\frac{x}{2}\right) + \cos^{-1}\left(\frac{y}{3}\right) = \theta$ , then sum of the digits in maximum value of  $9x^2 - 12xy \cos \theta + 4y^2$  is 7.
53. If Rolle's theorem is applicable to the function,  $f(x) = \frac{\ln x}{x}$  over the interval  $[a, b]$  where  $a, b \in \mathbb{I}^+$ , then the value of  $a+b$  is equal to 9.
54. Let  $a \in \left(0, \frac{\pi}{2}\right)$ , then the reciprocal of  $\lim_{a \rightarrow 0} \frac{1}{a^3} \int_0^a \ln(1 + \tan a \tan x) dx$  is equal to 2.
55. The absolute term in  $P(x) = \sum_{r=1}^n \left(x - \frac{1}{r}\right) \left(x - \frac{1}{r+1}\right) \left(x - \frac{1}{r+2}\right)$  as  $n$  approaches to infinity is  $k$ .  
 Then the value of  $4 - 6k$  is equal to 1.
56. The number of words in which the letters of the word DECISIONS be arranged so that letter  $N$  is somewhere in between  $I$ 's is  $\frac{k!}{12}$  where  $k$  is 7.

SPACE FOR ROUGH WORK

$\frac{92}{2}$        $\frac{7!}{2! \times 2}$        $\frac{1N1}{1}$       DECISIONS       $\frac{7!}{2! \cdot 2!}$   
 $\cos^{-1}\left(\frac{x}{2}\right) + \cos^{-1}\left(\frac{y}{3}\right) = \theta$   
 $\cos^{-1} \frac{xy - \sqrt{x^2-1} \sqrt{y^2-1}}{2} = \cos \theta$   
 $\frac{xy}{2} - \frac{\sqrt{x^2-1} \sqrt{y^2-1}}{2} = \cos \theta$   
 $9n^2 - 6ny \left[ \frac{ny}{2} - \sqrt{n^2-2} \sqrt{y^2-2} \right]$   
 $9n^2 - 3n^2y^2 - 6ny \frac{1}{2} \frac{4y^2}{2} + 4y^2$



57. Let  $f: (-1, 1) \rightarrow \mathbb{R}$  be a differentiable function with  $f(0) = -1$  and  $f'(0) = 1$ .  
Let  $g(x) = (f(2f(x) + 2))^2$ , then  $|g'(0)|$  is equal to         .
58. If  $a_1$  is the value of  $a$  for which function  $f(x) = x^2 + \frac{a}{x}$  has a local minimum at  $x = 2$  and  $a_2$  is the value of  $a$  for which  $f''(1)$  vanishes, then the value of  $\frac{a_1 + a_2}{3}$  is equal to 6.
59. The number of values of  $\theta$  between  $-\pi$  and  $\frac{3\pi}{2}$  that satisfies the equation  $5\cos 2\theta + 2\cos^2 \frac{\theta}{2} + 1 = 0$  is equal to 5.
60. If  $\alpha, \beta$  are two distinct real roots of the equation  $ax^3 + x - 1 - a = 0$  ( $a \neq -1, 0$ ), none of which is equal to unity, then the value of  $\lim_{x \rightarrow (1/\alpha)} \frac{(1+a)x^3 - x^2 - a}{(e^{1-\alpha x} - 1)(x-1)}$  is  $\frac{a\ell(k\alpha - \beta)}{\alpha}$ . The value of  $k\ell$  is 7.

SPACE FOR ROUGH WORK

$$a \pm \frac{a}{a_1} \quad 2n \pm \frac{-a}{n^2} \quad 4 - \frac{a}{4} < 0$$

$$\frac{2+a}{\frac{1}{n^2}} = 2+a$$

$$\frac{18}{3}$$

$$-\pi \quad -\frac{\pi}{2} \quad 0 \quad \frac{\pi}{2} \quad \pi \quad \frac{3\pi}{2}$$

$$\begin{aligned} 5(2\cos^2 \theta - 1) + 2\cos^2 \frac{\theta}{2} + 1 &= 0 & f(0) &= -1 & f'(0) &= 1 \\ 10\cos^2 \theta + 2\cos^2 \frac{\theta}{2} - 4 &= 0 & g(x) &= (f(2f(x) + 2))^2 \\ 20\cos^2 \frac{\theta}{2} + 2\cos^2 \frac{\theta}{2} - 14 &= 0 & |g'(0)| &= ? \\ 22\cos^2 \frac{\theta}{2} &= 14 \\ \cos^2 \frac{\theta}{2} &= \frac{14}{22} = \frac{7}{11} \\ \cos \frac{\theta}{2} &= \pm \sqrt{\frac{7}{11}} \end{aligned}$$