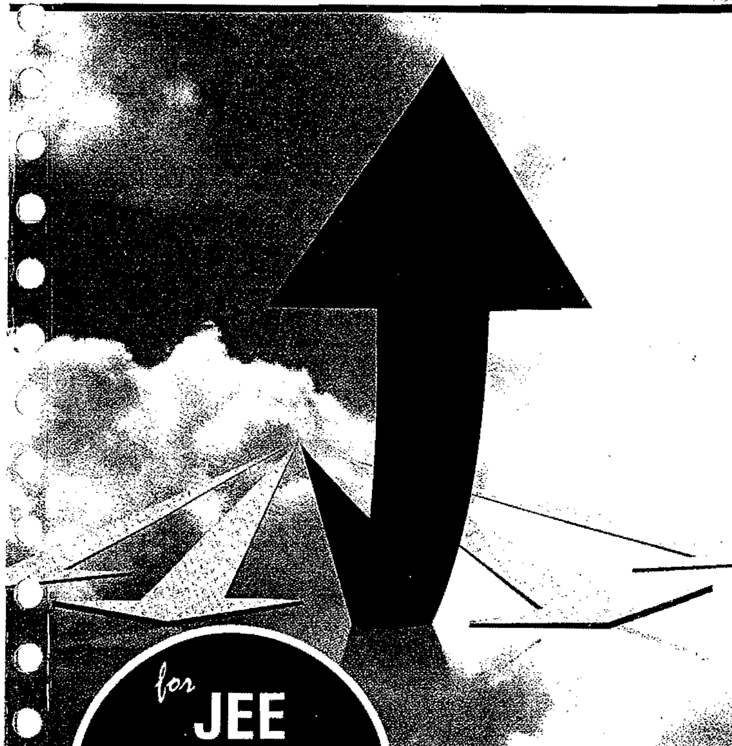
Resonance
Educating for better tomorrow
Distance Learning Programmes Division (DLPD)

RANK BOOSTER
Redefined



1000+
Performance Boosters..

for JEE (ADVANCED) 2015

Best problems to revise, strengthen & clarify the concepts.

- Topic Wise 1000+ Questions.
- 6 Part Syllabus Test.
- 3 Full Syllabus Test.
- Covers Class XI & Class XII Syllabus.
- Complete Solutions.

PHYSICS

Contents

SECTION-I (TOPIC WISE PROBLEMS)

Contents

SECTION-II (PRACTICE TEST PAPERS)

Answers & Solutions

SECTION-I (TOPIC WISE PROBLEMS)

Answers & Solutions

SECTION-II (PRACTICE TEST PAPERS)

Particular	Page No.

6 PTs (PART TEST)

SECTION-I
TOPIC WISE PROBLEMS

TOPIC 1 UNIT DIMENSIONS AND ERROR ANALYSIS

SECTION - I : STRAIGHT OBJECTIVE TYPE

1.1 In a system of units if force (F), acceleration (A) and time (T) are taken as fundamental units, then the dimensional formula of energy is :
(A) FA^2T (B) FAT^2 (C) FA^2T^3 (D) FAT

1.2 $\frac{E^2}{\mu_0}$ has the dimensions (E = electric flux, μ_0 = permeability of free space)
(A) $[M^2L^3T^{-2}A^2]$ (B) $[MLT^{-4}]$ (C) $[ML^3T^{-2}]$ (D) $[M^{-1}L^2TA^{-2}]$

1.3 In the equation $\int \frac{dt}{\sqrt{2at-t^2}} = ax \sin^{-1}\left[\frac{t}{a}-1\right]$ The value of x is :
(A) 1 (B) – 1 (C) 0 (D) 2

1.4 The dimensions of the quantity $\hbar c$ $\left(\text{where } \hbar = \frac{h}{2\pi}\right)$ is :
(A) $[ML^2T^{-1}]$ (B) $[MLT^{-1}]$ (C) $[ML^3T^{-2}]$ (D) $[ML^3T^{-1}]$

1.5 A particle of mass m is executing oscillations about the origin on the x-axis. Its potential energy is $U(x) = K|x|^3$, where K is a positive constant. If the amplitude of oscillation is a, then its time period T is :
(A) proportional to $\frac{1}{\sqrt{a}}$ (B) independent of a (C) proportional to $\sqrt{a}$ (D) proportional to $a^{3/2}$

1.6 In the formula $X = 3YZ^2$, X and Z have dimensions of capacitance and magnetic induction respectively. What are the dimensions of Y in MKSQ system ?
(A) $[M^{-3}L^{-1}T^3Q^4]$ (B) $[M^{-3}L^{-2}T^4Q^4]$ (C) $[M^{-2}L^{-2}T^4Q^4]$ (D) $[M^{-3}L^{-2}T^3Q^1]$

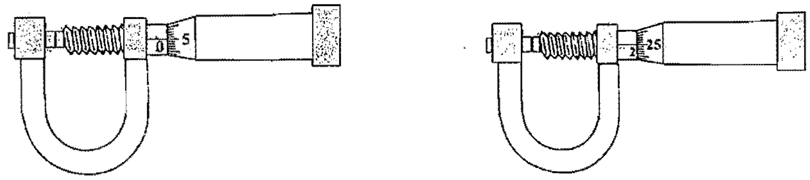
1.7 The dimensions of $\frac{1}{2}\varepsilon_0 E^2$ (ε_0 : permittivity of free space ; E : electric field) is :
(A) $[MLT^{-1}]$ (B) $[ML^{-1}T^{-2}]$ (C) $[MLT^{-2}]$ (D) $[ML^2T^{-1}]$

1.8 In the relation $P = \frac{\alpha}{\beta} e^{\frac{\alpha Z}{k\theta}}$

P is pressure, Z is distance, k is Boltzmann constant and θ is the temperature. The dimensional formula of β will be :
(A) $[M^0L^2T^0]$ (B) $[M^1L^2T^1]$ (C) $[M^1L^0T^{-1}]$ (D) $[M^0L^2T^{-1}]$

1.9 Which of the following sets have different dimensions ?
(A) Pressure, Young's modulus, Stress
(B) Emf, Potential difference, Electric potential
(C) Heat, Work done, Energy
(D) Dipole moment, Electric flux, Electric field

1.10 The number of circular divisions on the shown screw gauge is 50. It moves 0.5 mm on main scale for one complete rotation and main scale has 1/2 mm marks . The diameter of the ball is :



(A) 2.25 mm (B) 2.20 mm (C) 1.20 mm (D) 1.25 mm

SECTION - II : MORE THAN ONE CHOICE TYPE

1.11 Let $[\varepsilon_0]$denote the dimensional formula of the permittivity of the vacuum and $[\mu_0]$ that of the permeability of the vacuum. If M = mass, L = length, T = time and I = electric current, then :

(A) $[\varepsilon_0] = [M^{-1} L^{-3} T^2 I]$ (B) $[\varepsilon_0] = [M^{-1} L^{-3} T^4 I^2]$

(C) $[\mu_0] = [MLT^{-2} I^2]$ (D) $[\mu_0] = [ML^2 L^{-1} I]$

1.12 The SI nit of the inductance, the henry can by written as :

(A) weber/ ampere (B) volt-second/ampere

(C) joule/(ampere)^2 (D) ohm-second

SECTION - III : ASSERTION AND REASON TYPE

1.13 **Statement - 1 :** Unit of torque is joule

Statement - 2 : Unit of torque should be N-m and that is called joule.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

1.14 **Statement - 1 :** If two physical quantities have same dimensions, than that can be certainly added or substracted.

Statement - 2 : If the dimensions of both the quantities are same then both the physical quantities should be similar.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

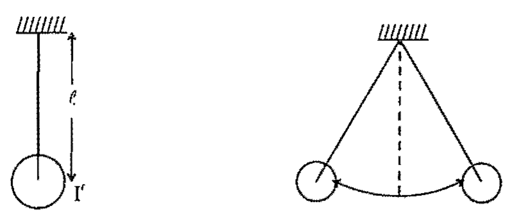
(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

SECTION - IV : COMPREHENSION TYPE

Comprehension # 1

Determining the value of 'g' using a simple pendulum



In this experiment. a small spherical bob is hanged with a cotton thread. This arrangement is called sample pendulum. The bob is displaced slightly and allowed to oscillate. To find time period, time taken for 50 oscillations is noted using a stop watch.

Theoretically $T = 2\pi\sqrt{\frac{L}{g}}$ $\Rightarrow$ $g = 4\pi^2 \frac{L}{T^2}$ ....(1)

where L = Equivalent length of pendulum = length of thread (ℓ) + radius (r) of bob,
T = time period of the simple pendulum
so 'g' can be easily determined by equation ...(1).

Graphical method to find 'g' :

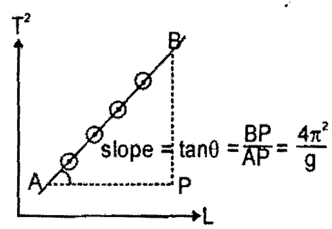
$$T^2 = \left(\frac{4\pi^2}{g}\right)L \quad \ldots\ldots\ldots\ldots(2)$$

so, $T^2 \propto L$

* Find T for different values of L.

* Plot T^2 v/s L curve. From equation (2), it should be a straight line, with slope = $\left(\frac{4\pi^2}{g}\right)$.

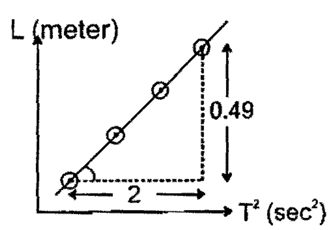
Find slope of T^2 v/s L graph and equate it to $\left(\frac{4\pi^2}{g}\right)$ and get 'g'.



1.15 In certain observation we got ℓ = 23.2 cm, r = 1.32 cm and time taken for 10 oscillation was 10.0 sec. Estimate the value of 'g' in proper significant figure. (take π^2 = 10)

(A) 9.8 m/s² (B) 9.80 m/s² (C) 9.800 m/s² (D) none of these

1.16 For different values of L, we get different values of 'T'. The curve between L v/s T^2 is shown. Estimate 'g' from this curve. (take π^2 = 10)

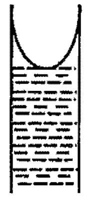


(A) 9.7 m/s² (B) 9.6 m/s² (C) 9.8 m/s² (D) 10 m/s²

Comprehension # 2

Working : Resonance tube is a 100 cm tube. Initially it is filled with water. To increase the length of air column in the tube, water level is lowered. The air column is forced with a tuning fork of frequency f_0. Let at length ℓ_1, we get a first resonance (loud voice) then

$$\ell_{eq_1} = \frac{V}{4f_0}$$

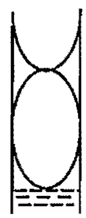


$$\Rightarrow \quad \ell_1 + \varepsilon = \frac{V}{4f_0} \quad ....(i)$$

where ε is end correction

If we further lower the water level, the noise becomes moderate. But at ℓ_2. We, again get a loud noise (second resonance) then

$$\ell_{eq_2} = \frac{3V}{4f_0}$$



$$\Rightarrow \quad \ell_2 + \varepsilon = \frac{3V}{4f_0} \quad ....(ii)$$

From (i) and (ii)

$$V = 2f_0 (\ell_2 - \ell_1)$$

Observation table :

Room temperature is 27°C

		Position of water level (cm)			
Freq. of tuning fork in (Hz) (f_0)	Resonance	Water level is falling	Water level is rising	Mean resonant length	Speed of sound $V = 2f_0(l_2 - l_1)$
330 Hz	1st Resonance	23.9	24.1	l_1 = ...	V = ...
	2nd Resonance	73.9	74.1	l_2 = ...	

1.17 Speed of sound calculated is roughly

(A) 340 m/sec (B) 380 m/sec (C) 430 m/sec (D) 330 m/s

1.18 In the previous question, speed of sound at 0°C is roughly

(A) 324 m/sec (B) 380 m/sec (C) 430 m/sec (D) 314 m/s

1.19 What should be minimum length of tube, so that third resonance can also be heard.

(A) ℓ_3 = 421 cm (B) ℓ_3 = 214 cm (C) ℓ_3 = 124 cm (D) None of these

1.20 From equation (i) and (ii) end correction can be calcualted. Estimate the diameter of the tube imparical formula ($\varepsilon \approx 0.3d$)

(A) 2.5 cm (B) 3.3 cm (C) 5.2 cm (D) None of these

SECTION - V : MATRIX - MATCH TYPE

1.21. Some physical quantities are given in **Column I** and some possible SI units in which these quantities may be expressed are given in **Column II**. Match the physical quantities in **Column I** with the units in **Column II**.

	Column I		Column II
(A)	GM_eM_s G - universal gravitational constant, M_e - mass of the earth, M_s - mass of the Sun	(p)	(volt) (coulomb) (metre)
(B)	$\frac{3RT}{M}$ R - universal gas constant, T - absolute temperature, M - molar mass	(q)	(kilogram) $(\text{metre})^3$ $(\text{second})^{-2}$
(C)	$\frac{F^2}{q^2B^2}$ F - force, q - charge, B - magnetic field	(r)	$(\text{metre})^2$ $(\text{second})^{-2}$
(D)	$\frac{GM_e}{R_e}$ G - universal gravitational constant, M_e - mass of the earth R_e - radius of the earth	(s)	(farad) $(\text{volt})^2$ $(\text{kg})^{-1}$

SECTION - VI : INTEGER TYPE

1.22 In some observations, value of 'g' are coming as 9.81, 9.80, 9.82, 9.79, 9.78, 9.84, 9.79, 9.78, 9.79 and 9.80 m/s^2. Calculate absolute errors and percentage error in g. is $\frac{x}{100}$% then x is.

1.23 From Meter Bridge, resistivity of a wire comes from

$$\rho = \frac{\pi D^2 s}{4L} \quad \frac{\ell}{100-\ell}$$

where ℓ is balance length, D is diameter of wire, S is resistance, L is total length of wire. Find the value of ℓ (in cm) corresponding to max. permissible error in ρ.

1.24 A physical quantity x is calculated from the relation $x = \frac{a^2b^3}{c\sqrt{d}}$. If % error in a, b, c and d are 2%, 1%, 3% and 4% respectively, what is the percentage error in x.

TOPIC 2 RECTILINEAR MOTION

SECTION - I : STRAIGHT OBJECTIVE TYPE

2.1 A body starts from rest and is uniformly accelerated for 30 s. The distance travelled in the first 10 s is x_1, next 10 s is x_2 and the last 10 s is x_3. Then $x_1 : x_2 : x_3$ is :

(A) 1 : 2 : 4 (B) 1 : 2 : 5 (C) 1 : 3 : 5 (D) 1 : 3 : 9

2.2 A ball is dropped from the top of a building. The ball takes 0.5 s to fall past the 3 m length of a window some distance from the top of the building. If the velocities of the ball at the top and at the bottom of the window are v_T and v_B respectively, then (take g = 10 m/s²) :

(A) $v_T + v_B = 12\ ms^{-1}$ (B) $v_T - v_B = 4.9\ m\ s^{-1}$ (C) $v_B v_T = 1\ ms^{-1}$ (D) $\frac{v_B}{v_T} = 1\ ms^{-1}$

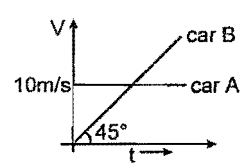
2.3 A point moves in a straight line under the retardation av^2. If the initial velocity is u, the distance covered in ' t ' seconds is :

(A) a u t (B) $\frac{1}{a}\ell n\,(a\,u\,t)$ (C) $\frac{1}{a}\ell n\,(1 + a\,u\,t)$ (D) $a\,\ell n\,(a\,u\,t)$

2.4 A particle is thrown upwards from ground. It experiences a constant resistance force which can produce retardation of 2 m/s². The ratio of time of ascent to the time of descent is [g = 10 m/s²] :

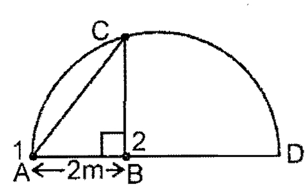
(A) 1 : 1 (B) $\sqrt{\frac{2}{3}}$ (C) $\frac{2}{3}$ (D) $\sqrt{\frac{3}{2}}$

2.5 Initially car A is 10.5 m ahead of car B. Both start moving at time t = 0 in the same direction along a straight line. The velocity time graph of two cars is shown in figure. The time when the car B will catch the car A, will be :



(A) t = 21 sec (B) t = $2\sqrt{5}$ sec (C) 20 sec. (D) None of these

2.6 A semicircle of radius R = 5m with diameter AD is shown in figure. Two particles 1 and 2 are at points A and B on shown diameter at t = 0 and move along segments AC and BC with constant speeds u_1 and u_2 respectively. Then the value of $\frac{u_1}{u_2}$ for both particles to reach point C simultaneously will be :

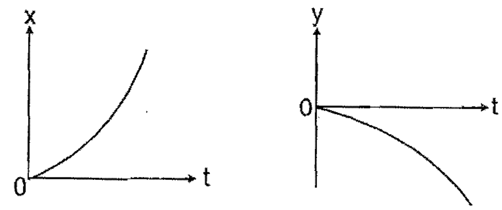


(A) $\frac{5\sqrt{2}}{4}$ (B) $\frac{2\sqrt{2}}{5}$ (C) $2\sqrt{2}$ (D) $\sqrt{\frac{5}{4}}$

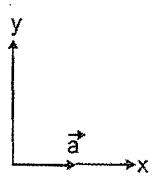
2.7 Two bikes A and B start from a point. A moves with uniform speed 40 m/s and B starts from rest with uniform acceleration 2 m/s^2. If B starts at t = 0 and A starts from the same point at t = 10 s, then the time during the journey in which A was ahead of B is :

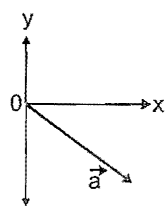
(A) 20 s (B) 8 s
(C) 10 s (D) A was never ahead of B

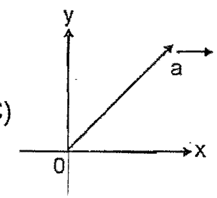
2.8 Graphs I and II give coordinates x(t) and y(t) of a particle moving in the x-y plane Acceleration of the particle is constant and the graphs are drawn to the same scale. Which of the vector shown in options best represents the acceleration of the particle :

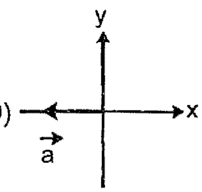


(I) (II)

(A) 

(B) 

(C) 

(D) 

2.9 An insect moving along a straight line, (without returning) travels in every second distance equal to the magnitude of time elapsed. Assuming acceleration to be constant, and the insect starts at t = 0. Find the magnitude of initial velocity of insect.

(A) $\frac{1}{2}$ unit (B) $\frac{1}{4}$ unit (C) $\frac{3}{2}$ unit (D) **1** unit

2.10 A stone is dropped from the top of building and at the same time a second stone is thrown vertically upward from the bottom of the building with a speed of 20 ms^{-1}. They pass each other 3 seconds later. Find the height of the building.

(A) 40 m (B) 60 m (C) 65 m (D) 80 m

2.11 The position vector of a particle is given as $\vec{r} = (t^2 - 4t + 6)\hat{i} + (t^2)\hat{j}$. The time after which the velocity vector and acceleration vector becomes perpendicular to each other is equal to

(A) 1sec (B) 2 sec (C) 1.5 sec (D) not possible

2.12 Each of the four particles move along x axis. Their coordinates (in metres) as function of time (in seconds) are given by

Particle 1 : $x(t) = 3.5 - 2.7t^3$ Particle 2 : $x(t) = 3.5 + 2.7t^3$
Particle 3 : $x(t) = 3.5 + 2.7t^2$ Particle 4 : $x(t) = 3.5 - 3.4t - 2.7t^2$

which of these particles is speeding up for t > 0?

(A) All four (B) only 1 (C) only 1, 2 and 3. (D) only 2, 3 and 4.

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

2.13 Mark the correct statements:

(A) Average velocity of a particle moving on a straight line was zero in a time interval but its instantaneous velocity was never zero in that interval (Consider finite acceleration only)

(B) Average velocity of a particle was zero in a time interval but its speed was never zero in that interval (Consider finite acceleration only)

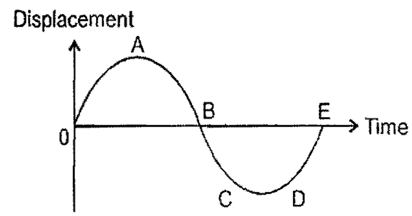
(C) We can have a situation where magnitude of acceleration is non-zero, but rate of change of speed is zero

(D) Average speed of particle in a given time interval is always greater than or equal to the magnitude of the average velocity in the same time interval.

2.14 Which of the following statements are true for a moving body ?
(A) if its speed changes, its velocity must change and it must have some acceleration.
(B) if its velocity changes, its speed must change and it must have some acceleration.
(C) if its velocity changes, its speed may or may not change, and it must have some acceleration.
(D) if its speed changes but direction of motion does not change, its velocity may remain constant.

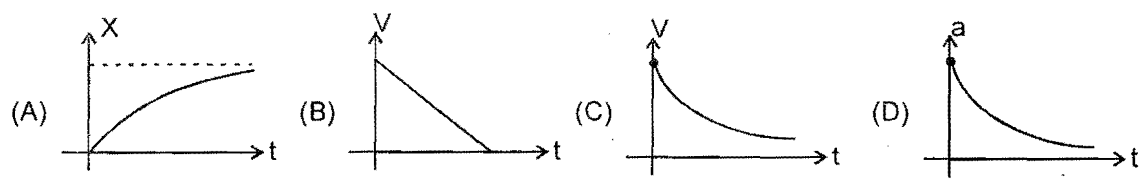
2.15 If velocity of the particle is given by $v = \sqrt{x}$, where x denotes the position of the particle and initially particle was at x = 4, then which of the following are correct?
(A) at t = 2 sec, the position of the particle is at x = 9.
(B) Particle's acceleration at t = 2 sec. is 1 m/s^2
(C) Particle's acceleration is $\frac{1}{2}$ m/s^2 throughout the motion.
(D) Particle will never go in negative direction from it's starting position.

2.16 A particle has a rectilinear motion and the figure gives its displacement as a function of time. Which of the following statements are true with respect to the motion ?



(A) In the motion between O and A the velocity is positive and acceleration is negative.
(B) Between A and B the velocity and acceleration are positive .
(C) Between B and C the velocity is negative and acceleration is positive .
(D) Between D and E the acceleration is positive.

2.17 A rabit is moving in straight line towards a carrot, slowing down its speed so that in each second it moves half the remaining distance from his nose to a carrot. If the total distance travelled by the rabbit in time t is X, its instanteneous speed V and magnitude of its instanteneous acceleration 'a' then which of the following graph(S) is/are best representing the motion ?



2.18 A particle moves with an initial velocity v_0 and retardation βv, where v is its velocity at any time t (β is a positive constant).
(A) the particle will cover a total distance of v_0/β
(B) the particle will continue to move for a very long time
(C) the particle will stop shortly
(D) the velocity of particle will become $v_0/2$ after time $1/\beta$.

SECTION - III : ASSERTION AND REASON TYPE

2.19 **Statement 1 :** Magnitude of average velocity is equal to average speed.
Statement 2 : Magnitude of instantaneous velocity is equal to instantaneous speed.
(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1.
(C) Statement-1 is True, Statement-2 is False.
(D) Statement-1 is False, Statement-2 is True.

2.20 **Statement 1 :** When velocity of a particle is zero then acceleration of particle is also zero.
Statement 2 : Acceleration is equal to rate of change of velocity.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.
(C) Statement-1 is True, Statement-2 is False.
(D) Statement-1 is False, Statement-2 is True.

SECTION - IV : COMPREHENSION TYPE

Comprehension # 1

A particle moves along x-axis and its acceleration at any time t is $a = 2\sin(\pi t)$, where t is in seconds and a is in m/s². The initial velocity of particle (at time t = 0) is u = 0.

2.21 Then the distance travelled (in meters) by the particle from time t = 0 to t = 1 s will be :

(A) $\frac{2}{\pi}$ (B) $\frac{1}{\pi}$ (C) $\frac{4}{\pi}$ (D) None of these

2.22 Then the distance travelled (in meters) by the particle from time t = 0 to t = t will be :

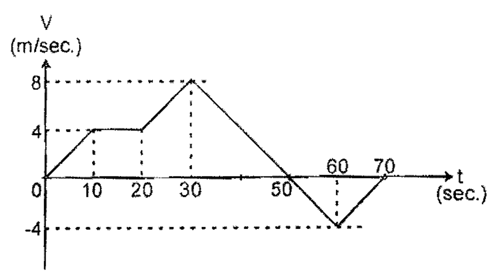
(A) $\frac{2}{\pi^2}\sin\pi t - \frac{2t}{\pi}$ (B) $-\frac{2}{\pi^2}\sin\pi t + \frac{2t}{\pi}$ (C) $\frac{2t}{\pi}$ (D) None of these

2.23 Then the magnitude of displacement (in meters) by the particle from time t = 0 to t = t will be :

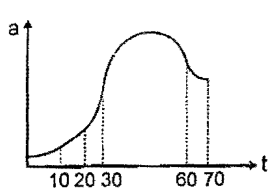
(A) $\frac{2}{\pi^2}\sin\pi t - \frac{2t}{\pi}$ (B) $-\frac{2}{\pi^2}\sin\pi t + \frac{2t}{\pi}$ (C) $\frac{2t}{\pi}$ (D) None of these

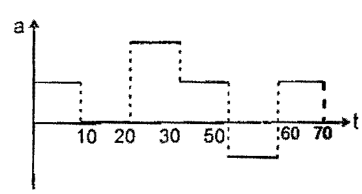
Comprehension # 2

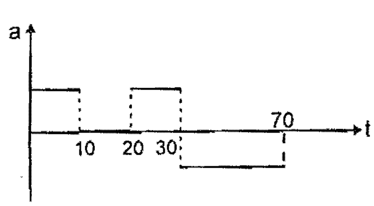
A car is moving on a straight road. The velocity of the car varies with time as shown in the figure. Initially (at t = 0), the car was at x = 0, where, x is the position of the car at any time 't'.

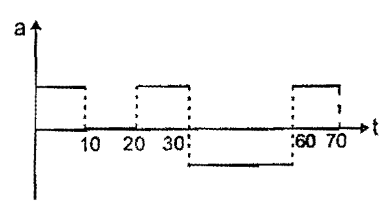


2.24 The variation of acceleration (a) with time (t) will be best represented by :

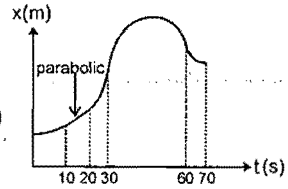
(A)


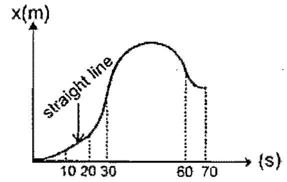
(B)


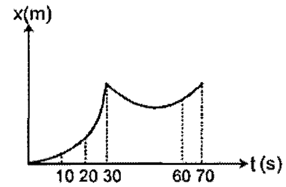
(C)


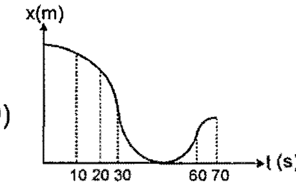
(D)


2.25 The displacement time graph will be best represented by :

(A)


(B)


(C)


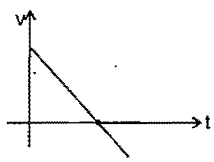
(D)


2.26 The maximum displacement from the starting position will be :
(A) 200 m (B) 250 m (C) 160 m (D) 165 m

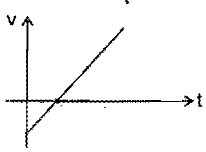
SECTION - V : MATRIX - MATCH TYPE

2.27 The velocity time graph for a particle moving along a straight line is given in each situation of column-I. In the time interval $\infty > t > 0$, match the graph in column-I with corresponding statements in column-II.

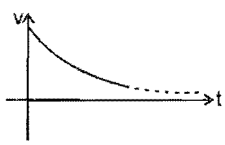
Column-I **Column-II**

(A)


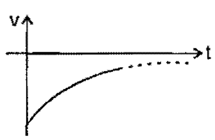
(p) speed of particle is continously decreasing.

(B)


(q) magnitude of acceleration of particle is decreasing with time.

(C)


(r) direction of acceleration of particle does not change.

(D)


(s) magnitude of acceleration of particle is increasing with time.

(t) magnitude of acceleration of particle does not change.

2.28 The equation of motion of the particle is described in column I. At t = 0, particle is at origin and at rest. Match the column I with the statements in column II .

	Column I		**Column II**
(A)	$x = (3t^2 + 2t)$m	(p)	velocity of particle at t = 1 s is 8 m/s.
(B)	$v = 8t$ m/s	(q)	particle moves with uniform acceleration.
(C)	$a = 16\,t$	(r)	particle moves with variable acceleration.
(D)	$v = 6t - 3t^2$	(s)	acceleration of the particle at t = 1sec is $2m/s^2$
		(t)	particle will change its direction some time.

SECTION - VI : INTEGER TYPE

2.29 A railway track runs parallel to a road until a turn brings the road to railway crossing. A cyclist rides along the road everyday at a constant speed 20 km/hr. He normally meets a train that travels in same direction at the crossing. One day he was late by 25 minutes and met the train 10 km before the railway crossing. The speed of the train (in km/hr) is 30N then the N is.

TOPIC 3 PROJECTILE MOTION

SECTION - I : STRAIGHT OBJECTIVE TYPE

3.1 A body is thrown horizontally with a velocity $\sqrt{2gh}$ from the top of a tower of height h. It strikes the level ground through the foot of the tower at a distance x from the tower. The value of x is :

(A) h (B) $\frac{h}{2}$ (C) 2h (D) $\frac{2h}{3}$

3.2 It was calculated that a shell when fired from a gun with a certain velocity and at an angle of elevation $\frac{5\pi}{36}$ rad should strike a given target. In actual practice, it was found that a hill just prevented the trajectory. At what angle of elevation should the gun be fired to hit the target?

(A) $\frac{5\pi}{36}$ rad (B) $\frac{11\pi}{36}$ rad (C) $\frac{7\pi}{36}$ rad (D) $\frac{13\pi}{36}$ rad.

3.3 A ball is projected horizontally with a speed v from the top of a plane inclined at an angle 45° with the horizontal. How far from the point of projection will the ball strike the plane?

(A) $\frac{v^2}{g}$ (B) $\sqrt{2}\,\frac{v^2}{g}$ (C) $\frac{2v^2}{g}$ (D) $\sqrt{2}\left[\frac{2v^2}{g}\right]$

3.4 Consider a boy on a trolley who throws a ball with speed 20 m/s at an angle 37° with respect to trolley in direction of motion of trolley which moves horizontally with speed 10 m/s then what will be maximum distance travelled by ball parallel to road :

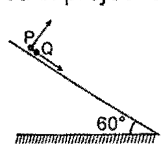
(A) 20.2 m (B) 12 m (C) 31.2 m (D) 62.4 m

3.5 A particle is projected up the inclined plane such that its component of velocity along the incline is 10 m/s. Time of flight is 2 sec and maximum height above the incline is 5 m. Then velocity of projection will be :

(A) 10 m/s (B) $10\sqrt{2}$ m/s (C) $5\sqrt{5}$ m/s (D) none

3.6 Two men A and B, A standing on the extended floor nearby a building and B is standing on the roof of the building. Both throw a stone each towards each other. Then which of the following will be correct:
(A) stone will hit A, but not B
(B) stone will hit B, but not A
(C) stone will not hit either of them, but will collide with each other
(D) none of these

3.7 A particle P is projected from a point on the surface of smooth inclined plane (see figure). Simultaneously another particle Q is released on the smooth inclined plane from the same position. P and Q collide on the inclined plane after t = 4 second. The speed of projection of P is : (in m/s)



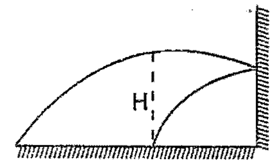
(A) 5 m/s (B) 10 m/s (C) 15 m/s (D) 20 m/s

3.8 A particle is projected from a point (0, 1) on Y-axis (assume + Y direction vertically upwards) aiming towards a point (4, 9). It fell on ground along x axis in 1 sec. Taking g = 10 m/s² and all coordinate in metres. Find the X-coordinate where it fell :

(A) (3, 0) (B) (4, 0) (C) (2, 0) (D) $(2\sqrt{5}, 0)$

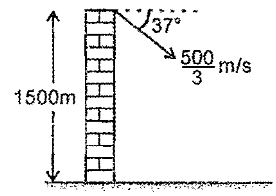
3.9 Velocity of a stone projected, 2 second before it reaches the maximum height, makes angle 53° with the horizontal then the velocity at highest point will be : (in m/s)
(A) 20 m/s (B) 15 m/s (C) 25 m/s (D) 80/3 m/s

3.10 A stone is projected from a horizontal plane. It attains maximum height 'H' & strikes a stationary smooth wall & falls on the ground vertically below the maximum height. Assume the collision to be elastic, the height of the point on the wall where ball will strike is :



(A) $\frac{H}{2}$ (B) $\frac{H}{4}$ (C) $\frac{3H}{4}$ (D) None of these

3.11 A particle is projected from a tower as shown in figure, then the distance from the foot of the tower where it will strike the ground will be : (take g = 10 m/s²)

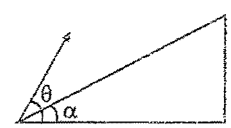


(A) 4000/3 m
(B) 5000/3 m
(C) 2000 m
(D) 3000 m

3.12 Distance between a frog and an insect on a horizontal plane is 10 m. Frog can jump with a maximum speed of $\sqrt{10}$ m/s. g =10 m/s². Minimum number of jumps required by the frog to catch the insect is :
(A) 5 (B) 10 (C) 100 (D) 50

3.13 A particle starts from the origin at t = 0 and moves in the x-y plane with constant acceleration 'a' in the y direction. Its equation of motion is $y = bx^2$. The x component of its velocity (at t = 0) is :
(A) variable (B) $\sqrt{\frac{2a}{b}}$ (C) $\frac{a}{2b}$ (D) $\sqrt{\frac{a}{2b}}$

3.14 A projectile is fired at an angle θ with the horizontal. Find the condition under which it lands perpendicular on an inclined plane inclination α as shown in figure.



(A) $\sin\alpha = \cos(\theta - \alpha)$ (B) $\cos\alpha = \sin(\theta - \alpha)$
(C) $\tan\theta = \cot(\theta - \alpha)$ (D) $\cot(\theta - \alpha) = 2\tan\alpha$

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

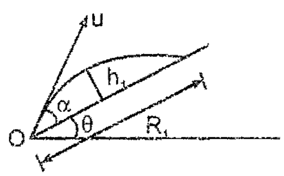
3.15 A particle is projected at an angle θ from ground with speed u (g = 10 m/s²) :
(A) if u = 10 m/s and θ = 30°, then time of flight will be 1 sec.
(B) if u = $10\sqrt{3}$ m/s and θ = 60°, then time of flight will be 3 sec.
(C) if u = $10\sqrt{3}$ m/s and θ = 60°, then after 2 sec velocity becomes perpendicular to initial velocity.
(D) if u = 10 m/s and θ = 30°, then velocity never becomes perpendicular to initial velocity during its flight.

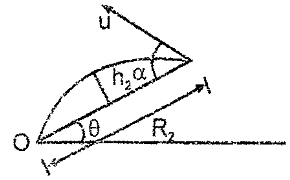
3.16 A particle is projected vertically upwards with a velocity u from a point O . When it returns to the point of projection :
(A) its average velocity is zero (B) its displacement is zero
(C) its average speed is u/2 (D) its average speed is u.

3.17 A particle of mass m moves along a curve $y = x^2$. When particle has x – co-ordinate as 1/2 and x component of velocity as 4m/s then:
(A) the position coordinate of particle are (1/2, 1/4)
(B) the velocity of particle will be along the line 4x – 4y – 1 = 0.
(C) the magnitude of velocity at that instant is $4\sqrt{2}$ m/s
(D) the magnitude of angular momentum of particle about origin at that position is 0.

3.18 A stone is projected from level ground at time t=0. Let v_x and v_y are the horizontal and vertical components of velocity at any time t ; x and y are displacements along horizontal and vertical from the point of projection at any time t. Then:
(A) v_y – t graph is a straight line
(B) x – t graph is a straight line passing through origin
(C) y – t graph is a straight line passing through origin
(D) v_x – t graph is a straight line

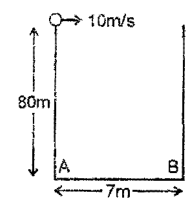
3.19 Two balls are thrown from an inclined plane at angle of projection α with the plane one up the incline plane and other down the incline as shown in the figure. If R_1 & R_2 be their respective ranges and h_1 & h_2 be their respective maximum height then





(A) $h_1 = h_2$
(B) $R_2 - R_1 = T_1^2$
(C) $R_2 - R_1 = g \sin\theta\, T_2^2$
(D) $R_2 - R_1 = g \sin\theta\, T_1^2$

3.20 A ball is projected horizontally from top of a 80 m deep well with velocity 10 m/s. Then particle will fall on the bottom at a distance of (all the collisions with the wall are elastic) :



(A) 5 m from A
(B) 5 m from B
(C) 2 m from A
(D) 2 m from B

SECTION - III : ASSERTION AND REASON TYPE

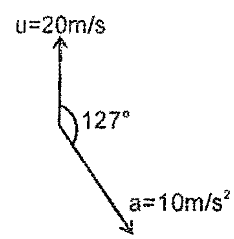
3.21 **Statement-1 :** Two stones are simultaneously projected from level ground from same point with same speed but different angles with horizontal. Both stones move in same vertical plane. Then the two stones may collide in mid air.
Statement-2 : For two stones projected simultaneously from same point with same speed at different angles with horizontal, their trajectories may intersect at some point.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

3.22 **Statement-1 :** During flight under action of gravity, the change in velocity of a projectile in same time intervals is same. (Neglect air friction)
Statement- 2 : Neglecting air friction, the acceleration of projectile is constant during flight.
(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

SECTION - IV : COMPREHENSION TYPE

Comprehension

The direction of velocity of a particle at time t=0 is as shown in the figure and has magnitude u = 20m/s. The acceleration of particle is always constant and has magnitude 10m/s². The angle between its initial velocity and acceleration is 127°. (Take sin 37° = 3/5)



3.23 The instant of time at which acceleration and velocity are perpendicular is :
(A) 0.6 sec.
(B) 1.2 sec.
(C) 2.4 sec.
(D) None of these

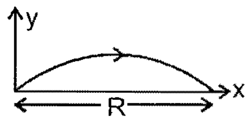
3.24 The instant of time at which speed of particle is least :
(A) 0.6 sec. (B) 1.2 sec. (C) 2.4 sec. (D) None of these

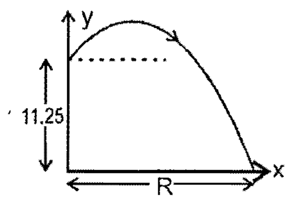
3.25 The instant of time t at which acceleration of particle is perpendicular to its displacement (displacement from t = 0 till that instant t) is :
(A) 0.6 sec. (B) 1.2 sec. (C) 2.4 sec. (D) None of these

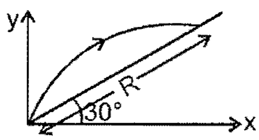
SECTION - V : MATRIX - MATCH TYPE

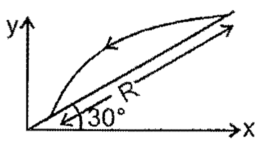
3.26 In the column-I, the path of a projectile (initial velocity 10 m/s and angle of projection with horizontal 60° in all cases) is shown in different cases. Range 'R' is to be matched in each case from column-II. Take g = 10 m/s^2. Arrow on the trajectory indicates the direction of motion of projectile.
Match each entry of column–I with its corresponding entry in column–II

Column-I

(A) 

(B) 

(C) 

(D) 

Column-II

(p) $R = \frac{15\sqrt{3}}{10}$ m

(q) $R = \frac{40}{3}$ m

(r) $R = 5\sqrt{3}$ m

(s) $R = \frac{20}{3}$ m

(t) R = 100 m

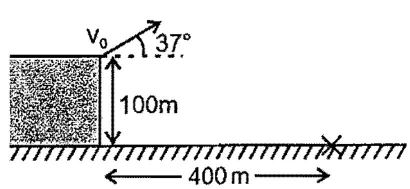
3.27 A ball is thrown at an angle 75° with the horizontal at a speed of 20 m/s towards a high wall at a distance d. If the ball strikes the wall, its horizontal velocity component reverses the direction without change in magnitude and the vertical velocity component remains same. Ball stops after hitting the ground. Match the statement of column I with the distance of the wall from the point of throw in column II .

Column I		Column II
(A) Ball strikes the wall directly	(p)	d = 8 m
(B) Ball strikes the ground at x = 12 m from the wall	(q)	d = 10 m
(C) Ball strikes the ground at x = 10 m from the wall	(r)	d = 15 m
(D) Ball strikes the ground at x = 5 m from the wall	(s)	d = 25 m
	(t)	d = 30 m

SECTION - VI : INTEGER TYPE

3.28 A stone is dropped from a height of 45 m from horizontal level ground. There is horizontal wind blowing due to which horizontal acceleration of the stone becomes 10 m/s^2. (Take g= 10m/s^2) Find time taken by stone to reach the ground. (in m/s)

3.29 A projectile is fired into the air from the edge of a 100 m high cliff at an angle of 37° above the horizontal. The projectile hits a target 400 m away from the base of the cliff. If initial velocity of the projectile, v_0 is $x\sqrt{5}$ m/s then x is ? (Neglect air friction and assume x-axis to be horizontal and y-axis to be vertical).



TOPIC 4 RELATIVE MOTION

SECTION - I : STRAIGHT OBJECTIVE TYPE

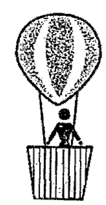
4.1 A train is standing on a platform , a man inside a compartment of the train drops a stone . At the same instant train starts to move with constant acceleration . The path of the particle as seen by the person who drops the stone is :
(A) parabola
(B) straight line for sometime & parabola for the remaining time
(C) straight line
(D) variable path that cannot be defined

4.2 A man wearing a hat of extended length 12 cm is running in rain falling vertically downwards with speed 10 m/s. The maximum speed with which man can run, so that rain drops does not fall on his face (the length of his face below the extended part of the hat is 16 cm) will be :
(A) 7.5 m/s (B) 13.33 m/s (C) 10 m/s (D) zero

4.3 A car starts with constant acceleration $a = 2 m/s^2$ at $t = 0$. Two coins are released from the car at $t = 3$ & $t = 4$. Each coin takes 1 second to fall on ground. Then the distance between the two coins will be : (Assume coin sticks to the ground)
(A) 9 m (B) 7 m (C) 15 m (D) 2m

4.4 A swimmer crosses the river along the line making an angle of 45° with the direction of flow. Velocity of the river is 5 m/s. Swimmer takes 6 seconds to cross the river of width 60 m. The velocity of the swimmer with respect to water will be :
(A) 10 m/s (B) 12 m/s (C) $5\sqrt{5}$ m/s (D) $10\sqrt{2}$ m/s

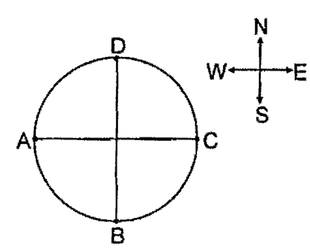
4.5 A man in a balloon, throws a stone downwards with a speed of 5 m/s with respect to balloon . The balloon is moving upwards with a constant acceleration of $5 m/s^2$. Then velocity of the stone relative to the man after 2 second is :
(A) 10 m/s (B) 30 m/s (C) 15 m/s (D) 35 m/s



4.6 Three stones A, B and C are simultaneously projected from same point with same speed. A is thrown upwards, B is thrown horizontally and C is thrown downwards from a building. When the distance between stone A and C becomes 10 m, then distance between A and B will be:
(A) 10 m (B) 5 m (C) $5\sqrt{2}$ m (D) $10\sqrt{2}$ m

4.7 A projectile A is projected from ground. An observer B running on ground with uniform velocity of magnitude 'v' observes A to move along a straight line. The time of flight of A as measured by B is T. Then the range R of projectile on ground is :
(A) $R = vT$ (B) $R < vT$
(C) $R > vT$ (D) information insufficient to draw inference

4.8 Two aeroplanes fly from their respective position 'A' and 'B' starting at the same time and reach the point 'C' (along straight line) simultaneously when wind was not blowing. On a windy day they head towards 'C' but both reach the point 'D' simultaneously in the same time which they took to reach 'C'. Then the wind is blowing in :
(A) North-East direction
(B) North-West direction
(C) Direction making an angle $0 < \theta < 90$ with North towards West.
(D) North direction



4.9 A particle is thrown up inside a stationary lift of sufficient height. The time of flight is T. Now it is thrown again with same initial speed v_0 with respect to lift. At the time of second throw, lift is moving up with speed v_0 and uniform acceleration g upward (the acceleration due to gravity). The new time of flight is:

(A) $\frac{T}{4}$ (B) $\frac{T}{2}$ (C) T (D) 2T

4.10 A swimmer crosses a river with minimum possible time 10 second. And when he reaches the other end starts swimming in the direction towards the point from where he started swimming. Keeping the direction fixed the swimmer crosses the river in 15 sec. The ratio of speed of swimmer with respect to water and the speed of river flow is (Assume constant speed of river & swimmer) :

(A) $\frac{3}{2}$ (B) $\frac{9}{4}$ (C) $\frac{2}{\sqrt{5}}$ (D) $\frac{\sqrt{5}}{2}$

4.11 A taxi leaves the station X for station Y every 10 minutes. Simultaneously, a taxi also leaves the station Y for station X every 10 minutes. The taxis move at the same constant speed and go from X to Y or vice versa in 2 hours. How many taxis coming from the other side will meet each taxi enroute from Y to X:

(A) 11 (B) 12 (C) 23 (D) 24

4.12 Consider a collection of a large number of particles each with speed v. The direction of velocity is randomly distributed in the collection. The magnitude of the relative velocity between a pair of particles averaged over all the pairs in the collection is :

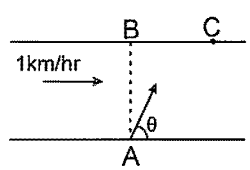
(A) greater than v (B) less than v (C) equal to v (D) we can't say anything

4.13 An aeroplane is flying in geographic meridian vertical plane at an angle of 30^0 above with the horizontal (north) and wind is blowing from west. A package is dropped from an aeroplane. The velocity of the wind if package hits a kite flying in the space with a position vector $\vec{R} = (400\sqrt{3}\hat{i} + 80\hat{j} + 200\hat{k})$ m with respect to the point of dropping.(Here $\hat{i}$ and $\hat{j}$ are the unit vectors along north and vertically up respectively and $\hat{k}$ be the unit vector due east. :

(A) 50 m/sec (B) 25 m/sec (C) 20 m/sec (D) 10 m/sec

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

4.14 A river is flowing with a speed of 1 km/hr. A swimmer wants to go to point 'C' starting from 'A'. He swims with a speed of 5 km/hr, at an angle θ w.r.t. the river. If AB = BC = 400 m. Then :



(A) the value of θ is 53° (B) time taken by the man is 6 min
(C) time taken by the man is 8 min (D) the value of θ is 45°

4.15 A swimmer who can swim in a river with speed mv (with respect to still water) where v is the speed of river current, jumps into the river from one bank to cross the river: m in a position contant]

(A) If $m \le 1$ he can not reach a point on other bank directly opposite to his starting point
(B) If $m < 1$ he can not cross the river
(C) If $m > 1$ then only he can reach a point on other bank
(D) He can reach the other bank at some point, whatever be the value of m.

4.16 A man is standing on a road and observes that rain is falling at angle 45° with the vertical. The man starts running on the road with constant acceleration 0.5 m/s^2. After a certain time from the start of the motion, it appears to him that rain is still falling at angle 45° with the vertical, with speed $2\sqrt{2}$ m/s. Motion of the man is in the same vertical plane in which the rain is falling. Then which of the following statement(s) are true:

(A) It is not possible

(B) Speed of the rain relative to the ground is 2 m/s.

(C) Speed of the man when he finds rain to be falling at angle 45° with the vertical, is 4m/s.

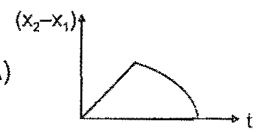
(D) The man has travelled a distance 16m on the road by the time he again finds rain to be falling at angle 45°.

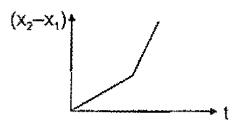
4.17 Two stones are thrown vertically upwards simultaneously from the same point on the ground with initial speed u_1 = 30 m/sec and u_2 = 50 m/sec. Which of the curve represents correct variation (for the time interval in which both reach the ground) of

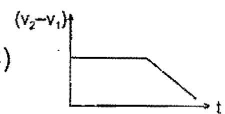
$(x_2 - x_1)$ = the relative position of second stone with respect to first with time (t).

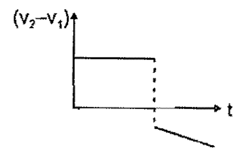
$(v_2 - v_1)$ = the relative velocity of second stone with respect to first with time (t).

Assume that stones do not rebound after hitting the ground

(A) 

(B) 

(C) 

(D) 

SECTION - III : ASSERTION AND REASON TYPE

4.18 **Statement-1 :** The magnitude of velocity of two boats relative to river is same. Both boats start simultaneously from same point on one bank may reach opposite bank simultaneously moving along different paths.

Statement-2 : For boats to cross the river in same time. The component of their velocity relative to river in direction normal to flow should be same.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

4.19 **Statement-1 :** Three projectiles are moving in different paths in the air. Vertical component of relative velocity between any of the pair does not change with time as long as they are in air. Neglect the effect of air friction.

Statement-2 : Relative acceleration between any of the pair of projectiles is zero.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

4.20 **Statement-1 :** Two stones are projected with different velocities from ground from same point and at same instant of time. Then these stones cannot collide in mid air. (Neglect air friction)

Statement-2 : If relative acceleration of two particles initially at same position is always zero, then the distance between the particle either remains constant or increases continuously with time.

(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement 1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True.

SECTION - IV : COMPREHENSION TYPE

Comprehension # 1

A swimmer can swim with a speed v in still water.

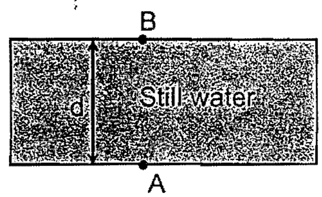


fig (i)

4.21 If the swimmer crosses a swimming pool of width 'd' from A to directly opposite point B on other side in time t_1 as shown in figure (i) and in a flowing river (river velocity 'u') of same width d from A′ to directly opposite point B′ on other bank in time t_2, then (t_1/t_2) is equal to : (Assume v > u)

(A) $\sqrt{1-\frac{v^2}{u^2}}$ (B) $\sqrt{1+\frac{u^2}{v^2}}$ (C) $\sqrt{1-\frac{u^2}{v^2}}$ (D) 1

4.22 If the minimum time taken by swimmer in swimming pool to reach opposite bank is t_1' and minimum. time to reach opposite bank in river is t_2', then the ratio $\frac{t_1'}{t_2'}$ will have a value :

(A) 1 (B) $\frac{\sqrt{v^2+u^2}}{v}$ (C) $\frac{\sqrt{v^2-u^2}}{u}$ (D) $\frac{u}{v}$

4.23 If the time taken by swimmer to reach opposite point on other bank in river is T_1 and the time taken to travel an equal distance upstream (against the water current) in the river is T_2, then ratio $\frac{T_2}{T_1}$ will have a value :

(A) $\sqrt{\frac{1-u/v}{1+u/v}}$ (B) $\sqrt{\frac{1+u/v}{1-u/v}}$ (C) $\frac{\sqrt{v^2-u^2}}{(v+u)}$ (D) $\frac{\sqrt{v^2-u^2}}{v}$

Comprehension # 2

Raindrops are falling with a velocity $10\sqrt{2}$ m/s making an angle of 45° with the vertical. The drops appear to be falling vertically to a man running with constant velocity. The velocity of rain drops change such that the rain drops appear to be falling vertically with now the velocity it appeared earlier to the same person running with same velocity.

4.24 The magnitude of velocity of man with respect to ground is:

(A) $10\sqrt{2}$ m/s (B) $10\sqrt{3}$ m/s (C) 20 m/s (D) 10 m/s

4.25 After the velocity of rain drops change, the magnitude of velocity of raindrops with respect to ground is:

(A) 20 m/s (B) $20\sqrt{3}$ m/s (C) 10 m/s (D) $10\sqrt{3}$ m/s

4.26 The angle (in degrees) between the initial and the final velocity vectors of the raindrops with respect to the ground is:

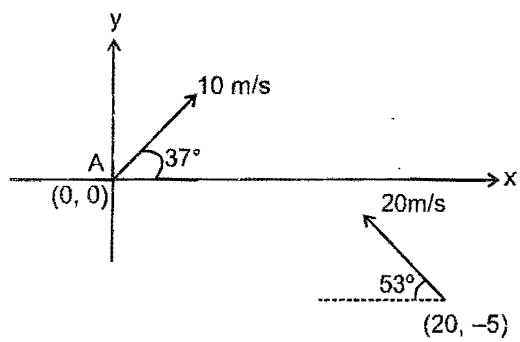
(A) 8 (B) 15 (C) 22.5 (D) 37

SECTION - V : MATRIX - MATCH TYPE

4.27 Two particles A and B moving in x-y plane are at origin at t = 0 sec. The initial velocity vectors of A and B are $\vec{u}_A = 8\hat{i}$ m/s and $\vec{u}_B = 8\hat{j}$ m/s. The acceleration of A and B are constant and are $\vec{a}_A = -2\hat{i}$ m/s^2 and $\vec{a}_B = -2\hat{j}$ m/s^2. Column I gives certain statements regarding particle A and B. Column II gives corresponding results. Match the statements in column I with corresponding results in Column II.

	Column–I		Column–II
(A)	The time (in seconds) at which velocity of A relative to B is zero	(p)	$16\sqrt{2}$
(B)	The distance (in metres) between A and B when their relative velocity is zero.	(q)	$8\sqrt{2}$
(C)	The time (in seconds) after t = 0 sec, at which A and B are at same position	(r)	8
(D)	The magnitude of relative velocity of A and B at the instant they are at same position.	(s)	4
		(t)	6 secconds

4.28 Two particles A & B are projected as shown in fig in x–y plane. Under the effect of force which provide a constant acceleration a = 11 m/s^2 in negative y-direction. Then match situation in column-I with the corresponding results in column-II (All positions are given in metre) ($\vec{V}_{AB}$ = velocity of A w.r.t. B; $\vec{r}_{AB}$ = Position of A w.r.t. B).



	Column–I		Column–II
(A)	Separation between the two particles is minimum at ....sec.	(p)	0
(B)	Minimum separation between the two particles is ....m.	(q)	0.5
(C)	Time when velocities of both particles are perpendicular each other at .... sec.	(r)	0.9
(D)	At the time of minimum separation $\vec{V}_{AB} \cdot \vec{r}_{AB}$ =	(s)	2
		(t)	$2\sqrt{5}$

SECTION - VI : INTEGER TYPE

4.29 When two bodies move uniformly towards each other, the distance between them diminishes by 16 m every 10 s. If velocity of one body is reversed the distance between them will decrease 3 m every 5 s. Itspeed of second body is $\frac{10}{x}$ m/s then x is

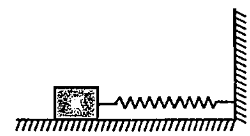
4.30 A swimmer jumps from a bridge over a canal and swims 1km upstream. After that first km, he passes a floating cork. He continues swimming for half an hour and then turns around and swims back to the bridge. The swimmer and the cork reach the bridge at the same time. The swimmer has been swimming at a constant speed. If speed of water in canal is X km/hr., then X is

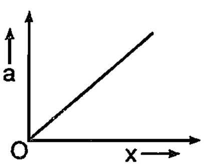
TOPIC 5

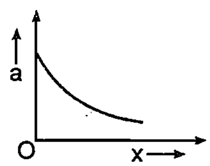
NEWTON'S LAW OF MOTION

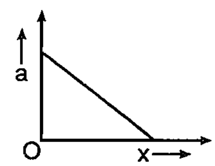
SECTION - I : STRAIGHT OBJECTIVE TYPE

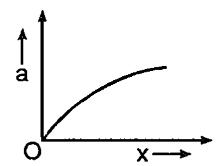
5.1 A light spring is compressed and placed horizontally between a vertical fixed wall and a block free to slide over a smooth horizontal table top as shown in the figure. The system is released from rest. The graph which represents the relation between the magnitude of acceleration 'a' of the block and the distance 'x' travelled by it (as long as the spring is compressed) is :



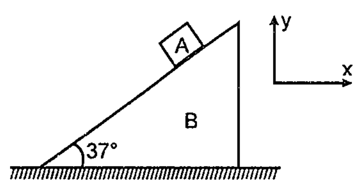
(A) 

(B) 

(C) 

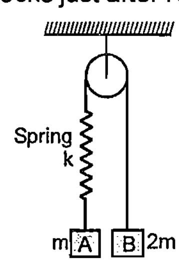
(D) 

5.2 In the figure shown the acceleration of A is, $\vec{a}_A = 15\hat{i}+15\hat{j}$ then the acceleration of B is: (A remains in contact with B)



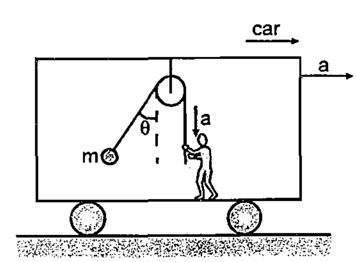
(A) $6\hat{i}$ (B) $-15\hat{i}$ (C) $-10\hat{i}$ (D) $-5\hat{i}$

5.3 Two blocks A and B of masses m & 2m respectively are held at rest such that the spring is in natural length. Find out the accelerations of both the blocks just after release :



(A) $g\downarrow, g\downarrow$ (B) $\frac{g}{3}\downarrow, \frac{g}{3}\uparrow$ (C) 0, 0 (D) $g\downarrow, 0$

5.4 A bob is hanging over a pulley inside a car through a string. The second end of the string is in the hand of a person standing in the car. The car is moving with constant acceleration 'a' directed horizontally as shown in figure. Other end of the string is pulled with constant acceleration 'a' vertically downward. The tension in the string is equal to :



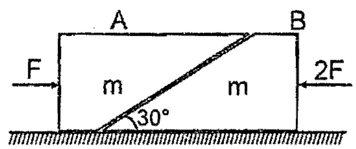
(A) $m\sqrt{g^2+a^2}$ (B) $m\sqrt{g^2+a^2} - ma$

(C) $m\sqrt{g^2+a^2} + ma$ (D) $m(g + a)$

5.5 Inside a horizontally moving box, an experimenter finds that when an object is placed on a smooth horizontal table and is released, it moves with an acceleration of 10 m/s². In this box if 1 kg body is suspended with a light string, the tension in the string in equilibrium position. (w.r.t. experimenter) will be. (Take g = 10 m/s²)

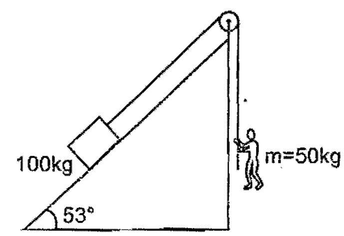
(A) 10 m/s². (B) $10\sqrt{2}$ m/s². (C) 20 m/s². (D) zero

5.6 Two blocks 'A' and 'B' each of mass 'm' are placed on a smooth horizontal surface. Two horizontal force F and 2F are applied on both the blocks 'A' and 'B' respectively as shown in figure. The block A does not slide on block B. Then the normal reaction acting between the two blocks is :



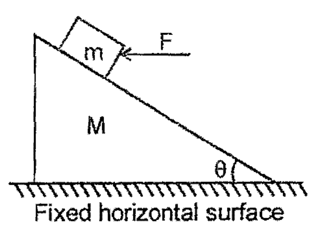
(A) F (B) F/2 (C) $\frac{F}{\sqrt{3}}$ (D) 3F

5.7 In the arrangement shown, by what acceleration the boy must go up so that 100 kg block remains stationary on the wedge. The wedge is fixed and friction is absent everywhere. (Take g = 10 m/s²)



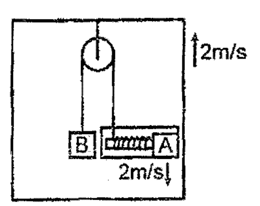
(A) 2 m/s² (B) 4 m/s² (C) 6 m/s² (D) 8 m/s²

5.8 A block of mass m lies on wedge of mass M, which lies on fixed horizontal surface. The wedge is free to move on the horizontal surface. A horizontal force of magnitude F is applied on block as shown, neglecting friction at all surfaces, the value of force F such that block has no relative motion w.r.t. wedge will be : (where g is acceleration due to gravity)



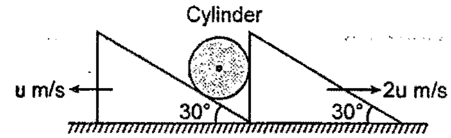
(A) (M + m) g tanθ (B) (M + m) g cotθ (C) $\frac{m}{M}$ (M + m) g tanθ (D) $\frac{m}{M}$ (M + m) g cotθ

5.9. In the figure shown the velocity of lift is 2 m/s while string is winding on the motor shaft with velocity 2 m/s and block A is moving downwards with a velocity of 2 m/s, then find out the velocity of block B.



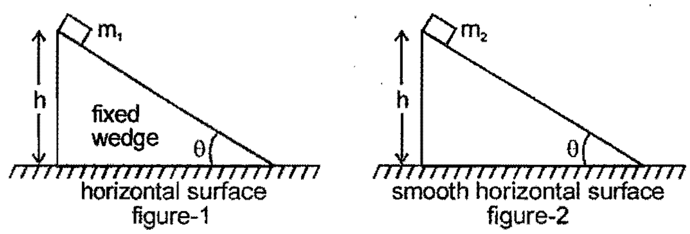
(A) 2 m/s ↑ (B) 2 m/s ↓ (C) 4 m/s ↑ (D) 8 m/s ↑

5.10 System is shown in the figure. Assume that cylinder remains in contact with the two wedges. The velocity of cylinder is -



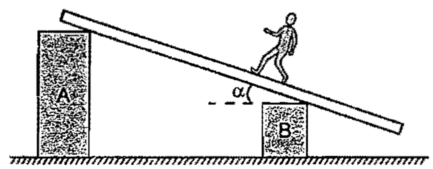
(A) $\sqrt{19-4\sqrt{3}}\,\frac{u}{2}$ m/s (B) $\frac{\sqrt{13}u}{2}$ m/s (C) $\sqrt{3}u$ m/s (D) $\sqrt{7}u$ m/s

5.11 A block of mass m_1 lies on top of fixed wedge as shown in figure-1 and another block of mass m_2 lies on top of wedge which is free to move as shown in figure-2. At time t = 0, both the blocks are released from rest from a vertical height h above the respective horizontal surface on which the wedge is placed as shown. There is no frcition between block and wedge in both the figures. Let T_1 and T_2 be the time taken by block in figure-1 and block in figure-2 respectively to just reach the horizontal surface, then :



(A) $T_1 > T_2$ (B) $T_1 < T_2$ (C) $T_1 = T_2$ (D) Data insufficient

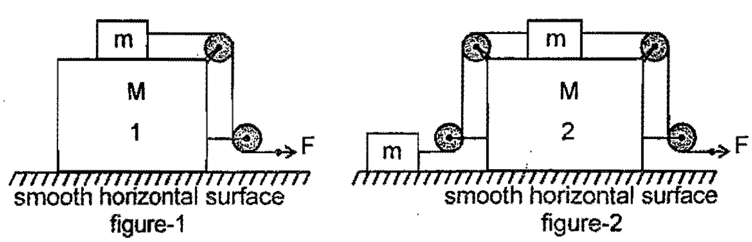
5.12 A plank is held at an angle α to the horizontal (Fig.) on two fixed supports A and B. The plank can slide against the supports (without friction) because of its weight Mg. Acceleration and direction in which a man of mass m should move so that the plank does not move.



(A) $g \sin\alpha\left(1+\frac{m}{M}\right)$ down the incline (B) $g \sin\alpha\left(1+\frac{M}{m}\right)$ down the incline

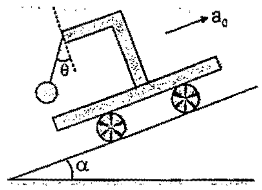
(C) $g \sin\alpha\left(1+\frac{m}{M}\right)$ up the incline (D) $g \sin\alpha\left(1+\frac{M}{m}\right)$ up the incline

5.13 In the situation shown in figure all the string are light and inextensible and pullies are light. There is no friction at any surface and all block are of cuboidal shape. A horizontal force of magnitude F is applied to right most free end of string in both cases of figure 1 and figure 2 as shown. At the instant shown, the tension in all strings are non zero. Let the magnitude of acceleration of large blocks (of mass M) in figure 1 and figure 2 are a_1 and a_2 respectively. Then :



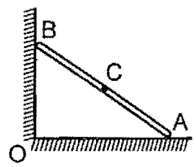
(A) $a_1 = a_2 \neq 0$ (B) $a_1 = a_2 = 0$ (C) $a_1 > a_2$ (D) $a_1 < a_2$

.14 A pendulum of mass m hangs from a support fixed to a trolley. The direction of the string when the trolley rolls up a plane of inclination α with acceleration a_0 is



(A) $\theta = \tan^{-1} \alpha$ (B) $\theta = \tan^{-1}\left(\frac{a_0}{g}\right)$ (C) $\theta = \tan^{-1}\left(\frac{g}{a_0}\right)$ (D) $\theta = \tan^{-1}\left(\frac{a_0 + g\sin\alpha}{g\cos\alpha}\right)$

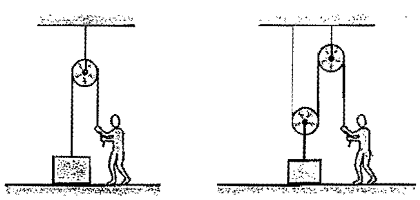
5.15 A rod of length 2ℓ is moving such that its ends A and B move in contact with the horizontal floor and vertical wall respectively as shown in figure. O is the intersection point of the vertical wall and horizontal floor. The velocity vector of the centre of rod C is always directed along tangent drawn at C to the



(A) circle of radius $\frac{\ell}{2}$ whose centre lies at O (B) circle of radius ℓ whose centre lies at O

(C) circle of radius 2ℓ whose centre lies at O (D) None of these

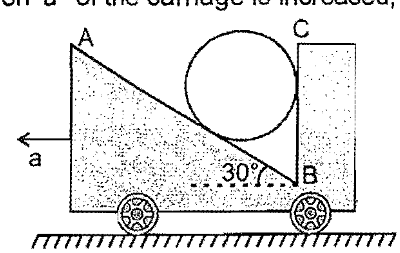
5.16 In the figure shown, a person wants to raise a block lying on the ground to a height h. In both the cases if time required is same then in which case he has to exert more force. Assume pulleys and strings light.



(A) (i) (B) (ii)

(C) same in both (D) cannot be determined

5.17 A cylinder rests in a supporting carriage as shown. The side AB of carriage makes an angle 30° with the horizontal and side BC is vertical. The carriage lies on a fixed horizontal surface and is being pulled towards left with an horizontal acceleration '*a*'. The magnitude of normal reactions exerted by sides AB and BC of carriage on the cylinder be N_{AB} and N_{BC} respectively. Neglect friction everywhere. Then as the magnitude of acceleration '*a*' of the carriage is increased, pick up the correct statement :



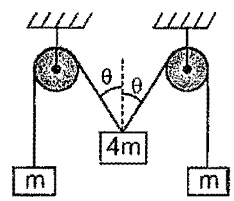
(A) N_{AB} increases and N_{BC} decreases. (B) Both N_{AB} and N_{BC} increase.

(C) N_{AB} remains constant and N_{BC} increases. (D) N_{AB} increases and N_{BC} remains constant.

5.18 A car is moving on a plane inclined at 30° to the horizontal with an acceleration of 9.8 m/s² parallel to the plane upward. A bob is suspended by a string from the roof. The angle in degrees which the string makes with the vertical is : (Assume that the bob does not move relative to car) [g = 9.8 m/s²]

(A) 20°   (B) 30°   (C) 45°   (D) 60°

5.19 In the figure shown, the pulleys and strings are massless. The acceleration of the block of mass 4m just after the system is released from rest is ($\theta = \sin^{-1}\frac{3}{5}$)



(A) $\frac{2g}{5}$ downward   (B) $\frac{2g}{5}$ upwards   (C) $\frac{5g}{11}$ upwards   (D) $\frac{5g}{11}$ downwards

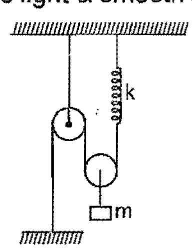
5.20 Five persons A, B, C, D & E are pulling a cart of mass 100 kg on a smooth surface and cart is moving with acceleration 3 m/s² in east direction. When person 'A' stops pulling, it moves with acceleration 1m/s² in the west direction. When person 'B' stops pulling, it moves with acceleration 24 m/s² in the north direction. The magnitude of acceleration of the cart when only A & B pull the cart keeping their directions same as the old directions, is :

(A) 26 m/s²   (B) $3\sqrt{71}$ m/s²   (C) 25 m/s²   (D) 30 m/s²

5.21 A body of mass 32 kg is suspended by a spring balance from the roof of a vertically operating lift and going downward from rest. At the instants the lift has covered 20 m and 50 m, the spring balance showed 30 kg & 36 kg respectively. The velocity of the lift is:

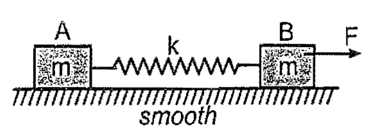
(A) decreasing at 20 m & increasing at 50 m
(B) increasing at 20 m & decreasing at 50 m
(C) continuously decreasing at a constant rate throught the journey
(D) continuously increasing at constant rate throughout the journey

5.22 Mass m shown in the figure is in equilibrium. If it is displaced further by x and released find its acceleration just after it is released. Take pulleys to be light & smooth and strings light.



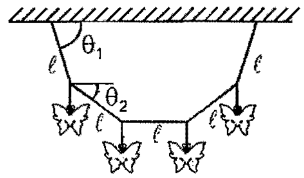
(A) $\frac{4kx}{5m}$ (B) $\frac{2kx}{5m}$ (C) $\frac{4kx}{m}$ (D) none of these

5.23 Initially the spring is undeformed. Now the force 'F' is applied to 'B' as shown in the figure . When the displacement of 'B' w.r.t. 'A' is 'x' towards right in some time then the relative acceleration of 'B' w.r.t. 'A' at that moment is :



(A) $\frac{F}{2m}$ (B) $\frac{F - kx}{m}$ (C) $\frac{F - 2kx}{m}$ (D) none of these

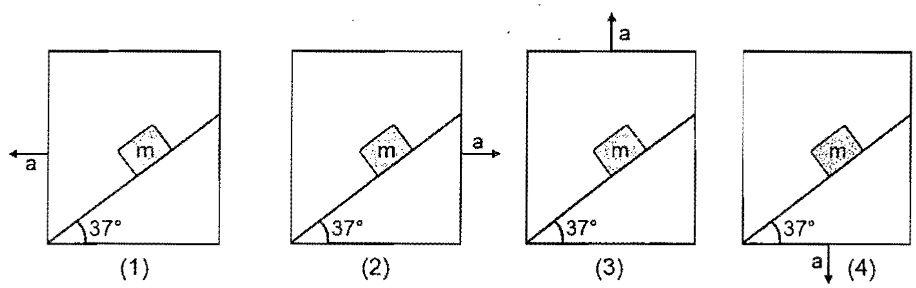
5.24 Four identical metal butterflies are hanging from a light string of length 5ℓ at equally placed points as shown in the figure . The ends of the string are attached to a horizontal fixed support. The middle section of the string is horizontal. The relation between the angle θ_1 and θ_2 is given by.



(A) $\sin\theta_1 = 2\sin\theta_2$ (B) $2\cos\theta_1 = \sin\theta_2$
(C) $\tan\theta_1 = 2\tan\theta_2$ (D) $\theta_2 < \theta_1$ and no other conclusion can be derived.

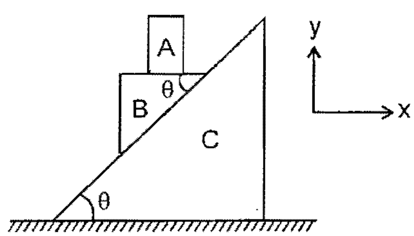
SECTION - II : MULTIPLE CORRECT ANSWER TYPE

5.25 A block of mass m is placed on a wedge. The wedge can be accelerated in four manners marked as (1), (2), (3) and (4) as shown. If the normal reactions in situation (1), (2), (3) and (4) are N_1, N_2, N_3 and N_4 respectively and acceleration with which the block slides on the wedge in situations are b_1, b_2, b_3 and b_4 respectively then :



(A) $N_3 > N_1 > N_2 > N_4$ (B) $N_4 > N_3 > N_1 > N_2$
(C) $b_2 > b_3 > b_4 > b_1$ (D) $b_2 > b_3 > b_1 > b_4$

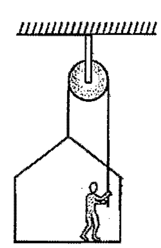
5.26 In the figure shown all the surface are smooth. All the blocks A, B and C are movable, x-axis is horizontal and y-axis vertical as shown. Just after the system is released from the position as shown.



Horizontal Surface

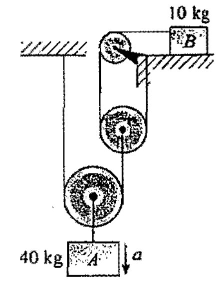
(A) Acceleration of 'A' relative to ground is in negative y-direction
(B) Acceleration of 'A' relative to B is in positive x-direction
(C) The horizontal acceleration of 'B' relative to ground is in negative x-direction.
(D) The acceleration of 'B' relative to ground directed along the inclined surface of 'C' is greater than $g\sin\theta$.

5.27 A painter is applying force himself to raise him and the box with an acceleration of 5 m/s^2 by a massless rope and pulley arrangement as shown in figure. Mass of painter is 100 kg and that of box is 50 kg. If $g = 10\ m/s^2$, then :



(A) tension in the rope is 1125 N
(B) tension in the rope is 2250 N
(C) force of contact between the painter and the floor is 375 N
(D) none of these

5.28 Figure shows two blocks A and B connected to an ideal pulley string system. In this system when bodies are released then : (neglect friction and take g = 10 m/s^2)



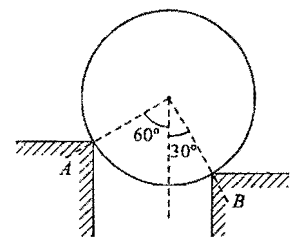
(A) Acceleration of block A is 1 m/s^2

(B) Acceleration of block A is 2 m/s^2

(C) Tension in string connected to block B is 40 N

(D) Tension in string connected to block B is 80 N

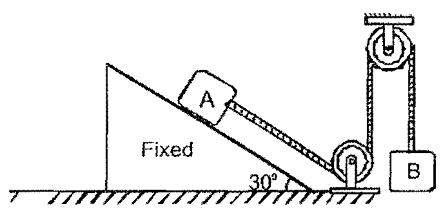
5.29 A cylinder of mass M and radius R is resting on two corner edges A and B as shown in the figure . The normal reaction at the edges A and B are : (Neglect friction)



(A) $N_A = \sqrt{2}N_B$ (B) $N_B = \sqrt{3}N_A$

(C) $N_A = \frac{Mg}{2}$ (D) $N_B = \frac{2\sqrt{3}Mg}{5}$

5.30 Two blocks A and B of equal mass m are connected through a massless string and arranged as shown in the figure . The wedge is fixed on horizontal surface. Friction is absent everywhere. When the system is released from rest.

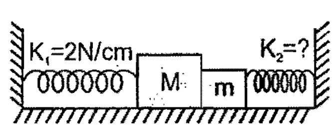


Starting Case

(A) tension in string is $\frac{mg}{2}$ (B) tension in string is $\frac{mg}{4}$

(C) acceleration of A is g/2 (D) acceleration of A is $\frac{3}{4}g$

5.31 Two blocks of mass M and m, are used to compress two different massless springs as shown in the figure. The left spring is compressed by 3 cm, while the right spring is compressed by an unknown amount. The system is at rest and all contact surfaces are smooth. Which of the following statements are true ?

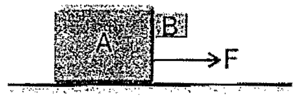


smooth horizontal surface

(A) The force exerted on block of mass m by the right spring is 6 N to the left.
(B) The force exerted on block of mass m by the right spring is impossible to determine.
(C) The net force on block of mass m is zero.
(D) The normal force exerted by block of mass m on block of mass M is 6 N.

SECTION - III : ASSERTION AND REASON TYPE

5.32 **Statement-1 :** Block A is moving on horizontal surface towards right under action of force F . All surfaces are smooth. At the instant shown the force exerted by block A on block B is equal to net force on block B.



Statement-2 : From Newton's third law, the force exerted by block A on B is equal in magnitude to force exerted by block B on A.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

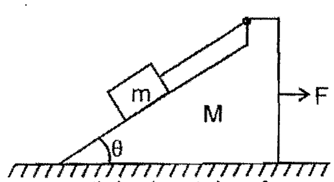
5.33 **Statement-1 :** A man standing in a lift which is moving upward, will feel his weight to be greater than when the lift was at rest.
Statement-2 : If the acceleration of the lift is 'a' upward, then the man of mass m shall feel his weight to be equal to normal reaction (N) exerted by the lift given by $N = m(g+a)$ (where g is acceleration due to gravity)
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

5.34 **Statement-1 :** According to the Newton's third law of motion, the magnitude of the action and reaction force in an action reaction pair is same only in an inertial frame of reference.
Statement-2 : Newton's laws of motion are applicable in every inertial reference frame.
(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True.

SECTION - IV : COMPREHENSION TYPE

Comprehension # 1

A light inextensible string connects a block of mass m and top of wedge of mass M. The string is parallel to inclined surface and the inclined surface makes an angle θ with horizontal as shown in the figure. All surfaces are smooth. Now a constant horizontal force of minimum magnitude F is applied to wedge towards right such that the normal reaction on block exerted by wedge just becomes zero.



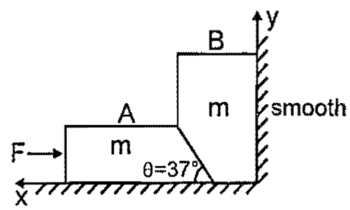
5.35 The magnitude of acceleration of wedge is
(A) g tanθ (B) g cotθ (C) g sinθ (D) g cosθ

5.36 The magnitude of tension in string is
(A) mg secθ (B) mg cosecθ (C) mg tanθ (D) mg cotθ

5.37 The magnitude of net horizontal force on wedge is :
(A) Mg cotθ (B) (M + m)g secθ (C) (M + m)g cotθ (D) Mg cosecθ

Comprehension # 2

Two smooth blocks are placed at a smooth corner as shown in the figure. Both the blocks are having mass m. We apply a force F on the small block m. Block A presses the block B in the normal direction, due to which pressing force on vertical wall will increase, and pressing force on the horizontal wall decrease, as we increase F. (θ = 37° with horizontal). As soon as the pressing force on the horizontal wall by block B becomes zero, it will loose the contact with the ground. If the value of F is further increased, the block B will accelerate in upward direction and simultaneously the block A will move toward right.



5.38 What is minimum value of F, to lift block B from ground :

(A) $\frac{25}{12}$ mg (B) $\frac{5}{4}$ mg (C) $\frac{3}{4}$ mg (D) $\frac{4}{3}$ mg

5.39 If both the blocks are stationary, the force exerted by ground on block A is :

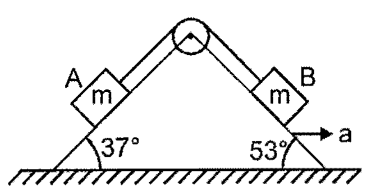
(A) mg + $\frac{3F}{4}$ (B) mg − $\frac{3F}{4}$ (C) mg + $\frac{4F}{3}$ (D) mg − $\frac{4F}{3}$

5.40 If acceleration of block A is a rightward, then acceleration of block B will be :

(A) $\frac{3a}{4}$ upwards (B) $\frac{4a}{3}$ upwards (C) $\frac{3a}{5}$ upwards (D) $\frac{4a}{5}$ upwards

Comprehension # 3

Two blocks A and B of equal masses m kg each are connected by a light thread, which passes over a massless pulley as shown in the figure. Both the blocks lie on wedge of mass m kg. Assume friction to be absent everywhere and both the blocks to be always in contact with the wedge. The wedge lying over smooth horizontal surface is pulled towards right with constant acceleration a (m/s²). (g is acceleration due to gravity)



5.41 Normal reaction (in N) acting on block B is

(A) $\frac{m}{5}(3g + 4a)$ (B) $\frac{m}{5}(3g - 4a)$ (C) $\frac{m}{5}(4g + 3a)$ (D) $\frac{m}{5}(4g - 3a)$

5.42 Normal reaction (in N) acting on block A.

(A) $\frac{m}{5}(3g + 4a)$ (B) $\frac{m}{5}(3g - 4a)$ (C) $\frac{m}{5}(4g + 3a)$ (D) $\frac{m}{5}(4g - 3a)$

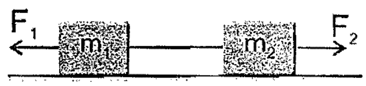
5.43 The maximum value of acceleration a (in m/s²) for which normal reactions acting on the block A and block B are nonzero.

(A) $\frac{3}{4}g$ (B) $\frac{4}{3}g$ (C) $\frac{3}{5}g$ (D) $\frac{5}{3}g$

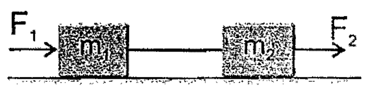
SECTION - V : MATRIX - MATCH TYPE

5.44 Column-I gives four different situations involving two blocks of mass m_1 and m_2 placed in different ways on a smooth horizontal surface as shown. In each of the situations horizontal forces F_1 and F_2 are applied on blocks of mass m_1 and m_2 respectively and also $m_2 F_1 < m_1 F_2$. Match the statements in column I with corresponding results in column-II .

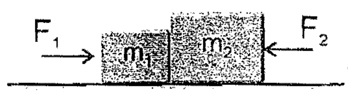
Column I

(A) 

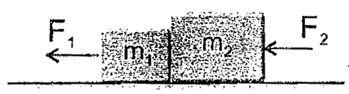
. Both the blocks are connected by massless inelastic string. The magnitude of tension in the string is

(B) 

. Both the blocks are connected by massless inelastic string. The magnitude of tension in the string is

(C) 

. The magnitude of normal reaction between the blocks is

(D) 

. The magnitude of normal reaction between the blocks is

Column II

(p) $\frac{m_1 m_2}{m_1 + m_2}\left(\frac{F_1}{m_1} - \frac{F_2}{m_2}\right)$

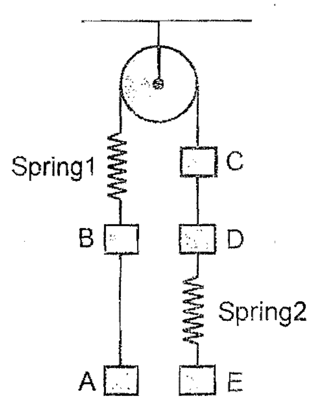
(q) $\frac{m_1 m_2}{m_1 + m_2}\left(\frac{F_1 - F_2}{m_1 + m_2}\right)$

(r) $\frac{m_1 m_2}{m_1 + m_2}\left(\frac{F_2}{m_2} - \frac{F_1}{m_1}\right)$

(s) $m_1 m_2 \left(\frac{F_1 + F_2}{m_1 + m_2}\right)$

(t) $\frac{m_1 m_2}{m_1 + m_2}\left(\frac{F_1}{m_1} + \frac{F_2}{m_2}\right)$

5.45 The system shown below is initially in equilibrium. Masses of the blocks A, B, C, D and E are respectively 3m, 3m, 2m, 2m and 2m. Match the conditions in column-I with the effects in column-II .



Column-I

(A) After spring 2 is cut, tension in string AB

(B) After spring 2 is cut, tension in string CD

(C) After string between C and pulley is cut, tension in string AB

(D) After string between C and pulley is cut, tension in string CD

Column-II

(p) increases

(q) decreases

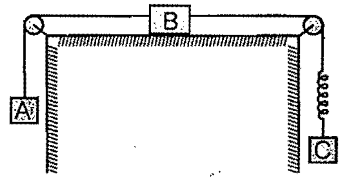
(r) decreases and then increases

(s) zero

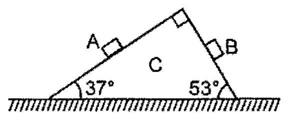
(t) remain constant

SECTION - VI : INTEGER TYPE

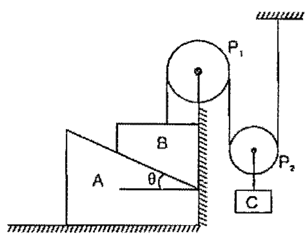
5.46 In the figure shown all the contacts are smooth. Strings and spring are light. Initially 'A' is held by someone and 'B' and 'C' are at rest and in equilibrium also. Find out the acceleration of block C in m/s^2 just after the block 'A' is released. Masses of A, B and C are M, M and 2M respectively.



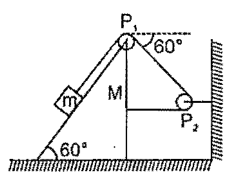
5.47 In the figure shown blocks 'A' and 'B' are kept on a wedge 'C'. A, B and C each have mass m. All surfaces are smooth. Find the acceleration of C.



5.48 In the figure shown P_1 and P_2 are massless pulleys. P_1 is fixed and P_2 can move. Masses of A, B and C are $\frac{9m}{64}$, 2m and m respectively . All contacts are smooth and the string is massless. $\theta = \tan^{-1}\left(\frac{3}{4}\right)$. Find the acceleration of block C in m/s^2.



5.49 In the arrangement shown in the Fig., a block of mass m = 2 kg lies on a wedge of mass M = 8 kg. The initial acceleration of the wedge (if the surfaces are smooth) given by $\frac{3\sqrt{3}g}{x}$ m/s^2 then x is.

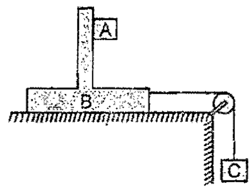


TOPIC 6

FRICTION

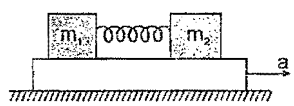
SECTION - I : STRAIGHT OBJECTIVE TYPE

6.1 In the arrangement shown in the figure mass of the block B and A are 2 m, 8 m respectively. Surface between B and floor is smooth. The block B is connected to block C by means of a pulley. If the whole system is released then the minimum value of mass of the block C so that the block A remains stationary with respect to B is : (Co-efficient of friction between A and B is μ.)



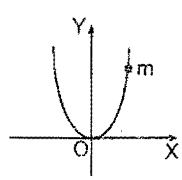
(A) $\frac{m}{\mu}$ (B) $\frac{2m}{\mu+1}$ (C) $\frac{10\,m}{1-\mu}$ (D) $\frac{10m}{\mu-1}$

6.2 Two block of masses m_1 and m_2 are connected with a massless unstretched spring and placed over a plank moving with an acceleration 'a' as shown in figure. The coefficient of friction between the blocks and platform is μ.



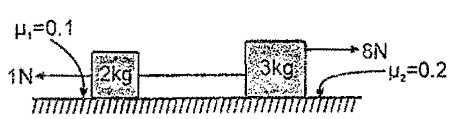
(A) spring will be stretched if $a > \mu g$
(B) spring will be compressed if $a \le \mu g$
(C) spring will neither be compressed nor be stretched for $a \le \mu g$
(D) spring will be in its natural length under all conditions

6.3 A bead of mass m is located on a parabolic wire with its axis vertical and vertex directed towards downward as in figure and whose equation is $x^2 = ay$. If the coefficient of friction is μ, the highest distance above the x-axis at which the particle will be in equilibrium is



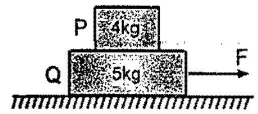
(A) μa (B) $\mu^2 a$ (C) $\frac{1}{4}\mu^2 a$ (D) $\frac{1}{2}\mu a$

6.4 In the shown arrangement if f_1, f_2 and T be the frictional forces on 2 kg block, 3kg block & tension in the string respectively, then their values are:



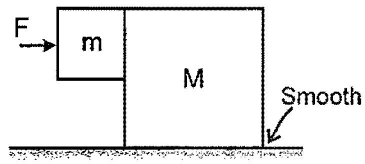
(A) 2 N, 6 N, 3.2 N (B) 2 N, 6 N, 0 N
(C) 1 N, 6 N, 2 N (D) data insufficient to calculate the required values.

6.5 The coefficient of friction between 4kg and 5 kg blocks is 0.2 and between 5 kg block and ground is 0.1 respectively. Choose the correct statement :



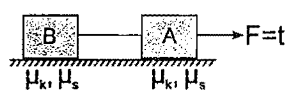
(A) Minimum force needed to cause system to move is 17 N
(B) When force is 4N static friction at all surfaces is 4N to keep system at rest
(C) Maximum acceleration of 4kg block is 2m/s²
(D) Slipping between 4kg and 5 kg blocks start when F is 17N

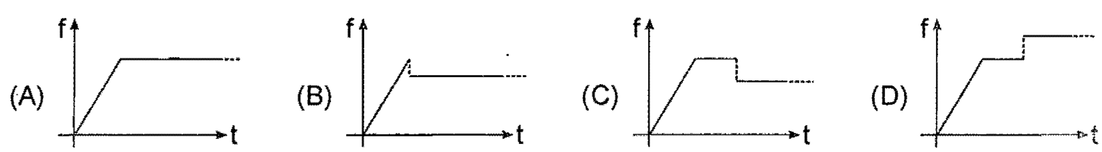
6.6 The two blocks, m = 10 kg and M = 50 kg are free to move as shown. The coefficient of static friction between the blocks is 0.5 and there is no friction between M and the ground. A minimum horizontal force F is applied to hold m against M that is equal to :



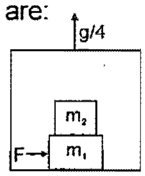
(A) 100 N (B) 50 N (C) 240 N (D) 180 N

6.7 A force F = t is applied to a block A as shown in figure, where t is time in seconds. The force is applied at t = 0 seconds when the system was at rest. Which of the following graph correctly gives the frictional force between A and horizontal surface as a function of time t.[Assume that at t = 0, tension in the string connecting the two blocks is zero].





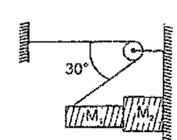
6.8 A plank of mass m_1 = 8 kg with a bar of mass m_2 = 2 kg placed on its rough surface, lie on a smooth floor of elevator ascending with an acceleration g/4. The coefficient of friction is μ = 1/5 between m_1 and m_2 . A horizontal force F = 30 N is applied to the plank. Then the acceleration of bar and the plank in the reference frame of elevator are:



(A) 3.5 m/s², 5 m/s² (B) 5 m/s², $\frac{50}{8}$ m/s² (C) 2.5 m/s², $\frac{25}{8}$ m/s² (D) 4.5 m/s², 4.5m/s²

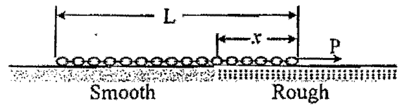
6.9 A block of mass 1 kg lies on a horizontal surface in a truck. The coefficient of static friction between the block and the surface is 0.6 If the acceleration of the truck is 5 m/s², the frictional force acting on the block is :
(A) 5 N (B) 6 N (C) 10 N (D) 15 N

6.10 Two blocks with masses M_1 and M_2 of 10 kg and 20 kg respectively are placed as in fig. Coefficient of friction μ = 0.2 between all surfaces, then tension in string and acceleration of M_2 block will be :



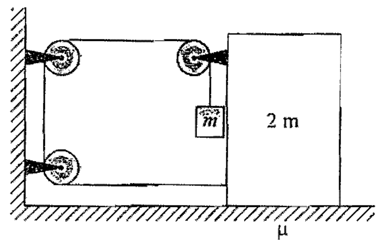
(A) 250 N, 3 m/s² (B) 200 N, 6 m/s² (C) 306 N, 4.7 m/s² (D) 400 N, 6.5 m/s²

6.11 A chain of length L is placed on a horizontal surface as shown in figure. At any instant x is the length of chain on rough surface and the remaining portion lies on smooth surface. Initially x = 0. A horizontal force P is applied to the chain (as shown in the figure). In the duration x changes from x = 0 to x = L, for chain to move with constant speed :



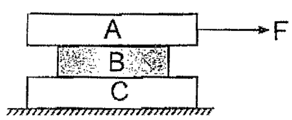
(A) the magnitude of P should increase with time
(B) the magnitude of P should decrease with time
(C) the magnitude of P should increase first and then decrease with time
(D) the magnitude of P should decrease first and then increase with time

6.12 In the system shown in figure the friction coefficient between ground and bigger block is μ. There is no friction between both the blocks. The string connecting both the block is light; all three pulleys are light and frictionless. Then the minimum limiting value of μ so that the system remains in equilibrium is



(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{3}{2}$

6.13 Given m_A = 20 kg, m_B = 10 kg, m_C = 20 kg. Between A and B μ_1 = 0.3, between B and C μ_2 = 0.3 and between C and ground μ_3 = 0.1. The least horizontal force F to start the motion of any part of the system of three blocks resting upon one another as shown in figure is (g = 10 m/s²)



(A) 60 N (B) 90 N (C) 80 N (D) 50 N

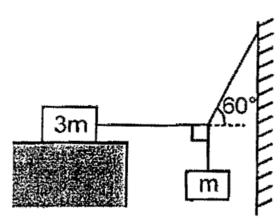
6.14 A 1.5 kg box is initially at rest on a horizontal surface when at t = 0 a horizontal force $\vec{F} = (1.8t)\hat{i}$N (with t in seconds), is applied to the box. The acceleration of the box as a function of time t is given by :

$\vec{a} = 0$ for $0 \le t \le 2.85$

$\vec{a} = (1.2t - 2.4)\hat{i}$ m/s² for $t > 2.85$

The coefficient of kinetic friction between the box and the surface is :
(A) 0.12 (B) 0.24 (C) 0.36 (D) 0.48

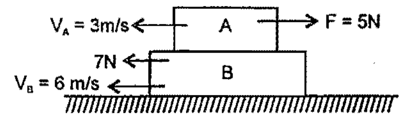
6.15 A mass *m* is supported as shown in the figure by ideal strings connected to a rigid wall and to a mass 3*m* at rest on a fixed horizontal surface. The string connected to larger mass is horizontal, that connected to smaller mass is vertical and the one connected to wall makes an angle 60° with horizontal. Then the minimum coefficient of static friction between the larger mass and the horizontal surface that permits the system to remain in equilibrium in the situation shown is :



(A) $\frac{1}{\sqrt{3}}$ (B) $\frac{1}{3\sqrt{3}}$

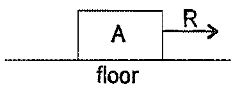
(C) $\frac{\sqrt{3}}{2}$ (D) $\sqrt{\frac{3}{2}}$

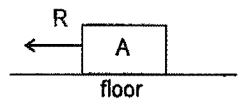
6.16 In the following figure, find the direction of friction on the blocks and ground respectively.

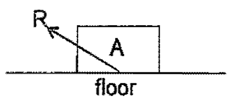


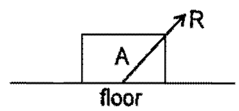
(A) Block A (left), block B(right due to block A, right due to ground)
(B) Block A (right), block B(left due to block A, left due to ground)
(C) Block A (right), block B(left due to block A, right due to ground)
(D) Block A (left), block B(left due to block A, left due to ground)

6.17 A box 'A' is lying on the horizontal floor of the compartment of a train running along horizontal rails from left to right. At time 't', it decelerates. Then the reaction R by the floor on the box is given best by :

(A) 

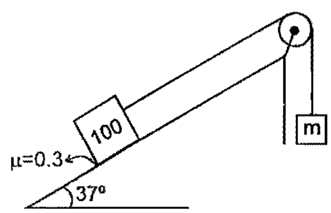
(B) 

(C) 

(D) 

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

6.18 The value of mass m for which the 100 kg block remains in static equilibrium is



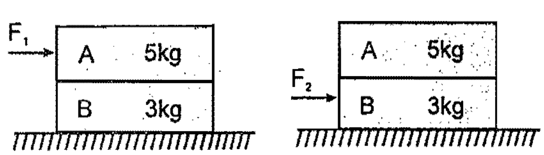
(A) 35 kg (B) 37 kg (C) 83 kg (D) 85 kg

6.19 The force F_1 that is necessary to move a body up an inclined plane is double the force F_2 that is necessary to just prevent it from sliding down, then :

(A) $F_2 = w \sin(\theta - \phi) \sec\phi$ (B) $F_1 = w \sin(\theta - \phi) \sec\phi$
(C) $\tan\phi = 3\tan\theta$ (D) $\tan\theta = 3\tan\phi$

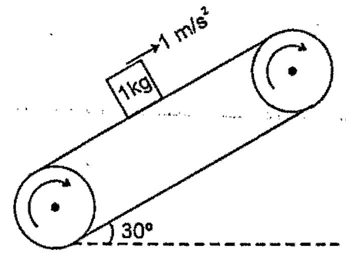
Where ϕ = angle of friction
θ = angle of inclined plane
w = weight of the body

6.20 A block A (5 kg) rests over another block B (3 kg) placed over a smooth horizontal surface. There is friction between A and B. A horizontal force F_1 gradually increasing from zero to a maximum is applied to A so that the blocks move together without relative motion. Instead of this another horizontal force F_2, gradually increasing from zero to a maximum is applied to B so that the blocks move together without relative motion. Then

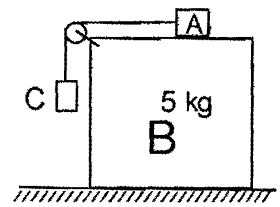


(A) F_1 (max) = F_2 (max) (B) F_1 (max) > F_2 (max)
(C) F_1 (max) < F_2 (max) (D) F_1 (max): F_2 (max) = 5: 3

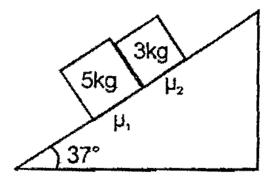
6.21 A block of mass 1 kg is stationary with respect to a conveyer belt that is accelerating with 1 m/s^2 upwards at an angle of 30° as shown in figure. Which of the following is/are correct ;
(A) Force of friction on block is 6 N upwards.
(B) Force of friction on block is 1.5 N upwards.
(C) Contact force between the block & belt is 10.5 N.
(D) Contact force between the block & belt is $5\sqrt{3}$ N.



6.22 All the blocks shown in the figure are at rest. The pulley is smooth and the strings are light. Coefficient of friction at all the contacts is 0.2. A frictional force of 10 N acts between A and B.The block A is about to slide on block B.The normal reaction and frictional force exerted by the ground on the block B is :
(A) The normal reaction exerted by the ground on the block B is 110N
(B) The normal reaction exerted by the ground on the block B is 50 N
(C) The frictional force exerted by the ground on the block B is 20N
(D) The frictional force exerted by the ground on the block B is zero

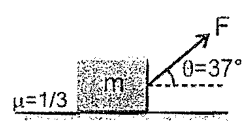


6.23 Two blocks of masses 5 kg and 3kg are placed in contact over an inclined surface of angle 37°, as shown. μ_1 is friction coefficient between 5kg block and the surface of the incline and similarly, μ_2 is friction coefficient between the 3kg block and the surface of the incline. After the release of the blocks from the inclined surface,
(A) if μ_1 = 0.5 and μ_2 = 0.3 then 5 kg block exerts 3N force on the 3 kg block
(B) if μ_1 = 0.5 and μ_2 = 0.3 then 5 kg block exters 8 N force on the 3 kg block
(C) if μ_1 = 0.3 and μ_2 = 0.5 then 5 kg block exerts 1 N force on the 3kg block.
(D) if μ_1 = 0.3 and μ_2 = 0.5 then 5 kg block exerts no force on the 3kg block.

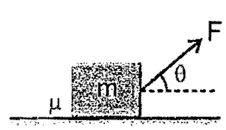


SECTION - III : ASSERTION AND REASON TYPE

6.24 **Statement-1 :** A block of mass m is placed at rest on rough horizontal surface. The coefficient of friction between the block and horizontal surface is $\mu = \frac{1}{3}$. The minimum force F applied at angle θ = 37° (as shown in figure) to pull the block horizontally is not equal to μmg. (Take $\sin 37° = \frac{3}{5}$, $\cos 37° = \frac{4}{5}$)

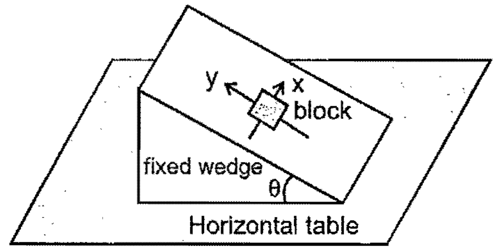


Statement-2 : For a block of mass m placed on rough horizontal surface, the minimum horizontal force required to pull the block is μmg. The minimum force F applied at angle θ (as shown in figure) to pull the block horizontally may be less than μmg. (Where μ is co-efficient of friction).



6.25 **Statement-1 :** A body is lying at rest on a rough horizontal surface. A person accelerating with acceleration $a\hat{i}$ (where a is positive constant and $\hat{i}$ is a unit vector in horizontal direction) observes the body. With respect to him, the block experiences a kinetic friction.
Statement-2 : Whenever there is relative motion between the contact surfaces then kinetic friction acts.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

6.26 **Statement-1 :** A fixed wedge of inclination θ lies on horizontal table. x and y axes are drawn on inclined surface as shown, such that x axis is horizontal and y-axis is along line of greatest slope. A block of mass m is placed (at rest) on inclined surface at origin. The coefficient of friction between block and wedge is μ, such that tanθ = μ. Then a force F > μmg cosθ applied to block parallel to inclined surface and along x-axis can move the block along x-axis.



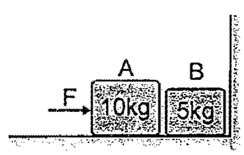
Statement-2 : To move the block placed at rest on rough inclined surface along the inclined surface, the net force on block (except frictional force) should be greater than μN. (N = normal reaction on block).

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

SECTION - IV : COMPREHENSION TYPE

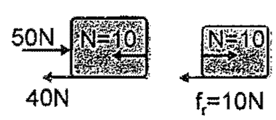
Comprehension # 1

Two bodies A and B of masses 10 kg and 5 kg are placed very slightly separated as shown in figure. The coefficients of friction between the floor and the blocks are as $\mu_s = \mu_k = 0.4$. Block A is pushed by an external force F. The value of F can be changed. When the welding between block A and ground breaks, block A will start pressing block B and when welding of B also breaks, block B will start pressing the vertical wall –



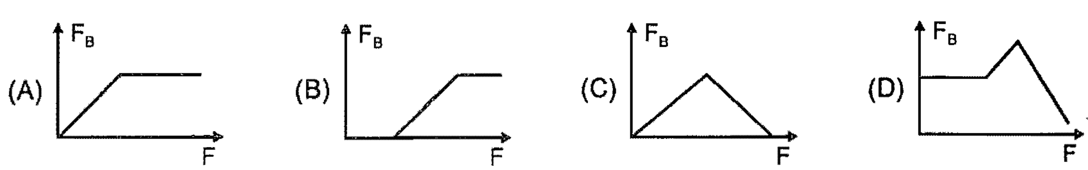
6.27 If F = 20 N, with how much force does block A presses the block B
(A) 10 N (B) 20 N (C) 30 N (D) Zero

6.28 If F = 50 N, the friction force acting between block B and ground will be :



(A) 10 N (B) 20 N (C) 30 N (D) None

6.29 The force of friction acting on B varies with the applied force F according to curve :

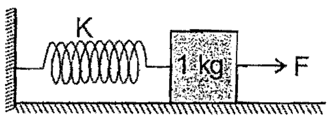


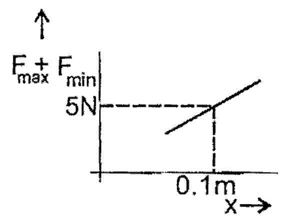
Comprehension # 2

A block of mass 1 kg is placed on a rough horizontal surface. A spring is attached to the block whose other end is joined to a rigid wall,as shown in the figure. A horizontal force is applied on the block so that it remains at rest while the spring is elongated by x. $x \geq \frac{\mu mg}{k}$. Let F_{max} and F_{min} be the maximum and minimum values of force F for which the block remains in equilibrium. For a particular x,

$F_{max} - F_{min} = 2$ N.

Also shown is the variation of $F_{max} + F_{min}$ versus x, the elongation of the spring.





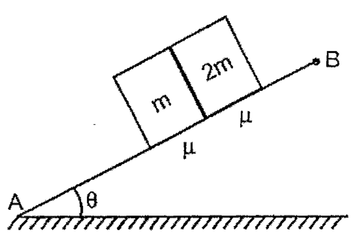
6.30 The coefficient of friction between the block and the horizontal surface is :
(A) 0.1 (B) 0.2 (C) 0.3 (D) 0.4

6.31 The spring constant of the spring is:
(A) 25 N/m (B) 20 N/m (C) 2.5 N/m (D) 50 N/m

6.32 The value of F_{min} , if x = 3 cm is :
(A) 0 (B) 0.2N (C) 5N (D) 1N

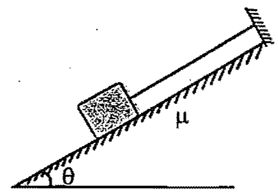
SECTION - V : MATRIX - MATCH TYPE

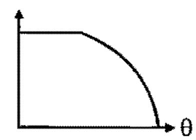
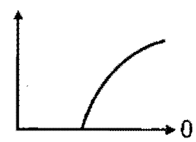
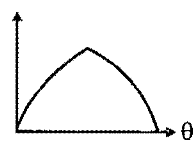
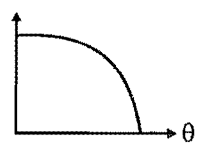
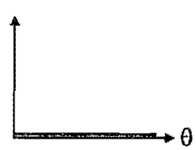
6.33 Two blocks of mass m and 2m are slowly just placed in contact with each other on a rough fixed inclined plane as shown.Initially both the blocks are at rest on inclined plane. The coefficient of friction between either block and inclined surface is μ. There is no friction between both the blocks. Neglect the tendency of rotation of blocks on the inclined surface. Column I gives four situation. Column II gives condition under which statements in column I are true. Match the statement in column I with corresponding conditions in column II.



Column I	Column II
(A) The magnitude of acceleration of both blocks are same if	(p) $\mu = 0$
(B) The normal reaction between both the blocks is zero if	(q) $\mu > 0$
(C) The net reaction exerted by inclined surface on each block make same angle with inclined surface (AB) if	(r) $\mu > \tan\theta$
(D) The net reaction exerted by inclined surface on block of mass 2m is double that of net reaction exerted by inclined surface on block of mass m if	(s) $\mu < \tan\theta$
	(t) $\mu = \tan\theta$

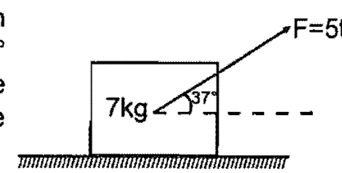
6.34 A block of mass m is put on a rough inclined plane of inclination θ, and is tied with a light thread shown. Inclination θ is increased gradually from θ = 0° to θ = 90°. Match the columns according to corresponding curve.



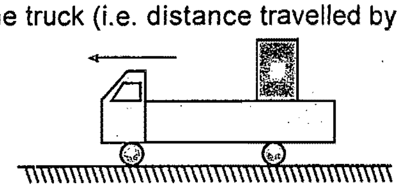
	Column I		**Column II**
(A)	Tension in the thread versus θ	(p)	
(B)	Normal reaction between the block and the incline versus θ	(q)	
(C)	friction force between the block and the incline versus θ	(r)	
(D)	Net interaction force between the block and the incline versus θ	(s)	
		(t)	

SECTION - VI : INTEGER TYPE

6.35 A block of 7 kg is placed on a rough horizontal surface and is pulled through a variable force F(in N) = 5t, where 't' is time in second, at an angle of 37° with the horizontal as shown in figure. The coefficient of static friction of the block with the surface is one. If the force starts acting at t = 0 s, Find the time (in sec.) at which the block starts to slide. (Take g = 10 m/s^2) :

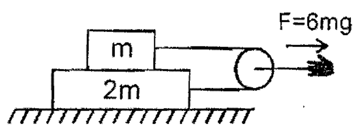


6.36 The rear side of a truck is open and a box of 40 kg mass is placed 5 m away from the open end as shown. The coefficient of friction between the box & the surface below it is 0.15. On a straight road, the truck starts from rest and accelerates with 2 ms^{-2} At what distance (in m.) from the starting point does the box fall off the truck (i.e. distance travelled by the truck)? [Ignore the size of the box]

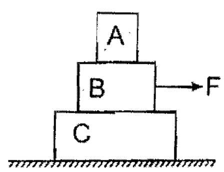


6.37 A block of mass 5 kg is placed on a rough horizontal surface of a moving compartment. It is seen by an observer sitting inside the compartment, that a force of 10 N is required in horizontal direction to move the box in a direction parallel to the motion of compartment while a force of 20 N is required in horizontal direction to move the box in opposite direction. If coefficient of friction between the surface of the block and the surface is $\frac{X}{10}$ then find out value of X.

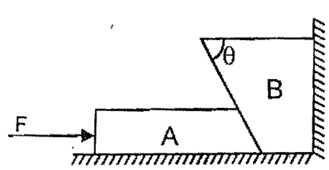
6.38 A block of mass m is placed on top of a block of mass 2m which in turn is placed on fixed horizontal surface. The coefficient of friction between all surfaces is $\mu = 1$. A massless string is connected to each mass and wraps halfway around a massless and frictionless pulley, as shown in the figure. The pulley is pulled by horizontal force of magnitude F = 6 mg towards right as shown. If the magnitude of acceleration of pulley is $\frac{X}{2}$ m/s^2, fill the value of X. (Take g = 10 m/s^2)



6.39 In the figure shown, the coefficient of static friction between C and ground is 0.5, coefficient of static friction between A and B is 0.25, coefficient of static friction between B and C is zero. Find the minimum value of force 'F(in Newton)', to cause sliding between A and B. Masses of A, B and C are respectively 2 kg, 4 kg and 5 kg.



6.40 In the figure shown, the coefficient of static friction between B and the wall is 2/3 and the coefficient of kinetic friction between B and the wall is 1/3. Other contacts are smooth. Find the minimum force 'F' required to lift B, up. It is $\frac{xmg}{2}$ then x is. Mass of A is 2m and the mass of B is m. Take $\tan\theta = 3/4$.



TOPIC 7

WORK, POWER & ENERGY

SECTION - I : STRAIGHT OBJECTIVE TYPE

7.1 Work done by static friction on an object:
(A) may be positive (B) must be negative (C) must be zero (D) none of these

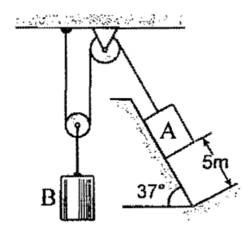
7.2 A man places a chain (of mass 'm' and length 'ℓ') on a table slowly. Initially the lower end of the chain just touches the table. The man drops the chain when half of the chain is in vertical position. Then work done by the man in this process is :

(A) $-mg\,\frac{\ell}{2}$ (B) $-\frac{mg\ell}{4}$ (C) $-\frac{3mg\ell}{8}$ (D) $-\frac{mg\ell}{8}$

7.3 The potential energy of a particle of mass m free to move along x-axis is given by $U = \frac{1}{2}kx^2$ for $x < 0$ and $U = 0$ for $x \geq 0$ (x denotes the x-coordinate of the particle and k is a positive constant). If the total mechanical energy of the particle is E, then its speed at $x = -\sqrt{\frac{2E}{k}}$ is

(A) zero (B) $\sqrt{\frac{2E}{m}}$ (C) $\sqrt{\frac{E}{m}}$ (D) $\sqrt{\frac{E}{2m}}$

7.4 The blocks A and B shown in the figure have masses $M_A = 5$ kg and $M_B = 4$ kg. The system is released from rest. The speed of B after A has travelled a distance 1 m along the incline is:



(A) $\frac{\sqrt{3}}{2}\sqrt{g}$ (B) $\frac{\sqrt{3}}{4}\sqrt{g}$ (C) $\frac{\sqrt{g}}{2\sqrt{3}}$ (D) $\frac{\sqrt{g}}{2}$

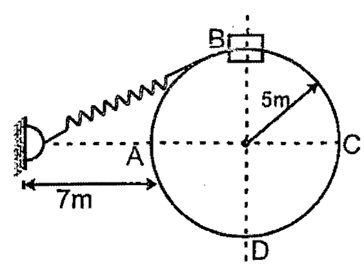
7.5 Of the sentences given

(i) Internal forces acting on the system cannot change $\frac{1}{2}mv_{cm}^2$, where m is the total mass of the system.

(ii) Internal forces acting on a system cannot change kinetic energy of system with respect to centre of mass

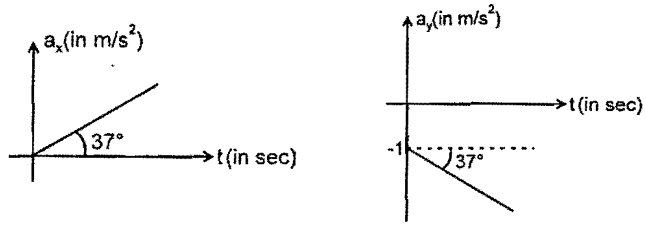
(A) both (i) and (ii) are correct
(B) only (i) is correct
(C) only (ii) is correct
(D) Both (i) and (ii) are wrong

7.6 A collar 'B' of mass 2 kg is constrained to move along a horizontal smooth and fixed circular track of radius 5 m. The spring lying in the plane of the circular track and having spring constant 200 N/m is undeformed when the collar is at 'A'. If the collar starts from rest at 'B', the normal reaction exerted by the track on the collar when it passes through 'A' is :



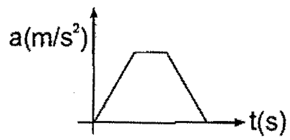
(A) 360 N (B) 720 N
(C) 1440 N (D) 2880 N

7.7 In the figure the variation of components of acceleration of a particle of mass 1 kg is shown w.r.t. time. The initial velocity of the particle is $\vec{u} = (-3\hat{i} + 4\hat{j})$ m/s. The total work done by the resultant force on the particle in time interval from t = 0 to t = 4 seconds is :

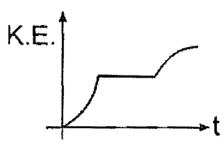


(A) 22.5 J (B) 10 J (C) 0 (D) None of these

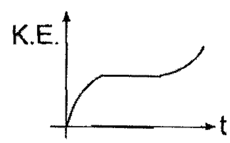
7.8 For a particle moving on a straight line the variation of acceleration with time is given by the graph as shown. Initially the particle was at rest. Then the corresponding kinetic energy of the particle versus time graph will be



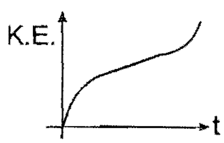
(A)



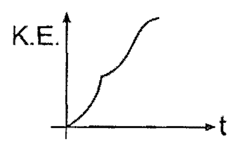
(B)



(C)



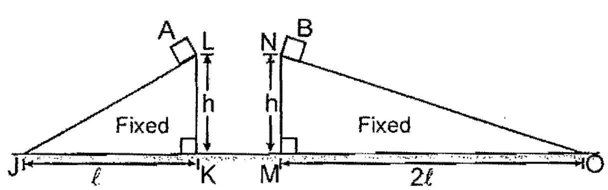
(D)



7.9 The potential energy (in SI units) of a particle of mass 2 kg in a conservative field is U = 6x – 8y. If the initial velocity of the particle is $\vec{u} = -1.5\hat{i} + 2\hat{j}$ then the total distance travelled by the particle in first two seconds is :

(A) 10 m (B) 12 m (C) 15 m (D) 18 m

7.10 Two identical blocks A and B are placed on two inclined planes as shown in diagram. Neglect air resistance and other friction



Read the following statements and choose the correct options.

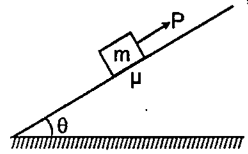
Statements I : Kinetic energy of 'A' on sliding to J will be greater than the kinetic energy of B on falling to M.

Statements II : Acceleration of 'A' will be greater than acceleration of 'B' when both are released to slide on inclined plane

Statements III : Work done by external agent to move block slowly from position B to O is negative

(A) only statement I is true (B) only statement II is true
(C) only I and III are true (D) only II and III are true

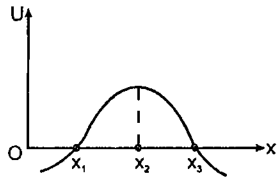
7.11 A block of mass m is being pulled up the rough incline by an agent delivering constant power P. The coefficient of friction between the block and the incline is μ. The maximum speed of the block during the course of ascent is :



(A) $v = \dfrac{P}{mg\sin\theta + \mu mg\cos\theta}$ (B) $v = \dfrac{P}{mg\sin\theta - \mu mg\cos\theta}$

(C) $v = \dfrac{2P}{mg\sin\theta - \mu mg\cos\theta}$ (D) $v = \dfrac{3P}{mg\sin\theta - \mu mg\cos\theta}$

7.12 In the figure shown the potential energy U of a particle is plotted against its position 'x' from origin. Then which of the following statement is correct.

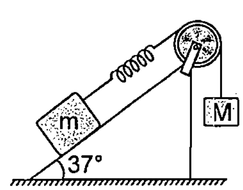


(A) x_1 is in stable equilibrium (B) x_2 is in stable equilibrium
(C) x_3 is in stable equilibrium (D) none of these

7.13 One end of an unstretched vertical spring is attached to the ceiling and an object attached to the other end is slowly lowered to its equilibrium position. If S be gain in spring energy & G be loss in gravitational potential energy in the process, then
(A) S = G (B) S = 2G
(C) G = 2S (D) None of these

7.14 The potential energy function associated with the force $\vec{F} = 4xy\,\hat{i} + 2x^2\hat{j}$ is :
(A) $U = -2x^2y$ (B) $U = -2x^2y$ + constant
(C) $U = 2x^2y$ + constant (D) not defined

7.15 A block of mass m is attached with a massless spring of force constant k. The block is placed over a fixed rough inclined surface for which the coefficient of friction is $\mu = \frac{3}{4}$. The block of mass m is initially at rest. The block of mass M is released from rest with spring in unstretched state. The minimum value of M required to move the block up the plane is (neglect mass of string and pulley and friction in pulley.)

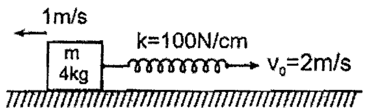


(A) $\frac{3}{5}$m (B) $\frac{4}{5}$m (C) $\frac{6}{5}$m (D) $\frac{3}{2}$m

7.16 The potential energy for a force field $\vec{F}$ is given by U (x,y) = cos (x + y). The force acting on a particle at position given by coordinates $\left(0, \frac{\pi}{4}\right)$ is :-

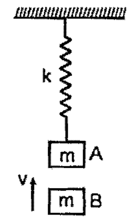
(A) $-\frac{1}{\sqrt{2}}(\hat{i}+\hat{j})$ (B) $\frac{1}{\sqrt{2}}(\hat{i}+\hat{j})$ (C) $\left(\frac{1}{2}\hat{i}+\frac{\sqrt{3}}{2}\hat{j}\right)$ (D) $\left(\frac{1}{2}\hat{i}-\frac{\sqrt{3}}{2}\hat{j}\right)$

7.17 The spring block system lies on a smooth horizontal surface. The free end of the spring is being pulled towards right with constant speed $v_0 = 2$m/s. At t = 0 sec, the spring of constant k = 100 N/cm is unstretched and the block has a speed 1 m/s to left. The maximum extension of the spring is



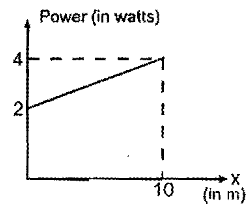
(A) 2 cm (B) 4 cm (C) 6 cm (D) 8 cm

7.18 Block 'A' is hanging from a vertical spring and is at rest. Block 'B' strikes the block 'A' with velocity 'v' and sticks to it. Then the value of 'v' for which the spring just attains natural length is :



(A) $\sqrt{\frac{60\,mg^2}{k}}$ (B) $\sqrt{\frac{6\,mg^2}{k}}$ (C) $\sqrt{\frac{10\,mg^2}{k}}$ (D) none of these

7.19 A particle A of mass $\frac{10}{7}$ kg is moving in the positive x direction. Its initial position is x = 0 & initial velocity is 1 m/s. The velocity at x = 10 is in m/s (use the graph given)



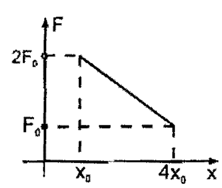
(A) 4 m/s (B) 2 m/s (C) $3\sqrt{2}$ m/s (D) 100/3 m/s

7.20 A fire hose has a diameter of 2.5 cm and is required to direct a jet of water to a height of at least 40 m. The minimum power of the pump needed for this hose is :
(A) 21.5 kW (B) 40 kW (C) 36.5 kW (D) 48 kW

7.21 A particle is projected vertically upwards with a speed of 16 m/s, after some time, when it again passes through the point of projection, its speed is found to be 8 m/s. It is known that the work done by air resistance is same during upward and downward motion. Then the maximum height attained by the particle is (Take g = 10 m/s²) :
(A) 8 m (B) 4.8 m (C) 17.6 m (D) 12.8 m

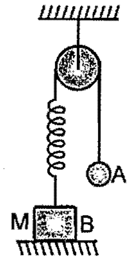
7.22 An engine can pull 4 coaches at a maximum speed of 20 m/s. Mass of the engine is twice the mass of every coach. Assuming resistive forces to be proportional to the weight, approximate maximum speeds in m/s of the engine when it pulls 6 coaches are (power of engine remains constant) :
(A) 8.5 m/s and 15 m/s respectively (B) 6.5 m/s and 8 m/s respectively
(C) 8.5 m/s and 13 m/s respectively (D) 10.5 m/s and 15 m/s respectively

7.23 A particle of mass m moving along a straight line experiences force F which varies with the distance x travelled as shown in the figure. If the velocity of the particle at x_0 is $\sqrt{\frac{2F_0x_0}{m}}$, then velocity at $4x_0$ is:-



(A) $2\sqrt{\frac{2F_0x_0}{m}}$ (B) $2\sqrt{\frac{F_0x_0}{m}}$ (C) $\sqrt{\frac{F_0x_0}{m}}$ (D) none of these

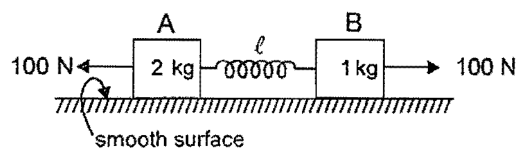
7.24 In the figure, the ball A is released from rest when the spring is at its natural length. For the block B, of mass M to leave contact with the ground at some stage, the minimum mass of A must be:



(A) 2 M (B) M (C) M/2

(D) A function of M and the force constant of the spring.

7.25 In the figure shown initially spring is in unstretched state & blocks are at rest. Now 100 N force is applied on block A & B as shown in the figure. After some time velocity of 'A' becomes 2 m/s & that of 'B' is 4 m/s & block A displaced by amount 10 cm and spring is stretched by amount 30 cm. Then work done by spring (in joule) force on A will be :



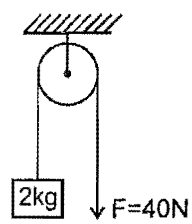
(A) 9/3 J (B) – 6 J (C) 6 J (D) None of these

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

7.26 Which of the following is/are conservative force(s) ?

(A) $\vec{F} = 2r^3\hat{r}$ (B) $\vec{F} = -\frac{5}{r}\hat{r}$ (C) $\vec{F} = \frac{3(x\hat{i}+y\hat{j})}{(x^2+y^2)^{3/2}}$ (D) $\vec{F} = \frac{3(y\hat{i}+x\hat{j})}{(x^2+y^2)^{3/2}}$

7.27 A block of mass 2 kg is hanging over a smooth and light pulley through a light string. The other end of the string is pulled by a constant force F = 40 N. The kinetic energy of the particle increase 40 J in a given interval of time. Then : ($g = 10\ m/s^2$)



(A) tension in the string is 40 N

(B) displacement of the block in the given interval of time is 2 m

(C) work done by gravity is – 20 J

(D) work done by tension is 80 J

7.28 There are two massless springs A and B of spring constant K_A and K_B respectively and $K_A > K_B$. If W_A and W_B be denoted as work done on A and work done on B respectively, then

(A) If they are compressed to same distance, $W_A > W_B$

(B) If they are compressed by same force (upto equilibrium state) $W_A < W_B$

(C) If they are compressed by same distance, $W_A = W_B$

(D) If they are compressed by same force (upto equilibrium state) $W_A > W_B$

7.29 Work done by a force on an rigid object having no rotational motion will be zero, if :

(A) the force is always perpendicular to acceleration of object.

(B) the object is at rest relative to ground but the point of application of force moves on the object.

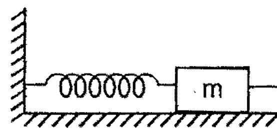
(C) the force is always perpendicular to velocity of object.

(D) The point of application of force is fixed relative to ground but the object moves.

7.30 The potential energy (in joules) of a particle of mass 1kg moving in a plane is given by V = 3x + 4y, the position coordinates of the point being x and y, measured in metres. If the particle is at rest at (6, 4) ; then
(A) its acceleration is of magnitude $5 m/s^2$
(B) its speed when it crosses the y–axis is 10m/s
(C) it crosses the y-axis (x = 0) at y = –4
(D) it moves in a striaght line passing through the origin (0, 0)

SECTION - III : ASSERTION AND REASON TYPE

7.31 **Statement-1 :** One end of ideal massless spring is connected to fixed vertical wall and other end to a block of mass m initially at rest on smooth horizontal surface. The spring is initially in natural length. Now a horizontal force F acts on block as shown. Then the maximum extension in spring is equal to maximum compression in spring.



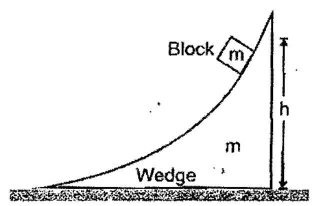
Statement-2 : To compress and to expand an ideal unstretched spring by equal amount, same work is to be done on spring.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

7.32 **Statement-1 :** work done by friction is always negative
Statement-2 : If frictional force acts on a body its K.E. may decrease.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

SECTION - IV : COMPREHENSION TYPE

Comprehension # 1

A block of mass m slides down a wedge of mass m as shown. The whole system is at rest, when the height of the block is h = 10 m. Above the ground. The wedge surface is smooth and gradually flattens. There is no friction between wedge and ground.



Smooth ground

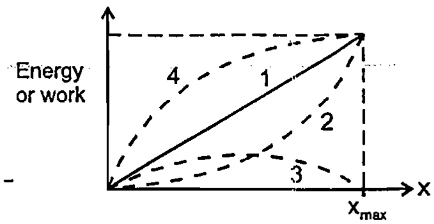
7.33 As the block slides down, which of the following quantities associated with the system remains conserved?
(A) Total linear momentum of the system of wedge and block
(B) Total mechanical energy of the complete system
(C) Total kinetic energy of the system
(D) Both linear momentum as well as mechanical energy of the system

7.34 If there would have been friction between wedge and block, which of the following quantities would still remain conserved ?
(A) Linear momentum of the system along horizontal direction
(B) Linear momentum of the system along vertical direction
(C) Linear momentum of the system along a tangent to the curved surface of the wedge
(D) Mechanical energy of the system

7.35 If there is no friction any where, the speed of the wedge, as the block leaves the wedge is :
(A) $10 ms^{-1}$ (B) $20 ms^{-1}$ (C) $30 ms^{-1}$ (D) None of these

Comprehension # 2

A spring lies along an x axis attached to a wall at one end and a block at the other end. The block rests on a frictionless surface at x = 0. A force of constant magnitude F is applied to the block that begins to compress the spring, until the block comes to a maximum displacement x_{max}.



7.36 During the displacement, which of the curves shown in the graph best represents the kinetic energy of the block.

(A) 1 (B) 2 (C) 3 (D) 4

7.37 During the displacement, which of the curves shown in the graph best represents the work done on the spring block system by the applied force.

(A) 1 (B) 2 (C) 3 (D) 4

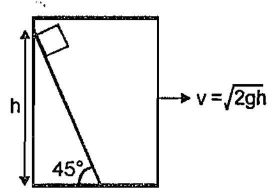
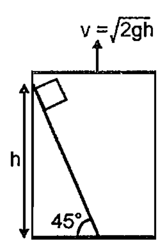
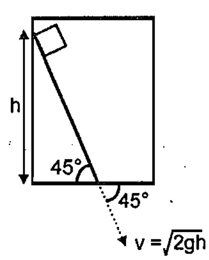
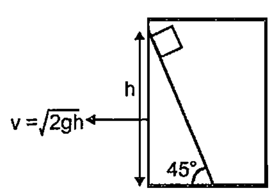
7.38 During the first half of the motion, applied force transfers more energy to the

(A) kinetic energy (B) potential energy

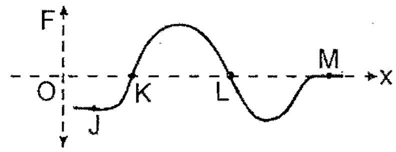
(C) equal to both (D) depends upon mass of the block

SECTION - V : MATRIX - MATCH TYPE

7.39 Figure shows four situations in which a small block of mass 'm' is released from rest (with respect to smooth fixed wedge) as shown in figure. Column-II shows work done by normal reaction on the block (with respect to an observer who is stationary on ground) till block reaches at the bottom of inclined wedge, match the appropriate column.

Column–I	Column–II
(A) 	(p) Positive
(B) 	(q) Negative
(C) 	(r) equal to mgh in magnitude
(D) 	(s) equal to zero

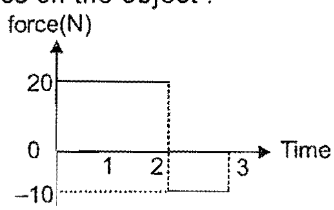
7.40 A particle moving along x-axis is being acted upon by one dimensional conservative force F. In the F–x curve shown, four points J, K, L, M are marked on the curve. Column II gives different type of equilibrium for the particle at different positions. Column I gives certain positions on the force position graphs. Match the positions in Column-I with the corresponding nature of equilibrium at these positions.



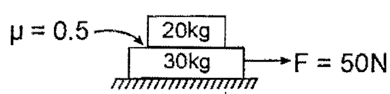
	Column I		Column II
(A)	Point J is position of	(p)	Neutral equilibrium
(B)	Point K is position of	(q)	Unstable equilibrium
(C)	Point L is position of	(r)	Stable equilibrium
(D)	Point M is position of	(s)	No equilibrium
		(t)	Equilibrium

SECTION - VI : INTEGER TYPE

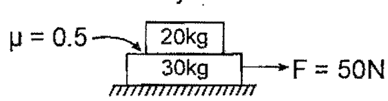
7.41 Starting at rest, a 5 kg object is acted upon by only one force as indicated in the figure. Find the total work done (in Joule) by the force on the object :-



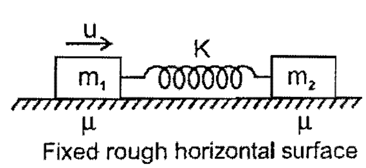
7.42 A small block of mass 20 kg rests on a bigger block of mass 30 kg, which lies on a smooth horizontal plane. Initially the whole system is at rest. The coefficient of friction between the blocks is 0.5. A horizontal force F = 50 N is applied on the lower block then. Find the work done (in Joule) by frictional force on upper block in t = 2sec. (magnitude in joule)



7.43 In previous is the magnitude of work done by frictional force on upper and lower block ?



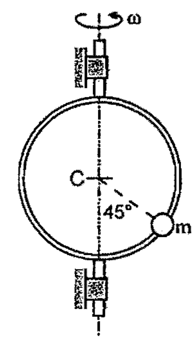
7.44 The blocks of mass m_1 = 1 kg and m_2 = 2 kg are connected by a spring, rest on a rough horizontal surface. The spring is unstreched. The spring constant of spring is K = 2 N/m. The coefficient of friction between blocks and horizontal surface is $\mu = \frac{1}{2}$. Now the left block is imparted a velocity u towards right as shown. Then what is the largest value of u (in m/s) such that the block of mass m_2 never moves.(Take g=10m/s^2)



TOPIC 8 CIRCULAR MOTION

SECTION - I : STRAIGHT OBJECTIVE TYPE

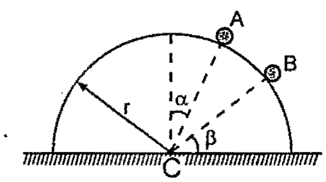
8.1 A small bead of mass m = 1 kg is carried by a circular hoop having centre at C and radius r = 1 m which rotates about a fixed vertical axis (as shown). The coefficient of friction between bead and hoop is μ = 0.5. The maximum angular speed of the hoop for which the bead does not have relative motion with respect to hoop: initial position of bead is shown in figure :



(A) $(5\sqrt{2})^{1/2}$ (B) $(10\sqrt{2})^{1/2}$

(C) $(15\sqrt{2})^{1/2}$ (D) $(30\sqrt{2})^{1/2}$

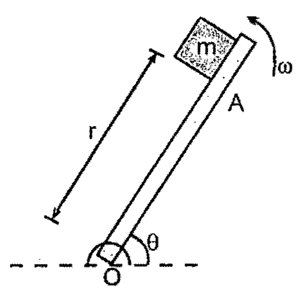
8.2 A particle initially at rest starts moving from point A on the surface of a fixed smooth hemisphere of radius r as shown. The particle looses its contact with hemisphere at point B. C is centre of the hemisphere. The equation relating α and β is :



(A) $3 \sin \alpha = 2 \cos \beta$ (B) $2 \sin \alpha = 3 \cos \beta$

(C) $3 \sin \beta = 2 \cos \alpha$ (D) $2 \sin \beta = 3 \cos \alpha$

8.3 The member OA rotates about a horizontal axis through O with a constant counter clockwise velocity ω = 3 rad/sec. As it passes the position θ = 0, a small mass m is placed upon it at a radial distance r = 0.5 m. If the mass is observed to slip at θ = 37°, the coefficient of friction between the mass & the member is ______.



(A) $\frac{3}{16}$ (B) $\frac{9}{16}$ (C) $\frac{4}{9}$ (D) $\frac{5}{9}$

8.4 Two particles A & B separated by a distance 2R are moving counter clockwise along the same circular path of radius R each with uniform speed v. At time t = 0, A is given a tangential acceleration of magnitude $a = \frac{72 v^2}{25 \pi R}$

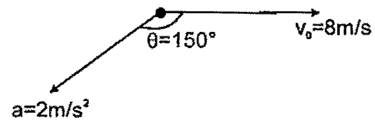
(A) the time lapse for the two bodies to collide is $\frac{6\pi R}{5V}$

(B) the angle covered by A is $\frac{11\pi}{6}$ (When A collied C with B)

(C) angular velocity of A is $\frac{11V}{5R}$

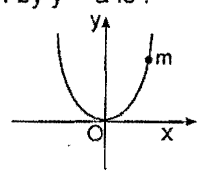
(D) radial acceleration of A is $\frac{289 v^2}{5R}$

8.5 The figure shows the velocity and acceleration of a point like body at the initial moment of its motion. The acceleration vector of the body remains constant. The minimum radius of curvature of trajectory of the body is (in m.)



(A) 2 m (B) 4 m (C) 8 m (D) 16 m

8.6 A bead of mass m is located on a parabolic wire with its axis vertical and vertex at the origin as shown in figure and whose equation is $x^2 = 4ay$. The wire frame is fixed and the bead can slide on it without friction. The bead is released from the point y = 4a on the wire frame from rest. The tangential acceleration of the bead when it reaches the position given by y = a is :

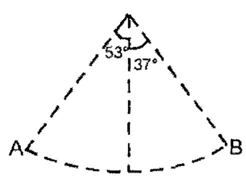


(A) $\frac{g}{2}$ (B) $\frac{\sqrt{3}g}{2}$ (C) $\frac{g}{\sqrt{2}}$ (D) $\frac{g}{\sqrt{5}}$

8.7 A particle is moving in a circular path. The acceleration and momentum vectors at an instant of time are $\vec{a} = 2\hat{i} + 3\hat{j}$ m/s² and $\vec{P} = 6\hat{i} - 4\hat{j}$ kgm/s. Then the motion of the particle is

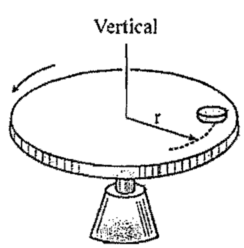
(A) uniform circular motion
(B) circular motion with tangential acceleration
(C) circular motion with tangential retardation
(D) we cannot say anything from $\vec{a}$ and $\vec{P}$ a time only

8.8 A section of fixed smooth circular track of radius 20 m. in vertical plane is shown in the figure. A block is released from position A and leaves the track at B. The radius of curvature of its trajectory when it just leaves the track at B is: (in m.)



(A) R (B) $\frac{R}{4}$ (C*) $\frac{R}{2}$ (D) none of these

8.9 A small coin of mass 40 g is placed on the horizontal surface of a rotating disc. The disc starts from rest and is given a constant angular acceleration $\alpha = 2$ rad/s². The coefficient of static friction between the coin and the disc is $\mu_s = 3/4$ and coefficient of kinetic friction is $\mu_k = 0.5$. The coin is placed at a distance r = 1 m from the centre of the disc. The magnitude of the resultant force on the coin exerted by the disc just before it starts slipping on the disc is :



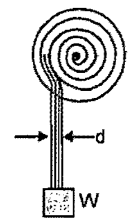
(A) 0.2 N (B) 0.3 N (C) 0.4 N (D) 0.5 N

8.10 A ring of mass 2π kg and of radius 0.25m is making 300rpm about an axis through its centre perpendicular to its plane. The tension (in newtons) developed in the ring is:

(A) 50 (B) 100 (C) 175 (D) 250

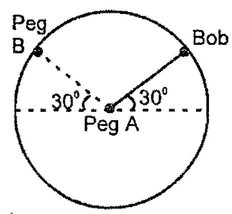
8.11 A car driver going at some speed suddenly finds a wide wall at a distance r. To avoid hitting the wall he should.
(A) apply the brakes
(B) should turn the car in a circle of radius r.
(C) apply the brakes and also turn the car in a circle of radius r.
(D) jump on the back seat

8.12 A weight W attached to the end of a flexible rope of diameter d=0.75cm is raised vertically by winding the rope on a reel as shown. If the reel is turned uniformly at the rate of 2 r.p.s. What is the tension in rope. The inertia of rope may be neglected.



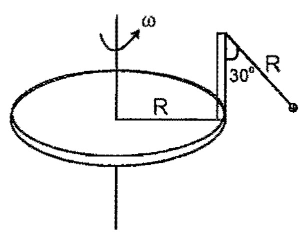
(A) 1.019W (B) 0.51W
(C) 2.04W (D) W

8.13 A bob is attached to one end of a string other end of which is fixed at peg A. The bob is taken to a position where string makes an angle of 30^0 with the horizontal. On the circular path of the bob in vertical plane there is a peg 'B' at a symmetrical position with respect to the position of release as shown in the figure. If v_c and v_a be the minimum speeds in clockwise and anticlockwise directions respectively, given to the bob in order to hit the peg 'B' then ratio $v_c : v_a$ is equal to :



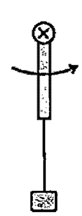
(A) 1 : 1 (B) $1 : \sqrt{2}$ (C) 1 : 2 (D) 1 : 4

8.14 A disc of radius R has a light pole fixed perpendicular to the disc at the circumference which in turn has a pendulum of length R attached to its other end as shown in figure. The disc is rotated with a constant angular velocity ω. The string is making an angle 30^0 with the rod. Then the angular velocity ω of disc is:



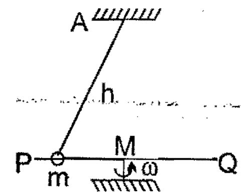
(A) $\left(\frac{\sqrt{3}g}{R}\right)^{1/2}$ (B) $\left(\frac{\sqrt{3}g}{2R}\right)^{1/2}$ (C) $\left(\frac{g}{\sqrt{3}R}\right)^{1/2}$ (D) $\left(\frac{2g}{3\sqrt{3}R}\right)^{1/2}$

8.15 One end of a light rod of length 1 m is attached with a string of length 1m. Other end of the rod is attached at point O such that rod can move in a vertical circle. Other end of the string is attached with a block of mass 2kg. The minimum velocity that must be given to the block in horizontal direction so that it can complete the vertical circle is ($g = 10\ m/s^2$).



(A) $4\sqrt{5}$ (B) $5\sqrt{5}$ (C) 10 (D) $3\sqrt{5}$

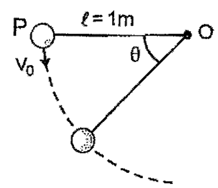
8.16 A smooth rod PQ rotates in a horizontal plane about its mid point M which is h = 0.1 m vertically below a fixed point A at a constant angular velocity 14 rad/s. A light elastic string of natural length 0.1 m force contant 1.47 N/cm has one end fixed at A and its other end attached to a ring of mass m = 0.3 kg which is free to slide along the rod. When the ring is stationary relative to rod, then inclination of string with vertical, tension in string.



(A) $\cos\theta = 3/5$, T = 9.8 N (B) $\theta = 60$, T = 0

(C) $\cos\theta = 2/5$, T = 4.9 N (D) $\theta = 30$, T = 0

8.17 The sphere at P is given a downward velocity v_0 and swings in a vertical plane at the end of a rope of ℓ = 1m attached to a support at O. The rope breaks at angle 30° from horizontal, knowing that it can withstand a maximum tension equal to three times the weight of the sphere. Then the value of v_0 will be : ($g = \pi^2$ m/s^2)

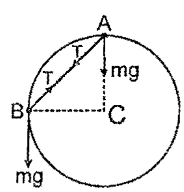


(A) $\frac{g}{2}$ m/s (B) $\frac{2g}{3}$ m/s (C) $\sqrt{\frac{3g}{2}}$ m/s (D) $\frac{g}{3}$ m/s

8.18 A simple pendulum is oscillating in a vertical plane. If resultant acceleration of bob of mass m at a point A is in horizontal direction, find the tangential force at this point in terms of tension T and mg.

(A) mg (B) $\frac{mg}{T}\sqrt{T^2-(mg)^2}$ (C) $\frac{mg}{T}\sqrt{(mg)^2+T^2}$ (D) $\frac{T}{mg}\sqrt{(mg)^2+T^2}$

8.19 Objects A and B each of mass m are connected by light inextensible cord. They are constrained to move on a frictionless ring in a vertical plane as shown in figure. The objects are released from rest at the positions shown. The tension in the cord just after release will be

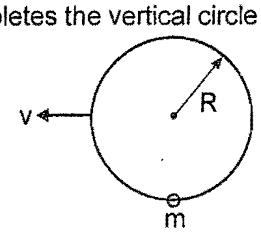


(A) $mg\sqrt{2}$ (B) $\frac{mg}{\sqrt{2}}$ (C) $\frac{mg}{2}$ (D) $\frac{mg}{4}$

8.20 A circular curve of a highway is designed for traffic moving at 72 km/h. If the radius of the curved path is 100 m, the correct angle of banking of the road should be given by :

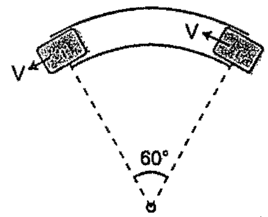
(A) $\tan^{-1}\frac{2}{3}$ (B) $\tan^{-1}\frac{3}{5}$ (C) $\tan^{-1}\frac{2}{5}$ (D) $\tan^{-1}\frac{1}{4}$

8.21 A ring of radius R lies in vertical plane. A bead of mass 'm' can move along the ring without friction. Initially the bead is at rest at the bottom most point on ring. The minimum constant horizontal speed v with which the ring must be pulled such that the bead completes the vertical circle



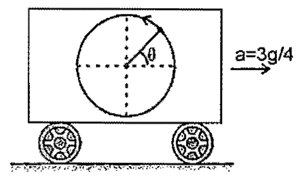
(A) $\sqrt{3gR}$ (B) $\sqrt{4gR}$ (C) $\sqrt{5gR}$ (D) $\sqrt{5.5gR}$

8.22 A car moves around a curve at a constant speed. When the car goes around the arc subtending 60° at the centre, then the ratio of magnitude of instantaneous acceleration to average acceleration over the 60° arc is :



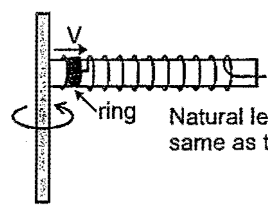
(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{6}$ (C) $\frac{2\pi}{3}$ (D) $\frac{5\pi}{3}$

8.23 A bus is moving with a constant acceleration a = 3g/4 towards right. In the bus, a ball is tied with a rope and is rotated in vertical circle as shown in the figure. The tension in the rope will be minimum, when the rope makes an angle θ = ______ .



(A) 53° (B) 37° (C) 180 – 53° (D) 180 + 37°

8.24 A ring attached with a spring is fitted in a smooth rod. The spring is fixed at the outer end of the rod. The mass of the ring is 3kg & spring constant of spring is 300 N/m. The ring is given a velocity 'V' towards the outer end of the rod. And the rod is set to be rotating with an angular velocity ω. Then ring will move with constant speed with respect to the rod if :

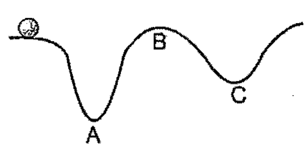


(A) angular velocity of rod is increased continuously
(B) ω = 10 rad/s
(C) angular velocity of rod is decreased continuously.
(D) constant velocity of ring is not possible

8.25 A uniform rod of mass m and length ℓ is rotating with constant angular velocity ω about an axis which passes through its one end and perpendicular to the length of rod. The area of cross section of the rod is A and its Young's modulus is Y. Neglect gravity. The strain at the mid point of the rod is :

(A) $\frac{m\omega^2 \ell}{8AY}$ (B) $\frac{3m\omega^2 \ell}{8AY}$ (C) $\frac{3m\omega^2 \ell}{4AY}$ (D) $\frac{m\omega^2 \ell}{4AY}$

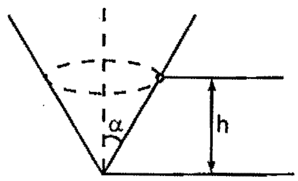
8.26 A body moves along an uneven surface with constant speed at all points. The normal reaction of the road on the body is :



(A) maximum at A (B) maximum at B
(C) minimum at C (D) the same at A, B & C

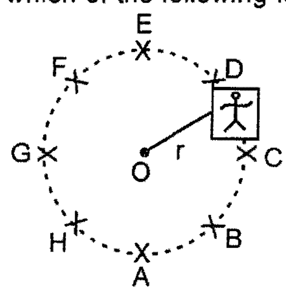
SECTION - II : MULTIPLE CORRECT ANSWER TYPE

8.27 A particle is describing circular motion in a horizontal plane in contact with the smooth inside surface of a fixed right circular cone with its axis vertical and vertex down. The height of the plane of motion above the vertex is h and the semivertical angle of the cone is α. The period of revolution of the particle:



(A) increases as h increases
(B) decreases as h increases
(C) increases as α increases
(D) decreases as α increases

8.28 A machine, in an amusement park, consists of a cage at the end of one arm, hinged at O. The cage revolves along a vertical circle of radius r (ABCDEFGH) about its hinge O, at constant linear speed $v = \sqrt{gr}$. The cage is so attached that the man of weight 'w' standing on a weighing machine, inside the cage, is always vertical. Then which of the following is/are correct



(A) the weight reading at A is greater than the weight reading at E by 2 w.
(B) the weight reading at G = w
(C) the ratio of the weight reading at E to that at A = 0
(D) the ratio of the weight reading at A to that at C = 2.

8.29 A small sphere of mass m suspended by a thread is first taken aside so that the thread forms the right angle with the vertical and then released, then :

(A) total acceleration of sphere as a function of θ is $g\sqrt{1+3\cos^2\theta}$
(B) thread tension as a function of θ is $T = 3mg \cos\theta$
(C) the angle θ between the thread and the vertical at the moment when the total acceleration vector of the sphere is directed horizontally is $\cos^{-1} 1/\sqrt{3}$
(D) thread tension at the moment when the vertical component of the sphere's velocity is maximum will be mg

8.30 On a circular table, A and B are moving on the circumference. Man A runs behind man B to catch him. A runs with constant angular speed ω_1 with respect to table and B runs at constant tangential speed v_2 with respect to ground. If it is found that the table rotates 30° in the opposite direction in every one second and the initial angular separation between A and B is 30°, then A catches B after : (Radius of table is 3 m).

(A) 0.5 sec, if $\omega_1 = \frac{5\pi}{6}$ rad/s and $v_2 = 3.14$ m/s

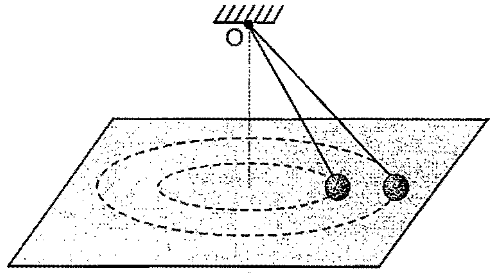
(B) 0.5 sec, if $\omega_1 = \frac{4\pi}{3}$ rad/s and $v_2 = 3.14$ m/s

(C) 0.5 sec, if $\omega_1 = \frac{4\pi}{3}$ rad/s and $v_2 = 6.28$ m/s

(D) A can not catch B within 0.5 s, if $\omega_1 = \frac{\pi}{6}$ rad/s and $v_2 = 6.28$ m/s

SECTION - III : ASSERTION AND REASON TYPE

8.31 **Statement-1 :** Two small spheres are suspended from same point O on roof with strings of different lengths. Both spheres move along horizontal circles as shown. Then both spheres may move along circles in same horizontal plane.



Statement-2 : For both spheres in statement-1 to move in circular paths in same horizontal plane, their angular speeds must be same.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

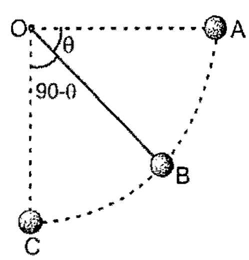
8.32 **Statement-1 :** A ball tied by thread is undergoing circular motion (of radius R) in a vertical plane. (Thread always remains in vertical plane). The difference of maximum and minimum tension in thread is independent of speed (u) of ball at the lowest position ($u > \sqrt{5gR}$)

Statement-2 : For a ball of mass m tied by thread undergoing vertical circular motion (of radius R), difference in maximum and minimum magnitude of centripetal acceleration of the ball is independent of speed (u) of ball at the lowest position ($u > \sqrt{5gR}$).

SECTION - IV : COMPREHENSION TYPE

Comprehension # 1

One end of a light string of length L is connected to a ball and the other end is connected to a fixed point O. The ball is released from rest at t = 0 with string horizontal and just taut. The ball then moves in vertical circular path as shown.The time taken by ball to go from position A to B is t_1 and from B to lowest position C is t_2. Let the velocity of ball at B is $\vec{v}_B$ and at C is $\vec{v}_C$ respectively.



8.33 If $|\vec{v}_C| = 2|\vec{v}_B|$ then the value of θ as shown is

(A) $\cos^{-1}\frac{1}{4}$ (B) $\sin^{-1}\frac{1}{4}$ (C) $\cos^{-1}\frac{1}{2}$ (D) $\sin^{-1}\frac{1}{2}$

8.34 If $|\vec{v}_C| = 2|\vec{v}_B|$ then :

(A) $t_1 > t_2$ (B) $t_1 < t_2$ (C) $t_1 = t_2$ (D) Information insufficient

8.35 If $|\vec{v}_C - \vec{v}_B| = |\vec{v}_B|$, then the value of θ as shown is :

(A) $\cos^{-1}\left(\frac{1}{4}\right)^{1/3}$ (B) $\sin^{-1}\left(\frac{1}{4}\right)^{1/3}$ (C) $\cos^{-1}\left(\frac{1}{2}\right)^{1/3}$ (D) $\sin^{-1}\left(\frac{1}{2}\right)^{1/3}$

Comprehension # 2

A particle of mass M attached to an inextensible string is moving in a vertical circle of radius R about fixed point O. It is imparted a velocity u in horizontal direction at lowest position as shown in figure.

Following information is being given :

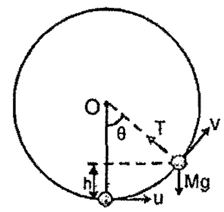
(i) Velocity at a height h can be calculated by using formula $v^2 = u^2 - 2gh$.

(ii) Particle will complete the circle if $u \geq \sqrt{5gR}$,

(iii) Particle will oscillates in lower half ($0° < \theta \leq 90°$) if $0 < u \leq \sqrt{2gR}$

(iv) The magnitude of tension at a height 'h' is calculated by using formula

$T = \frac{M}{R}\{u^2 + gR - 3gh\}$.



8.36 If R = 2m, M = 2 kg and u = 12 m/s. Then value of tension at lowest position is :
(A) 120 N (B) 164 N (C) 264 N (D) zero

8.37 Tension at highest point of its trajectory in above question will be :
(A) 100 N (B) 44 N (C) 144 N (D) 264 N

8.38 If M = 2 kg, R = 2m and u = 10 m/s the velocity of particle when θ = 60°,

(A) $2\sqrt{5}$ m/s (B) $4\sqrt{5}$ m/s (C) $5\sqrt{2}$ m/s (D) 5 m/s

SECTION - V : MATRIX - MATCH TYPE

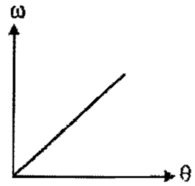
8.39 In column-I condition on velocity, force and acceleration of a particle is given. Resultant motion is described in column-II. $\vec{u}$ = initial velocity, $\vec{F}$ = resultant force and $\vec{v}$ = instantaneous velocity.

Column-I	Column-II
(A) $\vec{u} \times \vec{F} = 0$ and $\vec{F}$ = constant	(p) path will be circular path
(B) $\vec{u} \cdot \vec{F} = 0$ and $\vec{F}$ = constant	(q) speed will increase
(C) $\vec{v} \cdot \vec{F} = 0$ all the time and $\lvert\vec{F}\rvert$ = constant and the particle always remains in one plane.	(r) path will be straight line
(D) $\vec{u} = 2\hat{i} - 3\hat{j}$ and acceleration at all time $\vec{a} = 6\hat{i} - 9\hat{j}$	(s) path will be parabolic
	(t) Particle may retrace back

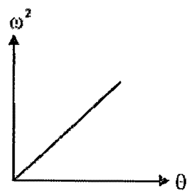
8.40 A particle is moving with speed $v = 2t^2$ on the circumference of circle of radius R. Match the quantities given in column-I with corresponding results in column-II

Column-I	Column-II
(A) Magnitude of tangential acceleration of particle	(p) decreases with time.
(B) Magnitude of Centripetal acceleration of particle	(q) increases with time
(C) Magnitude of angular speed of particle with respect to centre of circle	(r) remains constant
(D) Value of tan θ, where θ is angle between the total acceleration vector and centripetal acceleration vector of particle	(s) proportional to R
	(t) inversely proportional to R

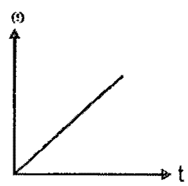
8.41 Each situation in column I gives graph of a particle moving in circular path. The variables ω, θ and t represent angular speed (at any time t), angular displacement (in time t) and time respectively. Column II gives certain resulting interpretation. Match the graphs in column I with statements in column II and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the OMR.

(A) 

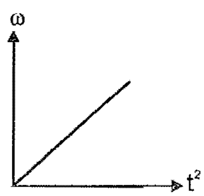
ω - θ graph

(B) 

ω^2 - θ graph

(C) 

ω - t graph

(D) 

ω - t^2 graph

(p) Angular acceleration of particle is uniform

(q) Angular acceleration of particle is non-uniform

(r) Angular acceleration of particle is directly proportional to t.

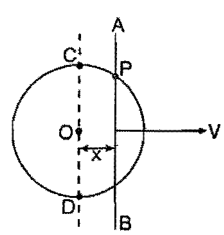
(s) Angular acceleration of particle is directly proportional to θ.

(t) Angular acceleration of particle is directly proportional to slope of the curve.

SECTION - VI : INTEGER TYPE

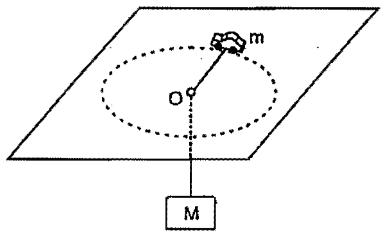
8.42 A ball is projected making an angle θ with the vertical. Consider a small part of the trajectory near the highest position and take it approximately to be a circular arc. What is the radius of this circle? This radius is called the radius of curvature (ROC) of the curve at the point.if u = 20 then ROC in m.

8.43 A rod AB is moving on a fixed circle of radius R with constant velocity 'v' as shown in figure. P is the point of intersection of the rod and the circle. At an instant the rod is at a distance $x = \frac{3R}{5}$ from centre of the circle. The velocity of the rod is perpendicular to the rod and the rod is always parallel to the diameter CD.



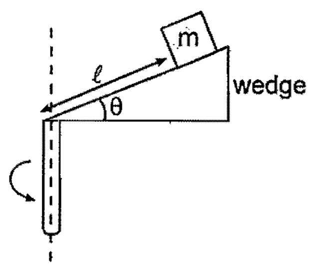
The speed of point of intersection P is $\frac{xv}{4}$ m./sec. then x is

8.44 A toy car of mass m can travel at a fixed speed. It moves in a circle on a fixed horizontal table. A string is connected to car and attached to a block of mass M = 5m that hangs as shown in figure (the portion of string below the table is always vertical). The coefficient of friction between the surface of table and tyres of the toy car is μ. The ratio of the maximum radius to the minimum radius for which the toy car can move in a circular path with centre O on table is $\frac{X}{4}$, then X is :



8.45 A squirrel climbing up a cylindrical post spirally makes the circle in a vertical height of four feet. If the top of the post is sixteen feet high and three feet in circumference, the distance travelled by it to reach the top is x in feet. Find the value of $\frac{x}{10}$.

8.46 A small wedge whose base is horizontal is fixed to a vertical rod as shown in the figure. $\theta = 45°$, $l = \frac{1}{5\sqrt{2}}$ m. The sloping side of the wedge is frictionless and the wedge is spun with a constant angular speed ω about vertical axis as shown in the figure. Find



The value of angular speed ω (rad/sec) for which the block of mass m just does not slide down the wedge?

TOPIC 9

CENTRE OF MASS

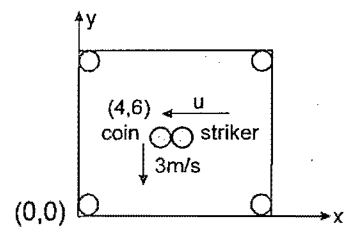
SECTION - I : STRAIGHT OBJECTIVE TYPE

9.1 Two particles A and B start moving due to their mutual interaction only. If at any time 't', $\vec{a}_A$ & $\vec{a}_B$ are their respective accelerations, $\vec{v}_A$ and $\vec{v}_B$ are their respective velocities and upto that time W_A and W_B are the work done on A & B respectively by the mutual force, m_A and m_B are their masses respectively, then which of the following is always correct.

(A) $\vec{v}_A + \vec{v}_B = 0$ (B) $m_A\vec{v}_A + m_B\vec{v}_B = 0$

(C) $W_A + W_B = 0$ (D) $\vec{a}_A + \vec{a}_B = 0$

9.2 On a smooth carom board, a coin moving in negative y–direction with a speed of 3 m/s is being hit at the point (4, 6) by a striker moving along negative x-axis. The line joining centres of the coin and the striker just before the collision is parallel to x-axis. After collision the coin goes into the hole located at the origin. Masses of the striker and the coin are equal. Considering the collision to be elastic, the initial and final speeds of the striker in m/s will be :



(A) (1.2, 0) (B) (2, 0) (C) (3, 0) (D) none of these

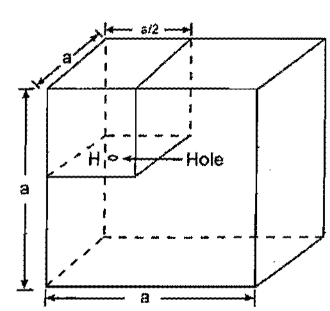
9.3 A train of mass M is moving on a circular track of radius 'R' with constant speed V. The length of the train is half of the perimeter of the track. The linear momentum of the train will be

(A) 0 (B) $\frac{2MV}{\pi}$ (C) MVR (D) MV

9.4 A canon shell moving along a straight line bursts into two parts. Just after the burst one part moves with momentum 20 Ns making an angle 30° with the original line of motion. The minimum momentum of the other part of shell just after the burst is :

(A) 0 Ns (B) 5 Ns (C) 10 Ns (D) 17.32 Ns

9.5 The figure shows a hollow cube of side 'a' of volume V. There is a small chamber of volume $\frac{V}{4}$ in the cube as shown. This chamber is completely filled by m kg of water. Water leaks through a hole H. Then the work done by gravity in this process assuming that the complete water finally lies at the bottom of the cube is :



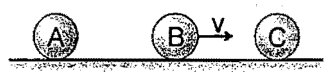
(A) $\frac{1}{2}$ mg a (B) $\frac{3}{8}$ mg a

(C) $\frac{5}{8}$ mga (D) $\frac{1}{8}$ mga

9.6 A balloon having mass 'm' is filled with gas and is held in hands of a boy. Then suddenly it get released and gas starts coming out of it with a constant rate. The velocities of the ejected gases is also constant 2 m/s with respect to the balloon. Find out the velocity of the balloon when the mass of gas is reduced to half. [Effect of atmosphere and gravity is neglected]

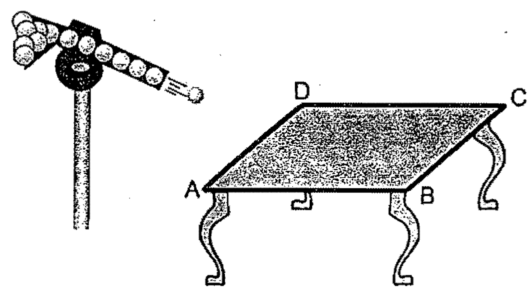
(A) $\ell n\, 2$ (B) $2\,\ell n\, 4$ (C) $2\,\ell n\, 2$ (D) none of these

9.7 Three balls A, B and C ($m_A = m_C = 4\,m_B$) are placed on a smooth horizontal surface. Ball B collides with ball C with an initial velocity n as shown in the figure. Total number of collisions between the balls will be : (All collisions are elastic)



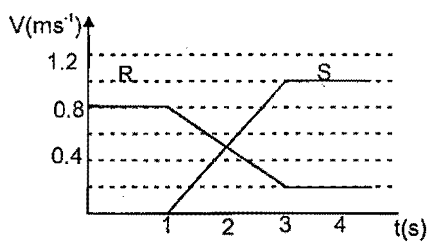
(A) one (B) two (C) three (D) four

9.8 A gun is firing 20 balls per seconds of mass 20 gm each on the smooth horizontal table surface ABCD. If the collision is perfectly elastic and balls are striking at the centre of table with a speed 5 m/sec at an angle of 60° with the vertical just before collision, then force exerted by one of the leg on ground is (assume total weight of the table is 0.2 kg and g = 10 m/s²) :



(A) 0.5 N (B) 1 N (C) 0.25 N (D) 0.75 N

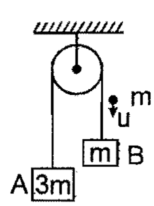
9.9 The diagram shows the velocity - time graph for two masses R and S that collided elastically. Which of the following statements is true?



I. R and S moved in the same direction after the collision.
II. The velocities of R and S were equal at the mid time of the collision.
III. The mass of R was greater than mass of S.

(A) I only (B) II only (C) I and II only (D) I, II and III

9.10 A system of two blocks A and B are connected by an inextensible massless string as shown in the figure . The pulley is massless and frictionless. Initially the system is at rest when, a bullet of mass 'm' moving with a velocity 'u' hits the block 'B' and gets embedded into it. The impulse imparted by tension force to the block of mass 3m is :

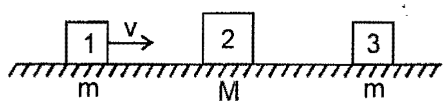


(A) $\frac{5mu}{4}$ (B) $\frac{4mu}{5}$ (C) $\frac{2mu}{5}$ (D) $\frac{3mu}{5}$

9.11 A stationary body explodes into four identical fragments such that three of them fly off mutually perpendicular to each other, each with same K.E., E_0. The energy of explosion will be:

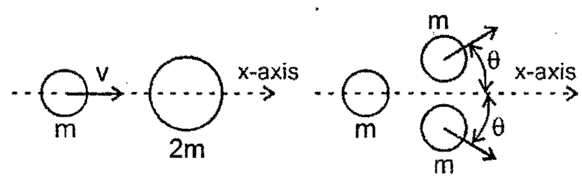
(A) $6E_0$ (B) $\frac{4E_0}{3}$ (C) $4E_0$ (D) $8E_0$

9.12 Three blocks are placed on smooth horizontal surface and lie on same horizontal straight line. Block 1 and block 3 have mass m each and block 2 has mass M (M > m). Block 2 and block 3 are initially stationary, while block 1 is initially moving towards block 2 with speed v as shown. Assume that all collisions are headon and perfectly elastic. What value of $\frac{M}{m}$ ensures that block 1 and block 3 have the same final speed ?



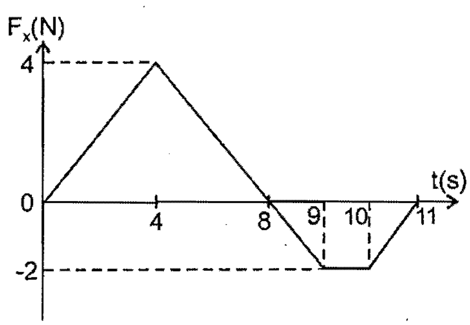
(A) $5+\sqrt{2}$ (B) $5-\sqrt{2}$ (C) $2+\sqrt{5}$ (D) $3+\sqrt{5}$

9.13 A particle of mass m is moving along the x-axis with speed v when it collides with a particle of mass 2m initially at rest. After the collision, the first particle has come to rest and the second particle has split into two equal-mass pieces that are shown in the figure. Which of the following statements correctly describes the speeds of the two pieces ? (θ > 0)



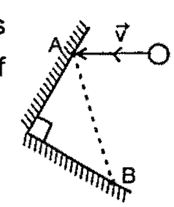
(A) Each piece moves with speed v.
(B) Each piece moves with speed v/2.
(C) One of the pieces moves with speed v/2, the other moves with speed greater than v/2
(D) Each piece moves with speed greater than v/2.

9.14 A 2 kg toy car can move along an x axis. Graph shows force F_x, acting on the car which begins at rest at time t = 0. The velocity of the car at t = 10 s is :



(A) $-\hat{i}$ m/s (B) $-1.5\,\hat{i}$ m/s (C) $6.5\,\hat{i}$ m/s (D) $13\,\hat{i}$ m/s

9.15 AB is an L shaped obstacle fixed on a horizontal smooth table. A ball strikes it at A, gets deflected and restrikes it at B. If the velocity vector before collision is $\vec{v}$ and coefficient of restitution of each collision is 'e', then the velocity of ball after its second collision at B is :



(A) $e^2\vec{v}$ (B) $-e^2\vec{v}$
(C) $-e\vec{v}$ (D) data insufficient

9.16 For a two-body system in absence of external forces, the kinetic energy as measured from ground frame is K_o and from center of mass frame is K_{cm}. Pick up the wrong statement

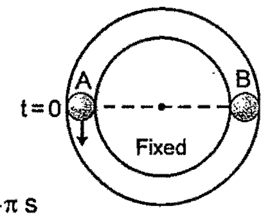
(A) The kinetic energy as measured from center of mass frame is least

(B) Only the portion of energy K_{cm} can be transformed from one form to another due to internal changes in the system.

(C) The system always retains at least $K_o - K_{cm}$ amount of kinetic energy as measured from ground frame irrespective of any kind of internal changes in the system.

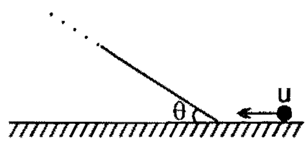
(D) The system always retains at least K_{cm} amount of kinetic energy as measured from ground frame irrespective of any kind of internal changes in the system

9.17 Particle 'A' moves with speed 10 m/s in a frictionless circular fixed horizontal pipe of radius 5 m and strikes with 'B' of double mass that of A. Coefficient of restitution is 1/2 and particle 'A' starts its journey at t = 0. The time at which second collision occurs is :



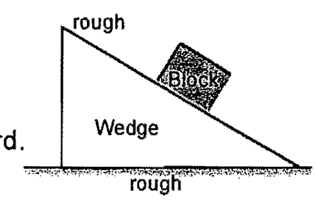
(A) $\frac{\pi}{2}$ s (B) $\frac{2\pi}{3}$ s (C) $\frac{5\pi}{2}$ s (D) 4π s

9.18 A particle of mass m is given initial speed u as shown in the figure. It transfers to the fixed inclined plane without a jump, that is, its trajectory changes sharply from the horizontal line to the inclined line. All the surfaces are smooth and $90° \geq \theta > 0°$. Then the height to which the particle shall rise on the inclined plane (assume the length of the inclined plane to be very large)



(A) increases with increase in θ (B) decreases with increase in θ
(C) is independent of θ (D) data insufficient

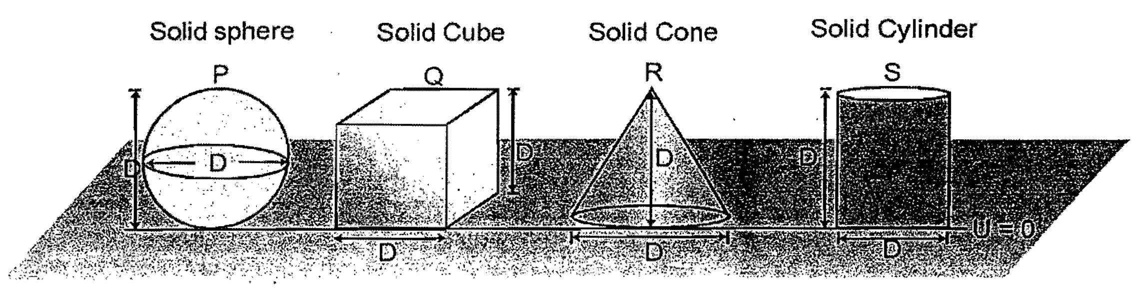
9.19 When a block is placed on a wedge as shown in the figure, the block starts sliding down and the wedge also start sliding on ground. All surfaces are rough. The centre of mass of (wedge + block) system will move



(A) leftward and downward. (B) right ward and downward.
(C) leftward and upwards. (D) only downward.

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

9.20 Assuming potential energy 'U' at ground level to be zero.



All objects are made up of same material.
U_P = Potential energy of solid sphere
U_Q = Potential energy of solid cube
U_R = Potential energy of solid cone
U_S = Potential energy of solid cylinder

(A) $U_S > U_P$ (B) $U_Q > U_S$ (C) $U_P > U_Q$ (D) $U_S > U_R$

9.21 A bag of mass M hangs by a long thread and a bullet (mass m) comes horizontally with velocity v and gets caught in the bag. Then for the combined system (bag + bullet) :

(A) Momentum is $mMv/(M + m)$ (B) KE is $(1/2)\ Mv^2$

(C) Momentum is mv (D) KE is $m^2v^2/2(M + m)$

9.22 A ball of mass m moving with a velocity v hits a massive wall of mass M (M >>m) moving towards the ball with a velocity u. An elastic impact lasts for a time Δt.

(A) The average elastic force acting on the ball is $\frac{m(u+v)}{\Delta t}$

(B) The average elastic force acting on the ball is $\frac{2m(u+v)}{\Delta t}$

(C) The kinetic energy of the ball increases by $2mu\ (u + v)$

(D) The kinetic energy of the ball remains the same after the collision.

9.23 A particle strikes a horizontal smooth floor with a velocity u making an angle θ with the floor and rebounds with velocity v making an angle ϕ with the floor. If the coefficient of restitution between the particle and the floor is e, then :

(A) the impulse delivered by the floor to the body is $mu(1 + e) \sin \theta$.

(B) $\tan \phi = e \tan \theta$.

(C) $v = u\sqrt{1-(1-e^2)\sin^2\theta}$.

(D) the ratio of the final kinetic energy to the initial kinetic energy is $(\cos^2 \theta + e^2 \sin^2 \theta)$

9.24 A smooth sphere A of mass m collides elastically with an identical sphere B at rest. The velocity of A before collision is 8 m/s in a direction making 60° with the line joining the centres at the time of impact. Which of the following is/are possible :

(A) the sphere A comes to rest after collision

(B) the sphere B will move with a speed of 8 m/s after collision

(C) the directions of motion of A and B after collision are at right angles

(D) the speed of B after collision is 4 m/s

SECTION - III : ASSERTION AND REASON TYPE

9.25 **Statement-1 :** No external force acts on system of two spheres which undergo a perfectly elastic head on collision. The minimum kinetic energy of this system is zero if the net momentum of this system is zero.

Statement-2 : In any two body system undergoing perfectly elastic head on collision, at the instant of maximum deformation, the complete kinetic energy of the system is converted to deformation potential energy of the system.

(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True.

9.26 **Statement-1 :** A sphere of mass m moving with speed u undergoes a perfectly elastic head on collision with another sphere of heavier mass M at rest (M > m), then direction of velocity of sphere of mass m is reversed due to collision [no external force acts on system of two spheres]

Statement-2 : During a collision of spheres of unequal masses, the heavier exerts more force on lighter mass in comparison to the force which lighter mass exerts on heavier mass.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

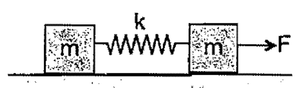
9.27 **Statement-1 :** A rocket launched vertically upward explodes at the highest point it reaches. The explosion produces three fragments with non-zero initial velocity. Then the initial velocity vectors of all the three fragments are in one plane.
Statement-2 : For sum of momentum of three particles to be zero all the three momentum vectors must be coplanar.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

9.28 **Statement-1 :** Two spheres undergo a perfectly elastic collision. The kinetic energy of system of both spheres is always constant. [There is no external force on system of both spheres].
Statement-2 : If net external force on a system is zero, the velocity of centre of mass remains constant.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

SECTION - IV : COMPREHENSION TYPE

Comprehension # 1

Two blocks of equal mass m are connected by an unstretched spring and the system is kept at rest on a frictionless horizontal surface. A constant force F is applied on the first block pulling it away from the other as shown in figure.



9.29 Then the displacement of the centre of mass at time t is :

(A) $\frac{Ft^2}{2m}$ (B) $\frac{Ft^2}{3m}$ (C) $\frac{Ft^2}{4m}$ (D) $\frac{Ft^2}{m}$

9.30 If the extension of the spring is x_0 at time t, then the displacement of the right block at this instant is :

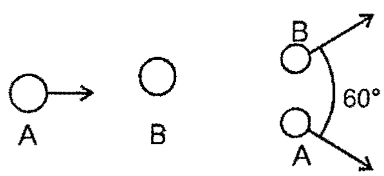
(A) $\frac{1}{2}\left(\frac{Ft^2}{2m}+x_0\right)$ (B) $-\frac{1}{2}\left(\frac{Ft^2}{2m}+x_0\right)$ (C) $\frac{1}{2}\left(\frac{Ft^2}{2m}-x_0\right)$ (D) $\left(\frac{Ft^2}{2m}+x_0\right)$

9.31 If the extension of the spring is x_0 at time t, then the displacement of the left block at this instant is :

(A) $\left(\frac{Ft^2}{2m}-x_0\right)$ (B) $\frac{1}{2}\left(\frac{Ft^2}{2m}+x_0\right)$ (C) $\frac{1}{2}\left(\frac{2Ft^2}{m}-x_0\right)$ (D) $\frac{1}{2}\left(\frac{Ft^2}{2m}-x_0\right)$

Comprehension # 2

A smooth ball 'A' moving with velocity 'V' collides with another smooth identical ball at rest. After collision both the balls move with same speed with angle between their velocities 60°. No external force acts on the system of balls.



9.32 The speed of each ball after the collision is

(A) $\frac{V}{2}$ (B) $\frac{V}{3}$ (C) $\frac{V}{\sqrt{3}}$ (D) $\frac{2V}{\sqrt{3}}$

9.33 If the kinetic energy lost is fully converted to heat then heat produced is

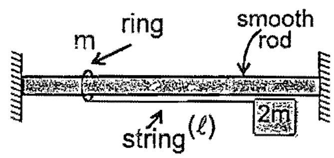
(A) $\frac{1}{3}mV^2$ (B) $\frac{2}{3}mV^2$ (C) 0 (D) $\frac{1}{6}mV^2$

9.34 The value of coefficient of restitution is

(A) 1 (B) $\frac{1}{3}$ (C) $\frac{1}{\sqrt{3}}$ (D) 0

Comprehension # 3

In given figure, the small block of mass 2m is released from rest when the string is in horizontal position,



9.35 Displacement of the ring when string makes an angle θ = 37° with the vertical will be:

(A) $\frac{4\ell}{15}$ (B) $\frac{\ell}{15}$ (C) $\frac{2\ell}{15}$ (D) none of these

9.36 Maximum possible velocity of ring of mass 'm' is (Assuming zero friction):

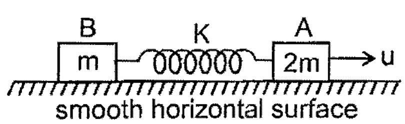
(A) $\sqrt{2g\ell}$ (B) $\sqrt{\frac{4g\ell}{3}}$ (C) $\sqrt{\frac{8g\ell}{3}}$ (D) none of these

9.37 Find the tension in the string when the block has maximum velocity.

(A) 12 mg (B) 14 mg (C) 8 mg (D) 20 mg

SECTION - V : MATRIX - MATCH TYPE

9.38 Two blocks A and B of mass m and 2m respectively are connected by a massless spring of spring constant K. This system lies over a smooth horizontal surface. At t = 0 the block A has velocity u towards right as shown while the speed of block B is zero, and the length of spring is equal to its natural length at that instant. In each situation of column I, certain statements are given and corresponding results are given in column II. Match the statements in column I with corresponding results in column II.

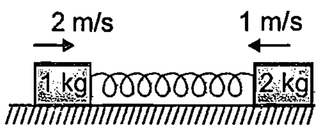


Column I	Column II
(A) The velocity of block A	(p) can never be zero
(B) The velocity of block B	(q) may be zero at certain instants of time
(C) The kinetic energy of system of two blocks	(r) is minimum at maximum compression of spring
(D) The potential energy of spring	(s) is maximum at maximum extension of spring

9.39 In all cases in column–I, the blocks are placed on the smooth horizontal surface.

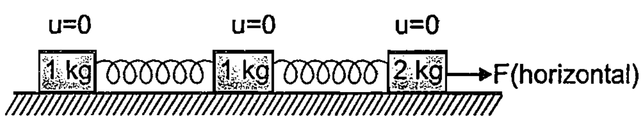
Column–I

(A) The initial velocities given to the blocks when spring is relaxed are as shown (friction is absent)



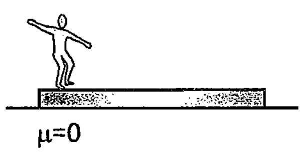
System (two blocks + spring)

(B) A constant force is applied on 2 kg block. Springs are initially relaxed & friction is absent



System (three blocks + two springs)

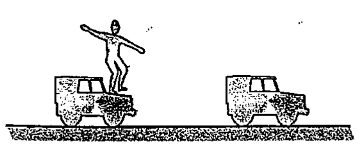
(C) There is no friction between plank and ground and initially system is at rest. Man starts moving on a large plank with constant velocity.



System (man and plank)

(D) Two trolleys are resting on a smooth horizontal

surface and a man standing on one of the trolleys jumps to the other with relative velocity of 4 m/s



System (two trolleys + man)

Column–II

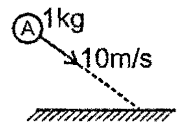
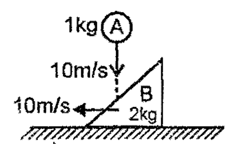
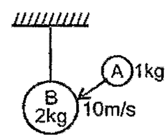
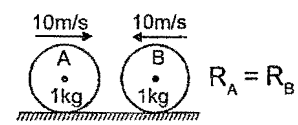
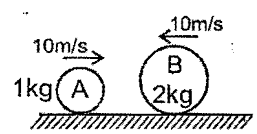
(p) Centre of mass of the complete system shown will not move horizontally

(q) Centre of mass of the complete system shown will move horizontally

(r) Mechanical energy of the system will be conserved

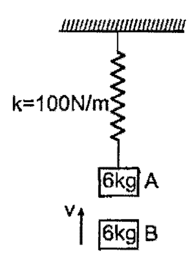
(s) Mechanical energy of the system will increase

9.40 In column–II different situations are shown in which one object collides with the another object. In each case friction is absent and neglect effect of non–impulsive forces. In column–I different direction are given. **You have to match the directions for each case in which momentum conservation can be applied on object A or object B or system A & B.** (Assume that objects do not bounce off the ground)

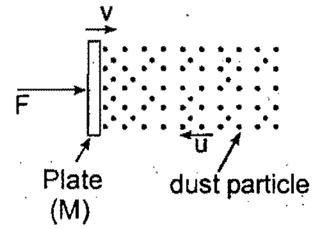
	Column-I		Column-II
(A)	Along the line of impact	(p)	
(B)	Perpendicular to line of impact	(q)	
(C)	In horizontal direction	(r)	
(D)	In vertical direction	(s)	
		(t)	

SECTION - VI : INTEGER TYPE

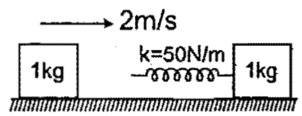
9.41 Block 'A' is hanging from a vertical spring and is at rest. Block 'B' strikes the block 'A' with velocity 'v' and sticks to it. Then the value of 'v' for which the spring just attains natural length is :



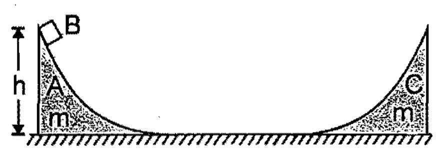
9.42 A plate of mass M is moved with constant velocity v against dust particles moving with velocity u in opposite direction as shown. The density of the dust is ρ and plate area is A. Find the force F required to keep the plate moving uniformly is $A\rho\,(u + v)^N$, then N is



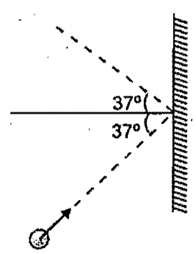
9.43 Each of the blocks shown in figure has mass 1 kg. The rear block moves with a speed of 2 m/s towards the front block kept at rest. The spring attached to the front block is light and has a spring constant 50 N/m. Find the maximum compression of the spring.(in cm.)



9.44 In the figure shown a small block B of mass m is released from the top of a smooth movable wedge A of the same mass m. The height of wedge A shown in figure is h = 100 cm. B ascends another movable smooth wedge C of the same mass. Neglecting friction any where find the maximum height (in cm) attained by block B on wedge C.



9.45 A particle moving on a smooth horizontal surface strikes a stationary wall. The angle of strike is equal to the angle of rebound & is equal to 37° and the coefficient of restitution with wall is $e = \frac{1}{5}$. Find the friction coefficient between wall and the particle in the form $\frac{X}{10}$ and fill value of X.:

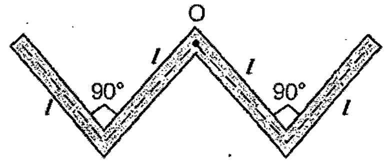


TOPIC 10

RIGID BODY DYNAMICS

SECTION - I : STRAIGHT OBJECTIVE TYPE

10.1 A uniform thin rod of length 4 l, mass 4m is bent at the points as shown in the figure. What is the moment of inertia of the rod about the axis passing through point O & perpendicular to the plane of the paper.

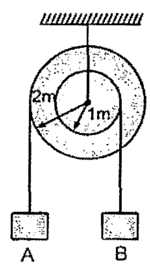


(A) $\frac{m\ell^2}{3}$ (B) $\frac{10m\ell^2}{3}$ (C) $\frac{m\ell^2}{12}$ (D) $\frac{m\ell^2}{24}$

10.2 Two points of a rod move with velocities 3 v & v perpendicular to the rod and in the same direction, separated by a distance 'r'. Then the angular velocity of the rod is :

(A) $\frac{3v}{r}$ (B) $\frac{4v}{r}$ (C) $\frac{5v}{r}$ (D) $\frac{2v}{r}$

10.3 In the pulley system shown, if radii of the bigger and smaller pulley are 2 m and 1 m respectively and the acceleration of block A is 5 m/s² in the downward direction, then the acceleration of block B will be:



(A) 0 m/s² (B) 5 m/s² (C) 10 m/s² (D) 5/2 m/s²

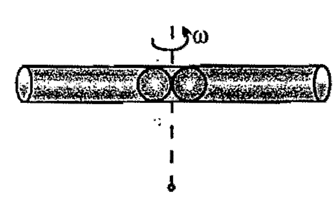
10.4 A uniform thin rod of mass 'm' and length L is held horizontally by two vertical strings attached to the two ends. One of the string is cut. Find the angular acceleration soon after it is cut :

(A) $\frac{g}{2L}$ (B) $\frac{g}{L}$ (C) $\frac{3g}{2L}$ (D) $\frac{2g}{L}$

10.5 A sphere is released on a smooth inclined plane from the top. When it moves down its angular momentum is :

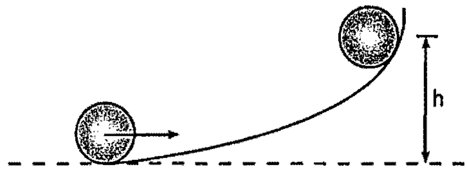
(A) conserved about every point
(B) conserved about the point of contact only
(C) conserved about the centre of the sphere only
(D) conserved about any point on a line parallel to the inclined plane and passing through the centre of the ball.

10.6 A smooth tube of certain mass is rotated in gravity free space and released. The two balls shown in the figure move towards ends of the tube. For the whole system which of the following quantity is not conserved ?



(A) Angular momentum (B) Linear momentum
(C) Kinetic energy (D) Angular speed

10.7 In the figure shown a ball rolls without sliding. On a horizontal surface. It ascends a curved track upto height h and returns. Value of h is h_1 for sufficiently rough curved track to avoid sliding and h_2 for smooth curved track, then :

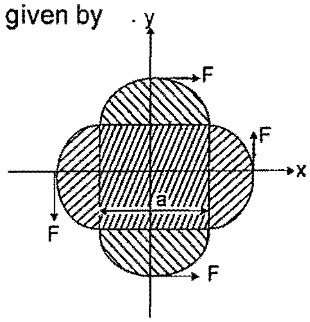


(A) $h_1 = h_2$ (B) $h_1 < h_2$ (C) $h_1 > h_2$ (D) $h_2 = 2h_1$

10.8 A uniform ring of radius R rolls without sliding. The radius of curvature of the path followed by any particle of the ring at the highest point of its path will be :
(A) R (B) 2 R (C) 4 R (D) none of these

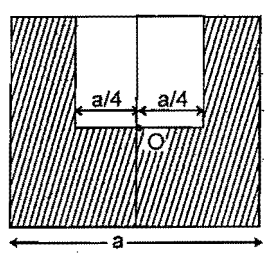
10.9 A uniform ring of radius R is given a back spin of angular velocity $V_0/2R$ and thrown on a horizontal rough surface with velocity of center to be V_0. The velocity of the centre of the ring when it starts pure rolling will be
(A) $V_0/2$ (B) $V_0/4$ (C) $3V_0/4$ (D) 0

10.10 A planar object made up of a uniform square plate and four semicircular discs of the same thickness and material is being acted upon by four forces of equal magnitude as shown in figure. The coordinates of point of application of forces is given by



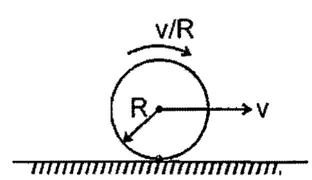
(A) (0, a) (B) (0, –a) (C) (a, 0) (D) (–a, 0)

10.11 A square plate of edge a/2 is cut out from a uniform square plate of edge 'a' as shown in figure. The mass of the remaining portion is M. The moment of inertia of the shaded portion about an axis passing through 'O' (centre of the square of side a) and perpendicular to plane of the plate is :



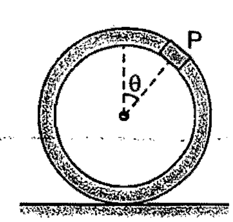
(A) $\frac{9}{64}Ma^2$ (B) $\frac{3}{16}Ma^2$ (C) $\frac{5}{12}Ma^2$ (D) $\frac{Ma^2}{6}$

10.12 A uniform disc is performing pure rolling on a smooth stationary surface with constant angular velocity as shown in figure. At any instant, for the lowermost point of the disc :



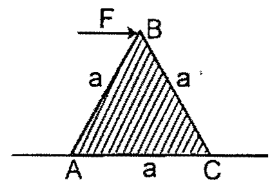
(A) Velocity is v, acceleration is zero
(B) Velocity is zero, acceleration is zero
(C) velocity is v, acceleration is $\frac{v^2}{R}$
(D) velocity is zero, acceleration is $\frac{v^2}{R}$

10.13 A small block of mass 'm' is rigidly attached at 'P' to a ring of mass '3m' and radius 'r'. The system is released from rest at $\theta = 90°$ and rolls without sliding. The angular acceleration of hoop just after release is –



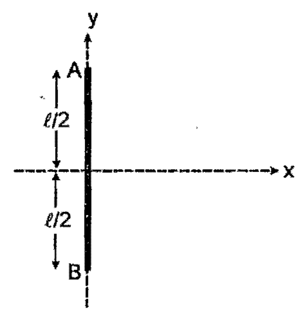
(A) $\frac{g}{4r}$ (B) $\frac{g}{8r}$ (C) $\frac{g}{3r}$ (D) $\frac{g}{2r}$

10.14 A uniform equilateral prism of mass m rests on a rough horizontal surface with coefficient of friction μ. A horizontal force F is applied on the prism as shown in the figure. If the coefficient of friction is sufficiently high so that the prism does not slide before toppling, then the minimum force required to topple the prism is :



(A) $\frac{mg}{\sqrt{3}}$ (B) $\frac{mg}{4}$ (C) $\frac{\mu mg}{\sqrt{3}}$ (D) $\frac{\mu mg}{4}$

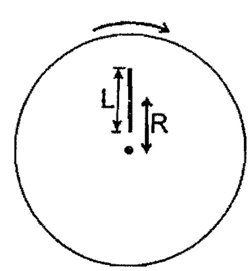
10.15 A uniform rod of mass m, length ℓ is placed over a smooth horizontal surface along y-axis and is at rest as shown in figure. An impulsive force F is applied for a small time Δt along x-direction at point A. The x–coordinate of end A of the rod when the rod becomes parallel to x–axis for the first time is (initially the coordinate of centre of mass of the rod is (0, 0)) :



(A) $\frac{\pi\ell}{12}$ (B) $\frac{\ell}{2}\left(1+\frac{\pi}{12}\right)$

(C) $\frac{\ell}{2}\left(1-\frac{\pi}{6}\right)$ (D) $\frac{\ell}{2}\left(1+\frac{\pi}{6}\right)$

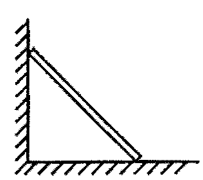
10.16 A uniform rod of mass M and length L lies radially on a disc rotating with angular speed ω in a horizontal plane about its axis. The rod does not slip on the disc and the centre of the rod is at a distance R from the centre of the disc. Then the kinetic energy of the rod is :



(A) $\frac{1}{2}m\omega^2\left(R^2+\frac{L^2}{12}\right)$ (B) $\frac{1}{2}m\omega^2R^2$

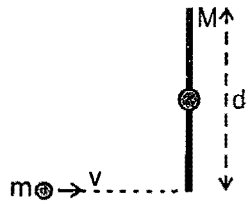
(C) $\frac{1}{24}m\omega^2L^2$ (D) None of these

10.17 A uniform rod of length ℓ is sliding such that one of its ends is always in contact with a vertical wall and its other end is always in contact with horizontal surface. Just after the rod is released from rest, the magnitude of acceleration of end points of the rod are a and b respectively. The angular acceleration of rod at this instant will be



(A) $\frac{a+b}{\ell}$ (B) $\frac{\sqrt{|a^2-b^2|}}{\ell}$ (C) $\frac{\sqrt{a^2+b^2}}{\ell}$ (D) None of these

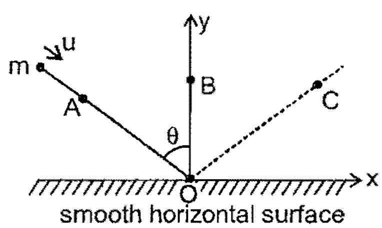
10.18 A particle of mass m is moving at speed v perpendicular to a rod of length d and mass M = 6m which pivots around a frictionless axle running through its centre. It strikes and sticks to the end of the rod. The moment of inertia of the rod about its centre is $Md^2/12$. Then the angular speed of the system just after the collision is



(A) 2v/3d. (B) 2v/d. (C) v/d. (D) 3v/2d.

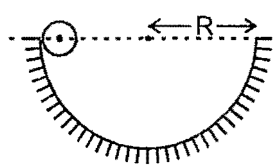
10.19 A particle falls freely near the surface of the earth. Consider a fixed point O (not vertically below the particle) on the ground. Then pickup the incorrect alternative
(A) Angular momentum of the particle about O is increasing.
(B) Torque of the gravitational force on the particle about O is decreasing.
(C) The moment of inertia of the particle about O is decreasing.
(D) The angular velocity of the particle about O is increasing.

10.20 A ball of mass m moving with constant velocity u collides with a smooth horizontal surface at O as shown. Neglect gravity and friction. The y-axis is drawn normal to the horizontal surface at the point of impact O and x-axis is horizontal as shown in the figure. About which point will the angular momentum of ball be conserved.



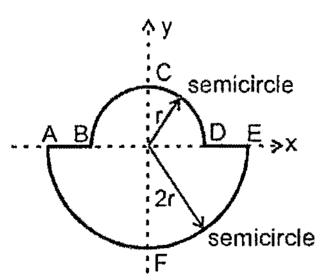
(A) point A (B) point B (C) point C (D) None of these

10.21 In the figure shown, a small solid spherical ball of mass 'm' can move without sliding in a fixed semicircular track of radius R in vertical plane. It is released from the top. The resultant force on the ball at the lowest point of the track is



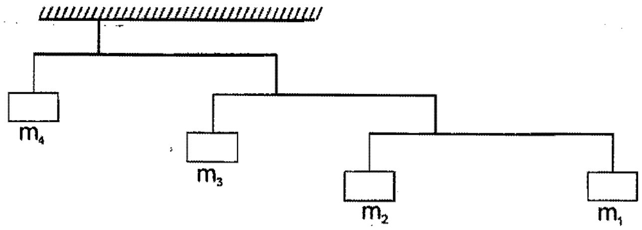
(A) $\frac{10mg}{7}$ (B) $\frac{17mg}{7}$ (C) $\frac{3mg}{7}$ (D) zero

10.22 A uniform thin rod is bent in the form of closed loop ABCDEFA as shown in the figure.



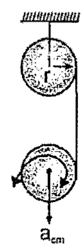
The ratio of moment of inertia of the loop about x-axis to that about y-axis is.
(A) > 1 (B) < 1 (C) = 1 (D) = 1/2

10.23 Figure shows an arrangement of masses hanging from a ceiling. In equilibrium, each rod is horizontal, has negligible mass and extends three times as far to the right of the wire supporting it as to the left. If mass m_4 is 48 kg then mass m_1 is equal to.



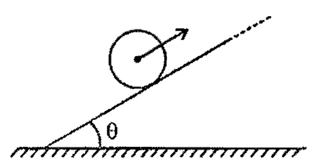
(A) 1 kg (B) 2 kg (C) 3 kg (D) 4 kg

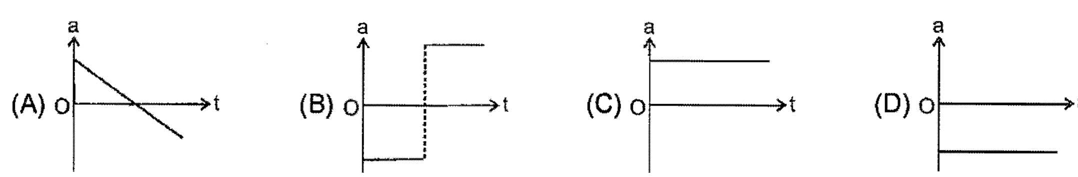
10.24 Two identical uniform discs of mass m and radius r are arranged as shown in the figure. If α is the angular acceleration of the lower disc and a_{cm} is acceleration of centre of mass of the lower disc, then relation among a_{cm}, α & r is :



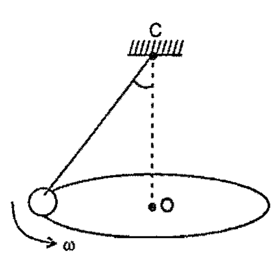
(A) $a_{cm} = \frac{\alpha}{r}$ (B) $a_{cm} = 2\alpha r$ (C) $a_{cm} = \alpha r$ (D) none of these

10.25 A uniform solid sphere rolls up (without slipping) the rough fixed inclined plane, and then back down. Which is the correct graph of acceleration 'a' of centre of mass of solid sphere as function of time t (for the duration sphere is on the incline) ? Assume that the sphere rolling up has a positive velocity.



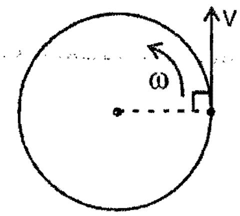


10.26 A conical pendulum consists of a simple pendulum moving in a horizontal circle as shown in the figure. C is the pivot, O the centre of the circle in which the pendulum bob moves and ω the constant angular velocity of the bob. If $\vec{L}$ is the angular momentum about point C, then



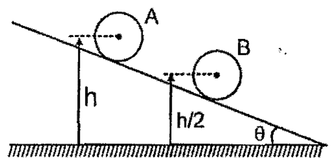
(A) $\vec{L}$ is constant
(B) only direction of $\vec{L}$ is constant
(C) only magnitude of $\vec{L}$ is constant
(D) none of the above

10.27 A child with mass m is standing at the edge of a playground merry-go-round (A large uniform disc which rotates in horizontal plane about a fixed vertical axis in parks) with moment of inertia I, radius R, and initial angular velocity ω as shown in figure. The child jumps off the edge of the merry-go-round with a velocity v with respect to the ground in direction tangent to periphery of the disc as shown. The new angular velocity of the merry-go-round is:



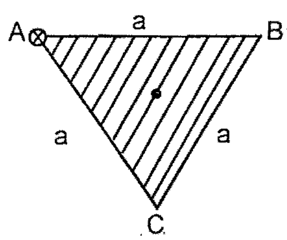
(A) $\sqrt{\frac{I\omega^2 - mv^2}{I}}$ (B) $\sqrt{\frac{(I+mR^2)\omega^2 - mv^2}{I}}$ (C) $\frac{I\omega - mvR}{I}$ (D) $\frac{(I+mR^2)\omega - mvR}{I}$

10.28 Two identical uniform solid spherical balls A & B of mass m each are placed on a fixed wedge as shown in figure. Ball B is kept at rest and it is released just before two balls collides. Ball A rolls down without slipping on inclined plane & collide elastically with ball B. The kinetic energy of ball A just after the collision with ball B is :



(A) $\frac{mgh}{7}$ (B) $\frac{mgh}{2}$ (C) $\frac{2mgh}{5}$ (D) $\frac{7mgh}{5}$

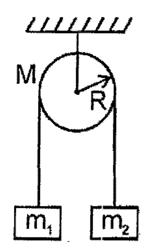
10.29 A uniform triangular plate ABC of moment of mass m and inertia I (about an axis passing through A and perpendicular to plane of the plate) can rotate freely in the vertical plane about point 'A' as shown in figure. The plate is released from the position shown in the figure. Line AB is horizontal. The acceleration of centre of mass just after the release of plate is :



(A) $\frac{mga^2}{\sqrt{3}I}$ (B) $\frac{mga^2}{4I}$ (C) $\frac{mga^2}{2\sqrt{3}I}$ (D) $\frac{mga^2}{3I}$

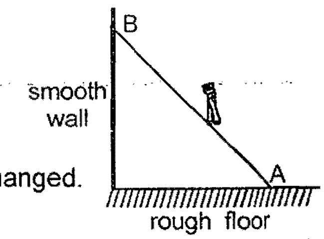
SECTION - II : MULTIPLE CORRECT ANSWER TYPE

10.30 In the shown figure, the pulley of mass M and radius R can rotate about its fixed horizontal axis (axle) without friction. Friction between light inextensible string and pulley is sufficient to prevent slipping of string over pulley. The masses of blocks are m_1 and m_2 such that $m_2 > m_1$. The system is released from rest as shown. Before the block of mass m_1 touches the pulley, pick up the correct statements.



(A) The magnitude of acceleration of any small length $d\ell$ of string is constant throughout the motion.
(B) Magnitude of force exerted by string on mass m_2 is larger as compared to that exerted by string on mass m_1.
(C) Accelerations of both blocks are same.
(D) The acceleration of small length $d\ell$ of string in contact with block of mass m_2 remains constant.

10.31 A ladder AB is supported by a smooth vertical wall and rough horizontal floor as shown. A boy starts moving from A to B slowly. The ladder remains at rest, then pick up the correct statement(s) :

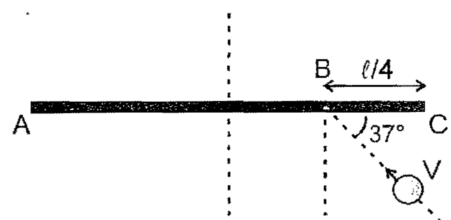


(A) Magnitude of normal reaction by wall on ladder at point B will increase.
(B) Magnitude of normal reaction by wall on ladder at point B will decrease.
(C) Magnitude of normal reaction by floor on ladder at point A will remain unchanged.
(D) Magnitude of friction force by floor on ladder at point A will increase.

10.32 A rigid body is in pure rotation, that is, undergoing fixed axis rotation. Then which of the following statement(s) are true ?
(A) You can find two points in the body in a plane perpendicular to the axis of rotation having same velocity
(B) You can find two points in the body in a plane perpendicular to the axis of rotation having same acceleration.
(C) Speed of all the particles lying on the curved surface of a cylinder whose axis coincides with the axis of rotation is same.
(D) Angular speed of the body is same as seen from any point in the body.

10.33 A uniform disc of mass 2kg and radius 1m is mounted on an axle supported on fixed frictionless bearings. A light cord is wrapped around the rim of the disc and mass of 1kg is tied to the free end. If it is released from rest ,
(A) the tension in the cord is 5N
(B) in first 4 seconds the angular displacement of the disc is 40 rad.
(C) the work done by the torque on the disc in first 4 sec. is 200J
(D) the increase in the kinetic energy of the disc in the first 4 seconds is 200J.

10.34 A uniform rod AC of length ℓ and mass m is kept on a horizontal smooth plane. It is free to rotate and move. A particle of same mass m moving on the plane with velocity v strikes the rod at point B making an angle 37^0 with the rod. The collision is elastic. After collision :



(A) The angular velocity of the rod will be $\frac{72}{55}\frac{v}{\ell}$

(B) The centre of the rod will travel a distance $\frac{\pi \ell}{3}$ in the time in which it makes half rotation

(C) Impulse of the impact force is $\frac{24mv}{55}$

(D) None of these

SECTION - III : ASSERTION AND REASON TYPE

10.35 **Statement-1 :** A uniform rigid disc rolls without slipping on a fixed rough horizontal surface with uniform angular velocity. Then the acceleration of lowest point on the disc is zero.
Statement-2 : For a uniform rigid disc rolling without slipping on a fixed rough horizontal surface, the velocity of the lowest point on the disc is always zero.
(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True.

10.36 **Statement-1 :** A uniform thin rod of length L is hinged about one of its end and is free to rotate about the hinge without friction. Neglect the effect of gravity. A force F is applied at a distance x from the hinge on the rod such that force is always perpendicular to the rod. As the value of x is increased from zero to L, the component of reaction by hinge on the rod perpendicular to length of rod increases.

Statement-2 : Under the conditions given in statement-1 as x is increased from zero to L, the angular acceleration of rod increases.

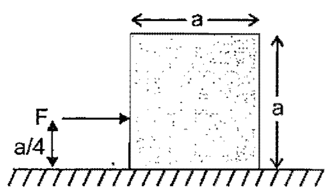
(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True.

10.37 **Statement-1 :** A uniform cubical block(of side a) undergoes translational motion on a smooth horizontal surface under action of horizontal force F as shown. Under the given condition, the horizontal surface exerts normal reaction non-uniformly on lower surface of the block.



Statement-2 : For the cubical block given in statement-1, the horizontal force F has tendency to rotate the cube about its centre in clockwise sense. Hence, the lower right edge of cube presses the horizontal surface harder in comparision to the force exerted by lower left edge of cube on horizontal surface.

(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True.

10.38 **Statement–1 :** A uniform solid sphere and a uniform hollow sphere of same radius and same material are released (at rest) from the top of a fixed inclined plane at the same time. They will reach the bottom simultaneously, if they roll with sliding.

Statement–2: In the situation of statement-1, the centres of both spheres have the same acceleration and they travel the same distance. Hence time taken is same.

(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True.

10.39 **Statement-1 :** The net momentum of a system of two moving particles is zero. Then at a particular instant of time, the net angular momentum of system of given two particle is same about any point.

Statement-2 : If net momentum of a system of two moving particle is zero, then angular momentum of system of given two particles is zero about any point.

(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True.

SECTION - IV : COMPREHENSION TYPE

Comprehension # 1

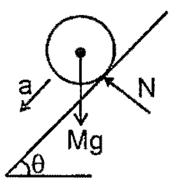
In this passage a brief idea is given of the motion of the rolling bodies on an inclined plane.
We will consider three cases : Objects are released on an incline plane

Case A : which is smooth. ;

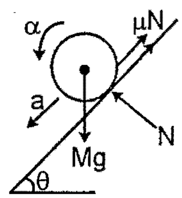
Case B : where friction is insufficient to provide pure rolling.

Case C : where friction is sufficient to provide pure rolling.

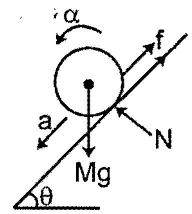
Force diagram for three cases are as follows : (where symbols have their usual meanings)



Case (A)
$\alpha = 0$



Case (B)
$a \neq \alpha R$



Case (C)
$a = \alpha R$

Equations for case (C) :

$Mg\sin\theta - f = Ma$

$fR = (Mk^2)\alpha$; where k = radius of gyration and f is force of friction.

$a = \alpha R$

on solving the above equations we will get

$$a = \frac{g\sin\theta}{\left(1+\frac{k^2}{R^2}\right)}$$

Object	Ring	Disc	Hollow sphere	Solid sphere
k	R	$\frac{R}{\sqrt{2}}$	$\sqrt{\frac{2}{3}}R$	$\sqrt{\frac{2}{5}}R$

To decide the minimum friction coefficient to provide pure rolling put

$$f = \mu\, Mg \cos\theta$$

And we will get
$$\mu_{min} = \frac{\tan\theta\left(\frac{k^2}{R^2}\right)}{\left(1+\frac{k^2}{R^2}\right)}$$

Equations for case (B) :

$Mg\sin\theta - \mu N = Ma$

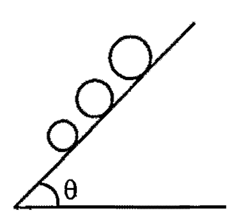
$\mu NR = Mk^2\alpha$

$N = Mg\cos\theta$

The K.E. of a rolling body can be expressed as :

$$K.E. = \frac{1}{2}MV_{CM}^2 + \frac{1}{2}I_{CM}\omega^2$$

10.40 Three solid uniform spheres are released on an inclined plane as shown in figure. The distance between the spheres remains constant during motion in :



(A) all three cases
(B) case 'A' & 'B'
(C) only case
(D) depends on the mass of the spheres

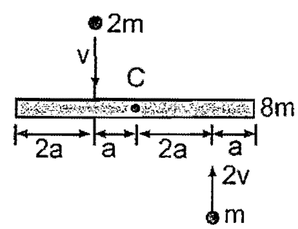
10.41 We have four objects : a solid sphere, a hollow sphere, a ring & a disc, all of same radius. When these are released on an inclined plane, it may happen that all of them do not perform pure rolling. But from the information of pure rolling, if one object can be confirmed to be purely rolling then it can be said that rest all will perform pure rolling. This object whose pure rolling confirms pure rolling of all other objects is :
(A) hollow sphere (B) solid sphere (C) ring (D) disc

10.42 If the four objects given in the above question are of same mass, same radius having the same friction coefficient & are released from the same height, then at the bottom the object which will have least kinetic energy for case 'B' will be then : -
(A) hollow sphere (B) solid sphere (C) ring (D) disc

10.43 Two children 'A' & 'B' use bicycles, having wheels of ring type and disc type respectively. During a race, bicycles are given the same velocity from the bottom of the inclined bridge to ascend the bridge without pedalling, then (assuming pure rolling) :
(A) both bicycles with reach upto same height
(B) bicycle of child 'A' will reach a greater height
(C) bicycle of child B will reach a greater height
(D) depends on the masses of bicycles and the child

Comprehension # 2

A uniform bar of length 6 a & mass 8 m lies on a smooth horizontal table. Two point masses m & 2 m moving in the same horizontal plane with speeds 2 v and v respectively strike the bar as shown in the figure & stick to the bar after collision.



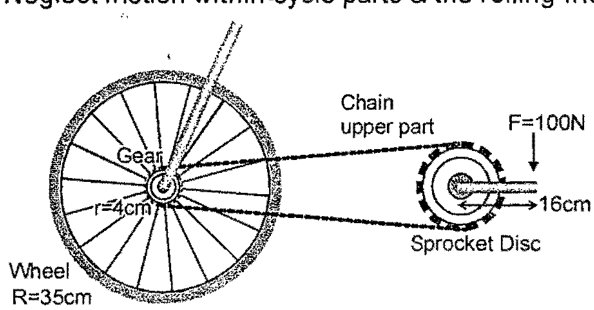
10.44 Velocity of the centre of mass of the system is
(A) $\frac{v}{2}$ (B) v (C) $\frac{2v}{3}$ (D) Zero

10.45 Angular velocity of the rod about centre of mass of the system is
(A) $\frac{v}{5a}$ (B) $\frac{v}{15a}$ (C) $\frac{v}{3a}$ (D) $\frac{v}{10a}$

10.46 Total kinetic energy of the system, just after the collision is
(A) $\frac{3}{5}mv^2$ (B) $\frac{3}{25}mv^2$ (C) $\frac{3}{15}mv^2$ (D) $3\,mv^2$

Comprehension # 3

A bicycle has pedal rods of length 16 cm connected to a sprocketed disc of radius 10 cm. The bicycle wheels are 70 cm in diameter and the chain runs over a gear of radius 4 cm. The speed of the cycle is constant and the cyclist applies 100 N force that is always perpendicular to the pedal rod, as shown in the figure. Assume tension in the lower part of chain is negligible. The cyclist is peddling at a constant rate of two revolutions per second. Assume that the force applied by other foot is zero when one foot is exerting 100 N force. Neglect friction within cycle parts & the rolling friction.



10.47 The tension in the upper portion of the chain is equal to
(A) 100 N (B) 120 N (C) 160 N (D) 240 N

10.48 Net torque on the rear wheel of the bicycle is equal to
(A) zero (B) 16 N-m (C) 6.4 N-m (D) 4.8 N-m

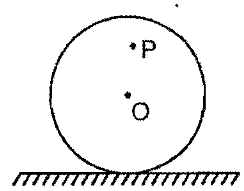
10.49 The power delivered by the cyclist is equal to
(A) 280 W (B) 100 W (C) 64 πW (D) 32 W

10.50 The speed of the bicycle is
(A) 6.4 π m/s (B) 3.5 π m/s (C) 2.8 π m/s (D) 5.6 π m/s

10.51 The net force of the friction on the rear wheel due to the road is :
(A) 100 N (B) 62 N (C) 32.6 N (D) 18.3 N

SECTION - V : MATRIX - MATCH TYPE

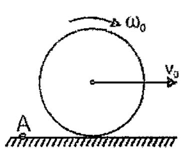
10.52 A uniform disc rolls without slipping on a rough horizontal surface with uniform angular velocity. Point O is the centre of disc and P is a point on disc as shown in figure. In each situation of column I a statement is given and the corresponding results are given in column-II. Match the statements in column-I with the results in column-II.



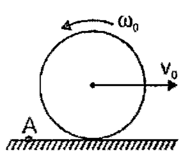
Column I	Column II
(A) The velocity of point P on disc	(p) Changes in magnitude with time
(B) The acceleration of point P on disc	(q) is always directed from that point (the point on disc given in column-I) towards centre of disc.
(C) The tangential acceleration of point P on disc	(r) is always zero
(D) The acceleration of point on disc which is in contact with rough horizontal surface	(s) is non-zero and remains constant in magnitude
	(t) Changes in direction with time.

10.53 In each situation of column-I, a uniform disc of mass m and radius R rolls on a rough fixed horizontal surface as shown. At t=0 (initially) the angular velocity of disc is ω_o and velocity of centre of mass of disc is v_o(in horizontal direction). The relation between v_o and ω_o for each situation and also initial sense of rotation is given for each situation in column-I . Then match the statements in column-I with the corresponding results in column-II .

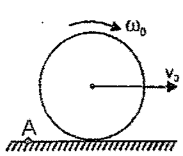
Column-I

(A) 

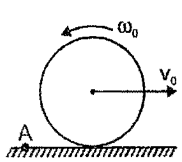
($v_o > R\,\omega_o$)

(B) 

($v_o > R\,\omega_o$)

(C) 

($v_o < R\,\omega_o$)

(D) 

($v_o < R\,\omega_o$)

Column-II

(p) The angular momentum of disc about point A (as shown in figure) remains conserved.

(q) The kinetic energy of disc after it starts rolling without slipping is less than its initial kinetic energy.

(r) In the duration disc rolls with slipping, the friction acts on disc towards left.

(s) Before rolling starts acceleration of the disc remains constant in magnitude and direction.

(t) Final angular velocity is independent of friction coefficient between disc and the surface.

0.54 Consider a system of particles (it may be rigid or non rigid). In the column-I some condition on force and torque is given. Column-II contains the effects on the system. (Letters have usual meaning)

	Column-I		Column-II
(A)	$\vec{F}_{res} = 0$	(p)	$\vec{P}_{system}$ will be constant
(B)	$\vec{\tau}_{res} = 0$	(q)	$\vec{L}_{system}$ will be constant
(C)	External force is absent	(r)	total work done by all forces will be zero
(D)	No nonconservative force acts.	(s)	total mechanical energy will be constant.
		(t)	total work done by all the forces may be non zero.

SECTION - VI : INTEGER TYPE

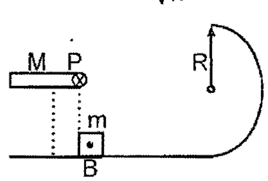
0.55 A uniform rod of length 75 cm is hinged at one of its ends and is free to rotate in vertical plane. It is released from rest when rod is horizontal. When the rod becomes vertical, it is broken at mid–point and lower part now moves freely. The distance of centre of lower part from hinge, when it again becomes vertical for the first time is r. Find the approximate value of 2r.

10.56 A solid homogeneous cylinder of height h=1m and base radius r=1m is kept vertically on a conveyer belt moving horizontally with an increasing velocity $v = 1 + t^2$. If the cylinder is not allowed to slip find the time when the cylinder is about to topple.

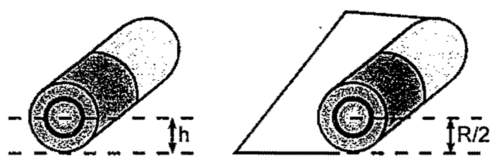
10.57 A billiard ball at rest is struck horizontally one tenth of the diameter below the top. If P be the linear impulse of the blow find the initial kinetic energy of the ball is $\frac{xp^2}{10M}$ then x is given by the mass of the ball is being M.

10.58 In the figure shown a block B of mass m can slide on a fixed horizontal smooth plane. A uniform solid sphere A of radius r of the same mass rolls without sliding on the block B. Find the angular acceleration of the sphere $\frac{xg}{r}$ then x is.

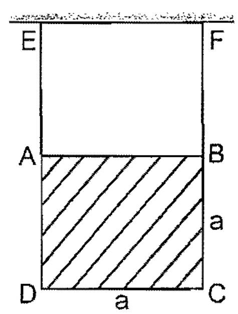
10.59 A uniform rod of length R and mass M is free to rotate about a horizontal axis passing through hinge P as in figure. First it is taken aside such that it becomes horizontal and then released. At the lowest point the rod hits the block B of mass m and stops. if ratio of mass of rod M to the mass of block m such that the block B completes the circle $\sqrt{x}$ then x is. Neglect any friction.



10.60 A carpet of mass M made of inextensible material is rolled along its length in the form of a cylinder of radius R and is kept on a rough floor. The carpet starts unrolling without sliding on the floor when a negligibly small push is given to it. the horizontal velocity of the axis of the cylindrical part of the carpet when its radius reduces to R/2.is $\sqrt{\frac{xgR}{3}}$ ms^{-1} then x is



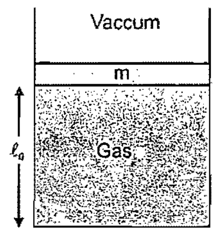
10.61 A thin uniform square plate ABCD of side 'a' and mass m is suspended in vertical plane as shown in the figure. AE and BF are two massless inextensible strings. The line AB is horizontal. Find the tension in the string AE just after BF is cut is $\frac{2mg}{x}$ then x is.



TOPIC 11 SIMPLE HARMONIC MOTION

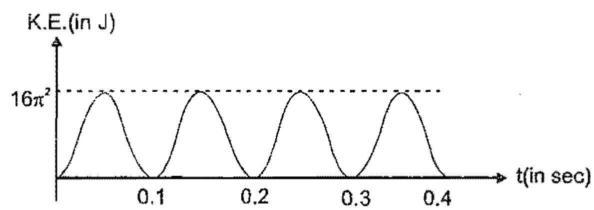
SECTION - I : STRAIGHT OBJECTIVE TYPE

11.1 A smooth piston of mass m and cross–section area A is in equilibrium on a diatomic gas filled in a cylindrical container and outside there is vaccum. The piston and the container are non–conducting and the height of gas column in the container is ℓ_0 at equilibrium. If the piston is displaced slightly down the equilibrium position, then the time period of its oscillation will be :



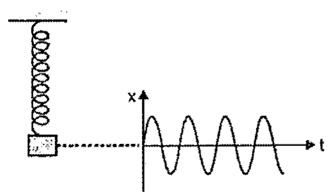
(A) $2\pi\sqrt{\frac{10\ell_0}{7g}}$ (B) $2\pi\sqrt{\frac{7\ell_0}{10g}}$ (C) $2\pi\sqrt{\frac{7\ell_0}{5g}}$ (D) $2\pi\sqrt{\frac{5\ell_0}{7g}}$

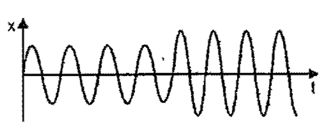
11.2 Kinetic energy of a particle executing SHM changes with time according to following graph mass of the particle is 2 kg. Then :

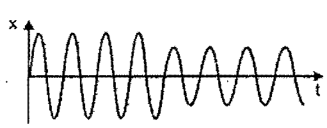


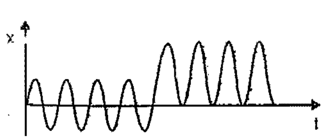
(A) time period of oscillation is 0.1 s
(B) frequency of oscillation is 5 Hz
(C) amplitude of oscillation is 0.6 m
(D) acceleration of the particle is zero at t = 0.1 s

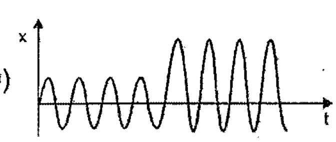
11.3 In the figure is shown a spring mass system oscillating in uniform gravity. If we neglect all dissipative force, it will keep on oscillating endlessly with constant amplitude and frequency. Accompanying graph shows how displacement x of the block from the equilibrium position varies with time t.
Now at a certain instant t = t_e when the block reaches its lowest position, gravity is switches off by some unknown mechanism. Which of the following graphs would correctly describes the changes taking place due to switching off the gravity ?



(A) 

(B) 

(C) 

(D) 

11.4 Two particles P and Q describe S.H.M. of same amplitude a, same frequency f along the same straight line from the same mean positon. The maximum distance between the two particles is $a\sqrt{2}$. The phase difference between the particle is :

(A) zero (B) $\pi/2$ (C) $\pi/6$ (D) $\pi/3$

11.5 A block of mass 'm' is suspended from a spring and executes vertical SHM of time period T as shown in figure. The amplitude of the SHM is A and spring is never in compressed state during the oscillation. The magnitude of minimum force exerted by spring on the block is :

(A) $mg - \frac{4\pi^2}{T^2} mA$ (B) $mg + \frac{4\pi^2}{T^2} mA$

(C) $mg - \frac{\pi^2}{T^2} mA$ (D) $mg + \frac{\pi^2}{T^2} mA$

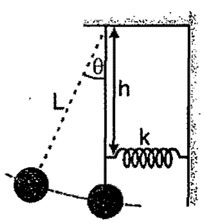
11.6 A particle performs S.H.M. of amplitude A along a straight line. When it is at a distance $\frac{\sqrt{3}}{2}$ A from mean position, its kinetic energy gets increased by an amount $\frac{1}{2} m\omega^2 A^2$ due to an impulsive force. Then its new amplitude becomes :

(A) $\frac{\sqrt{5}}{2}$ A (B) $\frac{\sqrt{3}}{2}$ A (C) $\sqrt{2}$ A (D) $\sqrt{5}$ A

11.7 A horizontal spring–block system of mass 2kg executes S.H.M. When the block is passing through its equilibrium position, an object of mass 1kg is put on it and the two move together. The new amplitude of vibration is (A being its initial amplitude):

(A) $\sqrt{\frac{2}{3}}$ A (B) $\sqrt{\frac{3}{2}}$ A (C) $\sqrt{2}$ A (D) $\frac{A}{\sqrt{2}}$

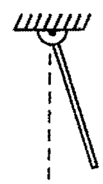
11.8 A pendulum of length L and bob of mass M has a spring of force constant k connected horizontally to it at a distance h below its point of suspension. The rod is in equilibrium in vertical position. The rod of length L used for vertical suspension is rigid and massless. The frequency of vibration of the system for small values of θ is :



(A) $\frac{1}{2\pi L}\sqrt{gL + \frac{kh}{m}}$ (B) $\frac{1}{2\pi L}\sqrt{\frac{mgL + k}{m}}$

(C) $2\pi\sqrt{\frac{mL^2}{mgL + kh}}$ (D) $\frac{1}{2\pi L}\sqrt{gL + \left(\frac{kh^2}{m}\right)}$

11.9 A metre stick swinging in vertical plane about a fixed horizontal axis passing through its one end undergoes small oscillation of frequency f_0. If the bottom half of the stick is cut off, then its new frequency of small oscillation would become :



(A) f_0 (B) $\sqrt{2}\, f_0$ (C) $2f_0$ (D) $2\sqrt{2}\, f_0$

11.10 Which of the following is greatest in SHM (assuming potential energy = 0 at mean position)

(A) Average kinetic energy with respect to space

(B) Average potential energy with respect to space

(C) Average kinetic energy with respect to time

(D) Average potential energy with respect to time

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

11.11 A particle is subjected to two simple harmonic motions along x and y directions according to, $x = 3 \sin 100 \pi t$; $y = 4 \sin 100 \pi t$.
(A) Motion of particle will be on ellipse traversing it in clockwise direction.
(B) Motion of particle will be on a straight line with slope 4/3.
(C) Motion will be a simple harmonic motion with amplitude 5.
(D) Phase difference between two motions is $\pi/2$.

11.12 A horizontal plank has a rectangular block placed on it. The plank starts oscillating vertically and simple harmonically with an amplitude of 40 cm. The block just loses contact with the plank when the latter is at momentary rest. Then :
(A) the period of oscillation is $\left(\frac{2\pi}{5}\right)$
(B) the block weighs double its weight, when the plank is at one of the positions of momentary rest.
(C) the block weighs 0.5 times its weight on the plank halfway up
(D) the block weighs 1.5 times its weight on the plank halfway down

SECTION - III : ASSERTION AND REASON TYPE

11.13 Assertion : Kinetic energy of SHM at mean position is equal to potential energy at ends for a particle moving in SHM.
Reason : Total energy in SHM is conserved.
(A) If both Assertion and Reason are true and the Reason is correct explanation of the Assertion.
(B) If both Assertion and Reason are true, but Reason is not correct explanation of the Assertion.
(C) if Assertion is true, but the Reason is false.
(D) if Assertion is false, but the Reason is true.

11.14 **Statement-1** : A SHM may be assumed as composition of many SHM's.
Statement-2 : Superposition of many SHM's (along same line) of same frequency will be a SHM.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

11.15 **Statement-1 :** A particle is moving along x-axis. The resultant force F acting on it is given by $F = -ax - b$. Where a and b are both positive constants. The motion of this particle is not SHM.
Statement-2 : In SHM resultant force must be proportional to the displacement from mean position.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

SECTION - IV : COMPREHENSION TYPE

Comprehension - 1

A small block of mass m is fixed at upper end of a massless vertical spring of spring constant $K = \frac{4mg}{L}$ and natural length '10L'. The lower end of spring is free and is at a height L from fixed horizontal floor as shown in the figure. The spring is initially unstressed and the spring-block system is released from rest from shown position.

11.16 At the instant speed of block is maximum, the magnitude of force exerted by spring on the block is

(A) $\frac{mg}{2}$ (B) mg (C) Zero (D) None of these

11.17 As the block is coming down, the maximum speed attained by the block is

(A) $\sqrt{gL}$ (B) $\sqrt{3gL}$ (C) $\frac{3}{2}\sqrt{gL}$ (D) $\sqrt{\frac{3}{2}gL}$

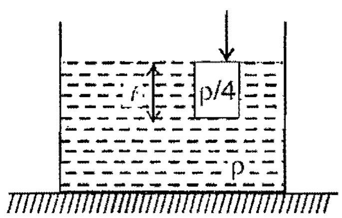
11.18 Till the block reaches its lowest position for the first time, the time duration for which the spring remains compressed is

(A) $\pi\sqrt{\frac{L}{2g}} + \sqrt{\frac{L}{4g}}\sin^{-1}\frac{1}{3}$ (B) $\frac{\pi}{4}\sqrt{\frac{L}{g}} + \sqrt{\frac{L}{4g}}\sin^{-1}\frac{1}{3}$

(C) $\pi\sqrt{\frac{L}{2g}} + \sqrt{\frac{L}{4g}}\sin^{-1}\frac{2}{3}$ (D) $\frac{\pi}{2}\sqrt{\frac{L}{2g}} + \sqrt{\frac{L}{4g}}\sin^{-1}\frac{2}{3}$

Comprehension - 2

A large tank of cross-section area A contains liquid of density ρ. A cylinder of density $\rho/4$ and length ℓ and cross- section area a (a <<A) is kept in equilibrium by applying an external vertically downward force as shown in the figure. The cylinder is just submerged in liquid. At t = 0 the external force is removed instantaneously. Assume that water level in the tank remains constant .



11.19 The acceleration of cylinder immediately after the external force is removed is
(A) g (B) 2g (C) 3g (D) zero

11.20 The speed of the cylinder when it reaches its equlibrium position is

(A) $\frac{1}{2}\sqrt{g\ell}$ (B) $\frac{3}{2}\sqrt{g\ell}$ (C) $\sqrt{2g\ell}$ (D) $2\sqrt{g\ell}$

11.21 After its release at t = 0, the time taken by cylinder to reach its equilibrium position for the first time is

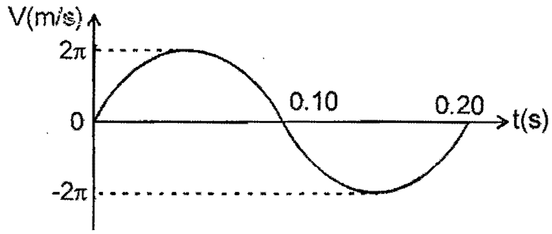
(A) $\frac{\pi}{8}\sqrt{\frac{\ell}{g}}$ (B) $\frac{\pi}{3}\sqrt{\frac{\ell}{g}}$ (C) $\frac{\pi}{4}\sqrt{\frac{\ell}{g}}$ (D) $\frac{\pi}{2}\sqrt{\frac{\ell}{g}}$

SECTION - V : MATRIX - MATCH TYPE

11.22 A particle of mass 2 Kg is moving on a straight line under the action of force F = (8 - 2x) N. The particle is released at rest from x = 6 m. For the subsequent motion match the following (All the values in the right cloumn are in their S.I. units.)

	Column I		Column II
(A)	Equilibrium postion is at x =	(p)	$\pi/4$
(B)	Amplitude of S.H.M is	(q)	$\pi/2$
(C)	Time taken to go directly from x = 2 to x = 4	(r)	4
(D)	Energy of S.H.M. is	(s)	6
		(t)	2

11.23 A simple harmonic oscillator consists of a block attached to a spring with k = 200 N/m. The block slides on a frictionless horizontal surface, with equilibrium point x = 0. A graph of the block's velocity v as a function of time t is shown. Correctly match the required information in the column I with the values given in the column II. (use $\pi^2 = 10$)

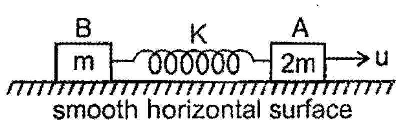


	Column—I		**Column—II**
(A)	The block's mass in kg	(p)	−0.20
(B)	The block's displacement at t = 0 in metres	(q)	0.0
(C)	The block's acceleration at t = 0.10 s in m/s^2	(r)	0.20
(D)	The block's maximum kinetic energy in Joule	(s)	4.0
		(t)	−200

11.24 In each situation of column-I, the x-coordinate of a particle moving along x-axis is given in terms of time t. (ω is a positive constant). Match the equation of motion given in column-I with the type of motion given in column-II

	Column—I		**Column—II**
(A)	$\sin \omega t - \cos \omega t$	(p)	SHM
(B)	$\sin^3 \omega t$	(q)	Periodic
(C)	$\sin \omega t + \sin 3\omega t + \sin 5\omega t$	(r)	Periodic but not SHM
(D)	$\exp(-\omega^2 t^2)$	(s)	Non periodic
		(t)	SHM but not periodic

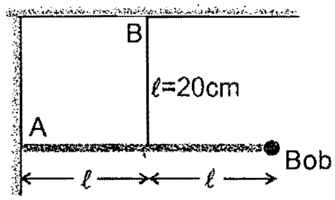
11.25 Two blocks A and B of mass m and 2m respectively are connected by a massless spring of spring constant K. This system lies over a smooth horizontal surface. At t = 0 the block A has velocity u towards right as shown in the figure while the speed of block B is zero and the length of spring is equal to its natural length at that instant. In each situation of column I, certain statements are given and corresponding results are given in column II.



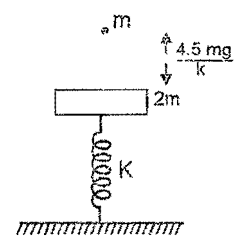
	Column—I		**Column—II**
(A)	The velocity of block A	(p)	can never be zero
(B)	The velocity of block B	(q)	may be zero at certain instants of time
(C)	The kinetic energy of system of two blocks	(r)	is minimum at maximum compression of spring
(D)	The potential energy of spring	(s)	is maximum at maximum extension of spring
		(t)	is maximum at maximum compression of spring

SECTION - VI : INTEGER TYPE

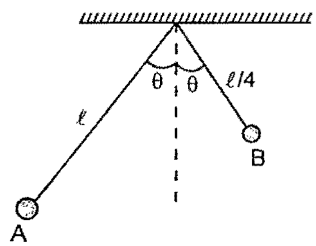
11.26 A weightless rigid rod of length 40 cm with a small iron bob at the end is hinged at point A to the wall so that it can rotate in all directions. The rod is kept in the horizontal position by a vertical inextensible string of length 20 cm, fixed at its mid point. The bob is displaced slightly, perpendicular to the plane of the rod and string. Find period of small oscillations of the system in the form $\frac{\pi X}{10}$ sec. and fill value of X. ($g = 10\ m/s^2$)



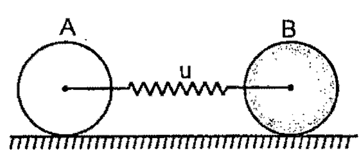
11.27 In the figure shown a plate of mass 60gm is at rest and in equilibrium. A particle of mass m = 30gm is released from height $\frac{4.5mg}{k}$ from the plate. The particle sticks to the plate. Neglecting the duration of collision find time from the collision of the particle and the plate to the moment when the spring has maximum compression. Spring has force constant 100 N/m. Calculate value of time in the form $X\pi$ ms (millisecond) and fill value of X.



11.28 Two simple pendulums A and B having lengths ℓ and $\ell/4$ respectively are released from the position as shown in the figure. Calculate the time after which the release of the two strings become parallel for the first time is $\frac{\pi}{x}\sqrt{\frac{l}{g}}$ then x is . Angle θ is very small



11.29 Two spheres A and B of the same mass m and the same radius are placed on a rough horizontal surface. A is a uniform hollow sphere and B is uniform solid sphere. A and B can roll without sliding on the floor. They are also tied centrally to a light spring of spring constant k as shown in figure. A and B are released when the extension in the spring is x_0. the amplitude of SHM of the spheres A is $\frac{x \cdot x_0}{46}$ then x is



11.30 A particle starts from point A, moves along a straight line path with an acceleration given by $a = 2(4 - x)$ where x is distance from point A. The particle stops at point B for a moment. Find the distance AB (in m). (All values are in S.I. units)

TOPIC 12 STRING WAVE

SECTION - I : STRAIGHT OBJECTIVE TYPE

12.1 A travelling wave $y = A\sin(kx - \omega t + \theta)$ passes from a heavier string to a lighter string. The reflected wave has amplitude 0.5 A. The junction of the strings is at x = 0. The equation of the reflected wave is:
(A) $y' = 0.5A\sin(kx + \omega t + \theta)$ (B) $y' = -0.5A\sin(kx + \omega t + \theta)$
(C) $y' = -0.5A\sin(\omega t - kx - \theta)$ (D) $y' = -0.5A\sin(kx + \omega t - \theta)$

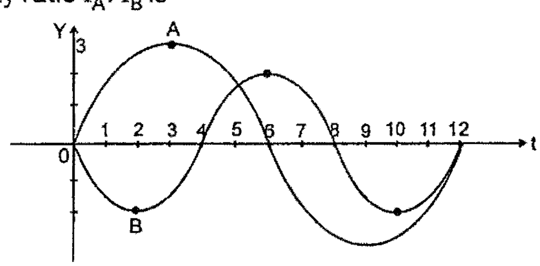
12.2 Which of the following travelling wave will produce standing wave, with node at x = 0, when superimposed on $y = A\sin(\omega t - kx)$
(A) $A\sin(\omega t + kx)$ (B) $A\sin(\omega t + kx + \pi)$
(C) $A\cos(\omega t + kx)$ (D) $A\cos(\omega t + kx + \pi)$

12.3 A wire of length 'ℓ' having tension T and radius 'r' vibrates with fundamental frequency 'f'. Another wire of the same metal with length '2ℓ' having tension 2T and radius 2r will vibrate with fundamental frequency :
(A) f (B) 2f (C) $\frac{f}{2\sqrt{2}}$ (D) $\frac{f}{2}\sqrt{2}$

12.4 A string of length 1.5 m with its two ends clamped is vibrating in fundamental mode. Amplitude at the centre of the string is 4 mm. Distances between the two points having amplitude 2 mm is:
(A) 1 m (B) 75 cm (C) 60 cm (D) 50 cm

12.5 Two particles of medium disturbed by the wave propagation are at $x_1 = 0$ and $x_2 = 1$cm. The respective displacements (in cm) of the particles can be given by the equations :
$y_1 = 2\sin 3\pi t$
$y_2 = 2\sin(3\pi t - \pi/8)$
The wave velocity is :
(A) 16 cm/sec (B) 24 cm/sec (C) 12 cm/sec (D) 8 cm/sec.

12.6 The displacement Vs time graph for two waves A and B which travel along the same string are shown in the figure. Their intensity ratio I_A/I_B is



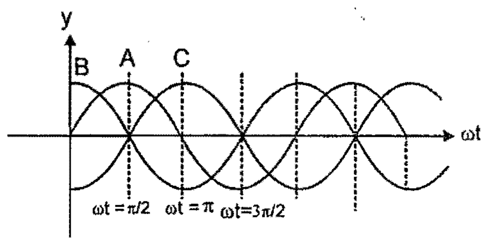
(A) $\frac{9}{4}$ (B) 1 (C) $\frac{81}{16}$ (D) $\frac{3}{2}$

12.7 At t = 0, a transverse wave pulse travelling in the positive x direction with a speed of 2 m/s in a wire is described by the function $y = \frac{6}{x^2}$, given that $x \neq 0$. Transverse velocity of a particle at x = 2m and t = 2 seconds is :
(A) 3 m/s (B) – 3 m/s (C) 8 m/s (D) – 8 m/s

12.8 Sinusoidal waves 5.00 cm in amplitude are to be transmitted along a string having a linear mass density equal to 4.00×10^{-2} kg/m. If the source can deliver a average power of 90 W and the string is under a tension of 100 N, then the highest frequency at which the source can operate is (take $\pi^2 = 10$) :

(A) 45.3 Hz (B) 50 Hz (C) 30 Hz (D) 62.3 Hz

12.9 The figure shows four progressive waves A, B, C & D. It can be concluded from the figure that with respect to wave A :



(A) the wave C is ahead by a phase angle of $\pi/2$ & the wave B lags behind by a phase angle $\pi/2$

(B) the wave C lags behind by a phase angle of $\pi/2$ & the wave B is ahead by a phase angle of $\pi/2$

(C) the wave C is ahead by a phase angle of π & the wave B lags behind by the phase angle of π

(D) the wave C lags behind by a phase angle of π & the wave B is ahead by a phase angle of π.

12.10 A 75 cm string fixed at both ends produces resonant frequencies 384 Hz and 288 Hz without there being any other resonant frequency between these two. Wave speed for the string is :

(A) 144 m/s (B) 216 m/s (C) 108 m/s (D) 72 m/s

12.11 A string of length 'ℓ' is fixed at both ends. It is vibrating in its 3rd overtone with maximum amplitude 'a'. The amplitude at a distance $\frac{\ell}{3}$ from one end is :

(A) a (B) 0 (C) $\frac{\sqrt{3}\,a}{2}$ (D) $\frac{a}{2}$

12.12 What is the percentage change in the tension necessary in a sonometer of fixed length to produce a note one octave lower (half of original frequency) than before

(A) 25% (B) 50% (C) 67% (D) 75%

12.13 A chord attached about an end to a vibrating fork divides it into 6 loops, when its tension is 36 N. The tension at which it will vibrate in 4 loops is :

(A) 24 N (B) 36 N (C) 64 N (D) 81 N

12.14 Two vibrating strings of same length, same cross section area and stretched to same tension are made of materials with densities ρ & 2ρ. Each string is fixed at both ends. If v_1 represents the fundamental mode of vibration of the one made with density ρ and v_2 for another, then v_1/v_2 is:

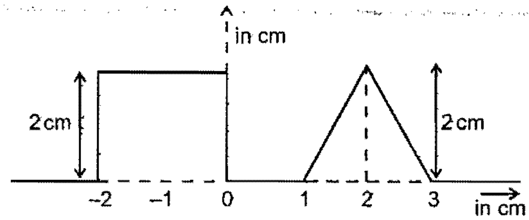
(A) $\frac{1}{2}$ (B) 2 (C) $\sqrt{2}$ (D) $\frac{1}{\sqrt{2}}$

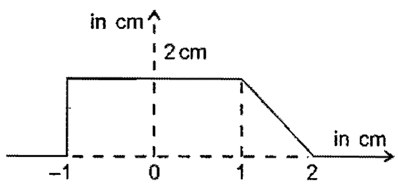
12.15 Which of the following function correctly represents the wave equation for finite values of x and t :

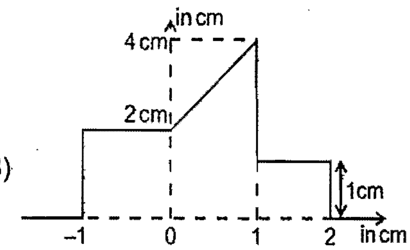
(A) $y = x^2 - t^2$ (B) $y = \cos x^2 \sin t$

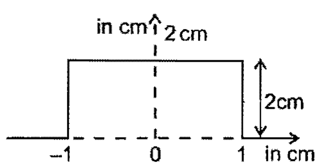
(C) $y = \log (x^2 - t^2) - \log(x - t)$ (D) $y = e^{2x} \sin t$

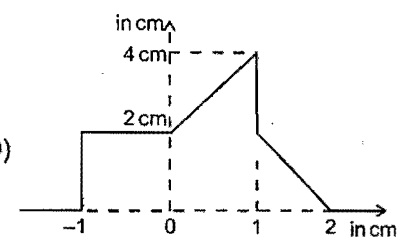
12.16 The figure shows at time t = 0 second, a rectangular and triangular pulse on a uniform wire are approaching each other. The pulse speed is 0.5 cm/s. The resultant pulse at t = 2 second is



(A) 

(B) 

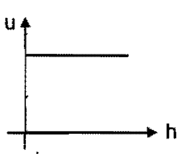
(C) 

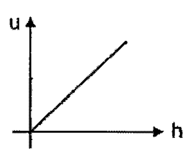
(D) 

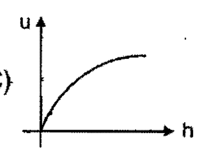
12.17 A loop of a string of mass per unit length μ and radius R is rotated about an axis passing through centre perpendicular to the plane with an angular velocity ω. A small disturbance is created in the loop having the same sense of rotation. The linear speed of the disturbance for a stationary observer is :

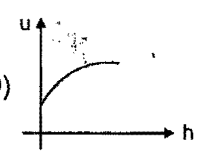
(A) ωR (B) $2\omega R$ (C) $3\omega R$ (D) zero

12.18 A uniform rope having mass m hangs vertically from a rigid support. A transverse wave pulse is produced at the lower end. The speed u of wave pulse varies with height h from the lower end as :

(A) 

(B) 

(C) 

(D) 

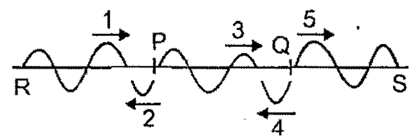
12.19 In a stationary wave that forms as a result of reflection of waves from an ostacle the ratio of the amplitude at an antinode to the amplitude at node is n. The fraction of energy reflected is :

(A) $\left(\frac{n-1}{n}\right)^2$ (B) $\left(\frac{n-1}{n+1}\right)^2$ (C) $\left(\frac{1}{n}\right)^2$ (D) $\left(\frac{n}{n+1}\right)^2$

12.20 The fundamental frequency of a sonometer wire of length ℓ is f_0. A bridge is now introduced at a distance of $\Delta\ell$ from the centre of the wire ($\Delta\ell << \ell$). The number of beats heard if both sides of the bridges are set into vibration in their fundamental modes are :

(A) $\frac{8f_0\Delta\ell}{\ell}$ (B) $\frac{f_0\Delta\ell}{\ell}$ (C) $\frac{2f_0\Delta\ell}{\ell}$ (D) $\frac{4f_0\Delta\ell}{\ell}$

12.21 There are three strings RP, PQ and QS as shown. Their mass and lengths are RP = (0.1Kg, 2m), PQ = (0.2 Kg, 3 m), QS = (0.15 Kg, 4 m) respectively. All the strings are under same tension. Wave-1 is incident at P. It is partly reflected (wave-2) and partly transmitted (wave-3). Now wave-3 is incident at Q. It is again partly transmitted (wave-5) and partly reflected (wave-4). Phase difference between wave-1 and wave :



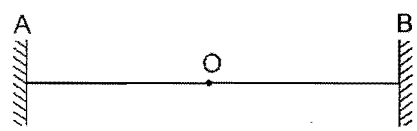
(A) 2 is π
(B) 4 is zero
(C) both (A) and (B) are correct
(D) both (A) and (B) are wrong

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

12.22 A wire of density 9×10^3 kg/m^3 is stretched between two clamps 1 m apart and is stretched to an extension of 4.9×10^{-4} metre. Young's modulus of material is 9×10^{10} N/m^2. Then :
(A) The lowest frequency of standing wave is 35 Hz
(B) The frequency of 1st overtone is 70 Hz
(C) The frequency of 1st overtone is 105 Hz
(D) The stress in the wire is 4.41×10^7 N/m^2

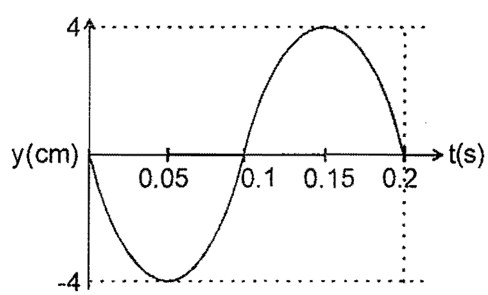
12.23 In a standing wave on a string :
(A) In one time period all the particles are simultaneously at rest twice.
(B) All the particles must be at their positive extremes simultaneously once in one time period .
(C) All the particles may be at their positive extremes simultaneously once in a time period.
(D) All the particles are never at rest simultaneously.

12.24 A wire, under tension between two fixed points *A* and *B*, executes transverse vibrations so that the midpoint O of *AB* is a node. Then :



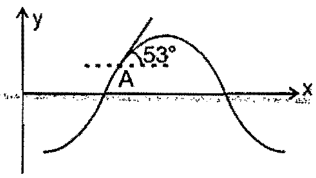
(A) All points of wire between *A* and *B* are in the same phase
(B) All points between *A* and *O* are in the same phase
(C) A point between *A* and *O* and a point between *O* and *B* may have a phase difference of $\pi/2$
(D) A point between *A* and *O* and a point between *O* and *B* may have a phase difference of π

12.25 For a certain transverse standing wave on a long string, an antinode is formed at x = 0 and next to it, a node is formed at x = 0.10 m. the displacement y(t) of the string particle at x = 0 is shown in figure.



(A) Transverse displacement of the particle at x = 0.05m and t = 0.05 s is $-2\sqrt{2}$ cm.
(B) Transverse displacement of the particle at x = 0.04 m and t = 0.025 s is $-2\sqrt{2}$ cm.
(C) Speed of the travelling waves that interfere to produce this standing wave is 2 m/s.
(D) The transverse velocity of the string particle at $x = \frac{1}{15}$ m and t = 0.1 s is 20π cm/s

12.26 y-x curve at an instant for a wave travelling along x axis on a string is shown. Slope at the point A on the curve, as shown, is 53°.



(A) Transverse velocity of the particle at point A is positive if the wave is travelling along positive x axis.
(B) Transverse velocity of the particle at point A is positive if the wave is travelling along negative x axis of the particle at point A
(C) Magnitude of transverse velocity of the particle at point A is greater than wave speed.
(D) Magnitude of transverse velocity of the particle at point A is lesser than wave speed.

12.27 For a certain stretched string, three consecutive resonance frequencies are observed as 105, 175, 245 Hz respectively. Then select the correct alternative (s) :
(A) The string is fixed at both ends
(B) The string is fixed at one end only
(C) The fundamental frequency is 35 Hz
(D) The fundamental frequency is 52.5 Hz

SECTION - III : ASSERTION AND REASON TYPE

12.28 **Statement-1 :** In a small segment of string carrying sinusoidal wave, total energy is conserved.
Statement-2 : Every small part moves in SHM and in SHM total energy is conserved.
(A) If both Assertion and Reason are true and the Reason is correct explanation of the Assertion.
(B) If both Assertion and Reason are true, but Reason is not correct explanation of the Assertion.
(C) if Assertion is true, but the Reason is false.
(D) if Assertion is false, but the Reason is true.

12.29 **Statement-1 :** Two waves moving in a uniform string having uniform tension cannot have different velocities.
Statement-2 : Elastic and inertial properties of string are same for all waves in same string. Moreover speed of wave in a string depends on its elastic and inertial properties only.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

12.30 **Statement-1 :** A standing wave pattern is formed in a string. The power transfer through a point (other than node and antinode) is zero always.
Statement-2 : At antinode tension is perpendicular to the velocity.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

SECTION - IV : COMPREHENSION TYPE

Comprehension - 1

In a standing wave experiment, a 1.2 kg horizontal rope is fixed in place at its two ends (x = 0 and x = 2.0 m) and made to oscillate up and down in the fundamental mode, at frequency 5.0 Hz. At t = 0, the point at x = 1.0 m has zero displacement and is moving upward in the positive direction of y axis with a transverse velocity 3.14 m/s.

12.31 Tension in the rope is :
(A) 60 N (B) 100 N (C) 120 N (D) 240 N

12.32 Speed of the participating travelling wave on the rope is
(A) 6 m/s (B) 15 m/s (C) 20 m/s (D) 24 m/s

12.33 What is the correct expression of the standing wave equation ?
(A) $(0.1)\sin(\pi/2)x \sin(10\pi)t$
(B) $(0.1)\sin(\pi)x \sin(10\pi)t$
(C) $(0.05)\sin(\pi/2)x \cos(10\pi)t$
(D) $(0.04)\sin(\pi)x \sin(10\pi)t$

Comprehension - 2

A sinusoidal wave is propagating in negative x–direction in a string stretched along x-axis. A particle of string at $x = 2$m is found at its mean position and it is moving in positive y direction at $t = 1$ sec. The amplitude of the wave, the wavelength and the angular frequency of the wave are 0.1meter, $\pi/4$ meter and 4π rad/sec respectively.

12.34 The equation of the wave is

(A) $y = 0.1 \sin(4\pi(t-1) + 8(x-2))$
(B) $y = 0.1 \sin((t-1) - (x-2))$
(C) $y = 0.1 \sin(4\pi(t-1) - 8(x-2))$
(D) none of these

12.35 The speed of particle at $x = 2$ m and $t = 1$sec is

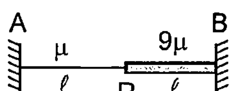
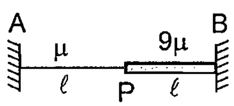
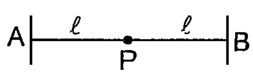
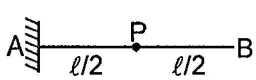
(A) 0.2π m/s (B) 0.6π m/s (C) 0.4π m/s (D) 0

12.36 The instantaneous power transfer through x=2 m and t= 1.125 sec, is

(A) 10 J/s (B) $\frac{4\pi}{3}$ J/s (C) $\frac{2\pi}{3}$ J/s (D) 0

SECTION - V : MATRIX - MATCH TYPE

12.37 **Match the column :**

	Column–I		Column–II
(A)	 Two strings each of length ℓ and linear mass density μ and 9μ are joined together and system is oscillated such that joint P is node T is tension in the strings. A and B are fixed ends.	(p)	Speed of component travelling wave is portion AP will be $\sqrt{\frac{T}{\mu}}$
(B)	 Two strings each of length ℓ and linear mass density μ and 9μ are joined together and system is oscillated such that joint P is antinode. T is tension in each string. A and B are fixed ends.	(q)	Speed of component travelling wave in the portion AP will be more than that in portion BP.
(C)	 P is the mid–point of the string fixed at both ends. T is tension in the string and μ is its linear mass density.	(r)	Frequency of oscillation of the system AB can be $\frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$
(D)	 T is the tension in the string fixed at A and B is free end. P is mid–point. μ is its the linear mass density.	(s)	Frequency of oscillation of the system AB can be $\frac{1}{4\ell}\sqrt{\frac{T}{\mu}}$
		(t)	Wavelength of the wave in the portion PB can be $\frac{2\ell}{3}$.

2.38 Match the statements in column-I with the statements in column-II.

	Column-I		Column-II
(A)	A tight string is fixed at both ends and sustaining standing wave	(p)	At the middle, antinode is formed in odd harmonic
(B)	A tight string is fixed at one end and free at the other end	(q)	At the middle, node is formed in even harmonic
(C)	A tight string is fixed at both ends and vibrating in four loops	(r)	the frequency of vibration is 300% more than its fundamental frequency
(D)	A tight string is fixed at one end and free at the other end, vibrating in 2nd overtone	(s)	Phase difference between SHMs of any two particles will be either π or zero.
		(t)	The frequency of vibration is 400% more than fundamental frequency.

SECTION - VI : INTEGER TYPE

12.39 A 40 cm long wire having a mass 3.2 gm and area of cross section 1 mm^2 is stretched between the support 40.05 cm apart. In its fundamental mode. It vibrate with a frequency 1000/64 Hz. the young's modulus of the wire is 10^x N/m^2 then x is.

12.40 A non–uniform string of mass 45 kg and length 1.5 m has a variable linear mass density given by $\mu = kx$, where x is the distance from one end of the string and k is a constant. Tension in the string is 15 N which is uniform. Find the time (in second) required for a pulse generated at one end of the string to travel to the other end.

12.41 The equation of a string wave is given by (all quantity expressed in S.I. units) $Y = 5 \sin 10\pi (t - 0.01x)$ along the x-axis. The magnitude of phase difference between the points separated by a distance of 10 m along x- axis is $x\pi$ then x is

TOPIC 13 — SOUND WAVES

SECTION - I : STRAIGHT OBJECTIVE TYPE

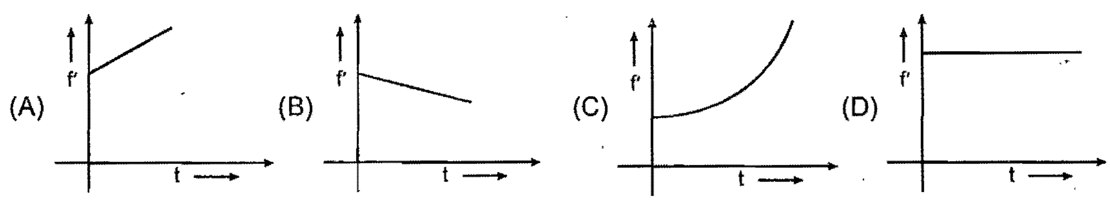
13.1 A closed organ pipe has length 'ℓ'. The air in it is vibrating in 3rd overtone with maximum amplitude 'a'. Find the amplitude at a distance of $\ell/7$ from closed end of the pipe.

(A) a (B) a/2 (C) $\sqrt{3}$ a/2 (D) a/$\sqrt{2}$

13.2 When a sound wave is reflected from a wall, the phase difference between the reflected and incident pressure wave is:

(A) 0 (B) π (C) $\pi/2$ (D) $\pi/4$

13.3 A source of frequency 'f' is stationary and an observer starts moving towards it at t = 0 with constant small acceleration. Then the variation of observed frequency f ' registered by the observer with time is best represented as :



13.4 A stationary observer receives sonic oscillations from two tuning forks, one of which approaches and the other recedes with same speed. As this takes place the observer hears the beat frequency of 2 Hz. Find the speed of each tuning fork, if their oscillation frequency is 680 Hz and the velocity of sound in air is 340 m/s.

(A) 1 m/s (B) 2 m/s (C) 0.5 m/s (D) 1.5 m/s

13.5 A source of sound of frequency 256 Hz is moving rapidly towards a wall with a velocity of 5 m/sec. If sound travels at a speed of 330 m/sec, then number of beats per second heard by an observer between the wall and the source is :

(A) 7.7 Hz (B) 9 Hz (C) 4 Hz (D) none of these

13.6 A point source is emitting sound in all directions. The ratio of distance of two points from the point source where the difference in loudness levels is 3 dB is: ($\log_{10} 2 = 0.3$)

(A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{2}}$ (C) $\frac{1}{4}$ (D) $\frac{2}{3}$

13.7 Two coherent sources of different intensities send waves which interfere. The ratio of the maximum intensity to the minimum intensity is 25. The intensities are in the ratio:

(A) 25: 1 (B) 5: 1 (C) 9: 4 (D) 625: 1

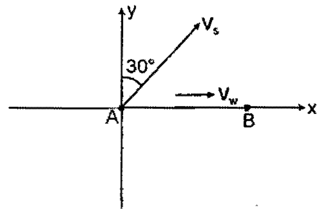
13.8 The frequency of a man's voice is 300 Hz and its wavelength is 1 meter. If the wavelength of a child's voice is 1.5 m, then the frequency of the child's voice is :

(A) 200 Hz (B) 150 Hz (C) 400 Hz (D) 350 Hz.

13.9 A sound wave of frequency 440 Hz is passing through air. An O_2 molecule (mass = 5.3×10^{-26} kg) is set in oscillation with an amplitude of 10^{-6} m. Its speed at the centre of its oscillation is :

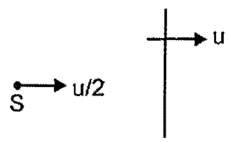
(A) 1.70×10^{-5} m/s (B) 17.0×10^{-5} m/s (C) 2.76×10^{-3} m/s (D) 2.77×10^{-5} m/s

13.10 In the figure shown a source of sound of frequency 510 Hz moves with constant velocity v_s = 20 m/s in the direction shown. The wind is blowing at a constant velocity v_w = 20 m/s towards an observer who is at rest at point B. Corresponding to the sound emitted by the source at initial position A, the frequency detected by the observer is equal to
(speed of sound relative to air = 330 m/s)



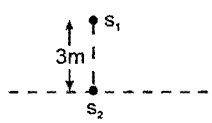
(A) 510 Hz (B) 500 Hz (C) 525 Hz (D) 550 Hz

13.11 A wall is moving with velocity u and a source of sound moves with velocity $\frac{u}{2}$ in the same direction as shown in the figure. Assuming that the sound travels with velocity 10u. The ratio of incident sound wavelength on the wall to the reflected sound wavelength by the wall, is equal to :



(A) 9:11 (B) 11:9 (C) 4:5 (D) 5:4

13.12 S_1 & S_2 are two coherent sources of sound having no initial phase difference. The velocity of sound is 330 m/s. No minima will be formed on the line passing through S_2 and perpendicular to the line joining S_1 and S_2 , if the frequency of both the sources is :

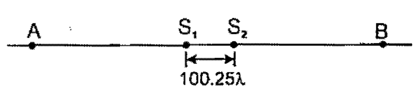


(A) 50 Hz (B) 60 Hz (C) 70 Hz (D) 80 Hz

13.13 Under similar conditions of temperature and pressure, In which of the following gases the velocity of sound will be largest.
(A) H_2 (B) N_2 (C) He (D) CO_2

13.14 When beats are produced by two progressive waves of nearly the same frequency, which one of the following is correct ?
(A) The particles vibrate simple harmonically, with the frequency equal to the difference in the component frequencies.
(B) The amplitude of vibration at any point changes simple harmonically with a frequency equal to the difference in the frequencies of the two waves.
(C) The frequency of beats depends upon the position, where the observer is
(D) The frequency of beats changes as the time progresses

13.15 S_1 and S_2 are two coherent sources of sound separated by distance 100.25 λ, where λ is the wave length of sound. S_1 leads S_2 in phase by π/2. A and B are two points on the line joining S_1 and S_2 as shown in figure. The ratio of amplitudes of source S_1 and S_2 are in ratio 1:2. The ratio of intensity at A to that of B $\left(\frac{I_A}{I_B}\right)$ is

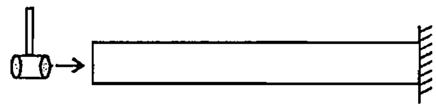


(A) ∞ (B) $\frac{1}{9}$ (C) 0 (D) 9

13.16 There is a set of four tuning forks, one with the lowest frequency vibrating at 550 Hz. By using any two tuning forks at a time, the following beat frequencies are heard: 1, 2, 3, 5, 7, 8. The possible frequencies of the other three forks are:

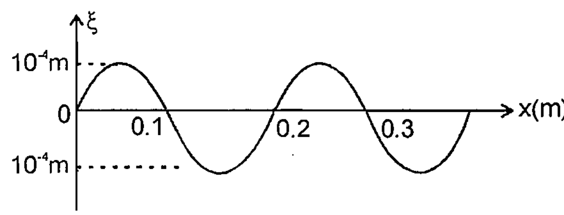
(A) 552, 553, 560 (B) 557, 558, 560
(C) 552, 553, 558 (D) 551, 553, 558

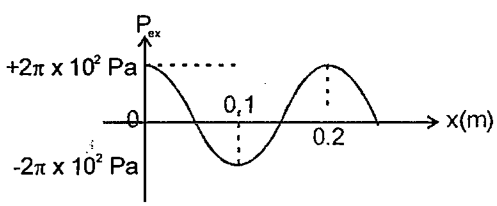
13.17 A 100 m long rod of density 10.0×10^4 kg/m^3 and having Young's modulus $Y = 10^{11}$ Pa, is clamped at one end. It is hammered at the other free end. The longitudinal pulse goes to right end, gets reflected and again returns to the left end. How much time, the pulse take to go back to initial point.

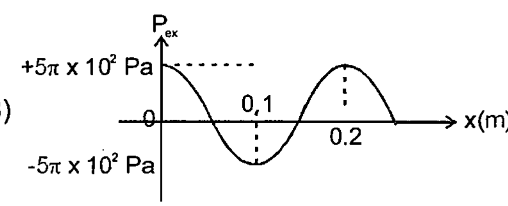


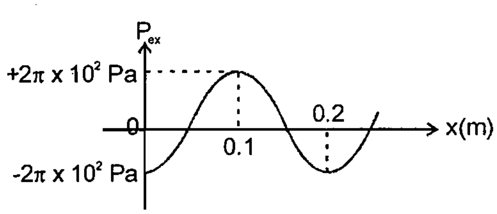
(A) 0.1 sec. (B) 0.2 sec. (C) 0.3 sec. (D) 2 sec.

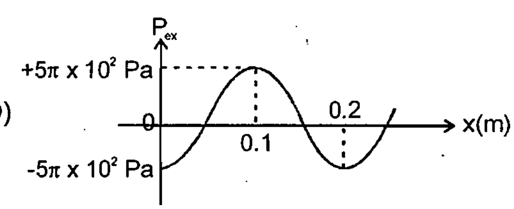
13.18 For a sound wave travelling towards +x direction, sinusoidal longitudinal displacement ξ at a certain time is given as a function of x. If Bulk modulus of air is $B = 5 \times 10^5$ N/m^2, the variation of pressure excess will be :



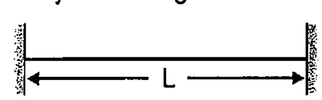
(A)


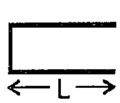
(B)


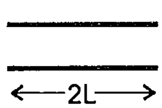
(C)


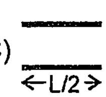
(D)


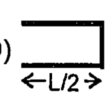
13.19 Figure shows a stretched string of length L and pipes of length L, 2L, L/2 and L/2 in options (A), (B), (C) and (D) respectively. The string's tension is adjusted until the speed of waves on the string equals the speed of sound waves in the air. The fundamental mode of oscillation is then set up on the string. In which pipe will the sound produced by the string cause resonance ?



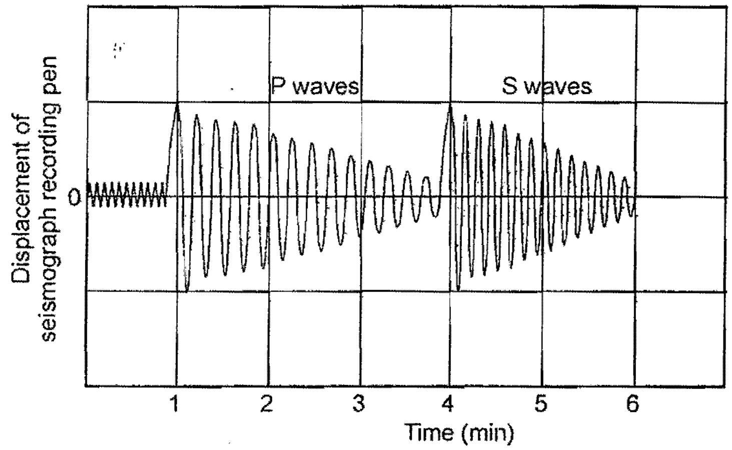
(A)


(B)


(C)


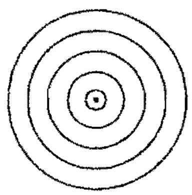
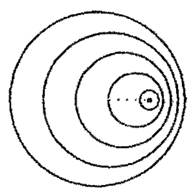
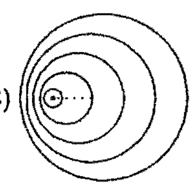
(D)


13.20 Earthquakes generate sound waves inside Earth. Unlike a gas, Earth can experience both transverse (S) and longitudinal (P) sound waves. Typically, the speed of S waves is about 4 km/s. A seismograph records P and S waves from an earthquake. The first P waves arrive 3.0 min before the first S wave (figure). Assuming the waves travel in a straight line, how far away does the earthquake occur ?
(Y_{earth} = 12.8 × 10^{10} pa, ρ_{earth} = 2000 kg/m^3)



(A) 1900 km (B) 1440 km (C) 1800 km (D) 1200 km

13.21 If the source is moving towards right, wave front of sound waves get modified to –

(A)  (B)  (C)  (D) None of these

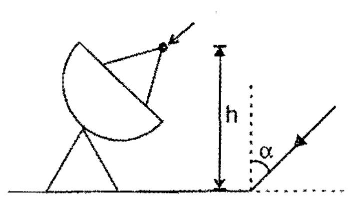
13.22 Equation of a stationary and a travelling waves are as follows $y_1 = a \sin kx \cos \omega t$ and $y_2 = a \sin(\omega t - kx)$. The phase difference between two points $x_1 = \frac{\pi}{3k}$ and $x_2 = \frac{3\pi}{2k}$ is ϕ_1 in the standing wave (y_1) and is ϕ_2 in travelling wave (y_2) then ratio $\frac{\phi_1}{\phi_2}$ is

(A) 1 (B) 5/6 (C) 3/4 (D) 6/7

13.23 In the resonance tube experiment, the first resonance is heard when length of air column is ℓ_1 and second resonance is heard when length of air column is ℓ_2. What should be the minimum length of the tube so that third resonance can also be heard.

(A) $2\ell_2 - \ell_1$ (B) $2\ell_1$ (C) $5\ell_1$ (D) $7\ell_1$

13.24 Radio waves coming at $\angle \alpha$ to vertical are received by a radar after reflection from a nearby water surface & directly. What can be height of antenna from water surface so that it records a maximum intensity (a maxima). (wavelength = λ) (Assume phase changes by π after reflection)



(A) $\frac{\lambda}{2\cos\alpha}$ (B) $\frac{\lambda}{2\sin\alpha}$ (C) $\frac{\lambda}{4\sin\alpha}$ (D) $\frac{\lambda}{4\cos\alpha}$

13.25 Microwaves from a transmitter are directed normally towards a plane reflector. A detector moves along the normal to the reflector. Between positions of 14 successive maxima, the detector travels a distance 0.14 m. If the velocity of light is 3×10^8 m/s, find the frequency of the transmitter -

(A) 1.5×10^{10} Hz (B) 10^{10} Hz (C) 3×10^{10} Hz (D) 6×10^{10} Hz

13.26 A man standing in front of a mountain at a certain distance beats a drum at regular intervals. The drumming rate is gradually increased and he finds that the echo is not heard distinctly when the rate becomes 40 per minute. He then moves nearer to the mountain by 90 meters and finds that the echo is again not heard when the drumming rate becomes 60 per minute

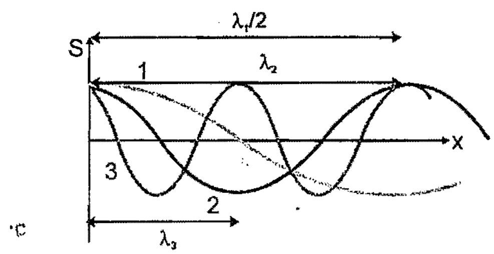
(a) The distance between the mountain and the initial position of the man is :

(A) 330 m (B) 300 m (C) 240 m (D) 270 m

(b) the velocity of sound is

(A) 330 m/s (B) 360 m/s (C) 300 m/s (D) 270 m/s

13.27 Figure shown is a graph, at a certain time t, of the displacement function S(x,t) of three sound waves 1,2 and 3 as marked on the curves that travel along x–axis through air. If P_1, P_2 and P_3 represent their pressure amplitudes respectively, then correct relation between them is :



(A) $P_1 > P_2 > P_3$ (B) $P_3 > P_2 > P_1$ (C) $P_1 = P_2 = P_3$ (D) $P_2 > P_3 > P_1$

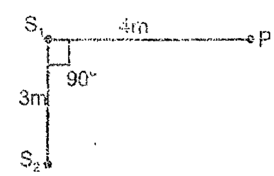
13.28 In Quincke's tube a detector detects minimum intensity. Now one of the tube is displaced by 5 cm. During displacement detector detects maximum intensity 10 times, then finally a minimum intensity (when displacement is complete). The wavelength of sound is :

(A) 10/9 cm (B) 1 cm (C) 1/2 cm (D) 5/9 cm

13.29 $x = x_1 + x_2$ (where $x_1 = 4 \cos \omega t$ and $x_2 = 3 \sin \omega t$) is the equation of motion of a particle along x-axis. The phase difference between x_1 and x is :

(A) 37° (B) 53° (C) 90° (D) none of these

13.30 S_1 and S_2 are two coherent sources of sound of frequency 110Hz each. They have no initial phase difference. The intensity at a point P due to S_1 is I_0 and due to S_2 is $4I_0$. If the velocity of sound is 330 m/s then the resultant intensity at P is

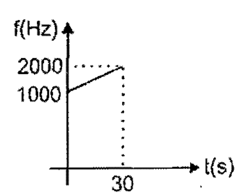


(A) I_0 (B) $9I_0$ (C) $3I_0$ (D) $8I_0$

13.31 A conveyor belt moves to the right with speed v = 300 m/min. A pieman puts pies on the belt at a rate of 20 per minute while walking with speed 30 m/min towards a receiver at the other end. The frequency with which they are received by the stationary receiver is:

(A) 26.67 / minute (B) 30 / minute (C) 22.22 / minute (D) 24 / minute

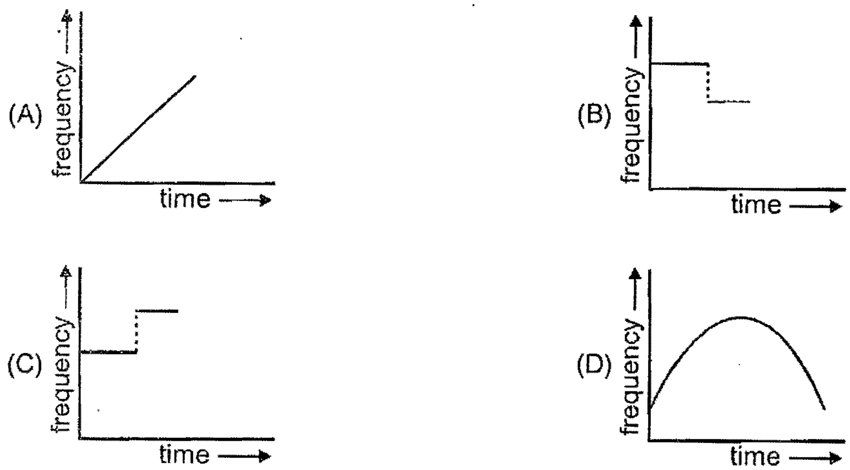
13.32 A detector is released from rest over a source of sound of frequency $f_0 = 10^3$ Hz. The frequency observed by the detector at time t is plotted in the graph. The speed of sound in air is : ($g = 10$ m/s^2)



(A) 330 m/s (B) 350 m/s (C) 300 m/s (D) 310 m/s

13.33 In the experiment for the determination of the speed of sound in air using the resonance column method, the length of the air column that resonates in the fundamental mode, with a tuning fork is 0.1 m. When this length is changed to 0.35 m, the same tuning fork resonates with the first overtone. Calculate the end correction :
(A) 0.012 m (B) 0.025 m (C) 0.05 m (D) 0.024 m

13.34 A roller skater carrying a portable stereo skates at constant speed towards an observer at rest. Which of the following accurately represents how the frequency perceived by the observer changes with time ?



SECTION - II : MULTIPLE CORRECT ANSWER TYPE

13.35 In a resonance tube experiment, a closed organ pipe of length 120 cm resonates when tuned with a tuning fork of frequency 340 Hz. If water is poured in the pipe then (given v_{air} = 340 m/sec.) :
(A) minimum length of water column to have the resonance is 45 cm.
(B) the distance between two successive nodes is 50 cm.
(C) the maximum length of water column to create the resonance is 95 cm.
(D) none of these

13.36 Two identical stretched wires are vibrated together. They produce 8 beats per second. When tension in one wire is changed then the beat frequency is increased. If T_1 and T_2 denote the tensions in the two wires at any instant and $T_1 > T_2$ (initially) then how the change may be performed.
(A) T_1 decreased (B) T_1 increased (C) T_2 decreased (D) T_2 increased

SECTION - III : ASSERTION AND REASON TYPE

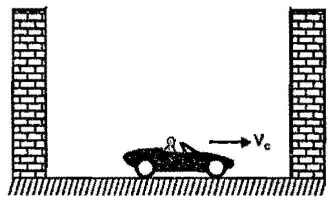
13.37 **Statement 1 :** Doppler formula for sound wave is symmetric with respect to the speed of source and speed of observer
Statement 2 : Motion of source with respect to stationary observer is not equivalent to the motion of an observer with respect to a stationary source.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

13.38 **Statement 1 :** The base of Laplace correction was that exchange of heat between the region of compression and rarefaction in air is negligible.
Statement 2 : Air is bad conductor of heat and velocity of sound in air is quite large.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

SECTION - IV : COMPREHENSION TYPE

Comprehension # 1

A driver is riding a car with velocity v_c between two vertical walls on a horizontal surface as shown in figure. A source of sound of frequency 'f' is situated on the car. ($v_c << v$, where v is the speed of sound in air)



13.39 Beat frequency observed by the driver corresponding to sound waves reflected from wall-1 and wall-2 (reflected waves corresponding to waves directly coming from source) :

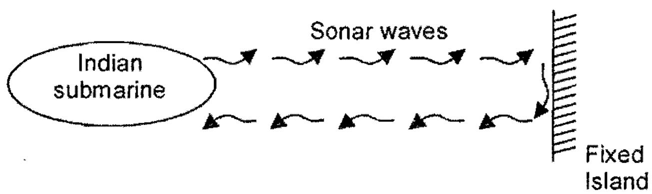
(A) $\frac{v_c}{v}f$ (B) $\frac{2v_c}{v}f$ (C) $\frac{v_c}{2v}f$ (D) $\frac{4v_c}{v}f$

13.40 Consider the sound wave observed by the driver directly from car has a wavelength λ_1 and the sound wave after reflection from wall-1 observed by the driver has wavelength λ_2 then $\frac{\lambda_1-\lambda_2}{\lambda_1+\lambda_2}$ is :

(A) $\frac{v_c}{v}$ (B) $\frac{2v_c}{v}$ (C) $\frac{v_c}{4v}$ (D) $\frac{4v_c}{v}$

Comprehension # 2

An Indian submarine is moving in "Arab Sagar" with a constant velocity. To detect enemy it sends out sonar waves which travel with velocity 1050 m/s in water. Initially the waves are getting reflected from a fixed island and the reflected waves are coming back to submarine. The frequency of reflected waves are detected by the submarine and found to be 10% greater than the sent waves.



Now an enemy ship comes in front, due to which the frequency of reflected waves detected by submarine becomes 21% greater than the sent waves.

13.41 The speed of Indian submarine is
(A) 10 m/sec (B) 50 m/sec (C) 100 m/sec (D) 20 m/sec.

13.42 The velocity of enemy ship should be :
(A) 50 m/sec. toward Indian submarine
(B) 50 m/sec. away from Indian submarine
(C) 100 m/sec. toward Indian submarine
(D) 100 m/sec. away from Indian submarine

13.43 If the wavelength received by enemy ship is λ' and wavelength of reflected waves received by submarine is λ'' then $\left(\frac{\lambda'}{\lambda''}\right)$ equals

(A) 1 (B) 1.1 (C) 1.2 (D) 2

13.44 Bulk modulus of sea water should be approximately ($\rho_{water} = 1000\ kg/m^3$)

(A) $10^8\ N/m^2$ (B) $10^9\ N/m^2$ (C) $10^{10}\ N/m^2$ (D) $10^{11}\ N/m^2$

Comprehension # 3

In sound wave, y (x, t) equation and ΔP (x, t) equation have a phase difference of $\frac{\pi}{2}$.

Pressure amplitude in ΔP (x, t) equation is equal to BAK.

y (x, t) equation of a longitudinal wave is given as:

$y = 10^{-2} \sin 2\pi \left(1000t + \frac{50}{17}x\right)$ (All SI units)

13.45 At t = 0, change in pressure is maximum at x = ...m.

(A) 0.34 (B) 0.255 (C) 0.085 (D) all of these

13.46 If density of the gas is 10^{-2} kg/m³, find the pressure amplitude :

(A) 200.62 N/m² (B) 421.24 N/m² (C) 100.26 N/m² (D) 21.36 N/m²

SECTION - V : MATRIX - MATCH TYPE

13.47 Match the columns I & II.

	Column-I		Column-II
(A)	Pitch	(p)	Number of harmonics present in the sound
(B)	Loudness	(q)	Intensity
(C)	Quality	(r)	Frequency
(D)	wave front	(s)	wave form
		(t)	locus of points vribrating in a phase

13.48 Match the Column I & II. :

	Column-I		Column-II
(A)	$y = 4 \sin(5x - 4t) + 3 \cos(4t - 5x + \pi/6)$	(p)	Particles at every position are performing SHM
(B)	$y = 10 \cos\left(t - \frac{x}{330}\right) \sin(100)\left(t - \frac{x}{330}\right)$	(q)	Equation of travelling wave
(C)	$y = 10 \sin(2\pi x - 120t) + 10 \cos(120t + 2\pi x)$	(r)	Equation of standing wave
(D)	$y = 10 \sin(2\pi x - 120t) + 8 \cos(118t - 59/30\pi x)$	(s)	Equation of Beats
		(t)	Initial displacement of particle at origin is zero

SECTION - VI : INTEGER TYPE

13.49 A 3 m long organ pipe open at both ends is driven to third harmonic standing wave. If the amplitude of pressure oscillation is 0.1 % of the mean atmospheric pressure ($P_0 = 10^5$ N/m²). if the amplitude of:

Particle oscillation is $\frac{x}{1089\pi}$ m then x is

13.50 In previous question density oscillation is $\frac{x}{1089}$ kgm⁻³ then x is

Speed of sound v = 330 m/s, density of air $\rho_0 = 1.0$ kg/m³.

13.51 In a car race sound signals emitted by the two cars are detected by the detector on the straight track at the end point of the race. Frequency observed are 330 Hz & 360 Hz and the original frequency is 300 Hz of both cars. Race ends with the separation of 100 m between the cars. Assume both cars move with constant velocity and velocity of sound is 330 m/s. Find the time taken by winning car (in sec.)

13.52 A bat emits ultrasonic sound of frequency 1000 kHz in air. If the sound meets a water surface, it gets partially reflected back and partially refracted (transmitted) in water. Difference of wavelength transmitted to wavelength reflected is x cm. then x is (speed of sound in air = 330 m/sec, Bulk modulus of water = 2.25×10^9, $\rho_{water} = 1000$ kg/m³)

TOPIC 14

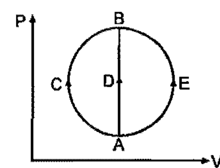
HEAT & THERMODYANMICS

SECTION - I : STRAIGHT OBJECTIVE TYPE

14.1 A diatomic ideal gas is heated at constant volume until the pressure is doubled and again heated at constant pressure until volume is doubled. The average molar heat capacity for whole process is:

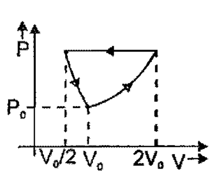
(A) $\frac{13R}{6}$ (B) $\frac{19R}{6}$ (C) $\frac{23R}{6}$ (D) $\frac{17R}{6}$

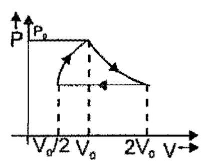
14.2 One mole of an ideal gas is taken from state A to state B by three different processes,
(a) ACB **(b)** ADB **(c)** AEB as shown in the P – V diagram.
The heat absorbed by the gas is :

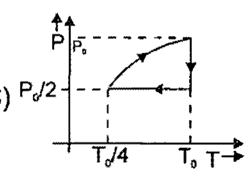


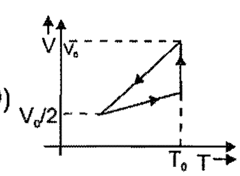
(A) greater in process (b) then in (a) (B) the least in process (b)
(C) the same in (a) and (c) (D) less in (c) then in (b)

14.3 One mole of an ideal gas at pressure P_0 and temperature T_0 is expanded isothermally to twice its volume and then compressed at constant pressure to $(V_0/2)$ and the gas is brought back to original state by a process in which P α V (Pressure is directly proportional to volume). The correct representation of process is :

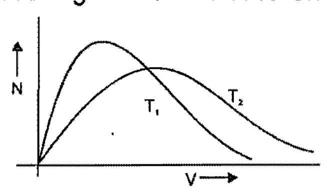
(A) 

(B) 

(C) 

(D) 

14.4 Maxwell's speed distribution curve is given for two different temperatures. For the given curves.



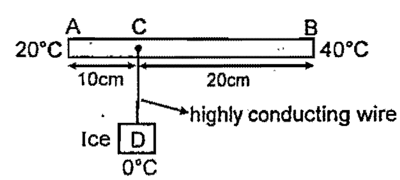
(A) $T_1 > T_2$ (B) $T_1 < T_2$ (C) $T_1 \le T_2$ (D) $T_1 = T_2$

14.5 There are two thin spheres A and B of the same material and same thickness. They emit radiation like black bodies. Radius of A is double that of B. A and B have same temperature T. When A and B are kept in a room of temperature T_0 (< T), the ratio of their rates of cooling (rate of fall of temperature) is:
[assume negligible heat exchange between A and B]

(A) 2 : 1 (B) 1 : 1 (C) 4 : 1 (D) 8 : 1

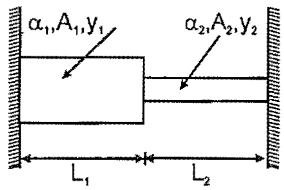
14.6 In the figure shown AB is a rod of length 30 cm and area of cross-section 1.0 cm² and thermal conductivity 336 S. I. units. The ends A & B are maintained at temperatures 20° C and 40 °C respectively. A point C of this rod is connected to a box D, containing ice at 0° C, through a highly conducting wire of negligible heat capacity. The rate at which ice melts in the box is :

[Assume latent heat of fusion for ice L_f = 80 cal/gm]



(A) 84 mg/s (B) 84 g/s
(C) 20 mg/s (D) 40 mg/s

14.7 Two elastic rods are joined between fixed supports as shown in the figure. Condition for no change in the lengths of individual rods with the increase of temperature
(α_1, α_2 = linear expansion co-efficient
A_1, A_2 = Area of rods
y_1, y_2 = Young modulus)



(A) $\frac{A_1}{A_2} = \frac{\alpha_1\ y_1}{\alpha_2\ y_2}$ (B) $\frac{A_1}{A_2} = \frac{L_1\ \alpha_1\ y_1}{L_2\ \alpha_2\ y_2}$ (C) $\frac{A_1}{A_2} = \frac{L_2\ \alpha_2\ y_2}{L_1\ \alpha_1\ y_1}$ (D) $\frac{A_1}{A_2} = \frac{\alpha_2\ y_2}{\alpha_1\ y_1}$

14.8 Four particles have speeds 1, 0, 2, 3 m/s. The root mean square speed of the particles is: (in m/s)

(A) 3.5 (B) $\sqrt{3.5}$ (C) 1.5 (D) $\sqrt{\frac{14}{3}}$

14.9 In a process the density of a gas remains constant. If the temperature is doubled, then the change in the pressure will be :

(A) 100 % increase (B) 200 % increase (C) 50 % decrease (D) 25 % decrease

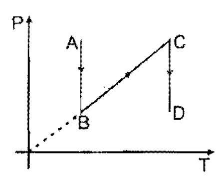
14.10 A steel rod of length 1m is heated from 25°C to 75°C keeping its length constant. The longitudinal strain developed in the rod is (Given : Coefficient of linear expansion of steel = 12×10^{-6}/°C)

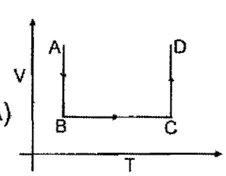
(A) 6×10^{-6} (B) -6×10^{-5} (C) -6×10^{-4} (D) zero

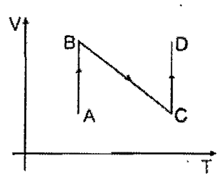
14.11 A rod of length ℓ and cross section area A has a variable thermal conductivity given by $k = \alpha T$, where α is a positive constant and T is temperature in kelvin. Two ends of the rod are maintained at temperatures T_1 and T_2 ($T_1 > T_2$). Heat current flowing through the rod will be

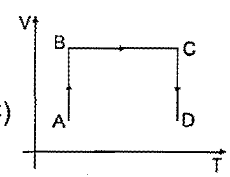
(A) $\frac{A\alpha(T_1^2 - T_2^2)}{\ell}$ (B) $\frac{A\alpha(T_1^2 + T_2^2)}{\ell}$ (C) $\frac{A\alpha(T_1^2 + T_2^2)}{3\ell}$ (D) $\frac{A\alpha(T_1^2 - T_2^2)}{2\ell}$

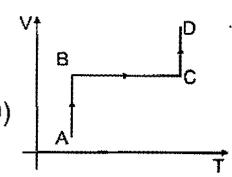
14.12 P-T diagram is shown below then choose the corresponding V-T diagram



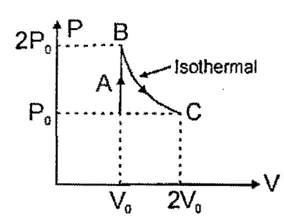
(A)


(B)


(C)


(D)


14.13 A diatomic ideal gas undergoes a thermodynamic change according to the P–V diagram shown in the figure. The total heat given to the gas is nearly :



(A) $2.5\ P_0V_0$ (B) $1.4\ P_0V_0$

(C) $3.9\ P_0V_0$ (D) $1.1\ P_0V_0$

14.14 On an X temperature scale, water freezes at – 125.0° X and boils at 375.0° X. On a Y temperature scale, water freezes at – 70.0°Y and boils at – 30.0°Y. The value of temperature on X-scale equal to the temperature of 50.0°Y on Y-scale is :

(A) 455.0° X (B) – 125.0° X (C) 1375.0° X (D) 1500.0° X

14.15 A solid spherical black body of radius r and uniform mass distribution is in free space. It emits power 'P' and its rate of cooling is R then

(A) $R P \propto r^2$ (B) $R P \propto r$ (C) $R P \propto 1/r^2$ (D) $R P \propto \frac{1}{r}$

14.16 A black body emits radiation at the rate P when its temperature is T. At this temperature the wavelength at which the radiation has maximum intensity is λ_0. If at another temperature T′ the power radiated is P′ and wavelength at maximum intensity is $\frac{\lambda_0}{2}$ then

(A) P′ T′ = 32PT (B) P′ T′ = 16PT (C) P′ T′ = 8PT (D) P′ T′ = 4PT

14.17 The emissive power of a black body at T = 300 K is 100 Watt/m². Consider a body B of area A = 10 m² coefficient of reflectivity r = 0.3 and coefficient of transmission t = 0.5. Its temperature is 300 K. Then which of the following is incorrect :

(A) The emissive power of B is 20 W/m² (B) The emissive power of B is 200 W/m²
(C) The power emitted by B is 200 Watts (D) The emissivity of B is = 0.2

14.18 There are four objects A, B, C and D. It is observed that A and B are in thermal equilibrium and C and D are also in thermal equilibrium. However, A and C are not in thermal equilibrium. We can conclude that :

(A) B and D are in thermal equilibrium
(B) B and D could be in thermal equilibrium
(C) B and D cannot be in thermal equilibrium
(D) The zeroth law of thermodynamics does not apply here because there are more than three objects

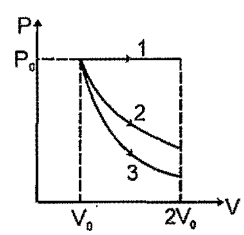
14.19 If H_C, H_K and H_F are heat required to raise the temperature of one gram of water by one degree in Celsius, Kelvin and Fahrenheit temperature scales respectively then :

(A) $H_K > H_C > H_F$ (B) $H_F > H_C > H_K$ (C) $H_K = H_C > H_F$ (D) $H_K = H_C = H_F$

14.20 Find the amount of work done to increase the temperature of one mole of an ideal gas by 30°C if it is expanding under the condition $V \propto T^{2/3}$.

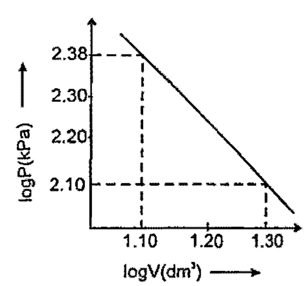
(A) 166.2 J (B) 136.2 (C) 126.2 J (D) none of these

14.21 A gas is expanded from volume V_0 to $2V_0$ under three different processes. Process 1 is isobaric process, process 2 is isothermal and process 3 is adiabatic. Let ΔU_1, ΔU_2 and ΔU_3 be the change in internal energy of the gas in these three processes. Then :



(A) $\Delta U_1 > \Delta U_2 > \Delta U_3$ (B) $\Delta U_1 < \Delta U_2 < \Delta U_3$
(C) $\Delta U_2 < \Delta U_1 < \Delta U_3$ (D) $\Delta U_2 < \Delta U_3 < \Delta U_1$

14.22 Logarithms of readings of pressure and volume for an ideal gas were plotted on a graph as shown in Figure. By measurring the gradient, it can be shown that the gas may be :



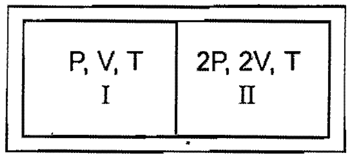
(A) monoatomic and undergoing an adiabatic change
(B) monoatomic and undergoing an isothermal change
(C) diatomic and undergoing an adiabatic change
(D) triatomic and undergoing an isothermal change.

14.23 A metallic sphere having radius 0.08 m and mass m = 10kg is heated to a temperature of 227°C and suspended inside a box whose walls are at a temperature of 27°C. The maximum rate at which its temperature will fall is :-
(Take e = 1, Stefan's constant $\sigma = 5.8 \times 10^{-8}$ Wm^{-2} K^{-4} and specific heat of the metal s = 90 cal/kg/deg J = 4.2 Joules/Calorie)

(A) .055 °C/sec (B) .066 °C/sec (C) .044 °C/sec (D) 0.03 °C/sec

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

14.24 A partition divides a container having insulated walls into two compartments I and II. The same gas fills the two compartments whose initial parameters are given. The partition is a conducting wall which can move freely without friction. Which of the following statement is/are correct, with reference to the final equilibrium position ?



(A) The Pressure in the two compartments are equal

(B) Volume of compartment I is $\frac{3V}{5}$

(C) Volume of compartment II is $\frac{12V}{5}$

(D) Final pressure in compartment I is $\frac{5P}{3}$

14.25 During an experiment, an ideal gas is found to obey a condition $\frac{P^2}{\rho}$ = constant [ρ = density of the gas]. The gas is initially at temperature T, pressure P and density ρ. The gas expands such that density changes to $\frac{\rho}{2}$

(A) The pressure of the gas changes to $\sqrt{2}$ P.

(B) The temperature of the gas changes to $\sqrt{2}$ T.

(C) The graph of the above process on the P-T diagram is parabola.

(D) The graph of the above process on the P-T diagram is hyperbola.

14.26 Pick the correct statements(s) :

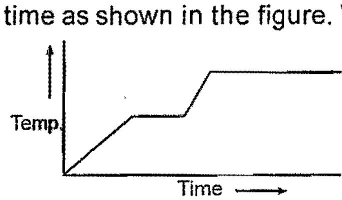
(A) The rms translational speed for all ideal-gas molecules at the same temperature is not the same but it depends on the molecular mass.

(B) Each particle in a gas has average translational kinetic energy and the equation $\frac{1}{2}mv^2_{rms} = \frac{3}{2}kT$ establishes the relationship between the average translational kinetic energy per particle and temperature of an ideal gas. It can be concluded that single particle has a temperature.

(C) Temperature of an ideal gas is doubled from 100°C to 200°C. The average kinetic energy of each particle is also doubled.

(D) It is possible for both the pressure and volume of a monoatomic ideal gas to change simultaneously without causing the internal energy of the gas to change.

14.27 Heat is supplied to a certain homogeneous sample of matter at a uniform rate. Its temperature is plotted against time as shown in the figure. Which of the following conclusions can be drawn?



(A) its specific heat capacity is greater in the solid state than in the liquid state.

(B) its specific heat capacity is greater in the liquid state than in the solid state.

(C) its latent heat of vaporization is greater than its latent heat of fusion.

(D) its latent heat of vaporization is smaller than its latent heat of fusion.

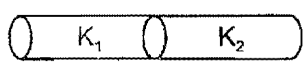
14.28 When the temperature of a copper coin is raised by 80 °C, its diameter increases by 0.2%,
(A) percentage rise in the area of a face is 0.4%
(B) percentage rise in the thickness is 0.4%
(C) percentage rise in the volume is 0.6%
(D) coefficient of linear expansion of copper is 0.25×10^{-4} / °C.

14.29 A vessel is partly filled with liquid. When the vessel is cooled to a lower temperature, the space in the vessel, unoccupied by the liquid remains constant. Then the volume of the liquid (V_L), volume of the vessel (V_V), the coefficients of cubical expansion of the material of the vessel (γ_v) and of the liquid (γ_L) are related as :
(A) $\gamma_L > \gamma_v$ (B) $\gamma_L < \gamma_v$ (C) $\gamma_v/\gamma_L = V_v/V_L$ (D) $\gamma_v/\gamma_L = V_L/V_v$

14.30 Two identical objects A and B are at temperatures T_A and T_B respectively. Both objects are placed in a room with perfectly absorbing walls maintained at a temperature T ($T_A > T > T_B$). The objects A and B attain the temperature T eventually. Select the correct statements from the following.
(A) A only emits radiation, while B only absorbs it until both attain the temperature T.
(B) A loses more heat by radiation than it absorbs, while B absorbs more radiation than it emits, until they attain the temperature T.
(C) Both A and B only absorb radiation, but do not emit it, until they attain the temperature T.
(D) Each object continues to emit and absorb radiation even after attaining the temperature T.

SECTION - III : ASSERTION AND REASON TYPE

14.31. Statement-1 : Two solid cylindrical rods of identical size and different thermal conductivity K_1 and K_2 are connected in series. Then the equivalent thermal conductivity of two rod system is less than the value of thermal conductivity of either rod.



Statement-2 : For two cylindrical rods of identical size and different thermal conductivity K_1 and K_2 connected in series, the equivalent thermal conductivity K is given by

$$\frac{2}{K} = \frac{1}{K_1} + \frac{1}{K_2}$$

(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True.

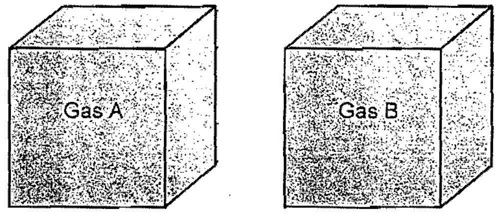
14.32 Statement-1 : As the temperature of the blackbody increases, the wavelength at which the spectral intensity (E_λ) is maximum decreases.
Statement-2 : The wavelength at which the spectral intensity will be maximum for a black body is proportional to the fourth power of its absolute temperature.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True.

14.33 Statement-1 : An ideal gas is enclosed within a container fitted with a piston. When volume of this enclosed gas is increased at constant temperature, the pressure exerted by the gas on the piston decreases.
Statement-2 : In the above situation the rate of molecules striking the piston decreases. If the rate at which molecules of a gas having same average speed striking a given area of the wall decreases, the pressure exerted by gas on the wall decreases.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

SECTION - IV : COMPREHENSION TYPE

Comprehension # 1

Two closed identical conducting containers are found in the laboratory of an old scientist. For the verification of the gas some experiments are performed on the two boxes and the results are noted.



Experiment 1. When the two containers are weighed $W_A = 225$ g , $W_B = 160$ g and mass of evacuated container $W_C = 100$ g.

Experiment 2. When the two containers are given same amount of heat same temperature rise is recorded. The pressure change found are

$\Delta P_A = 2.5$ atm. $\Delta P_B = 1.5$ atm.

Required data for unknown gas :

Mono (molar mass)	He 4g	Ne 20g	Ar 40 g	Kr 84 g	Xe 131 g	Rd 222 g
Dia (molar mass)	H_2 2g	F_2 19 g	N_2 28g	O_2 32g	Cl_2 71 g	

14.34 Identify the type of gas filled in container A and B respectively.
(A) Mono, Mono (B) Dia, Dia (C) Mono, Dia (D) Dia, Mono.

14.35 Identify the gas filled in the container A and B.
(A) N_2, Ne (B) He, H_2 (C) O_2 , Ar (D) Ar, O_2

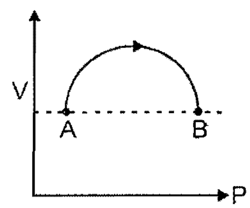
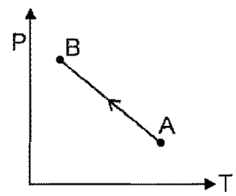
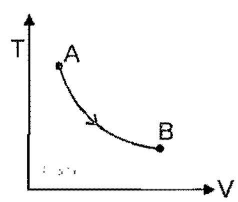
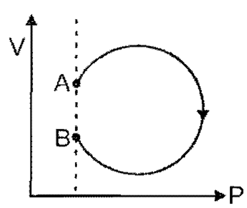
14.36 If the gases have initial temperature 300 K and they are mixed in an adiabatic container having the same volume as the previous containers. Now the temperature of the mixture is T and pressure is P. Then
(A) $P > P_A$, $T > 300$ K (B) $P > P_B$, $T = 300$ K
(C) $P < P_A$, $T = 300$ K (D) $P > P_A$, $T < 300$ K

SECTION - V : MATRIX - MATCH TYPE

14.37 An ideal monoatomic gas undergoes different types of processes which are described in column-I. Match the corresponding effects in column-II. The letters have usual meaning.

	Column-I		Column-II
(A)	$P = 2V^2$	(p)	If volume increases then temperature will also increase.
(B)	PV^2 = constant	(q)	If volume increases then temperature will decrease.
(C)	$C = C_V + 2R$	(r)	For expansion, heat will have to be supplied to the gas
(D)	$C = C_V - 2R$	(s)	If temperature increases then work done by gas is positive.
		(t)	If temperature decreases then work done by gas is positive

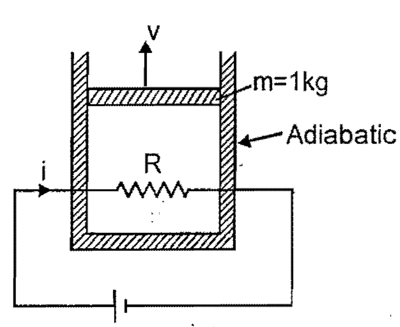
14.38 A sample of gas goes from state A to state B in four different manners, as shown by the graphs. Let W be the work done by the gas and ΔU be change in internal energy along the path AB. Correctly match the graphs with the statements provided.

Column-I		Column-II	
(A)		(p)	Both W and ΔU are positive
(B)		(q)	Both W and ΔU are negative
(C)		(r)	W is positive whereas ΔU is negative
(D)		(s)	W is negative whereas ΔU is positive
		(t)	Final temperature of an ideal gas is less than its initial temperature.

SECTION - VI : INTEGER TYPE

14.39 Figure shows the variation of internal energy (U) with the pressure (P) of 2.0 mole gas in cyclic process abcda. The temperature of gas at c and d are 300 and 500 K. Calculate the heat absorbed by the gas during the process 10xR ln2 then x is .

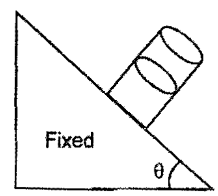
14.40 Current i = 2A flows through the resistance R = 10Ω. With what constant speed v (in m/s), must the piston move in upward direction so that temperature of ideal gas may remain unchanged. (g = 10 m/s²)



TOPIC 15 FLUID MECHANICS SURFACE TENSION, VISCOSITY & ELASTICITY

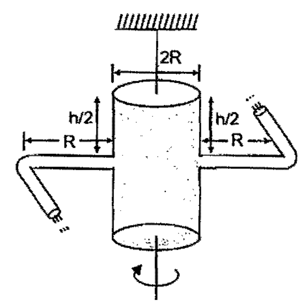
SECTION - I : STRAIGHT OBJECTIVE TYPE

15.1 A cylindrical vessel filled with water is released on an inclined surface of angle θ as shown in figure. The friction coefficient of surface with vessel is μ ($< \tan\theta$). Then the constant angle made by the surface of water with the incline will be :



(A) $\tan^{-1}\mu$
(B) $\theta - \tan^{-1}\mu$
(C) $\theta + \tan^{-1}\mu$
(D) $\cot^{-1}\mu$

15.2 A cylindrical container of radius 'R' and height 'h' is completely filled with a liquid. Two horizontal L shaped pipes of small cross-section area 'a' are connected to the cylinder as shown in the figure. Now the two pipes are opened and fluid starts coming out of the pipes horizontally in opposite directions. Then the torque due to ejected liquid on the system is:

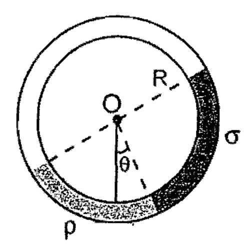


(A) $4\,a g h \rho R$
(B) $8\,a g h \rho R$
(C) $2\,a g h \rho R$
(D) none of these

15.3 A block of silver of mass 4 kg hanging from a string is immersed in a liquid of relative density 0.72. If relative density of silver is 10, then tension in the string will be:[take g = 10 m/s^2]
(A) 37.12 N (B) 42 N (C) 73 N (D) 21 N

15.4. A vessel contains oil (density = 0.8 gm/cm^3) over mercury (density = 13.6 gm/cm^3). A uniform sphere floats with half its volume immersed in mercury and the other half in oil. The density of the material of sphere in gm/cm^3 is:
(A) 3.3 (B) 6.4 (C) 7.2 (D) 12.8

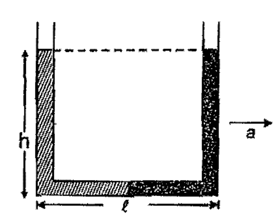
15.5 A small uniform tube is bent into a circular tube of radius R and kept in the vertical plane. Equal volumes of two liquids of densities ρ and σ ($\rho > \sigma$) fill half of the tube as shown in the figure. θ is the angle which the radius passing through the interface makes with the vertical.



(A) $\theta = \tan^{-1}\left(\frac{\rho-\sigma}{\rho+\sigma}\right)$
(B) $\theta = \tan^{-1}\left(\frac{\sigma-\rho}{\sigma+\rho}\right)$
(C) $\theta = \tan^{-1}\left(\frac{\rho}{\rho+\sigma}\right)$
(D) $\theta = \tan^{-1}\left(\frac{\rho}{\rho-\sigma}\right)$

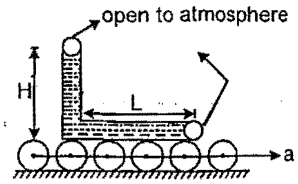
15.6 An open tank 10m long and 2m deep is filled up to 1.5 m height with oil of specific gravity 0.82. The tank is uniformly accelerated along its length from rest to a speed of 20 m/sec horizontally. The shortest time in which the speed may be attained without spilling any oil is : [g = 10 m/sec^2]
(A) 20 sec. (B) 18 sec. (C) 10 sec. (D) 5 sec.

15.7 A U-tube of base length "ℓ" filled with same volume of two liquids of densities ρ and 2ρ is moving with an acceleration "a" on the horizontal plane. If the height difference between the two surfaces (open to atmosphere) becomes zero, then the height h is given by:



(A) $\frac{a}{2g}\ell$ (B) $\frac{3a}{2g}\ell$ (C) $\frac{a}{g}\ell$ (D) $\frac{2a}{3g}\ell$

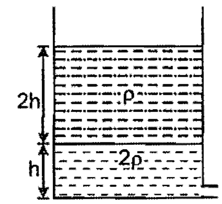
15.8 A narrow tube completely filled with a liquid is lying on a series of cylinders as shown in figure. Assuming no sliding between any surfaces, the value of acceleration of the cylinders for which liquid will not come out of the tube from anywhere is given by



(A) $\frac{gH}{2L}$ (B) $\frac{gH}{L}$

(C) $\frac{2gH}{L}$ (D) $\frac{gH}{\sqrt{2}L}$

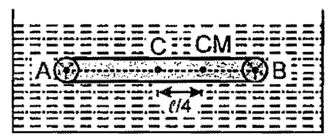
15.9 The velocity of the liquid coming out of a small hole of a vessel containing two different liquids of densities 2ρ and ρ as shown in figure is



(A) $\sqrt{6gh}$ (B) $2\sqrt{gh}$

(C) $2\sqrt{2gh}$ (D) $\sqrt{gh}$

15.10 A non uniform cylinder of mass m, length ℓ and radius r is having its cetnre of mass at a distance $\ell/4$ from the centre and lying on the axis of the cylinder. The cylinder is kept in a liquid of uniform density ρ. The moment of inertia of the rod about the centre of mass is I. The angular acceleration of point A relative to point B just after the rod is released from the position shown in figure is

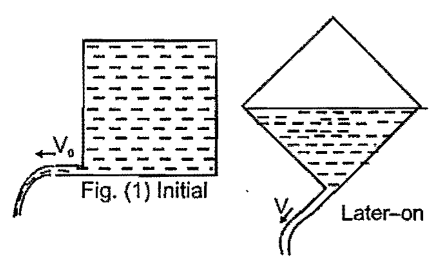


(A) $\frac{\pi\rho g \ell^2 r^2}{I}$ (B) $\frac{\pi\rho g \ell^2 r^2}{4I}$ (C) $\frac{\pi\rho g \ell^2 r^2}{2I}$ (D) $\frac{3\pi\rho g \ell^2 r^2}{4I}$

15.11 The coefficient of viscosity η of a liquid is defined as the tangential force on a layer in that liquid per unit area per unit velocity gradient across it. Then a sphere of radius 'a', moving through it under a constant force F attains a constant velocity 'V' given by : (where K is a numerical constant)

(A) $K Fa \eta$ (B) $K \frac{F}{a} \eta$ (C) $K \frac{F}{a\eta}$ (D) $K\eta \frac{a}{F}$

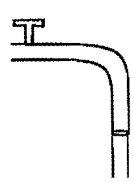
15.12 A square box of water has a small hole located in one of the bottom corner. When the box is full and placed on a level surface, complete opening of the hole results in a flow of water with a speed v_0, as shown in figure (1). When the box is still half empty, it is tilted by 45° so that the hole is at the lowest point. Now the water will flow out with a speed of



(A) V_0 (B) $V_0/2$

(C) $V_0/\sqrt{2}$ (D) $V_0/\sqrt[4]{2}$

15.13 A steady stream of water falls straight down from a pipe as shown. Assume the flow is incompressible, then



(A) the pressure in the water is higher at lower points in the stream.
(B) the pressure in the water is lower at lower points in the stream.
(C) the pressure in the water is the same at all points in the stream.
(D) pressure variation will depend upon density and exit speed of the water.

15.14 Water filled in an empty tank of area 10 A through a tap of cross sectional area A. The speed of water flowing out of tap is given by v (m/s) = $10\left(1 - \sin\frac{\pi}{30}t\right)$ where 't' is in seconds. The height of water level from the bottom of the tank at t = 15 sec will be

(A) 10 m (B) $15 + \frac{30}{\pi}$ m (C) $\frac{5}{4}$ m (D) $15 - \frac{30}{\pi}$ m

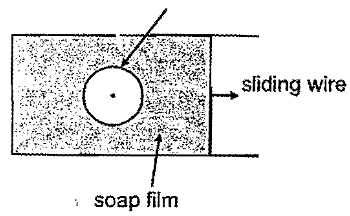
15.15 A mosquito with 8 legs stands on water surface and each leg makes depression of radius 'a'. If the surface tension and angle of contact are 'T' and zero respectively, then the weight of mosquito is

(A) 8 T . a (B) $16\pi T a$ (C) $\frac{Ta}{8}$ (D) $\frac{Ta}{16\pi}$

15.16 An isolated and charged spherical soap bubble has a radius 'r' and the pressure inside is atmospheric. If 'T' is the surface tension of soap solution, then charge on drop is:

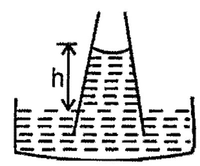
(A) $2\sqrt{\frac{2rT}{\varepsilon_0}}$ (B) $8\pi r\sqrt{2rT\varepsilon_0}$ (C) $8\pi r\sqrt{rT\varepsilon_0}$ (D) $8\pi r\sqrt{\frac{2rT}{\varepsilon_0}}$

15.17 The figure shows a soap film in which a closed elastic thread is lying. The film inside the thread is pricked. Now the sliding wire is moved out so that the surface area increases. The radius of the circle formed by elastic thread will



(A) increase
(B) decrease
(C) remains same
(D) data insufficient

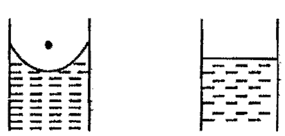
15.18 A capillary of the shape as shown is dipped in a liquid. Contact angle between the liquid and the capillary is 0° and effect of liquid inside the meniscus is to be neglected. T is surface tension of the liquid, r is radius of the meniscus, g is acceleration due to gravity and ρ is density of the liquid then height h in equilibrium is



(A) greater than $\frac{2T}{r\rho g}$ (B) equal to $\frac{2T}{r\rho g}$

(C) less than $\frac{2T}{r\rho g}$ (D) of any value depending upon actual values

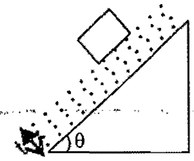
15.19 Shape of the meniscus formed by two liquids when capillaries are dipped in them are shown. In I it is hemispherical where as in II it is flat. Pick correct statement regarding contact angles formed by the liquids in both situations



(A) It is 180° in I and 90° in II. (B) It is 0° in I and 90° in II
(C) It is 90° in I and 0° in II (D) It is greater than 90° in I and equal to 90° in II.

15.20 A uniform rod of density ρ, and length '1' is having square cross-section of side 'a'. It is placed in a liquid of equal density ρ vertically along length in a tank having sufficient height of liquid. The surface tension of liquid is 'T' and angle of contact is 120°. Then :
(A) rod will float completely immersed inside the liquid :
(B) rod will sink to bottom of tank
(C) rod will float partially submerged with height $\frac{4T}{a\rho g}$ above liquid
(D) rod will float partially submerged with height $\frac{2T}{a\rho g}$ above liquid.

15.21 A cubical block of side 'a' and density 'ρ' slides over a fixed inclined plane with constant velocity 'v'. There is a thin film of viscous fluid of thickness 't' between the plane and the block. Then the coefficient of viscosity of the thin film will be :



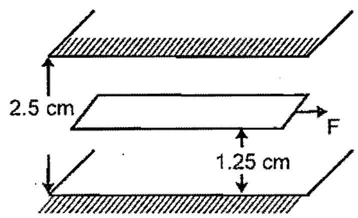
(A) $\eta = \frac{\rho\, agt \sin\theta}{v}$ (B) $\frac{\rho\, agt \tan^2\theta}{v}$

(C) $\frac{v}{\rho\, agt \sin\theta}$ (D) none of these

15.22 When a ball is released from rest in a very long column of viscous liquid, its downward acceleration is 'a'(just after release). Find its acceleration when it has acquired two third of the maximum velocity :

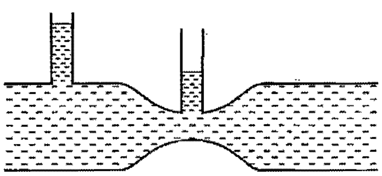
(A) $\frac{a}{3}$ (B) $\frac{2a}{3}$ (C) $\frac{a}{6}$ (D) none of these

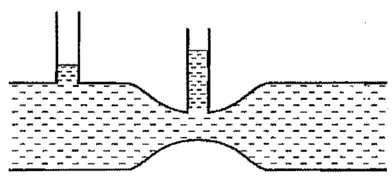
15.23 A space 2.5 cm wide between two large plane surfaces is filled with oil. Force required to drag a very thin plate of area 0.5 m^2 just midway the surfaces at a speed of 0.5 m/sec is 1N. The coefficient of viscosity in kg–sec/m^2 is :

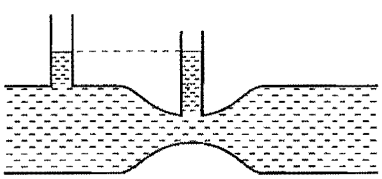


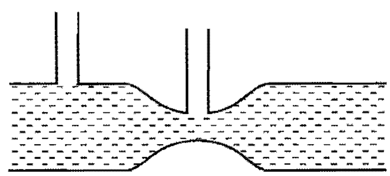
(A) 5×10^{-2} (B) 2.5×10^{-2} (C) 1×10^{-2} (D) 7.5×10^{-2}

15.24 For a fluid which is flowing steadily, the level in the vertical tubes is best represented by

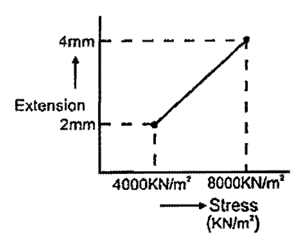
(A) 

(B) 

(C) 

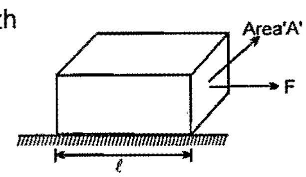
(D) 

15.25 In determination of young modulus of elasticity of wire, a force is applied and extension is recorded. Initial length of wire is '1 m'. The curve between extension and stress is depicted then young modulus of wire will be:



(A) 2×10^9 N/m^2 (B) 1×10^9 N/m^2
(C) 2×10^{10} N/m^2 (D) 1×10^{10} N/m^2

15.26 A block of mass 'M' area of cross-section 'A' & length 'ℓ' is placed on smooth horizontal floor. A force 'F' is applied on the block as shown. If 'y' is young modulus of material, then total extension in the block will be:



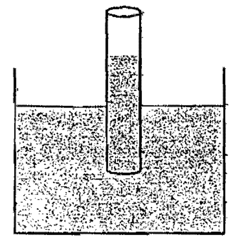
(A) $\frac{F\ell}{AY}$ (B) $\frac{F\ell}{2AY}$

(C) $\frac{F\ell}{3AY}$ (D) cannot extend

15.27 A uniform rod of mass m and length ℓ is rotating with constant angular velocity ω about an axis which passes through its one end and perpendicular to the length of rod. The area of cross section of the rod is A and its young's modulus is Y. Neglect gravity. The strain at the mid point of the rod is :

(A) $\frac{m\omega^2 \ell}{8AY}$ (B) $\frac{3m\omega^2 \ell}{8AY}$ (C) $\frac{3m\omega^2 \ell}{4AY}$ (D) $\frac{m\omega^2 \ell}{4AY}$

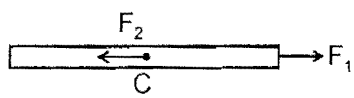
15.28 A long capillary tube of mass 'π' gm, radius 2mm and negligible thickness, is partially immersed in a liquid of surface tension 0.1 N/m. Take angle of contact zero and neglect buoyant force of liquid. The force required to hold the tube vertically, will be - (g = 10 m/s^2)



(A) 10.4 π mN
(B) 10.8 π mN
(C) 0.8 π mN
(D) 4.8 π mN

15.29 The elongation in a metallic rod hinged at one end and rotating in a horizontal plane becomes four times of the initial value. The angular velocity of rotation becomes :
(A) two times the initial value (B) half of initial value
(C) one third of initial value (D) four times the initial value.

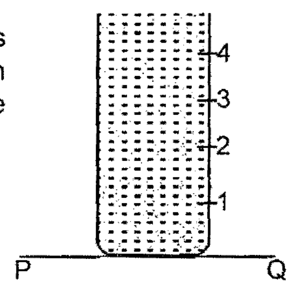
15.30 Two forces F_1 and F_2 act on a thin uniform elastic rod placed in space. Force F_1 acts at right end of the rod and F_2 acts exactly at centre of the rod as shown (both forces act parallel to length of the rod).



(i) F_1 causes extension of rod while F_2 causes compression of rod.
(ii) F_1 causes extension of rod and F_2 also causes extension of rod.
(iii) F_1 causes extension of rod while F_2 does not change length of rod.
The correct order of True / False in above statements is
(A) T F F (B) F T F (C) F F T (D) F F F

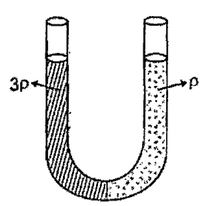
SECTION - II : MULTIPLE CORRECT ANSWER TYPE

15.31 A cylindrical vessel of 90 cm height is kept filled upto the brim. It has four holes 1, 2, 3, 4 which are respectively at heights of 20cm, 30 cm, 40 cm and 50 cm from the horizontal floor PQ. The water falling at the maximum horizontal distance from the vessel comes from

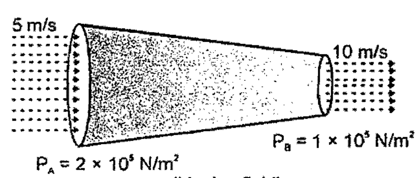


(A) hole number 4 (B) hole number 3
(C) hole number 2 (D) hole number 1.

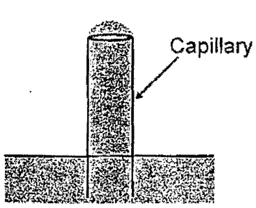
15.32 Which of the following is not possible ?

(A)


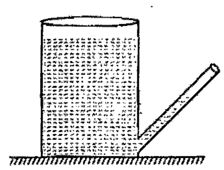
(Stationary fluid)

(B)


(Moving fluid)

(C)


(Stationary fluid)

(D)

(Stationary fluid)

SECTION - III : ASSERTION AND REASON TYPE

15.33 **Statement-1 :** Imagine holding two identical bricks under water. Brick A is completely submerged just below the surface of water, while brick B is at a greater depth. The magnitude of force exerted by the person (on the brick) to hold brick B in place is the same as magnitude of force exerted by the person (on the brick) to hold brick A in place.
Statement-2 : The magnitude of buoyant force on a brick completely submerged in water is equal to magnitude of weight of water it displaces and does not depend on depth of brick in water.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

15.34 **Statement-1 :** A uniform elastic rod lying on smooth horizontal surface is pulled by constant horizontal force of magnitude F as shown in figure (i). Another identical elastic rod is pulled vertically upwards by a constant vertical force of magnitude F (see figure ii). The extension in both rods will be same.

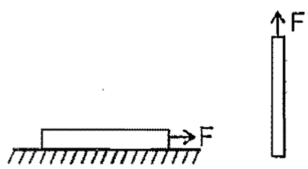


Fig.-i Fig.-ii

Statement-2 : In a uniform elastic rod, the extension depends only on forces acting at the ends of rod.
(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True.

SECTION - IV : COMPREHENSION TYPE

Comprehension : 1

If the container filled with liquid gets accelerated horizontally or vertically, pressure in liquid gets changed. In case of horizontally accelerated liquid (a_x), the free surface has the slope $\frac{a_x}{g}$. In case of vertically accelerated liquid (a_y) for calculation of pressure, effective g is used. A closed box with horizontal base 6m by 6m and a height 2m is half filled with liquid. It is given a constant horizontal acceleration g/2 and vertical downward acceleration g/2.

15.35 The angle of the free surface with the horizontal is equal to -
(A) 30° (B) $\tan^{-1}\frac{2}{3}$ (C) $\tan^{-1}\frac{1}{3}$ (D) 45°

15.36 Length of exposed portion of top of box is equal to-
(A) 2m (B) 3m (C) 4m (D) 2.5 m

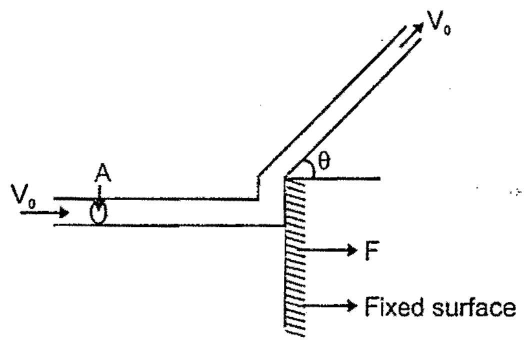
15.37 Water pressure at the bottom of centre of box is equal to – (atmospheric pressure = 10^5 N/m^2, density of water = 1000 kg/m^3, g = 10 m/sec^2)
(A) 1.1 MPa (B) 0.11 MPa (C) 0.101 MPa (D) 0.011 MPa

15.38 Maximum value of water pressure in the box is equal to
(A) 1.4 MPa (B) 0.12 MPa (C) 0.104 MPa (D) 0.014 MPa

15.39 What is the value of vertical acceleration of box for given horizontal acceleration (g/2), so that no part of bottom of box is exposed :
(A) g/2 upward (B) g/4 downward (C) g/4 upward (D) not possible

Comprehension 2

When jet of liquid strikes a fixed or moving surfaces, it exerts thrust on it due to rate of change of momentum.

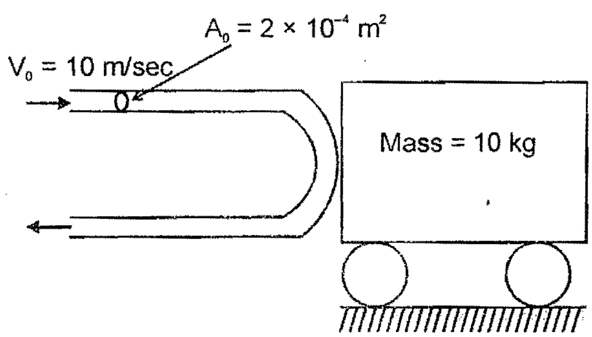


$F = (\rho AV_0).V_0 - (\rho AV_0).V_0 \cos\theta = \rho AV_0^2 [1 - \cos\theta]$

If surface is free and st arts moving due to thrust of liquid, then at any instant, the above equation gets modified based on relative change of momentum with respect to surface. Let any instant the velocity of surface is u, then above equation becomes –

$$F = \rho A(V_0 - u)^2 [1 - \cos\theta]$$

Based on above concept, in the below given figure, if the cart is frictionless and free to move in horizontal direction, then answer the following :



Given cross–section area of jet = 2×10^{-4} m² velocity of jet V_0 = 10 m/sec., density of liquid = 1000 kg/m³ , Mass of cart = M = 10 kg.

15.40 Initially (t = 0) the force on the cart is equal to :

(A) 20 N (B) 40 N (C) 80 N (D) zero

15.41 Velocity of cart at t = 10 sec. is equal to :

(A) 4 m/sec. (B) 6 m/sec. (C) 8 m/sec. (D) 5 m/sec.

15.42 In the above problem, what is the acceleration of cart at this instant –

(A) 1.6 m/sec² (B) 1 m/sec² (C) 0.64 m/sec² (D) 0.16 m/sec²

15.43 The time at which velocity of cart becomes 2 m/sec, is equal to

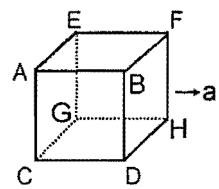
(A) 10/16 sec. (B) 2 sec. (C) 3.2 sec. (D) 4 sec.

15.44 The power supplied to the cart, when its velocity becomes 5 m/sec., is equal to :

(A) 100 W (B) 25 W (C) 50 W (D) 200 W

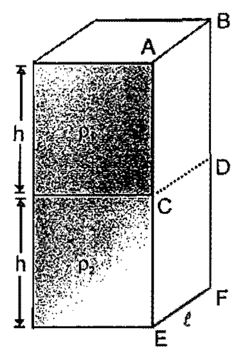
SECTION - V : MATRIX - MATCH TYPE

15.45 A cubical box is completely filled with mass m of a liquid and is given horizantal acceleration a as shown Match the force due to fluid pressure on the faces of the cube with their appropriate values (assume zero pressure at minimum pressure)



	Column I		Column II
(A)	force on face ABFE	(p)	$\frac{ma}{2}$
(B)	force on face BFHD	(q)	$\frac{mg}{2}$
(C)	force on face ACGE	(r)	$\frac{ma}{2}+\frac{mg}{2}$
(D)	force on face CGHD	(s)	$\frac{ma}{2}+mg$

15.46 A cuboid is filled with liquid of density ρ_2 upto height h and with liquid of density ρ_1, also upto height h as shown



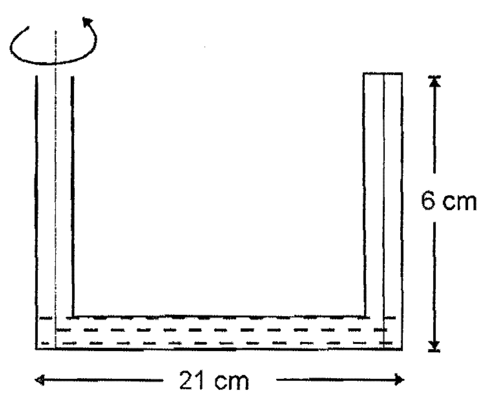
	Column I		Column II
(A)	Force on face ABCD due to liquid of density ρ_1	(p)	zero
(B)	Force on face ABCD due to liquid of density ρ_2	(q)	$\frac{\rho_1 gh^2\ell}{2}$
(C)	Force on face CDEF due to liquid of density ρ_1	(r)	$\rho_1 gh^2\ell$
(D)	Force on face CDEF due to liquid of density ρ_2	(s)	$\left[\rho_1 gh+\frac{\rho_2 gh}{2}\right]h\ell$
		(t)	$10\rho_1 gh^2 \ell^2$

SECTION - VI : INTEGER TYPE

15.47 A cube with a mass m = 20 g wettable by water floats on the surface of water. Each face of the cube a is 30 cm long. What is the distance between the lower face of the cube and the surface of the water (in mm.)?
[S.T of water = 70 dyn/cm, g = 10 m/sec^2, assume angle of contact with water is zero]

15.48 A piston of 20 cm diameter and 20 cm long moves down in a cylinder of diameter 20.0628 cm. The oil filling the annular space has a viscosity of 10 poise and the weight of the piston is 1 kg. If the speed is x/10 cm s^{-1} with which the piston slides down.then x is [g = 1000 cm/sec^2]

15.49 Length of horizontal arm of a uniform cross–section U–tube is l = 21 cm and ends of both of the vertical arms are open to surrounding of pressure 10500 N/m^2. A liquid of density ρ = 10^3 kg/m^3 is poured into the tube such that liquid just fills the horizontal part of the tube. Now one of the open ends is sealed and the tube is then rotated about a vertical axis passing through the other vertical arm with angular velocity ω_0 = 10 rad/sec. If length of each vertical arm be a = 6 cm. Calculate the length of air column in the sealed arm in cm. [g=10 m/sec^2]



15.50 A rod 1 m long is 10 cm^2 in area for a portion of its length and 5 cm^2 in area for the remainder . The strain energy of this stepped bar is 40 % of that a bar 10 cm^2 in area 1 m long under the same stress in 10cm^2 part. What is the length of the portion 10 cm^2 in area.

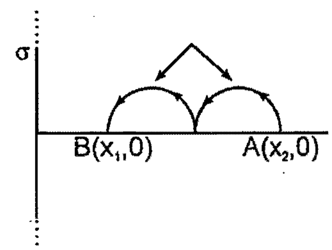
TOPIC 16 ELECTROSTATICS

SECTION - I : STRAIGHT OBJECTIVE TYPE

161 The electric field inside a sphere which carries a volume charge density proportional to the distance from the origin $\rho = \alpha r$ (α is a constant) is :

(A) $\frac{\alpha r^3}{4 \epsilon_0}$ (B) $\frac{\alpha r^2}{4 \epsilon_0}$ (C) $\frac{\alpha r^2}{3 \epsilon_0}$ (D) none of these

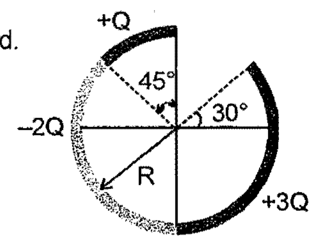
16.2 An infinitely long plate has surface charge density σ. As shown in the fig, a point charge q is moved from A to B. Net work done by electric field is:



(A) $\frac{\sigma q}{2 \epsilon_0}(x_1 - x_2)$ (B) $\frac{\sigma q}{2 \epsilon_0}(x_2 - x_1)$

(C) $\frac{\sigma q}{\epsilon_0}(x_2 - x_1)$ (D) $\frac{\sigma q}{\epsilon_0}(2 \pi r + r)$

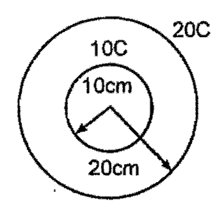
16.3 Figure shows three circular arcs, each of radius R and total charge as indicated. The net elecric potential at the centre of curvature is :



(A) $\frac{Q}{2\pi\varepsilon_0 R}$ (B) $\frac{Q}{4\pi\varepsilon_0 R}$

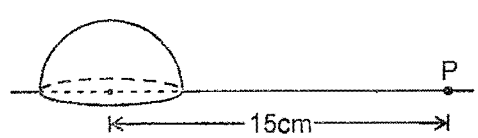
(C) $\frac{2Q}{\pi\varepsilon_0 R}$ (D) $\frac{Q}{\pi\varepsilon_0 R}$

16.4 Two concentric uniformly charged spheres of radius 10 cm & 20 cm are arranged as shown in the figure. Potential difference between the spheres is:



(A) 4.5×10^{11} V
(B) 2.7×10^{11} V
(C) 0
(D) none of these

16.5 Figure shows a solid hemisphere with a charge of 5 nC distributed uniformly throughout its volume. The hemisphere lies on a plane and point P is located on the plane, along a radial line from the centre of curvature at distance 15 cm. The electric potential at point P due to the hemisphere, is :



(A) 150 V (B) 300 V (C) 450 V (D) 600 V

16.6 A point charge Q is placed at a distance d from the centre of an uncharged conducting sphere of radius R. The potential of the sphere is (d > R) :

(A) $\frac{1}{4\pi \epsilon_0} \cdot \frac{Q}{(d-R)}$ (B) $\frac{1}{4\pi \epsilon_0} \cdot \frac{Q}{d}$ (C) $\frac{1}{4\pi \epsilon_0} \cdot \frac{Q}{R}$ (D) zero

16.7 A dipole of dipole moment p is kept at the centre of a ring of radius R and charge Q. The dipole moment has direction along the axis of the ring. The resultant force on the ring due to the dipole is:

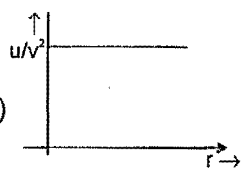
(A) zero (B) $\frac{kPQ}{R^3}$ (C) $\frac{2kPQ}{R^3}$

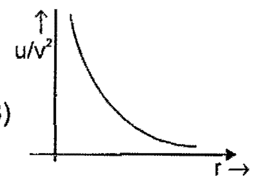
(D) $\frac{kPQ}{R^3}$ only if the charge is uniformly distributed on the ring.

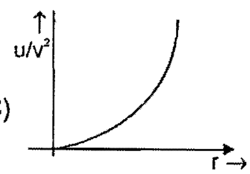
16.8 A conducting disc of radius R rotates about its axis with an angular velocity ω. Then the potential difference between the centre of the disc and its edge is (no magnetic field is present):

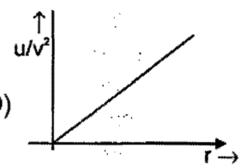
(A) zero (B) $\frac{m_e\omega^2 R^2}{2e}$ (C) $\frac{m_e\omega R^3}{3e}$ (D) $\frac{em_e\omega R^2}{2}$

16.9 At distance 'r' from a point charge, the ratio $\frac{u}{v^2}$ (where 'u' is energy density and 'v' is potential) is best represented by:

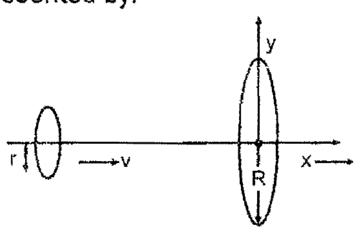
(A)


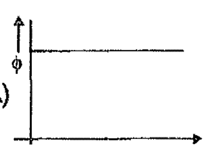
(B)


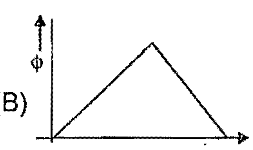
(C)


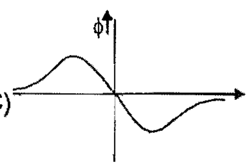
(D)


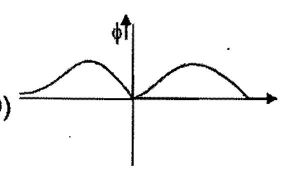
16.10 A ring of radius R is placed in the plane with its centre at origin and its axis along the x-axis and having uniformly distributed positive charge. A ring of radius r (<< R) and coaxial with the larger ring is moving along the axis with constant velocity then the variation of electrical flux (ϕ) passing through the smaller ring with Position will be best represented by:



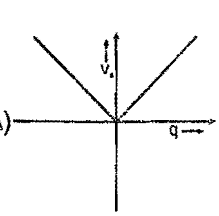
(A)


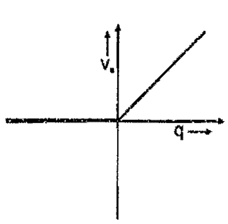
(B)


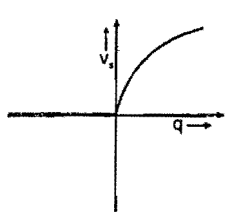
(C)


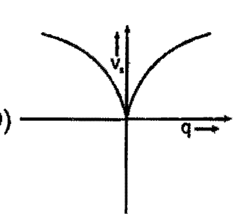
(D)


16.11 A negative charge Q is distributed uniformly in volume of a sphere of radius R and a point charge particle of charge q (may be negative or positive) is present on the surface of this sphere then the variation of escape velocity (v_s) of charge 'q' as a function of 'q' will be [neglect gravitational interaction]

(A)


(B)


(C)


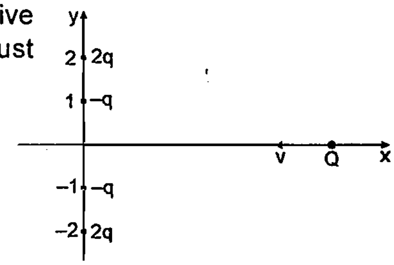
(D)


16.12 Electrical potential 'v' in space as a function of co-ordinates is given by, $v = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$. Then the electric field intensity at (1, 1, 1) is given by:

(A) $-(\hat{i} + \hat{j} + \hat{k})$ (B) $\hat{i} + \hat{j} + \hat{k}$ (C) zero (D) $\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$

16.13 Four charges are rigidly fixed along the Y axis as shown. A positive charge approaches the system along the X axis with initial speed just enough to cross the origin. Then its total energy at the origin is

(A) zero
(B) positive
(C) negative
(D) data insufficient

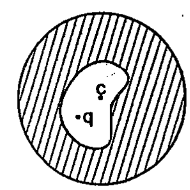


16.14 A dipole of dipole moment $\vec{P} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ is placed at point A (2, –3, 1). The electric potential due to this dipole at the point B (4, – 1, 0) is equal to (All the parameters specified here are in S.I. units.)

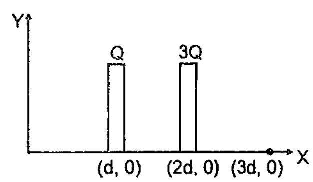
(A) 2×10^9 volts (B) -2×10^9 volts (C) 3×10^9 volts (D) -3×10^9 volts

16.15 The figure shows a charge q placed inside a cavity in an uncharged conductor. Now if an external electric field is switched on :

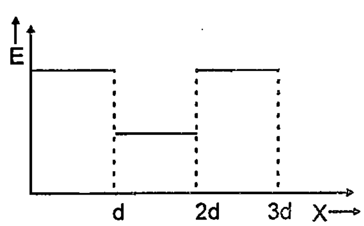
(A) only induced charge on outer surface will redistribute.
(B) only induced charge on inner surface will redistribute.
(C) both induced charge on outer and inner surface will redistribute.
(D) force on charge q placed inside the cavity will change.



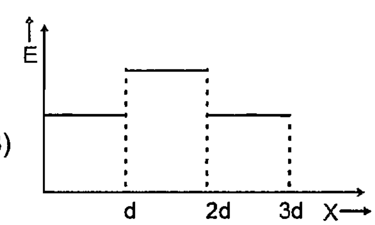
16.16 Two very large thin conducting plates having same cross-sectional area are placed as shown in figure. They are carrying charges 'Q' and '3Q' respectively. The variation of electric field as a function at x (for x = 0 to x = 3d) will be best represented by.



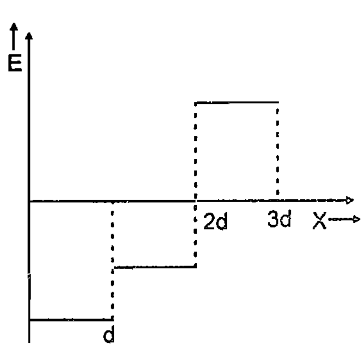
(A)



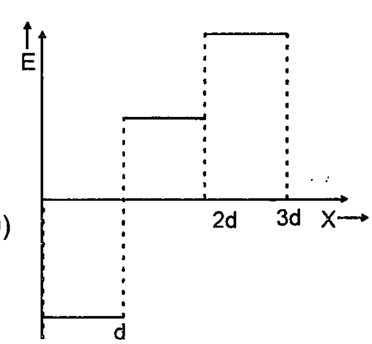
(B)



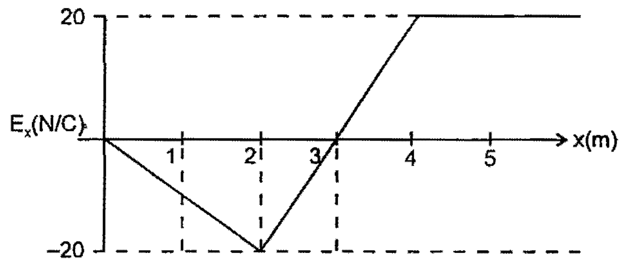
(C)



(D)

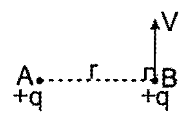


16.17 A graph of the x component of the electric field as a function of x in a region of space is shown. The Y and Z components of the electric field are zero in this region. If the electric potential is 10 V at the origin, then potential at x = 2.0 m is :



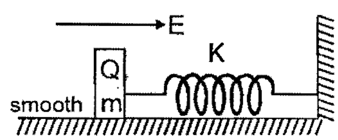
(A) 10 V (B) 40 V (C) – 10 V (D) 30 V

16.18 In the figure shown, A is a fixed charge. B (of mass m) is given a velocity V perpendicular to line AB. At this moment the radius of curvature of the resultant path of B is



(A) 0 (B) ∞ (infinity) (C) $\frac{4\pi \epsilon_0 r^2 mv^2}{q^2}$ (D) r

16.19 In the figure shown, initially the spring of negligible mass is in undeformed state and the block has zero velocity E is a uniform electric field, then : (K = spring constant)



(i) The maximum speed of the block will be $\frac{QE}{\sqrt{mK}}$

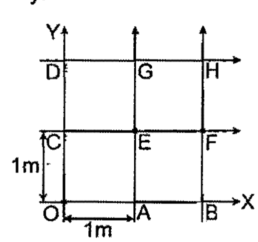
(ii) The maximum speed of the block will be $\frac{2QE}{\sqrt{mK}}$

(iii) The maximum compression of the spring will be $\frac{QE}{K}$

(iv) The maximum compression of the spring will be $\frac{2QE}{K}$

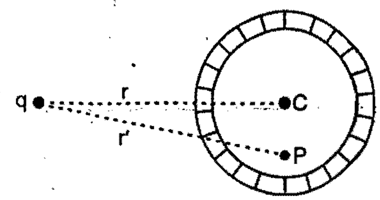
(A) only (i) and (iii) are correct (B) only (i) and (iv) are correct
(C) only (ii) and (iii) are correct (D) only (ii) and (iv) are correct

16.20 The grid (each square of 1m × 1m), represents a region in space containing a uniform electric field. If potentials at points O, A, B, C, D, E, F, G & H are respectively 0, – 1, – 2, 1, 2, 0, – 1, 1, and 0 volts, then find the electric field intensity.



(A) $(\hat{i}+\hat{j})$ V/m (B) $(\hat{i}-\hat{j})$ V/m (C) $(-\hat{i}+\hat{j})$ V/m (D) $(-\hat{i}-\hat{j})$ V/m

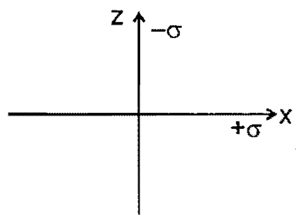
16.21 A neutral conducting spherical shell is kept near a charge q as shown. The potential at point P due to the induced charges is :

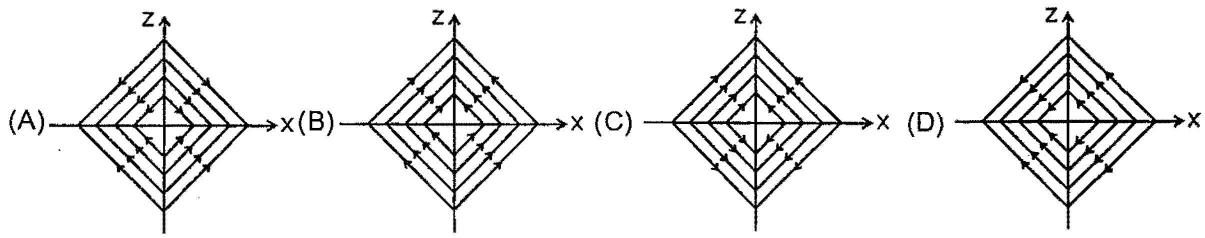


(A) $\frac{kq}{r}$ (B) $\frac{kq}{r'}$

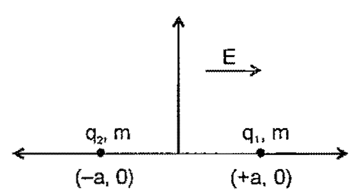
(C) $\frac{kq}{r}-\frac{kq}{r'}$ (D) $\frac{kq}{CP}$

16.22 Two infinitely large charged planes having uniform surface charge density $+\sigma$ and $-\sigma$ are placed along x-y plane and yz plane respectively as shown in the figure. Then the nature of electric lines of forces in x-z plane is given by :



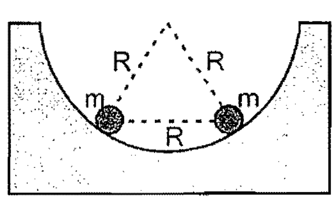


16.23 Two positively charged particles of charges q_1 and q_2 have mass m each. A uniform electric field having magnitude E exists in positive x direction as shown in the figure. The given two charged particles are released from rest at t = 0 as shown in the figure. If position of q_1 at t = 2 sec is given by coordinate (+2a, 0) then the x-coordinate of q_2 at t = 2 sec is (neglect gravitational interaction between the particles) -

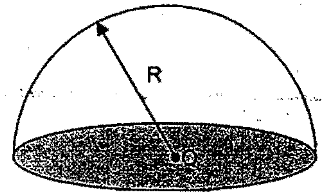


(A) $\frac{q_1+q_2}{m}E-2a$ (B) $\frac{q_1+q_2}{m}E-a$ (C) $2\left(\frac{q_1+q_2}{m}\right)E-2a$ (D) $2\left(\frac{q_1+q_2}{m}\right)E-a$

16.24 Two identical small balls each have a mass m and charge q. When placed in a hemispherical bowl of radius R with frictionless, non-conducting walls, the beads move and at equilibrium, the line joining the balls is horizontal and the distance between them is R (figure). Neglect any induced charge on the hemispherical bowl . Then the charge on each bead is: (here, $K=\frac{1}{4\pi\epsilon_0}$)



(A) $q=R\left(\frac{mg}{K\sqrt{3}}\right)^{1/2}$ (B) $q=\left(R\frac{mg}{K\sqrt{3}}\right)^{1/2}$ (C) $q=R\left(\frac{\sqrt{3}mg}{K}\right)^{1/2}$ (D) $q=\left(R\frac{\sqrt{3}mg}{K}\right)^{1/2}$



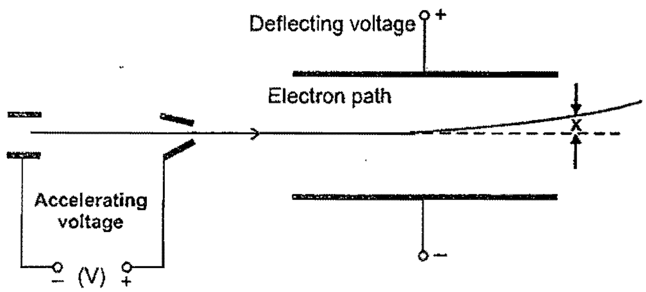
16.25 Charge Q coulombs is uniformly distributed throughout the volume of a solid hemisphere of radius R metres. Then the potential at centre O of the hemisphere in volts is :

(A) $\frac{1}{4\pi\varepsilon_o}\frac{3Q}{2R}$ (B) $\frac{1}{4\pi\varepsilon_o}\frac{3Q}{4R}$

(C) $\frac{1}{4\pi\varepsilon_o}\frac{Q}{4R}$ (D) $\frac{1}{4\pi\varepsilon_o}\frac{Q}{8R}$

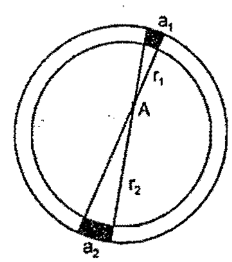
SECTION - II : MULTIPLE CORRECT ANSWER TYPE

16.26 The diagram shows part of an evacuated tube in which a stream of electrons from an electron gun passes between a pair of parallel large deflecting plates. The vertical displacement of the electron beam as it leaves the parallel plates is x. (Do not consider gravity and the electron enters the deflecting region parallel to the plates)



Which one of A to D below will change the displacement x of the beam as it leaves the parallel plates
(A) increasing the accelerating voltage
(B) increasing the deflecting voltage
(C) increasing the distance between the electron gun and the deflecting plates
(D) increasing the distance between the two deflecting plates

16.27 A particle of mass 2Kg and charge 1mC is projected vertically with a velocity $10 ms^{-1}$. There is a uniform horizontal electric field of 10^4 N/C.
(A) the horizontal range of the particle is 10m
(B) the time of flight of the particle is 2s
(C) the maximum height reached is 5m
(D) the horizontal range of the particle is 0.



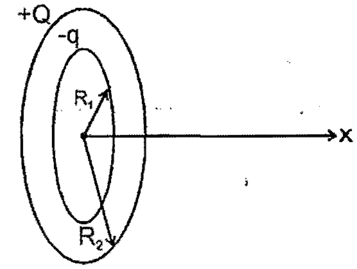
16.28 A wire having a uniform linear charge density λ, is bent in the form of a ring of radius R. Point A as shown in the figure, is in the plane of the ring but not at the centre. Two elements of the ring of lengths a_1 and a_2 subtend very small same angle at the point A. They are at distances r_1 and r_2 from the point A respectively.
(A) The ratio of charge of elements a_1 and a_2 is r_1/r_2.
(B) The element a_1 produced greater magnitude of electric field at A than element a_2.
(C) The elements a_1 and a_2 produce same potential at A.
(D) The direction of net electric field at A is towards element a_2.

16.29 Two infinite, parallel, non–conducting sheets carry equal positive charge density σ. One is placed in the yz plane at x = 0 and the other at x = a. Take potential V = 0 at x = 0.

(A) For $0 \le x \le a$, potential $V_x = 0$. (B) For $x \ge a$, potential $V_x = -\frac{\sigma}{\epsilon_0}(x-a)$

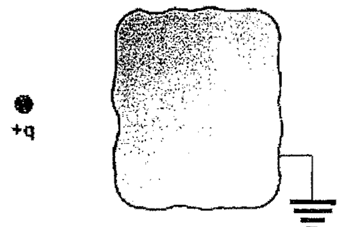
(C) For $x \ge a$, potential $V_x = \frac{\sigma}{\epsilon_0}(x-a)$ (D) For $x \le 0$ potential $V_x = \frac{\sigma}{\epsilon_0}x$

16.30 Two concentric rings of radii $R_1 = \sqrt{6}$ m and $R_2 = 4$m are placed in y-z plane with their centres at origin. They have uniform charge $-q$ and $+Q = 2\sqrt{2}$ q on the inner and outer rings respectively. Consider the electrostatic potential to be zero at infinity. Then



(A) The electric potential is zero at origin.
(B) The electric field intensity is zero at r = 2 m.
(C) A positive charged particle disturbed from origin along the x-axis will restore back to origin.
(D) Where potential is maximum on the x-axis, field intensity is zero.

16.31 In front of an earthed conductor a point charge + q is placed as shown in figure :



(A) On the surface of conductor the net charge is negative.
(B) On the surface of conductor at some points charges are negative and at some points charges may be positive distributed non uniformly
(C) Inside the conductor electric field due to point charge is non zero
(D) None of these

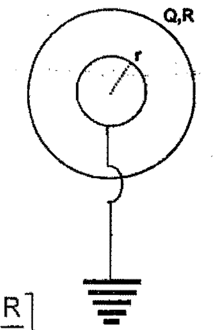
SECTION - III : ASSERTION AND REASON TYPE

16.32 **Statement 1 :** Electric field E at a point P is zero if potential at that point is zero.
Statement 2 : Potential difference between two points in space is zero if electric field at all points in space is zero.
(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True.

16.33 **Statement-1 :** For a non-uniformly charged thin circular ring with net charge zero, the electric field at any point on axis of the ring is zero.
Statement-2 : For a non-uniformly charged thin circular ring with net charge zero, the electric potential at each point on axis of the ring is zero.
(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True.

16.34 **Statement-1 :** A uniformly charged disc has a pin hole at its centre. The electric field at the centre of the disc is zero.
Statement-2 : Disc can be supposed to be made up of many rings. Also electric field at the centre of uniformly charged ring is zero..
(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True.

SECTION - IV : COMPREHENSION TYPE



Comprehension # 1

A metal ball (Neutral) with radius r is concentric with hollow metal sphere of radius 'R', having charge 'Q' as shown in figure, Now ball is connected with a very long wire to earth. Then :

16.35 Potential difference between sphere and metal ball, after grounding is :

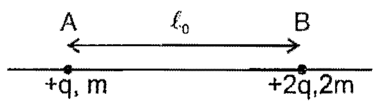
(A) $\frac{kQ}{R}$ (B) $-\frac{kQr}{R^2}$ (C) $\frac{kQ}{R}\left[1-\frac{r}{R}\right]$ (D) $\frac{kQ}{r}\left[1-\frac{R}{r}\right]$

16.36 After grounding :

(A) net electric field between sphere and ball is zero.
(B) electric field between ball and sphere is zero due to ball only.
(C) electric field between sphere and ball due to ball is non–zero.
(D) electric field between sphere and ball is non-zero, due to sphere

Comprehension # 2

Two positive point charges A and B have charge +q and +2q; mass m and 2m respectively as shown. Both the charges are released from rest when they are at a distance ℓ_0 apart. Neglect gravity and also assume the only force acting on either charge is the electrostatic force due to each other.



16.37 The speed of charge A at the instant separation between both charges is $2\ell_0$ is :

(A) $\sqrt{\frac{q^2}{12\pi\epsilon_0 m\ell}}$ (B) $\sqrt{\frac{q^2}{6\pi\epsilon_0 m\ell}}$ (C) $\sqrt{\frac{q^2}{4\pi\epsilon_0 m\ell}}$ (D) $\sqrt{\frac{q^2}{3\pi\epsilon_0 m\ell}}$

16.38 The work done by electrostatic force on charge A while the separation between both charges changes from ℓ_0 to $2\ell_0$ is.

(A) $\frac{q^2}{12\pi\epsilon_0 \ell}$ (B) $\frac{q^2}{6\pi\epsilon_0 \ell}$ (C) $\frac{q^2}{4\pi\epsilon_0 \ell}$ (D) $\frac{q^2}{24\pi\epsilon_0 \ell}$

16.39 Total work done by electrostatic force on charge A + charge B while the separation between both charges changes from ℓ_0 to $2\ell_0$ is.

(A) $\frac{q^2}{12\pi\epsilon_0 \ell}$ (B) $\frac{q^2}{6\pi\epsilon_0 \ell}$ (C) $\frac{q^2}{4\pi\epsilon_0 \ell}$ (D) $\frac{q^2}{24\pi\epsilon_0 \ell}$

Comprehension # 3

A charge q is divided into three equal parts and placed symmetrically on a circle of radius r. The same charge is divided into four equal parts and placed symmetrically on the same circle. The electric field intensities at the centre of the circle in two situations are zero.

16.40 The ratio of electric potentials at the centre in the two situations is

(A) $\frac{2}{\sqrt{3}}$ (B) $\frac{1}{1}$ (C) $\frac{4}{3}$ (D) $\frac{16}{9}$

16.41 The potential energy of the system in first situation where the charge is divided into three equal parts is :

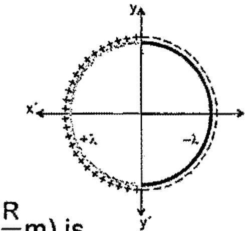
(A) $\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r}$ (B) $\frac{1}{36\pi\varepsilon_0}\frac{q^2}{r}$ (C) $\frac{1}{12\sqrt{3}\pi\varepsilon_0}\frac{q^2}{r}$ (D) $\frac{1}{12\pi\varepsilon_0}\frac{q^2}{r}$

16.42 If a charge (part charge) is removed from one location in both the situations, the ratio of magnitudes of the electric field intensities at the centre is

(A) $\frac{1}{2}$ (B) $\frac{1}{1}$ (C) $\frac{2}{3}$ (D) $\frac{4}{3}$

Comprehension # 4

A thin ring of radius R metres is placed in x-y plane such that its centre lies on origin. The half ring in region x< 0 carries uniform linear charge density + λ C/m and the remaining half ring in region x> 0 carries uniform linear charge density –λ C/m.



16.43 Then the electric potential (in volts) at point P whose coordinates are (0m, $+\frac{R}{2}$m) is

(A) $\frac{1}{4\pi\varepsilon_0}\frac{\lambda}{2}$ (B) 0 (C) $\frac{1}{4\pi\varepsilon_0}\frac{\lambda}{4}$ (D) cannot be determined

16.44 The direction of electric field at point P whose coordinates are (0m, $+\frac{R}{2}$m) is

(A) Along positive x-direction (B) Along negative x-direction
(C) Along negative y-direction (D) None of these

16.45 The dipole moment of the ring in C–m is

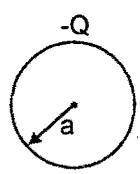
(A) $-(2\pi R^2\lambda)\,\hat{i}$ (B) $(2\pi R^2\lambda)\,\hat{i}$ (C) $-(4R^2\lambda)\,\hat{i}$ (D) $(4R^2\lambda)\,\hat{i}$

SECTION - V : MATRIX - MATCH TYPE

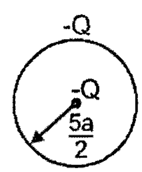
16.46 In each situation of column-I, some charge distributions are given with all details explained.The electrostatic potential energy and its nature is given in column -II. Then match situation in column-I with the corresponding results in column-II and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the OMR.

Column-I

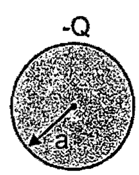
(A) A thin shell of radius a and having a charge – Q uniformly distributed over its surface as shown



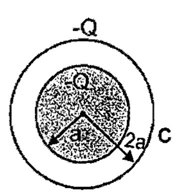
(B) A thin shell of radius $\frac{5a}{2}$ and having a charge – Q uniformly distributed over its surface and a point charge – Q placed at its centre as shown.



(C) A solid sphere of radius a and having a charge – Q uniformly distributed throughout its volume as shown.



(D) A solid sphere of radius a and having a charge – Q uniformly distributed throughout its volume. The solid sphere is surrounded by a concentric thin uniformly charged spherical shell of radius 2a and carrying charge –Q as shown



Column-II

(p) $\frac{1}{8\pi\epsilon_0}\frac{Q^2}{a}$ in magnitude

(q) $\frac{3}{20\pi\epsilon_0}\frac{Q^2}{a}$ in magnitude

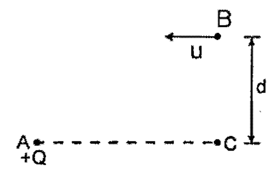
(r) $\frac{2}{5\pi\epsilon_0}\frac{Q^2}{a}$ in magnitude

(s) Positive in sign

(t) zero

SECTION - VI : INTEGER TYPE

16.47 A positive charge +Q is fixed at a point A. Another positively charged particle of mass m and charge +q is projected from a point B with velocity u as shown in the figure. The point B is at large distance from A and at distance 'd' from the line AC. The initial velocity is parallel to the line AC. The point C is at very large distance from A. Find the minimum distance (in meter) of +q from +Q during the motion. Take $Qq = 4\pi\epsilon_0 mu^2 d$ and $d = (\sqrt{2} - 1)$ meter.



16.48 Consider a cube of side a = 0.1 m placed such that its six faces are given by equations x = 0, x = +a, y = 0, y = +a, z = 0 and z = +a, placed in electric field given by $\vec{E} = x^2\hat{i} + y\hat{j}$ N/C. Find the electric flux crossing out of the cube in the unit of 10^{-4} N m²/C.

16.49 A solid sphere of radius 'R' has a cavity of radius $\frac{R}{2}$. The solid part has a uniform charge density 'ρ' and cavity has no charge. Find the electric potential at point 'A'. $\frac{\rho R}{x\varepsilon_0}$ then x is.

TOPIC 17 GRAVITATION

SECTION - I : STRAIGHT OBJECTIVE TYPE

17.1 A tunnel is dug along the diameter of the earth (Radius R & mass M). There is a particle of mass 'm' at the centre of the tunnel. The minimum velocity given to the particle so that it just reaches to the surface of the earth is :

(A) $\sqrt{\frac{GM}{R}}$ (B) $\sqrt{\frac{GM}{2R}}$ (C) $\sqrt{\frac{2GM}{R}}$

(D) it will reach with the help of negligible velocity.

17.2 A cavity of radius R/2 is made inside a solid sphere of radius R. The centre of the cavity is located at a distance R/2 from the centre of the sphere. The gravitational force on a particle of mass 'm' at a distance R/2 from the centre of the sphere on the line joining both the centres of sphere and cavity is (opposite to the centre of cavity).
[Here $g = GM/R^2$, where M is the mass of the sphere without cavity]

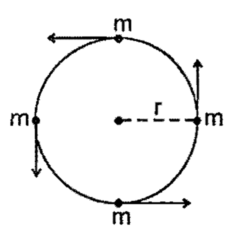
(A) $\frac{mg}{2}$ (B) $\frac{3\,mg}{8}$ (C) $\frac{mg}{16}$ (D) none of these

17.3 A satellite is launched in the equatorial plane in such a way that it can transmit signals upto 60° latitude on the earth. The angular velocity of the satellite is :

(A) $\sqrt{\frac{GM}{8R^3}}$ (B) $\sqrt{\frac{GM}{2R^3}}$ (C) $\sqrt{\frac{GM}{4R^3}}$ (D) $\sqrt{\frac{3\sqrt{3}GM}{8R^3}}$

17.4 A satellite is seen after each 8 hours over equator at a place on the earth when its sense of rotation is opposite to the earth. The time interval after which it can be seen at the same place when the sense of rotation of earth & satellite is same will be :
(A) 8 hours (B) 12 hours (C) 24 hours (D) 6 hours

17.5 Four similar particles of mass m are orbiting in a circle of radius r in the same angular direction because of their mutual gravitational attractive force. Velocity of a particle is given by



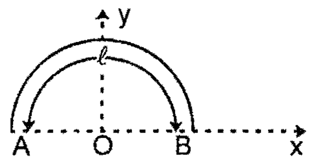
(A) $\left[\frac{Gm}{r}\left(\frac{1+2\sqrt{2}}{4}\right)\right]^{\frac{1}{2}}$ (B) $\sqrt[3]{\frac{Gm}{r}}$ (C) $\sqrt{\frac{Gm}{r}\left(1+2\sqrt{2}\right)}$ (D) $\left[\frac{1}{2}\frac{Gm}{r}\left(\frac{1+\sqrt{2}}{2}\right)\right]^{\frac{1}{2}}$

17.6 The gravitational potential of two homogeneous spherical shells A and B of same surface density at their respective centres are in the ratio 3 : 4. If the two shells coalesce into single one such that surface density remains same, then the ratio of potential at an internal point of the new shell to shell A is equal to :
(A) 3 : 2 (B) 4 : 3 (C) 5 : 3 (D) 5 : 6

17.7 A point P lies on the axis of a fixed ring of mass M and radius R, at a distance 2R from its centre O. A small particle starts from P and reaches O under gravitational attraction only. Its speed at O will be

(A) zero (B) $\sqrt{\frac{2GM}{R}}$ (C) $\sqrt{\frac{2GM}{R}(\sqrt{5}-1)}$ (D) $\sqrt{\frac{2GM}{R}(1-\frac{1}{\sqrt{5}})}$

17.8 Gravitational field at the centre of a semicircle formed by a thin wire AB of mass m and length ℓ is :



(A) $\frac{Gm}{\ell^2}$ along +x axis (B) $\frac{Gm}{\pi\ell^2}$ along +y axis

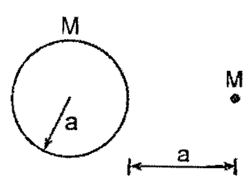
(C) $\frac{2\pi Gm}{\ell^2}$ along + x axis (D) $\frac{2\pi Gm}{\ell^2}$ along + y axis

17.9 The percentage change in the acceleration of the earth towards the sun from a total eclipse of the sun to the point where the moon is on a side of earth directly opposite to the sun is

(A) $\frac{M_s}{M_m}\frac{r_2}{r_1}\times 100$ (B) $\frac{M_s}{M_m}\left(\frac{r_2}{r_1}\right)^2\times 100$ (C) $2\left(\frac{r_1}{r_2}\right)^2\frac{M_s}{M_m}\times 100$ (D) $\left(\frac{r_1}{r_2}\right)^2\frac{M_m}{M_s}\times 100$

M_S = mass of the sun, M_M = mass of the moon, r_1 = earth sun distance, r_2 = earth moon distance.

17.10 A particle of mass M is at a distance 'a' from surface of a thin same spherical shell of uniform equal mass and having radius a.

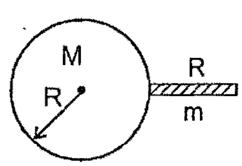


(A) Gravitational field & potential both are zero at centre of the shell
(B) Gravitational field is zero not only inside the shell but at a point outside the shell also
(C) Inside the shell, gravitational field alone is zero
(D) Neither gravitational field nor gravitational potential is zero inside the shell.

17.11 A small area is removed from a uniform spherical shell of mass M and radius R. Then the gravitational field intensity near the hollow portion is :

(A) $\frac{GM}{R^2}$ (B) $\frac{GM}{2R^2}$ (C) $\frac{3GM}{2R^2}$ (D) Zero

17.12 A uniform thin rod of mass m and length R is placed normally on surface of earth as shown. The mass of earth is M and its radius is R. Then the magnitude of gravitational force exerted by earth on the rod is



(A) $\frac{GMm}{2R^2}$ (B) $\frac{GMm}{4R^2}$ (C) $\frac{4GMm}{9R^2}$ (D) $\frac{GMm}{8R^2}$

17.13 Two particles of combined mass M, placed in space with certain separation, are released. Interaction between the particles is only of gravitational nature and there is no external force present. Acceleration of one particle with respect to the other when separation between them is R, has a magnitude :

(A) $\frac{GM}{2R^2}$ (B) $\frac{GM}{R^2}$

(C) $\frac{2GM}{R^2}$ (D) not possible to calculate due to lack of information

17.14 Maximum height reached by a rocket fired with a speed equal to 50% of the escape velocity from earth's surface is :

(A) R/2 (B) 16R/9 (C) R/3 (D) R/8

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

17.15 A double star is a system of two stars of masses m and 2m, rotating about their centre of mass only under their mutual gravitational attraction. If r is the separation between these two stars then their time period of rotation about their centre of mass will be proportional to

(A) $r^{3/2}$ (B) r (C) $m^{1/2}$ (D) $m^{-1/2}$

17.16 A satellite revolves around a planet in circular orbit of radius *R* (much larger than the radius of the planet) with a time period of revolution *T*. If the satellite is stopped and then released in its orbit (Assume that the satellite experiences gravitational force due to the planet only).

(A) It will fall on the planet

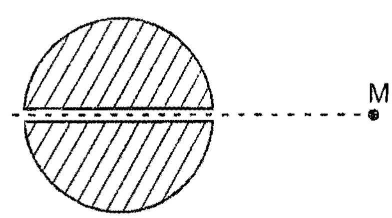
(B) The time of fall of the satellite is nearly $\frac{T}{\sqrt{8}}$

(C) The time of fall of the satellite on the planet is nearly $\frac{\sqrt{2}T}{8}$

(D) It cannot fall on the planet so time of fall of the satellite is meaningless

SECTION - III : ASSERTION AND REASON TYPE

17.17 **Statement-1 :** In free space a uniform spherical planet of mass M has a smooth narrow tunnel along its diameter. This planet and another superdense small particle of mass M start approaching towards each other from rest under action of their gravitational forces. When the particle passes through the centre of the planet, sum of kinetic energies of both the bodies is maximum.



Statement-2 : When the resultant of all forces acting on a particle or a particle like object (initially at rest) is constant in direction, the kinetic energy of the particle keeps on increasing.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

SECTION - IV : COMPREHENSION TYPE

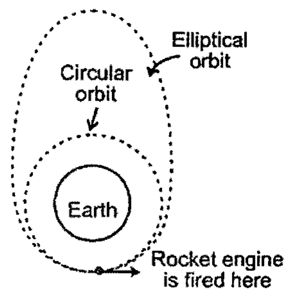
Comprehension

Changing from a circular to An elliptical orbit

Let us identify the system as the spacecraft and the Earth but not the portion of the fuel in the spacecraft that we use to change the orbit. In a given orbit, the mechanical energy of the spacecraft – Earth system is given by $E = -\frac{GMm}{2r}$.

This energy includes the kinetic energy of the spacecraft and the potential energy associated with the gravitational force between the spacecraft and the Earth. If the rocket engines are fired, the thrust force moves the spacecraft through a displacement. As a result, the mechanical energy of the spacecraft – Earth system increases.

The spacecraft has a new higher energy but is constrained to be in an orbit that includes the original starting point. It can not be in a higher energy circular orbit having a larger radius because this orbit would not contain the starting point. The only possibility is that the orbit is elliptical as shown in the figure.



$$E = -\frac{GMm}{2a}$$

17.18 If the spacecraft-earth system had initial energy $(-E_0)$, then the total mechanical energy of the system after firing the rocket will be :

(A) $-1.1\ E_0$ (B) $-0.9\ E_0$ (C) $-E_0$ (D) None of these

17.19 Semimajor axis of the new elliptical orbit is

(A) $\frac{6.7\times10^4}{9}$ Km (B) $\frac{6.4\times10^4}{9}$ Km (C) $\frac{7.1\times10^4}{9}$ km (D) $\frac{6.1\times10^4}{9}$ Km

17.20 Maximum height of the spacecraft above the surface of the Earth will be :

(A) $\frac{1.06\times10^4}{9}$ km. (B) $\frac{0.61\times10^4}{9}$ km (C) 300 km (D) $\frac{1.61\times10^4}{9}$ km

SECTION - V : MATRIX - MATCH TYPE

17.21 A satellite is revolving around the earth in a circular orbit of radius 'a' with velocity v_0. A particle is projected from the satellite in forward direction with relative velocity $v = \left[\sqrt{\frac{5}{4}} - 1\right] v_0$. During subsequent motion of particle, match the following :

	Column-I		Column-II
(A)	Total energy of particle	(p)	$-\frac{3GM_em}{a}$
(B)	Minimum distance of particle from the earth	(q)	$-\frac{5}{8}\frac{GM_em}{a}$
(C)	Maximum distance of particle from the earth	(r)	5a/3
		(s)	2a/3
		(t)	a

17.22 Let V and E denote the gravitational potential and gravitational field respectively at a point due to certain uniform mass distribution described in four different situations of column-I. Assume the gravitational potential at infinity to be zero. The value of E and V are given in column-II. Match the statement in column-I with results in column-II.

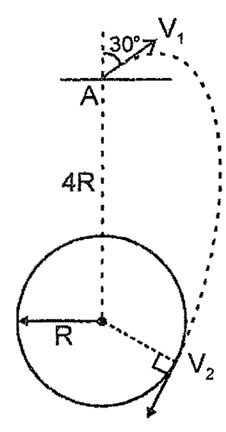
	Column-I		Column-II
(A)	At centre of thin spherical shell	(p)	$E = 0$
(B)	At centre of solid sphere	(q)	$E \neq 0$
(C)	A solid sphere has a non-concentric spherical cavity. At the centre of the spherical cavity	(r)	$V \neq 0$
(D)	At centre of line joining two point masses of equal magnitude	(s)	$V = 0$

SECTION - VI : INTEGER TYPE

17.23 Ravi can throw a ball at a speed on earth which can cross a river of width 10 m. Ravi reaches on an imaginary planet whose mean density is twice of the earth. if maximum possible radius of planet so that if Ravi throws the ball at same speed it may escape from planet.is x km. then x is. (Given radius of earth = 6.4×10^6 m.)

17.24 The gravitational field in a region is given by $\vec{E} = (3\hat{i} - 4\hat{j})$ N/kg. Find out the work done (in joule) in displacing a particle by 1 m along the line $4y = 3x + 9$.

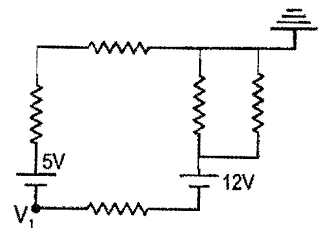
17.25 A particle is projected from point A, that is at a distance 4R from the centre of the earth, with speed V_1 in a direction making 30° with the line joining the centre of the earth and point A, as shown in the figure. Find the speed V_1 if particle passes grazing the surface of the earth. Consider gravitational interaction only between these two. (use $\frac{GM}{R} = 6.4 \times 10^7$ m²/s²) Express your answer in the form of $\frac{X}{\sqrt{2}}$ km/sec^{-1} and fill value of X.



TOPIC 18 CURRENT ELECTRICITY

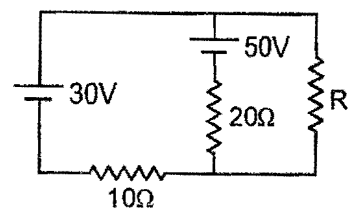
SECTION - I : STRAIGHT OBJECTIVE TYPE

18.1 In the circuit shown, each resistances is 2 Ω. The potential V_1 as indicated in the circuit, is equal to



(A) 11 V
(B) – 11V
(C) 9 V
(D) – 9 V

18.2 In the circuit shown, the value of R in ohm that will result in no current through the 30 V battery, is :



(A) 10 Ω
(B) 25 Ω
(C) 30 Ω
(D) 40 Ω

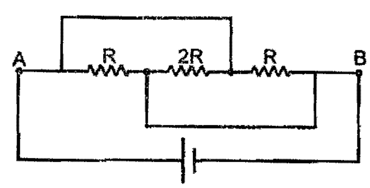
18.3 The maximum current in a galvanometer can be 10 mA. It's resistance is 10Ω. To convert it into an ammeter of 1 Amp. a resistor should be connected in.

(A) series, 0.1Ω (B) parallel, 0.1Ω (C) series, 100 Ω (D) parallel, 100Ω.

18.4 When a galvanometer is shunted with a 4Ω resistance, the deflection is reduced to one - fifth. If the galvanometer is further shunted with a 2Ω wire, the further reduction (find the ratio of decrease in current to the previous current) in the deflection will be (the main current remains the same).

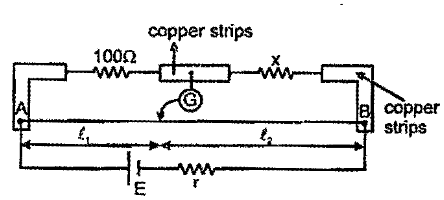
(A) (8/13) of the deflection when shunted with 4Ω only
(B) (5/13) of the deflection when shunted with 4Ω only
(C) (3/4) of the deflection when shunted with 4Ω only
(D) (3/13) of the deflection when shunted with 4Ω only

18.5 In the figure shown the current flowing through 2 R is :



(A) from left to right
(B) from right to left
(C) no current
(D) None of these

18.6 In a practical wheat stone bridge circuit as shown, when one more resistance of 100 Ω is connected is parallel with unknown resistance 'x', then ratio ℓ_1/ℓ_2 become '2'. ℓ_1 is balance length. AB is a uniform wire. Then value of 'x' must be:

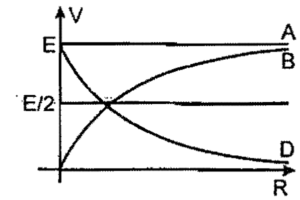


(A) 50 Ω
(B) 100 Ω
(C) 200 Ω
(D) 400 Ω

18.7 A battery of internal resistance 2 Ω is connected to a variable resistor whose value can vary from 4 Ω to 10 Ω. The resistance is initially set at 4 Ω. If the resistance is now increased then :

(A) power consumed by it will decrease
(B) power consumed by it will increase
(C) power consumed by it may increase or may decrease
(D) power consumed will first increase then decrease.

18.8 A cell of emf E having an internal resistance 'r' is connected to an external resistance R. The potential difference 'V' across the resistance R varies with R as shown by the curve :

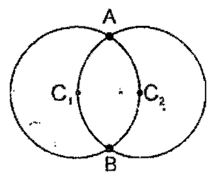


(A) A
(B) B
(C) C
(D) D

18.9 'n' identical light bulbs, each designed to draw P power from a certain voltage supply are joined in series and that combination is connected across that supply. The power consumed by one bulb will be-
(A) n P (B) P (C) P/n (D) P/n^2

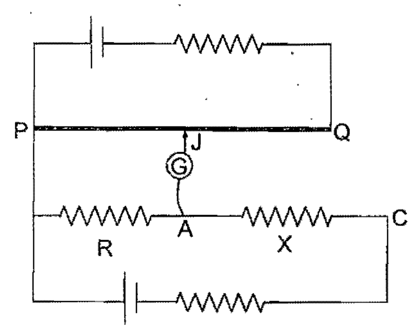
18.10 To get maximum current through a resistance of 2.5 Ω, one can use 'm' rows of cells, each row having 'n' cells. The internal resistance of each cell is 0.5 Ω. What are the values of n & m, if the total number of cells is 45.
(A) 3, 15 (B) 5, 9 (C) 9, 5 (D) 15, 3

18.11 Two circular rings of identical radii and resistance of 36Ω each are placed in such a way that they cross each other's centre C_1 and C_2 as shown in figure. Conducting joints are made at intersection points A and B of the rings. An ideal cell of emf 20 volts is connected across AB. The power delivered by cell is



(A) 80 watt (B) 100 watt (C) 120 watt (D) 200 watt

18.12 Circuit for the measurement of resistance by potentiometer is shown. The galvanometer is first connected at point A and zero deflection is observed at length PJ = 10 cm. In second case it is connect at point C and zero deflection is observed at a length 30 cm from P. Then the unknown resistance X is



(A) 2R
(B) $\frac{R}{2}$
(C) $\frac{R}{3}$
(D) 3R

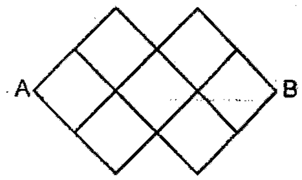
18.13 Two long coaxial and conducting cylinders of radius a and b are separated by a material of conductivity σ and a constant potential difference V is maintained between them, by a battery. Then the current, per unit length of the cylinder flowing from one cylinder the other is :

(A) $\frac{4\pi\sigma}{\ell n(b/a)}V$ (B) $\frac{4\pi\sigma}{(b+a)}V$ (C) $\frac{2\pi\sigma}{\ell n(b/a)}V$ (D) $\frac{2\pi\sigma}{(b+a)}V$

8.14 50 V battery is supplying current of 10 amp when connected to a resistor. If the efficiency of battery at this current is 25%. Then internal resistance of battery is :
(A) 2.5 Ω (B) 3.75 Ω (C) 1.25 Ω (D) 5 Ω

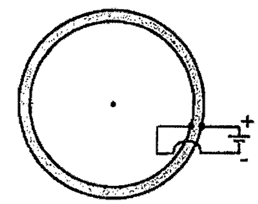
18.15 A battery is supplying power to a tape-recorder by cable of resistance of 0.02 Ω. If the battery is generating 50 W power at 5V, then power received by tape-recorder is : (neglect internal resistance of battery)
(A) 50 W (B) 45 W (C) 30 W (D) 48 W

.8.16 In the shown wire frame, each side of a square (the smallest square) has a resistance R. The equivalent resistance of the circuit between the points A and B is :



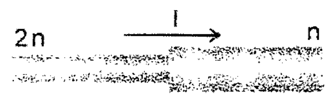
(A) R (B) 2R
(C) 4R (D) 8R

18.17 A spherical shell, made of material of electrical conductivity $\frac{10^9}{\pi}(\Omega\text{-m})^{-1}$, has thickness t = 2 mm and radius r = 10 cm. In an arrangement, its inside surface is kept at a lower potential than its outside surface.
The resistance offered by the shell is equal to -



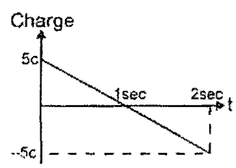
(A) $5\pi \times 10^{-12}\ \Omega$ (B) $2.5 \times 10^{-11}\ \Omega$
(C) $5 \times 10^{-12}\ \Omega$ (D) $5 \times 10^{-11}\ \Omega$

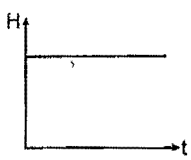
18.18 Two cylindrical rods of uniform cross-section area A and 2A, having free electrons per unit volume 2n and n respectively are joined in series. A current I flows through them in steady state. Then the ratio of drift velocity of free electron in left rod to drift velocity of electron in the right rod is $\left(\frac{v_L}{v_R}\right)$ is :

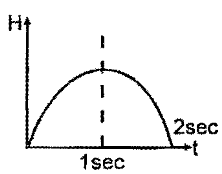


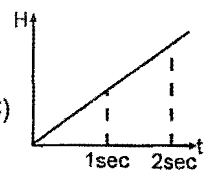
(A) $\frac{1}{2}$ (B) 1 (C) 2 (D) 4

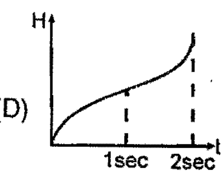
18.19 A charge passing through a resistor is varying with time as shown in the figure. The amount of heat generated in time 't' is best represented (as a function of time) by:



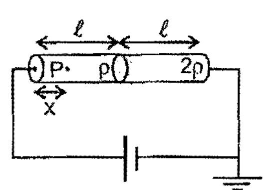
(A) 

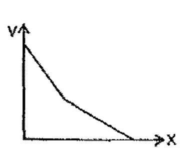
(B) 

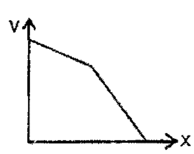
(C) 

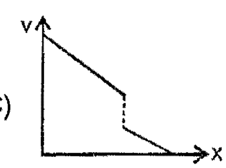
(D) 

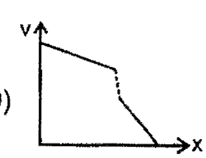
18.20 Two cylindrical rods of same cross-section area and same length are connected in series to an ideal cell as shown. The resistivity of left rod is ρ and that of right rod is 2ρ. Then the variation of potential at any point P distant x from left end of combined rod system is given by.



(A) 

(B) 

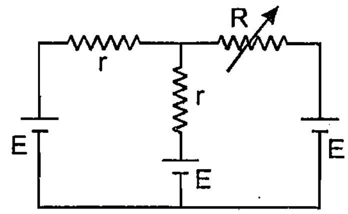
(C) 

(D) 

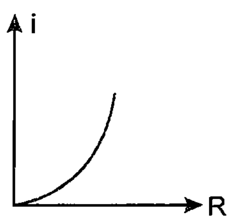
18.21 A copper sphere of 10cm diameter is lowered into a water filled hemispherical copper vessel of 20 cm diameter so that the sphere and the vessel becomes concentric. Electrical conductivity of water is $\sigma = 10^{-3}$ $(\Omega - m)^{-1}$. The electrical resistance between the sphere and the vessel is :

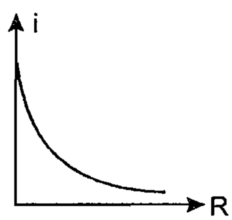
(A) 1591.6 Ω (B) 1450 Ω (C) 1682.4 Ω (D) 1489.6 Ω

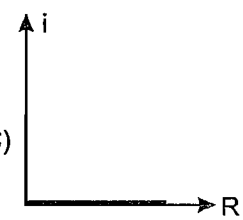
18.22 In the shown circuit the resistance R can be varied :

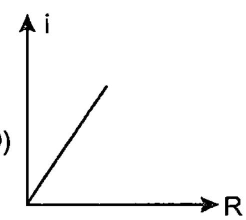


The variation of current through R against R is correctly plotted as :

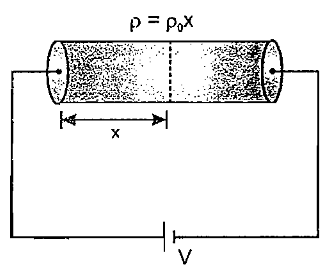
(A) 

(B) 

(C) 

(D) 

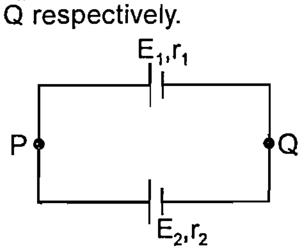
18.23 A cylindrical solid of length L and radius a is having varying resistivity given by $\rho = \rho_0 x$ where ρ_0 is a positive constant and x is measured from left end of solid. The cell shown in the figure is having emf V and negligible internal resistance. The electric field as a function of x is best described by :



(A) $\frac{2V}{L^2} \times x$ (B) $\frac{2V}{\rho_0 L^2} \times x$ (C) $\frac{V}{L^2} \times x$ (D) None of these

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

18.24 Two cells of unequal emfs E_1 and E_2 and internal resistances r_1 and r_2 are joined as shown in figure. V_P and V_Q are the potential at P and Q respectively.



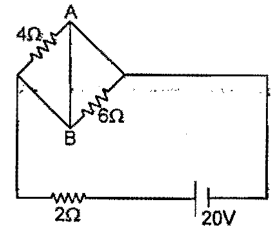
(A) The potential difference across both the cells will be equal

(B) One of the cell, will supply energy to the other cell.

(C) The potential difference across one of the cells will be greater than its emf.

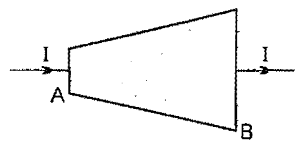
(D) $V_P - V_Q = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}$

18.25 In the circuit shown in the figure



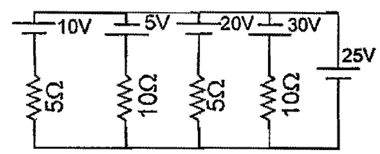
(A) power supplied by the battery is 200 watt
(B) current flowing in the circuit is 5 A
(C) potential difference across 4 Ω resistance is equal to the potential difference across 6Ω resistance
(D) current in wire AB is zero

18.26 In the figure a conductor of non–uniform cross-section is shown. A steady current I flows in it.



(A) The electric field at A is more than at B.
(B) The electric field at B is more than at A.
(C) The thermal power generated at A is more than at B in an element of small same width.
(D) The thermal power generated at B is more than at A in an element of small same width.

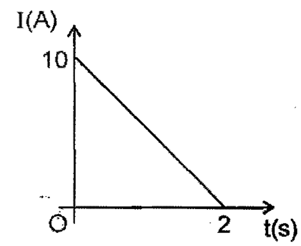
18.27 In the figure shown: (All batteries are ideal)



(A) current through 25 V cell is 20 A
(B) current through 25 V cell is 12.5 A
(C) power supplied by 20 V cell is 20 W
(D) power supplied by 20 V cell is – 20 W

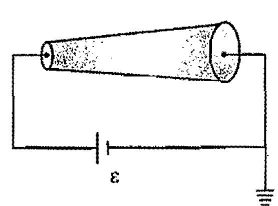
18.28 Consider a resistor of uniform cross sectional area connected to a battery of zero internal resistance. If the length of the resistor is doubled by stretching it then
(A) current will become four times.
(B) the electric field in the wire will become half.
(C) the thermal power produced by the resistor will become one fourth.
(D) the product of the current density and conductance will become half.

18.29 A variable current flows through a 1Ω resistor for 2 seconds. Time dependence of the current is shown in the graph.

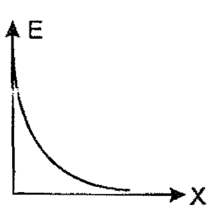


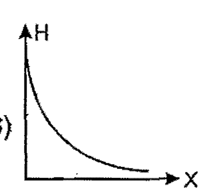
(A) Total charge flown through the resistor is 10 C.
(B) Average current through the resistor is 5A.
(C) Total heat produced in the resistor is 50 J.
(D) Maximum power during the flow of current is 100 W.

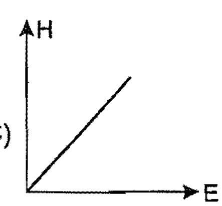
18.30 A conductor of truncated conical (frustum) is connected to a battery of emf ε as shown in the figure.

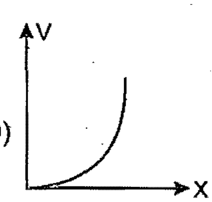


If at a section distant x from left end, electric field intensity, potential and rate of generation of heat per unit length are E, V and H respectively, then which of the following graph(s) is/are correct ?

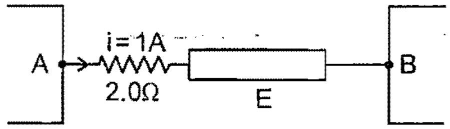
(A)


(B)


(C)


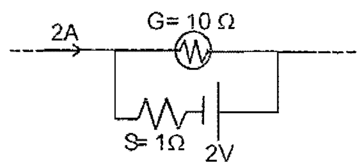
(D)


18.31 AB is part of a circuit as shown, that absorbs energy at a rate of 50 W. E is an emf device that has no internal resistance.



(A) Potential difference across AB is 48 V.
(B) Emf of the device is 48 V.
(C) Point B is connected to the positive terminal of E.
(D) Rate of conversion from electrical to chemical energy is 48 W in device E.

18.32 The galvanometer shown in the figure has resistance 10Ω. It is shunted by a series combination of a resistance S = 1Ω and an ideal cell of emf 2V. A current 2A passes as shown.



(A) The reading of the galvanometer is 1A
(B) The reading of the galvanometer is zero
(C) The potential difference across the resistance S is 1.5 V
(D) The potential difference across the resistance S is 2 V

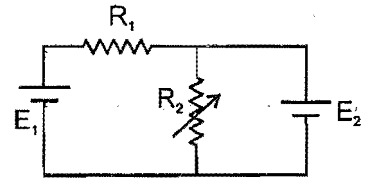
SECTION - III : ASSERTION AND REASON TYPE

18.33 **Statement-1 :** When an external resistor of resistance R (connected across a cell of internal resistance r) is varied, power consumed by resistance R is maximum when R = r.
Statement-2 : Power consumed by a resistor of constant resistance R is maximum when current through it is maximum.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

18.34 **Statement - 1 :** The current density $\vec{J}$ at any point in ohmic resistor is in direction of electric field $\vec{E}$ at that point.
Statement - 2 : A point charge when released from rest in a region having only electrostatic field always moves along electric lines of force.
(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True.

18.35 **Statement-1 :** A wire of uniform cross section and uniform resitivity is connected across an ideal cell. Now the length of the wire is doubled keeping volume of wire constant. The drift velocity of electrons after stretching the wire becomes one fourth of what it was before stretching the wire.
Statement-2 : If a wire (of uniform resitivity and uniform cross-section) of length ℓ_0 is stretched to length $n\ell_0$, then its resistance becomes n^2 times of what it was before stretching the wire(the volume of wire is kept constant in stretching process). Further at constant potential difference, current is inversely proportional to resistance. Finally drift velocity of free electron is directly proportional to current and inversely proportional to cross section area of current carrying wire..
(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True.

18.36 **Statement-1 :** In the circuit shown both cells are ideal and of fixed emf, the resistor of resistance R_1 has fixed resistance and the resistance of resistor R_2 can be varied(but R_2 is always non-zero). Then the electric power delivered to resistor of resistance R_1 is independent of value of resistance R_2.



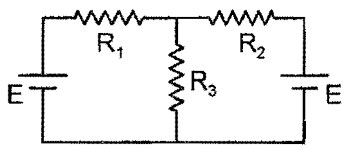
Statement-2 : If potential difference across a fixed resistance is unchanged, the power delivered to the resistor remains constant.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

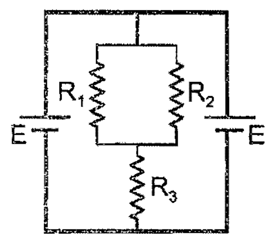
18.37 **Statement–1 :** The power delivered to a light bulb is more just after it is switched ON and the glow of the filament is increasing, as compared to when the bulb is glowing steadily, i.e., after some time of switching ON.
Statement–2 : As temperature increases, resistance of conductor increases.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True.

18.38 **Statement-1 :** For calculation of current in resistors of resistance R_1, R_2 and R_3 in the circuit shown in figure 1, the circuit can be redrawn as shown in figure 2 (this means that circuit shown in figure 2 is equivalent to circuit shown in figure 1). All the cells shown are ideal and identical.



(fig.1)



(fig.2)

Statement-2 : Whenever potential difference across two resistors is same, both resistors can be assumed as a combination of two resistors in parallel.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

SECTION - IV : COMPREHENSION TYPE

Comprehension # 1

Resistance value of an unknown resistor is calculated using the formula $R = \frac{V}{I}$ where V and I be the readings of the voltmeter and the ammeter respectively. Consider the circuits below. The internal resistances of the voltmeter and the ammeter (R_v and R_G respectively) are finite and non zero.

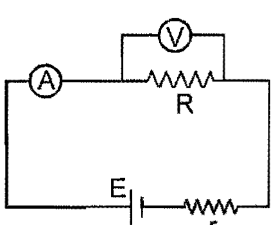


Figure (A)

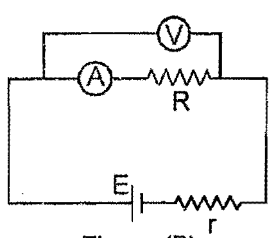


Figure (B)

Let R_A and R_B be the calculated values in the two cases A and B respectively.

18.39 The relation between R_A and the actual value R is

(A) $R > R_A$ (B) $R < R_A$

(C) $R = R_A$ (D) dependent upon E and r.

18.40 The relation between R_B and the actual value R is :

(A) $R < R_B$ (B) $R > R_B$

(C) $R = R_B$ (D) dependent upon E and r.

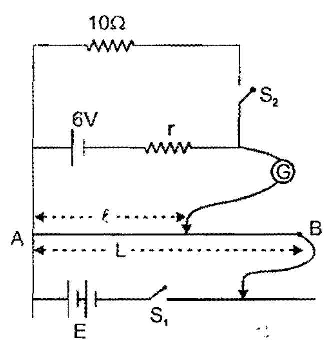
18.41 If the resistance of voltmeter is $R_v = 1\ k\ \Omega$ and that of ammeter is $R_G = 1\ \Omega$, the magnitude of the percentage error in the measurement of R (the value of R is nearly $10\ \Omega$) is :

(A) zero in both cases (B) non zero but equal in both cases

(C) more in circuit A (D) more in circuit B

Comprehension # 2

In the arrangement shown in the figure when the switch S_2 is open, the galvanometer shows no deflection for $\ell = L/2$. When the switch S_2 is closed, the galvanometer shows no deflection for $\ell = \frac{5}{12}L$. The internal resistance (r) of 6 V cell, and the emf E of the other battery are respectively. Wire AB is potentiometer wire and resistance of other conducting wires is negligible. (Internal resistance of cell E is negligible)



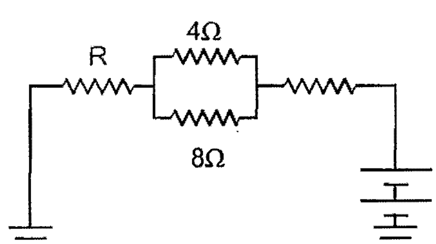
18.42 Calculate emf of cell E :

(A) 6 V (B) 5 V (C) 12 V (D) 10 V

18.43 Calculate the internal resistance 'r' :

(A) 1 Ω (B) 2 Ω (C) 3 Ω (D) zero

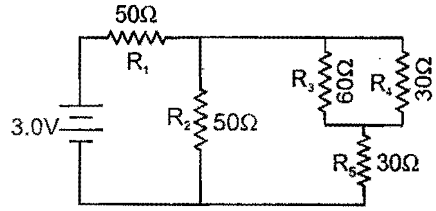
18.44 If the current in 8 Ω resistance is 2A then the current through resistance 'R' (in ampere) would be :



(A) 6 (B) 7 (C) 8 (D) 9

Comprehension # 3

In the circuit shown, the resistances are given in ohms and the battery is assumed ideal with emf equal to 3.0 volts.



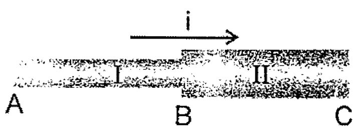
18.45 The resistor that dissipates maximum power.
(A) R_1 (B) R_2 (C) R_4 (D) R_5

18.46 The potential difference across resistor R_3 is
(A) 0.4 V (B) 0.6 V (C) 1.2 V (D) 1.5 V

18.47 The current passing through 3V battery is
(A) 10 mA (B) 30 mA (C) 40 mA (D) 60 mA

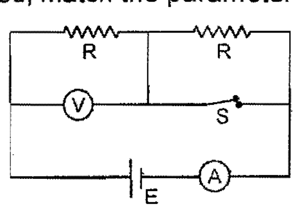
SECTION - V : MATRIX - MATCH TYPE

18.48 Column I gives physical quantities of a situation in which a current i passes through two rods I and II of equal length that are joined in series. The ratio of free electron density (n), resistivity (ρ) and cross-section area (A) of both are in ratio $n_1 : n_2 = 2 : 1$, $\rho_1 : \rho_2 = 2 : 1$ and $A_1 : A_2 = 1 : 2$ respectively. Column II gives corresponding results. Match the ratios in Column I with the values in Column II .



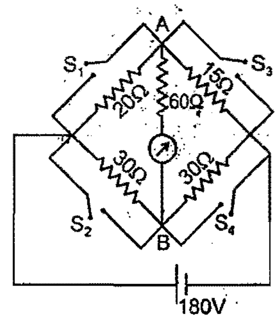
	Column I		Column II
(A)	$\frac{\text{Drift velocity of free electron in rod I}}{\text{Drift velocity of free electron in rod II}}$	(p)	0.5
(B)	$\frac{\text{Electric field in rod I}}{\text{Electric field in rod II}}$	(q)	1
(C)	$\frac{\text{Potential difference across rod I}}{\text{Potential difference across rod II}}$	(r)	2
(D)	$\frac{\text{Average time taken by free electron to move from A to B}}{\text{Average time taken by free electron to move from B to C}}$	(s)	4

18.49 In the circuit shown, battery, ammeter and voltmeter are ideal and the switch S is initially closed as shown. When switch S is opened, match the parameter of column I with the effects in column II.



	Column I		Column II
(A)	Equivalent resistance across the battery	(p)	Remains same
(B)	Power dissipated by left resistance R	(q)	Increases
(C)	Voltmeter reading	(r)	decreases
(D)	Ammeter reading	(s)	Becomes zero.

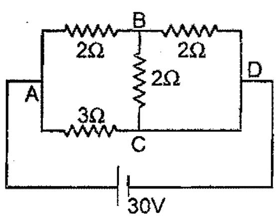
18.50 Consider the circuit shown. The resistance connected between the junction A and B is 60 Ω including the resistance of the galvanometer. The switches have no resistance when shorted and infinite resistance when opened. All the switches are initially open and they are closed as given in column I. Match the condition in column I with the direction of current through galvanometer and the value of the current through the battery in column II.



Column I	Column II
(A) Only switch S_1 is closed	(p) Current from A to B
(B) Only switch S_2 is closed	(q) Current from B to A
(C) Only switch S_3 is closed	(r) Current through the battery is 12.0 A
(D) Only switch S_4 is closed	(s) Current through the battery is 15.6 A
	(t) Current through the Galvanometer is 1.2 A

SECTION - VI : INTEGER TYPE

18.51 Find current in the branch CD of the circuit (in ampere).



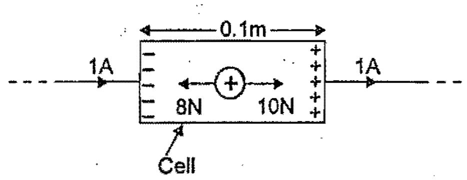
18.52 The circuit shown in the figure contains three resistors $R_1 = 100\Omega$, $R_2 = 50\Omega$ & $R_3 = 20\ \Omega$ and cells of emf's $E_1 = 2V$ & E_2. The ammeter indicates a current of 50mA. Determine the current(mA) in the resistor R_2:

18.53 All batteries are having emf 10 volt and internal resistance negligible. All resistors are in ohms. Calculate the current in the right most 2Ω resistor.

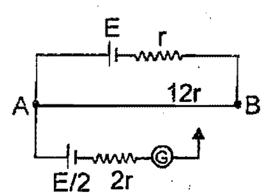
18.54 In the circuit diagram shown if the current through the 1 Ω resistor is $\frac{x}{2}$ A then x is.

18.55 The efficiency of a cell when connected to a resistance R is 60%. What will be its efficiency (in%) if the external resistance is increased to six times.

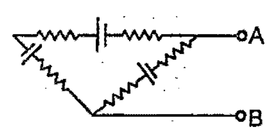
18.56 Figure shows a cell in which unit positive charge experience a constant non electric force of 10N and a constant electric force of 8N in directions shown in the figure. Find the emf of the cell, difference across the cell (in volt)



18.57 Consider the potentiometer circuit arranged as in figure. The potentiometer wire AB is 300 cm long. If the jockey touches the wire at a distance of 275 cm from A, then $\left(\frac{3E}{Nr}\right)$ current flow through galvanomerer find value of N.



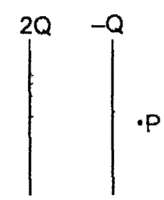
18.58 In the circuit shown all five resistors have the same value 200 ohms and each cell has an emf 3 volts. if the open circuit voltage is $\frac{x}{10}$ volt then x is.



TOPIC 19 CAPACITANCE

SECTION - I : STRAIGHT OBJECTIVE TYPE

19.1 In the figure shown the plates of a parallel plate capacitor have unequal charges. Its capacitance is 'C'. P is a point outside the capacitor and close to the plate of charge – Q. The distance between the plates is 'd', select incorrect alternative :



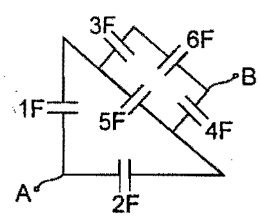
(A) A point charge at point 'P' will experience electric force due to capacitor

(B) The potential difference between the plates will be $\frac{3Q}{2C}$

(C) The energy stored in the electric field in the region between the plates is $\frac{9Q^2}{8C}$

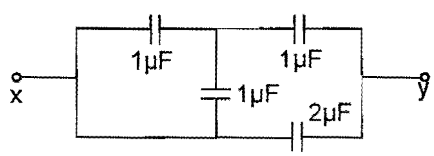
(D) The force on one plate due to the other plate is $\frac{Q^2}{2\pi \epsilon_0 d^2}$

19.2 In the figure shown the equivalent capacitance between 'A' and 'B' is :



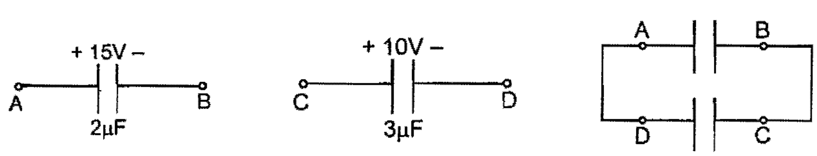
(A) 3.75 F
(B) 2 F
(C) 21 F
(D) 16 F

19.3 The equivalent capacitance between x & y is :



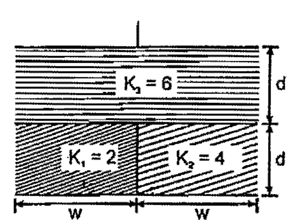
(A) $\frac{5}{6}$ μF (B) $\frac{7}{6}$ μF (C) $\frac{8}{3}$ μF (D) 1 μF

19.4 In the figure initial status of capacitance and their connection is shown. Which of the following is incorrect about this circuit :



(A) Final charge on each capacitor will be zero
(B) Final total electrical energy of the capacitors will be zero
(C) Total charge flown from A to D is 30μC
(D) Total charge flown from A to D is – 30μC

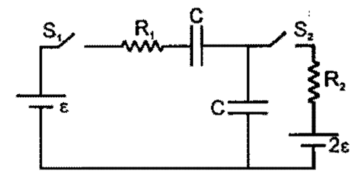
19.5 A parallel plate capacitor of capacitance C (without dielectrics) is filled by dielectric slabs as shown in the figure. Then the new capacitance of the capacitor is:



(A) 3.9 C (B) 4 C
(C) 2.4 C (D) 3 C

19.6 A capacitor (without dielectric) is discharging through a resistor. At some instant a dielectric is inserted between the plates, then
(A) Just after the insertion of the dielectric, current will increase.
(B) Just after the insertion of the dielectric, charge on capacitor will increase.
(C) Just after the insertion of the dielectric, energy stored in the capacitor will increase.
(D) after the insertion of the dielectric, time constant will increase

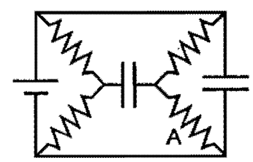
19.7 In the circuit shown, switch S_2 is closed first and is kept closed for a long time. Now S_1 is closed. Just after that instant the current through S_1 is:



(A) $\frac{\varepsilon}{R_1}$ towards right (B) $\frac{\varepsilon}{R_1}$ towards left

(C) zero (D) $\frac{2\varepsilon}{R_1}$

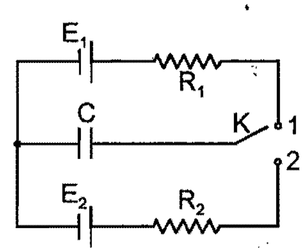
19.8 Each resistor in the following circuit has a resistance of 2MΩ and the capacitors have capacitances of 1μF. The battery voltage is 3V. The voltage across the resistor 'A' in the following circuit in steady state is :



(A) 0 V (B) 0.5 V
(C) 0.75 V (D) 1.5 V

19.9 An uncharged capacitor is connected in series with a resistor and a battery. The charging of the capacitor starts at t = 0. The rate at which energy in capacitor is stored :
(A) first increases then decreases (B) first decreases then increases
(C) remains constant (D) continuously decreases

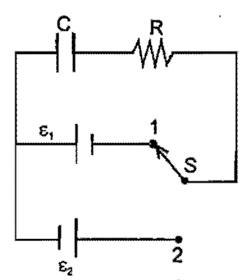
19.10 The key K (figure) is connected in turn to each of the contacts over short identical time intervals so that the change in the charge on the capacitor over each connection is small. The final charge q_r on the capacitor is :



(A) $\frac{(E_1R_2 + E_2R_1)C}{R_1 + R_2}$ (B) $\frac{(E_1 E_2) C}{E_1 + E_2}$

(C) $\frac{(E_1R_1 + E_2R_2) C}{R_1 + R_2}$ (D) none of these

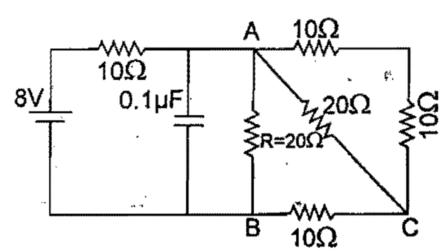
19.11 Initially switch S is connected to position 1 for a long time. The net amount of heat generated in the circuit after it is shifted to position 2 is



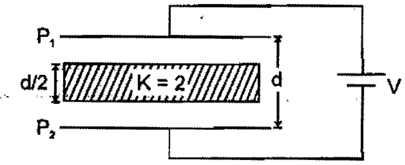
(A) $\frac{C}{2}(\varepsilon_1 + \varepsilon_2)\varepsilon_2$ (B) $C(\varepsilon_1 + \varepsilon_2)\varepsilon_2$

(C) $\frac{C}{2}(\varepsilon_1 + \varepsilon_2)^2$ (D) $C(\varepsilon_1 + \varepsilon_2)^2$

19.12 A capacitor of capacitance 0.1 μ F is connected to a battery of emf 8V as shown in the fig. Under steady state condition.
(A) Charge on the capacitor is 0.4 μ C.
(B) Charge on the capacitor is 0.2 μ C.
(C) Current in the resistor(R) between points A & B is 0.1 A.
(D) Current in the resistor(R) between point A & B is 0.4 A.



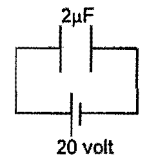
19.13 In the figure shown a parallel plate capacitor has a dielectric of width d/2 and dielectric constant K = 2. The other dimensions of the dielectric are same as that of the plates. The plates P_1 and P_2 of the capacitor have area 'A' each. The energy of the capacitor is :



(A) $\frac{\epsilon_0 AV^2}{3d}$ (B) $\frac{2\epsilon_0 AV^2}{d}$

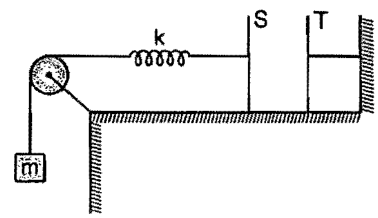
(C) $\frac{3}{2}\frac{\epsilon_0 AV^2}{d}$ (D) $\frac{2\epsilon_0 AV^2}{3d}$

19.14 In the figure a capacitor of capacitance 2μF is connected to a cell of emf 20 volt. The plates of the capacitor are drawn apart slowly to double the distance between them. The work done by the external agent on the plates is :



(A) – 200 μJ (B) 200 μJ (C) 400 μJ (D) – 400 μJ

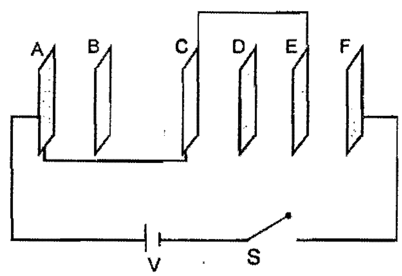
19.15 The plates S and T of an uncharged parallel plate capacitor are connected across a battery. The battery is then disconnected and the charged plates are now connected in a system as shown in the figure. The system shown is in equilibrium. All the strings and spring are insulating and massless. The magnitude of charge on one of the capacitor plates is:[Area of plates = A]



(A) $\sqrt{2mg A\epsilon_0}$ (B) $\sqrt{\frac{4mg A\epsilon_0}{k}}$

(C) $\sqrt{mg A\epsilon_0}$ (D) $\sqrt{\frac{2mg A\epsilon_0}{k}}$

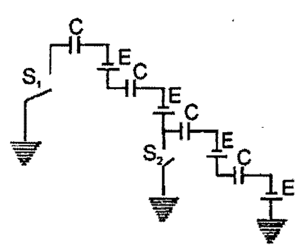
19.16 In the figure shown A, B, C, D, E, F are conducting plates each of area A and any two consecutive plates separated by a distance d. The net energy stored in the system after the switch S is closed is:



(A) $\frac{3\varepsilon_0 A}{2d}V^2$ (B) $\frac{5\varepsilon_0 A}{12d}V^2$

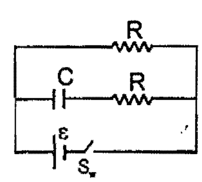
(C) $\frac{\varepsilon_0 A}{2d}V^2$ (D) $\frac{\varepsilon_0 A}{d}V^2$

19.17 In the given circuit, all the capacitors are initially uncharged. After closing the switch S_1 for a long time suddenly S_2 is also closed and kept closed for a long time. Total heat produced after closing S_2 will be :



(A) 4 $C\varepsilon^2$
(B) ½ $C\varepsilon^2$
(C) 2 $C\varepsilon^2$
(D) 0

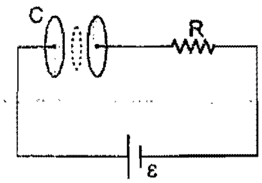
19.18 If at t = 0 the switch S_W is closed, then the charge on capacitor in the given circuit (initially uncharged) when the current through battery becomes 50% of its maximum value is (assume battery is ideal):



(A) $\frac{C\varepsilon}{3}$ (B) $\frac{C\varepsilon}{2}$

(C) $\frac{C\varepsilon}{4}$ (D) $C\varepsilon$

19.19 In the circuit shown the capacitor of capacitance C is initially uncharged. Now the capacitor is connected in the circuit as shown. The charge passed through an imaginary circular loop parallel to the plates (also circular) and having the area equal to half of the area of the plates, in one time constant is:



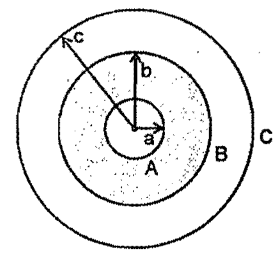
(A) $C\varepsilon\left(1-\frac{1}{e}\right)$ (B) $\frac{C\varepsilon}{2}\left(1-\frac{1}{e}\right)$ (C) $\frac{C\varepsilon}{4}$ (D) zero

19.20 A parallel plate capacitor (without dielectric) is charged by a battery and kept connected to the battery. A dielectric slab of dielectric constant ' k ' is inserted between the plates fully occupying the space between the plates. The energy density of electric field between the plates will be :

(A) increase k^2 times (B) decrease k^2 times

(C) increase k times (D) decrease k times

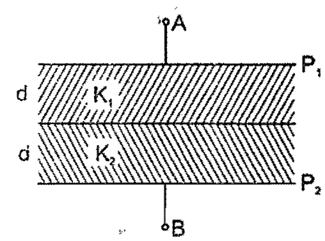
19.21 In the figure shown A and C are concentric conducting spherical shells of radius a and c respectively. A is surrounded by a concentric dielectric medium of inner radius a, outer radius b and dielectric constant k. If sphere A is given a charges Q, the potential at the outer surface of the dielectric is.



(A) $\frac{Q}{4\pi\epsilon_0 kb}$ (B) $\frac{Q}{4\pi\epsilon_0}\left(\frac{1}{a}+\frac{1}{k(b-a)}\right)$

(C) $\frac{Q}{4\pi\epsilon_0 b}$ (D) None of these

19.22 In the figure shown P_1 and P_2 are two conducting plates having charges of equal magnitude and opposite sign. Two dielectrics of dielectric constant K_1 and K_2 fill the space between the plates as shown in the figure. The ratio of electrical energy in 1st dielectric to that in the 2nd dielectric is



(A) 1 : 1 (B) $K_1 : K_2$

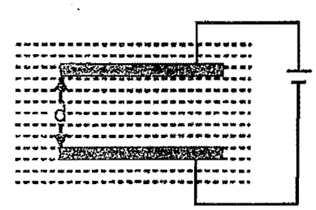
(C) $K_2 : K_1$ (D) $K_2^2 : K_1^2$

19.23 In the figure shown two long straight wires with the same cross-sections are arranged in air, parallel to one another. The distance between the axis of the wire is η times larger then the radius of wire's cross-section. Capacitance of the wires per unit length would be (Take $\eta >> 1$)

Top view of the arrangement

(A) $\frac{2\pi\epsilon_0}{\ln\eta}$ (B) $\frac{\pi\epsilon_0}{2\ln\eta}$ (C) $\frac{\pi\epsilon_0}{\ln\eta}$ (D) Information insufficient

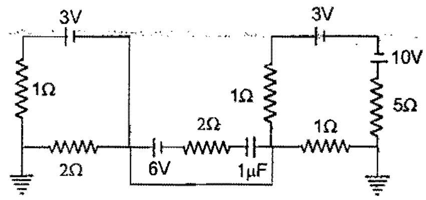
19.24 A parallel plate capacitor is immersed in a liquid dielectric having dielectric constant ε as shown in the figure. Find the force acting on a unit surface of the plate from the dielectric.



(A) $\frac{\varepsilon\varepsilon_0 V^2}{2d^2}$ (B) $\frac{\varepsilon_0(\varepsilon-1)V^2}{2d^2\times\varepsilon}$

(C) $\frac{\varepsilon V^2}{2d^2}$ (D) $\frac{\varepsilon(\varepsilon-1)\varepsilon_0 V^2}{2d^2}$

19.25 For the circuit shown in the figure, determine the charge of the capacitor in steady state.



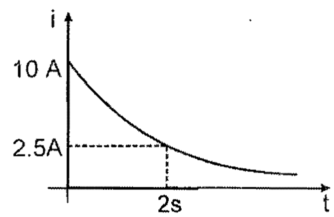
(A) $4\mu C$ (B) $6\mu C$ (C) $1\mu C$ (D) Zero

19.26 Two identical capacitors are charged to different potentials then they are connected to each other in such a way that the sum of charges of plates having positive polarity remains constant. Mark the correct statement.

(A) Sum of charges of plates having negative polarity remains constant.
(B) Mean of individual final potentials is different from mean of individual initial potentials.
(C) Total energy stored in two capacitors in final state may be equal to that in initial state.
(D) Heat dissipation in the circuit could be zero.

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

19.27 The figure shows, a graph of the current in a discharging circuit of a capacitor through a resistor of resistance 10 Ω.



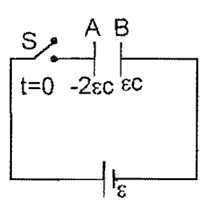
(A) The initial potential difference across the capacitor is 100 volt.

(B) The capacitance of the capacitor is $\frac{1}{10\ell n2}$ F.

(C) The total heat produced in the circuit will be $\frac{500}{\ell n2}$ joules.

(D) The thermal power in the resistor will decrease with a time constant $\frac{1}{2\ell n2}$ second.

19.28 A parallel plate capacitor of capacitance 'C' has charges on its plates initially as shown in the figure. Now at t = 0, the switch 'S' is closed. Select the correct alternative(s) for this circuit diagram.



(A) In steady state the charges on the outer surfaces of plates 'A' and 'B' will be same in magnitude and sign.
(B) In steady state the charges on the outer surfaces of plates 'A' and 'B' will be same in magnitude and opposite in sign.
(C) In steady state the charges on the inner surfaces of the plates 'A' and 'B' will be same in magnitude and opposite in sign.
(D) The work done by the cell by the time steady state is reached is $\frac{5\varepsilon^2 C}{2}$.

19.29 The plates of a parallel plate capacitor with no dielectric are connected to a voltage source. Now a dielectric of dielectric constant K is inserted to fill the whole space between the plates with voltage source remaining connected to the capacitor.
(A) the energy stored in the capacitor will become K-times
(B) the electric field inside the capacitor will decrease K-times
(C) the force of attraction between the plates will become K^2-times
(D) the charge on the capacitor will become K-times.

19.30 A parallel plate capacitor of capacitance 10 μF is connected to a cell of emf 10 Volt and fully charged. Now a dielectric slab (k = 3) of thickness equal to the gap between the plates, is very slowly inserted to completely fill in the gap, keeping the cell connected. During the filling process:
(A) the increase in charge on the capacitor is 200 μC.
(B) the heat produced is zero.
(C) energy supplied by the cell = increase in stored potential energy + work done on the person who is filling the dielectric slab.
(D) energy supplied by the cell = increase in stored potential energy + work done on the person who is filling the dielectric slab + heat produced.

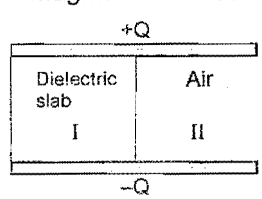
19.31 Capacitor C_1 of the capacitance 1 microfarad and capacitor C_2 of capacitance 2 microfarad are separately charged fully by a common battery. The two capacitors are then separately allowed to discharge through equal resistors at time t = 0.
(A) the current in each of the two discharging circuits is zero at t = 0.
(B) the current in the two discharging circuits at t = 0 are equal but non zero.
(C) the current in the two discharging circuits at t = 0 are unequal
(D) capacitor C_1 loses 50% of its initial charge sooner than C_2 loses 50% of its initial charge

SECTION - III : ASSERTION AND REASON TYPE

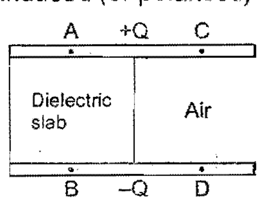
19.32 **Statement-1 :** If the potential difference across a plane parallel plate capacitor is doubled then the potential energy of the capacitor becomes four times under all conditions.

Statement-2 : The potential energy U stored in the capacitor is $U = \frac{1}{2}CV^2$, where C and V have usual meaning.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

19.33 **Statement-1 :** A charged plane parallel plate capacitor has half interplanar region (I) filled with dielectric slab. The other half region II has air. Then the magnitude of net electric field in region I is less than that in region II.



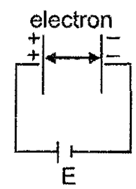
Statement-2 : In a dielectric medium induced (or polarised) charges tend to reduce the electric field.



(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True.

19.34 **Statement-1 :** A dielectric is inserted between the plates of an isolated fully-charged capacitor. The dielectric completely fills the space between the plates. The magnitude of electrostatic force on either metal plate decreases, as it was before the insertion of dielectric medium.
Statement-2 : Due to insertion of dielectric slab in an isolated parallel plate capacitor (the dielectric completely fills the space between the plates), the electrostatic potential energy of the capacitor decreases.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True.

19.35 **Statement - 1 :** During the charging of a capacitor using a battery, the electrons transferred from positive plate of capacitor to negative plate via dielectric medium in between the plates as shown.
Statement - 2 : The direction of electric field in between the capacitor plates is from positive plate to negative plate.

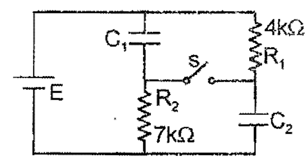


(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True.

SECTION - IV : COMPREHENSION TYPE

COMPREHENSION # 1

The switch s has been closed for long time and the electric circuit shown carries a steady current. Let $C_1 = 3.0\ \mu F$, $C_2 = 6.0\ \mu F$, $R_1 = 4.0\ k\Omega$, and $R_2 = 7.0\ k\Omega$. The power dissipated in R_2 is 2.8 W.



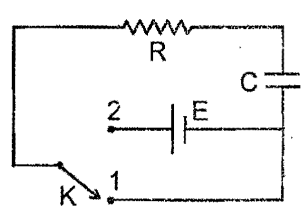
19.36 The power dissipated to the resistor R_1 is
(A) 2.8 W (B) 1.6 W (C) 4.9 W (D) 0

19.37 The charge on capacitors C_1 and C_2 are respectively.
(A) 940 μC, 940 μC (B) 440 μC, 440 μC (C) 240 μC, 840 μC (D) 840 μC, 240 μC

19.38 Long time after switch is opened, the charge on C_1 is :
(A) Zero (B) 420 μC (C) 240 μC (D) 660 μC

COMPREHENSION # 2

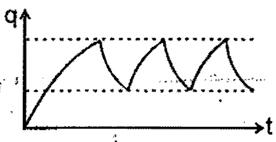
In the shown circuit involving a resistor of resistance R Ω, capacitor of capacitance C farad and an ideal cell of emf E volts, the capacitor is initially uncharged and the key is in position 1. At t = 0 second the key is pushed to position 2 for t_0 = RC seconds and then key is pushed back to position 1 for t_0 = RC seconds. This process is repeated again and again. Assume the time taken to push key from position 1 to 2 and vice versa to be negligible.

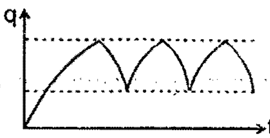


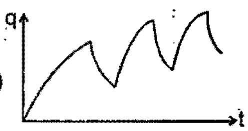
19.39 The charge on capacitor at t = 2RC second is
(A) CE (B) $CE\left(1-\frac{1}{e}\right)$ (C) $CE\left(\frac{1}{e}-\frac{1}{e^2}\right)$ (D) $CE\left(1-\frac{1}{e}+\frac{1}{e^2}\right)$

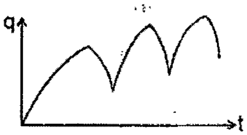
19.40 The current through the resistance at t = 1.5 RC seconds is
(A) $\frac{E}{e^2R}(1-\frac{1}{e})$ (B) $\frac{E}{eR}(1-\frac{1}{e})$ (C) $\frac{E}{R}(1-\frac{1}{e})$ (D) $\frac{E}{\sqrt{e}R}(1-\frac{1}{e})$

19.41 Then the variation of charge on capacitor with time is best represented by

(A) 

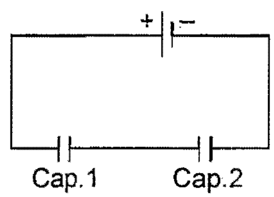
(B) 

(C) 

(D) 

SECTION - V : MATRIX - MATCH TYPE

19.42 Two identical capacitors are connected in series, and the combination is connected with a battery, as shown. Some changes in the capacitor 1 are now made independently after the steady state is achieved, listed in column-I. Some effects which may occur in new steady state due to these changes on the capacitor 2 are listed in column-II. Match the changes on capacitor 1 in column-I with corresponding effect on capacitor 2 in column-II.

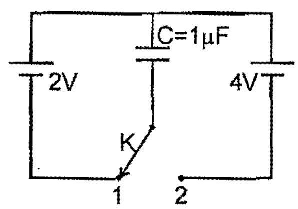


Column I	Column II
(A) A dielectric slab is inserted.	(p) Charge on the capacitor increases.
(B) Separation between plates increased.	(q) Charge on the capacitor decreases.
(C) A metal plate is inserted connecting both plates	(r) Energy stored in the capacitor increases.
(D) The left plate is grounded.	(s) Energy stored in capacitor is decreased
	(t) No change is occurred.

19.43 In each situation of column-I some changes are made to a charged capacitor under conditions of constant potential difference or constant charge. Condition of constant potential difference means that a cell is connected across the capacitor and condition of constant charge means that the capacitor is isolated. Match the conditions in column-I with corresponding results in column-II.

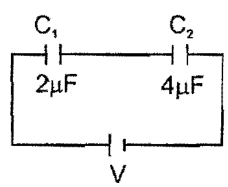
Column I	Column II
(A) For a capacitor maintained at constant potential difference, the separation between plates is increased.	(p) Then electric field inside the capacitor decreases in comparison to what it was before the change.
(B) For a capacitor maintained at constant charge, the separation between the plates is increased	(q) Then electric field inside the capacitor remains same.
(C) For a capacitor maintained at constant potential difference, area of the both the plates is doubled.	(r) Then potential energy stored in the capacitor decreases in comparison to what it was before the change.
(D) For a capacitor maintained at constant charge, area of both plates is doubled	(s) The potential energy stored in the capacitor increases in comparison to what it was before the change.
	(t) Capacitance of capacitor decreases

9.44 The circuit involves two ideal cells connected to a 1 μF capacitor via a key K. Initially the key K is in position 1 and the capacitor is charged fully by 2V cell. The key is pushed to position 2. Column I gives physical quantities involving the circuit after the key is pushed from position 1. Column II gives corresponding results. Match the statements in Column I with the corresponding values in Column II.



	Column I	**Column II**	
(A)	The net charge crossing the 4 volt cell in μC is	(p)	2
(B)	The magnitude of work done by 4 Volt cell in μJ is	(q)	6
(C)	The gain in potential energy of capacitor in μJ is	(r)	8
(D)	The net heat produced in circuit in μJ is	(s)	16

19.45 In the given figure, the separation between the plates of C_1 is slowly increased to double of its initial value then.

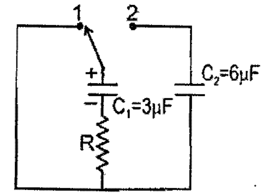


	Column-I	**Column-II**	
(A)	the potential difference across C_1	(p)	increases
(B)	the potential difference across C_2	(q)	decreases
(C)	the energy stored in C_1	(r)	increases by a factor of 6/5
(D)	the energy stored in C_2	(s)	decreases by a factor of 18/25
		(t)	decreases by a factor of 9/25

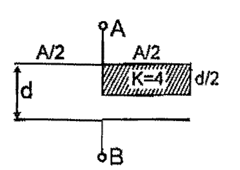
SECTION - VI : INTEGER TYPE

19.46 In the circuit shown the capacitors are initially uncharged. In a certain time the capacitor of capacitance 2μF gets a charge 20μC. In that time interval, find the heat produced in each resistor 6Ω in μ J

19.47 In the circuit shown a charged capacitor C_1 = 3 μF is discharged through R = 1 kΩ by putting the switch is position 1. When the current reaches I_0 = 2 A, the switch is thrown to position 2 to discharge through uncharged capacitor C_2 = 6 μF and steady state is allowed to reach. Find the heat dissipated (in Joules) in the resistor R after switch is thrown to position 2.



19.48 In the figure shown find the equivalent capacitance between terminals 'A' and 'B'.

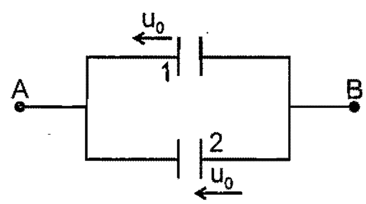


The letters have their usual meaning capacitance is $\frac{x\varepsilon_0 A}{10d}$ then x is.

19.49 A parallel plate capacitor is to be designed which is to be connected across 1 kV potential difference. The dielectric material which is to be filled between the plates has dielectric constant $K = 6\pi$ and dielectric strength 10^7 V/m. For safely the electric field is never to exceed 10% of the dielectric strength. With such specifications, if we want a capacitor of capacitance 50 pF, what minimum area (in mm^2) of plates is required for safe working ?

(use $\varepsilon_0 = \frac{1}{36\pi} \times 10^{-9}$ in MKS)

19.50 Two identical capacitor having plate separation d_0 are connected parallel to each other across points A and B as shown in the figure. A charge Q is imparted to the system by connecting a battery across A and B and battery is removed. Now first plate of first capacitor and second plate of second capacitor starts moving with constant velocity u_0 towards left. Find the magnitude of current flowing in the loop during this process.



19.51 Given that $C_A = 1\ \mu F$, $C_B = 2\ \mu F$ and $C_C = 2\ \mu F$. Initially each capacitor was charged to potential differences of $V_A = 10V$, $V_B = 40$ V and $V_C = 60$ V separately and are kept as shown in figure (a). Now they are connected as shown in figure (b). The + and - sign shown in figure (b) represent initial polarities. If total amount of heat produced in µJ is given by (3100 – N) by the time steady state is reached find value of N.

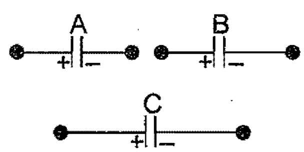


Fig. (a)

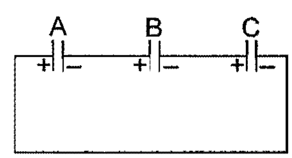
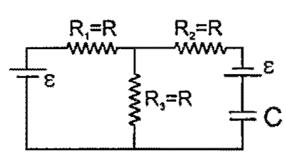


Fig. (b)

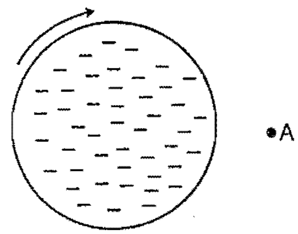
19.52 In the figure shown the capacitor is initially uncharged. The current in R_3 (= R) at time 't' is $\frac{E}{2R}\left(1-e^{\frac{-At}{BRC}}\right)$ find the value of (A+B).



TOPIC 20 MAGNETIC EFFECT OF CURRENT & MAGNETIC FORCE OF ON CHARGE

SECTION - I : STRAIGHT OBJECTIVE TYPE

20.1 The negatively and uniformly charged nonconducting disc as shown in the figure is rotated clockwise with great angular speed. The direction of the magnetic field at point A in the plane of the disc is :



(A) into the page (B) out of the page (C) up the page (D) down the page

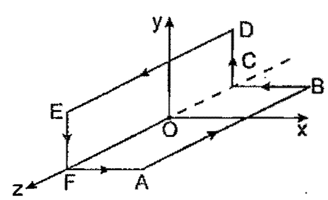
20.2 A particle is moving with velocity $\vec{v} = \hat{i} + 3\hat{j}$ and it produces an electric field at a point given by $\vec{E} = 2\hat{k}$. It will produce magnetic field at that point equal to (all quantities are in S.I. units)

(A) $\frac{6\hat{i} - 2\hat{j}}{c^2}$ (B) $\frac{6\hat{i} + 2\hat{j}}{c^2}$

(C) zero (D) can not be determined from the given data

20.3 Two observers moving with different velocities see that a point charge produces same magnetic field at the same point A. Their relative velocity must be parallel to $\vec{r}$, where $\vec{r}$ is the position vector of point A with respect to point charge. This statement is :
(A) true
(B) false
(C) nothing can be said
(D) true only if the charge is moving perpendicular to the $\vec{r}$

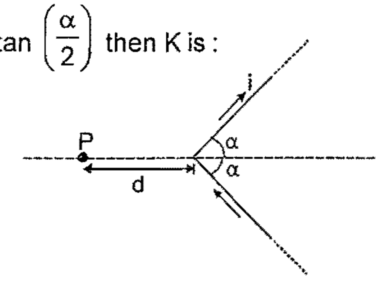
20.4 In the figure shown A B C D E F A was a square loop of side ℓ, but is folded in two equal parts so that half of it lies in x z plane and the other half lies in the y z plane. The origin 'O' is centre of the frame also. The loop carries current 'i'. The magnetic field at the centre is :



(A) $\frac{\mu_0 i}{2\sqrt{2}\,\pi \ell}(\hat{i} - \hat{j})$ (B) $\frac{\mu_0 i}{4\pi \ell}(-\hat{i} + \hat{j})$

(C) $\frac{\sqrt{2}\,\mu_0 i}{\pi \ell}(\hat{i} + \hat{j})$ (D) $\frac{\mu_0 i}{\sqrt{2}\,\pi \ell}(\hat{i} + \hat{j})$

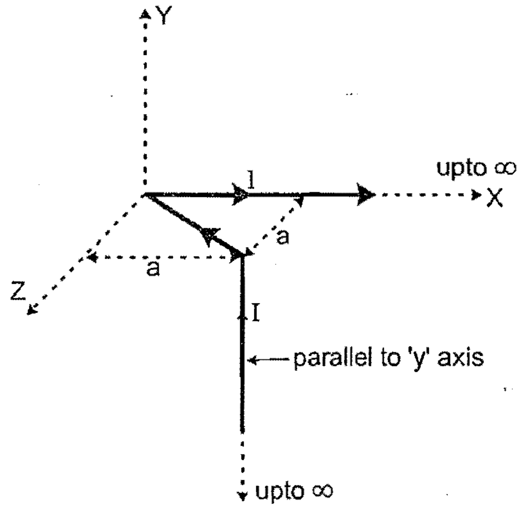
20.5 If the magnetic field at 'P' in the given figure can be written as K tan $\left(\frac{\alpha}{2}\right)$ then K is :



(A) $\frac{\mu_0 I}{4\pi d}$ (B) $\frac{\mu_0 I}{2\pi d}$

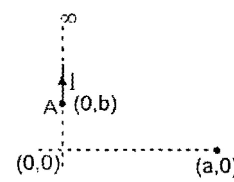
(C) $\frac{\mu_0 I}{\pi d}$ (D) $\frac{2\mu_0 I}{\pi d}$

20.6 The magnetic field at the origin due to the current flowing in the wire as shown in figure below is



(A) $-\frac{\mu_0 I}{8\pi a}(\hat{i}+\hat{k})$ (B) $\frac{\mu_0 I}{2\pi a}(\hat{i}+\hat{k})$ (C) $\frac{\mu_0 I}{8\pi a}(-\hat{i}+\hat{k})$ (D) $\frac{\mu_0 I}{4\pi a\sqrt{2}}(\hat{i}-\hat{k})$

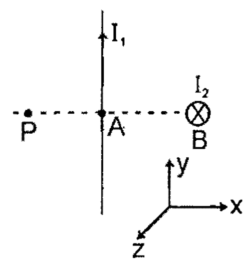
20.7 An infinitely long wire carrying current I is along Y axis such that its one end is at point A (0, b) while the wire extends upto + ∞. The magnitude of magnetic field strength at point (a, 0) is



(A) $\frac{\mu_0 I}{4\pi a}\left(1+\frac{b}{\sqrt{a^2+b^2}}\right)$ (B) $\frac{\mu_0 I}{4\pi a}\left(1-\frac{b}{\sqrt{a^2+b^2}}\right)$

(C) $\frac{\mu_0 I}{4\pi a}\left(\frac{b}{\sqrt{a^2+b^2}}\right)$ (D) None of these

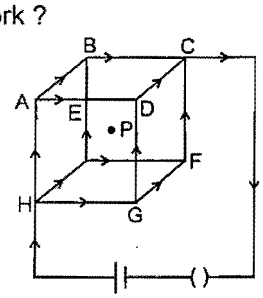
20.8 Two infinitely long linear conductors are arranged perpendicular to each other and are in mutually perpendicular planes as shown in figure. If I_1 = 2A along the y-axis and I_2 = 3A along negative z-axis and AP = AB = 1 cm. The value of magnetic field strength $\vec{B}$ at P is -



(A) $(3 \times 10^{-5}\,T)\,\hat{j} + (-4 \times 10^{-5}\,T)\,\hat{k}$ (B) $(3 \times 10^{-5}\,T)\,\hat{j} + (4 \times 10^{-5}\,T)\,\hat{k}$

(C) $(4 \times 10^{-5}\,T)\,\hat{j} + (3 \times 10^{-5}\,T)\,\hat{k}$ (D) $(-3 \times 10^{-5}\,T)\,\hat{j} + (4 \times 10^{-5}\,T)\,\hat{k}$

20.9 A steady current is set up in a cubic network composed of wires of equal resistance and length d as shown in figure. What is the magnetic field at the centre of cube P due to the cubic network ?

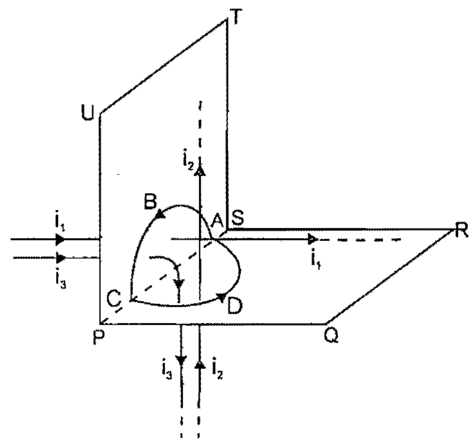


(A) $\frac{\mu_0}{4\pi}\frac{2I}{d}$ (B) $\frac{\mu_0}{4\pi}\frac{3I}{\sqrt{2}d}$

(C) 0 (D) $\frac{\mu_0}{4\pi}\frac{\theta\pi I}{d}$

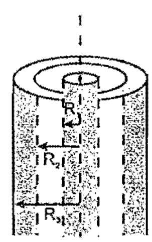
20.10 Figure shows an amperian path ABCDA. Part ABC is in vertical plane PSTU while part CDA is in horizontal plane PQRS. Direction of circulation along the path is shown by an arrow near point B and at D.

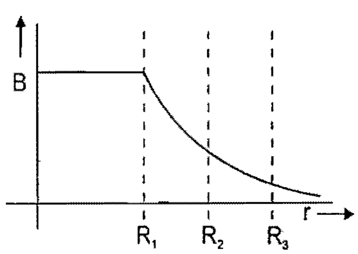
$\oint \vec{B}.d\vec{\ell}$ for this path according to Ampere's law will be :

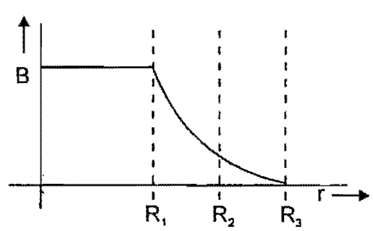


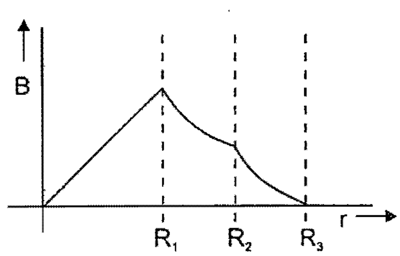
(A) $(i_1 - i_2 + i_3)\mu_0$ (B) $(-i_1 + i_2)\mu_0$ (C) $i_3\mu_0$ (D) $(i_1 + i_2)\mu_0$

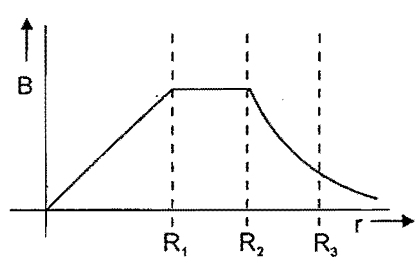
20.11 A coaxial cable is made up of two conductors. The inner conductor is solid and is of radius R_1 & the outer conductor is hollow of inner radius R_2 and outer radius R_3. The space between the conductors is filled with air. The inner and outer conductors are carrying currents of equal magnitudes and in opposite directions. Then the variation of magnetic field with distance from the axis is best plotted as:



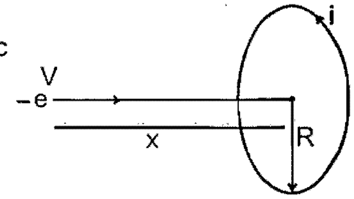
(A)


(B)


(C)


(D)


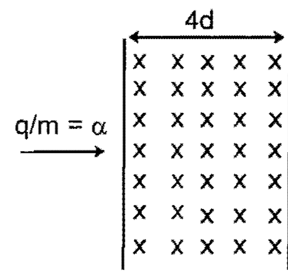
20.12 An electron moving with velocity V along the axis approaches a circular current carrying loop as shown in the figure. The magnitude of magnetic force on electron at this instant is :



(A) $\frac{\mu_0}{2}\frac{eviR^2x}{(x^2+R^2)^{3/2}}$ (B) $\mu_0 \frac{eviR^2x}{(x^2+R^2)^{3/2}}$

(C) $\frac{\mu_0}{4\pi}\frac{eviR^2x}{(x^2+R^2)^{3/2}}$ (D) 0

20.13 If a charged particle of charge to mass ratio $\frac{q}{m} = \alpha$ is entering in a uniform magnetic field of strength B which is extended up to 4d as shown in figure at a speed v = (2αd)(B), then which of the following is correct :



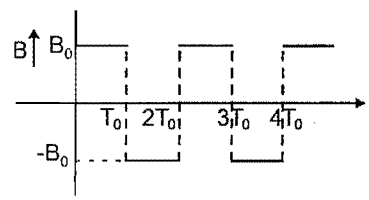
(A) angle subtended by charged particle at the centre of circular path is 2π.

(B) the charge will move on a circular path and will come out from magnetic field at a distance 4d from the point of insertion.

(C) the time for which particle will be in the magnetic field is $\frac{2\pi}{\alpha B}$.

(D) the charged particle will substend an angle of 90° at the centre of circular path

20.14 In a region magnetic field along x axis changes with time according to the given graph. If time period, pitch and radius of helix path are T_0, P_0 and R respectively then which of the following is incorrect if the particle is projected at an angle θ_0 with the positive x-axis in x-y plane :



(A) At $t = \frac{T_0}{2}$, co-ordinates of charge are $\left(\frac{P_0}{2}, 0, -2R_0\right)$.

(B) At $t = \frac{3T_0}{2}$, co-ordinates of charge are $\left(\frac{3P_0}{2}, 0, 2R_0\right)$.

(C) Two extremes from x-axis are at a distance $2R_0$ from each other.

(D) Two extremes from x-axis are at a distance $4R_0$ from each other.

20.15 A Positive point charge is moving in clockwise direction in a circle with constant speed. Consider the magnetic field produced by the charge at a point P (not centre of the circle) on the axis of the circle.

(A) it is constant in magnitude only

(B) it is constant in direction only

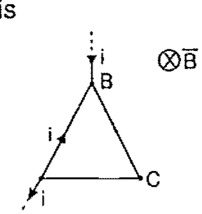
(C) it is constant in direction and magnitude both

(D) it is not constant in magnitude and direction both.

20.16 An α particle is moving along a circle of radius R with a constant angular velocity ω. Point A lies in the same plane at a distance 2R from the centre. Point A records magnetic field produced by α particle. If the minimum time interval between two successive times at which A records zero magnetic field is 't', the angular speed ω, in terms of t is -

(A) $\frac{2\pi}{t}$ (B) $\frac{2\pi}{3t}$ (C) $\frac{\pi}{3t}$ (D) $\frac{\pi}{t}$

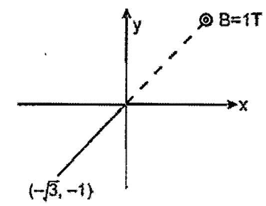
20.17 Figure shows an equilateral triangle ABC of side ℓ carrying currents, placed in uniform magnetic field B. The magnitude of magnetic force on triangle is



(A) $i\ell B$ (B) $2i\ell B$ (C) $3i\ell B$ (D) zero

20.18 There exists a uniform magnetic and electric field each of magnitude 1 T and 1 V/m respectively along positive y-axis. A charged particle of mass 1 kg and of charge 1 C is having velocity 1 m/sec along x-axis and is at origin at t = 0. Then the co-ordinates of particle at time π seconds will be:
(A) (0, 1, 2) (B) $(0, -\pi^2/2, -2)$ (C) $(2, \pi^2/2, 2)$ (D) $(0, \pi^2/2, 2)$

20.19 A uniform magnetic field of magnitude 1 T exists in region $y \geq 0$ is along $\hat{k}$ direction as shown. A particle of charge 1 C is projected from point $(-\sqrt{3}, -1)$ towards origin with speed 1 m/sec. If mass of particle is 1 kg, then co-ordinates of centre of circle in which particle moves are :

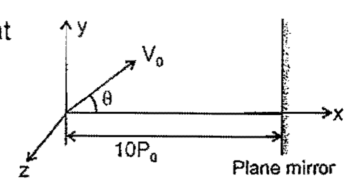


(A) $(1, \sqrt{3})$ (B) $(1, -\sqrt{3})$
(C) $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ (D) $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

20.20 A uniform magnetic field exists in region which forms an equilateral triangle of side a. The magnetic field is perpendicular to the plane of the triangle. A charge q enters into this magnetic field perpendicularly with speed v along perpendicular bisector of one side and comes out along perpendicular bisector of other side. The magnetic induction in the triangle is
(A) $\frac{mv}{qa}$ (B) $\frac{2mv}{qa}$ (C) $\frac{mv}{2qa}$ (D) $\frac{mv}{4qa}$

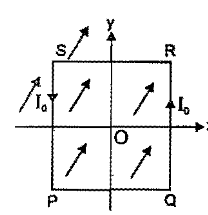
20.21 A positively charged particle having charge q and mass m enters with velocity $V\hat{j}$ at the origin in a magnetic field $B(-\hat{k})$ which is present in the space every where. The charge makes a perfectly inelastic collision with identical particle at rest but free to move at its maximum y-coordinate. After collision the combined charge will move on trajectory : (where $r = \frac{mV}{qB}$)
(A) $y = \frac{mv}{qB}(-\hat{i})$ (B) $(x + r)^2 + (y - r/2)^2 = r^2/4$
(C) $(x - r)^2 + (y - r)^2 = r^2$ (D) $(x - r)^2 + (y + r/2)^2 = r^2/4$

20.22 In the plane mirror, the co-ordinates of image of charged particle (initially at origin as shown) after two and half time periods are (initial velocity V_0 is in the xy-plane and the plane mirror is perpendicular to the x-axis. A uniform magnetic field $B\hat{i}$ exists in the whole space. P_0 is pitch of helix, R_0 is radius of helix).



(A) $17P_0, 0, -2R_0$ (B) $3P_0, 0, -2R_0$ (C) $17.5\ P_0, 0, -2R_0$ (D) $3P_0, 0, 2R_0$

20.23 A uniform, constant magnetic field $\vec{B}$ is directed at an angle of 45° to the x-axis in the xy-plane, PQRS is a rigid square wire frame carrying a steady current I_0, with its centre at the origin O. At time t = 0, the frame is at rest in the position shown in the figure, with its sides parallel to the x and y axes. Each side of the frame is of mass M and length L.. The torque $\vec{\tau}$ about O acting on the frame due to the magnetic field will be :



(A) $\vec{\tau} = \frac{BI_0L^2}{\sqrt{2}}\left(-\hat{i} + \hat{j}\right)$ (B) $\vec{\tau} = \frac{BI_0L^2}{\sqrt{2}}\left(\hat{i} - \hat{j}\right)$
(C) $\vec{\tau} = \frac{BI_0L^2}{\sqrt{2}}\left(\hat{i} + \hat{j}\right)$ (D) $\vec{\tau} = \frac{BI_0L^2}{\sqrt{2}}\left(-\hat{i} - \hat{j}\right)$

20.24 A ring of mass m, radius r having charge q uniformly distributed over it and free to rotate about its own axis is placed in a region having a magnetic field B parallel to its axis. If the magnetic field is suddenly switched off, the angular velocity acquired by the ring is

(A) $\frac{qB}{m}$ (B) $\frac{2qB}{m}$ (C) $\frac{qB}{2m}$ (D) None of these

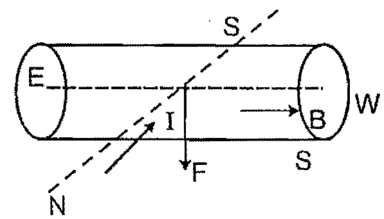
20.25 A uniform magnetic field of 1.5 T exists in a cylindrical region of radius 10.0 cm, it's direction being parallel to the axis along east to west. A current carrying wire in north south direction passes through this region. The wire intersects the axis and experience a force of 1.2 N downward. (as shown in figure). If the wire is turned from North South to north east–south west direction, then magnitude and direction of force is :

(A) 1.2 N, upward

(B) $1.2\sqrt{2}$ downward

(C) 1.2 N, downward

(D) $\frac{1.2}{\sqrt{2}}$ N, downward



20.26 In the above problem, if wire in north-south direction is lowered from the axis by a distance of 6 cm, then magnitude and direction of force is :

(A) 0.48 N, downward (B) 0.48 N, upward (C) 0.96 N, downward (D) 0.96 N, upward

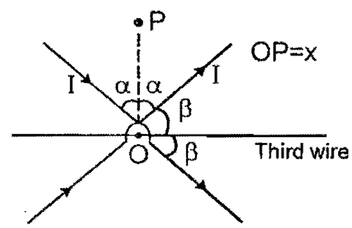
20.27 Three infinite current carrying conductors are placed as shown in figure. Two wires carry same current while current in third wire is unknown. The three wires do not intersect with each other and all of them are in the plane of paper. Which of the following is correct about a point 'P' which is also in the same plane :

(A) Magnetic field intensity at P is zero for all values of x.

(B) If the current in the third wire is $\frac{2I}{\sin\alpha}$ (left to right) then magnetic field will be zero at P for all values of x.

(C) If the current in the third wire is $\frac{2I}{\sin\alpha}$ (right to left) then magnetic field will be zero at P for all values of x.

(D) None of these



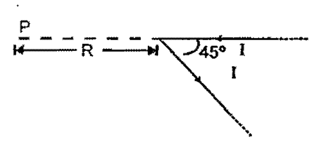
20.28 An insulating rod of length ℓ carries a charge q uniformly distributed on it. The rod is pivoted at its mid point and is rotated at a frequency f about a fixed axis perpendicular to rod and passing through the pivot. The magnetic moment of the rod system is

(A) Zero (B) $\pi q f \ell^2$ (C) $\frac{1}{12}\pi q f \ell^2$ (D) $\frac{1}{3}\pi q f \ell^2$

20.29 A long straight wire, carrying current I, is bent at its midpoint to form an angle of 45°. Magnetic field at point P, distance R from point of bending is equal to :

(A) $\frac{(\sqrt{2}-1)\mu_0 I}{4\pi R}$ (B) $\frac{(\sqrt{2}+1)\mu_0 I}{4\pi R}$

(C) $\frac{(\sqrt{2}+1)\mu_0 I}{4\sqrt{2}\pi R}$ (D) $\frac{(\sqrt{2}-1)\mu_0 I}{2\sqrt{2}\pi R}$



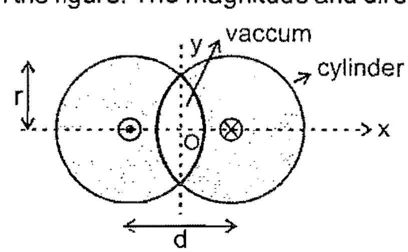
20.30 Two long cylinders (with axis parallel) are arranged as shown to form overlapping cylinders, each of radius r, whose centers are separated by a distance d. Current of density J (Current per unit area) flows into the plane of page along the right shaded part of one cylinder and an equal current flows out of the plane of the page along the left shaded part of the other, as shown in the figure. The magnitude and direction of magnetic field at point O (O is the origin of shown x-y axes) are

(A) $\frac{\mu_0}{2\pi}\pi J d$, in the + y-direction

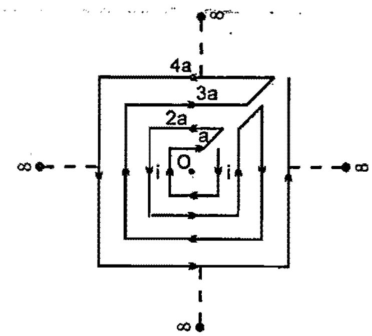
(B) $\frac{\mu_0}{2\pi} d^2 \frac{J}{r}$, in the + y-direction

(C) zero

(D) none of these



20.31 Determine the magnetic field at the centre of the current carrying wire arrangement shown in the figure. The arrangement extends to infinity. (The wires joining the successive squares are along the line passing through the centre)



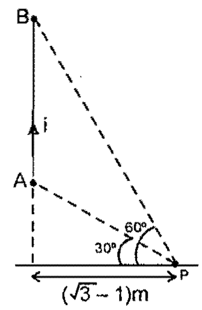
(A) $\frac{\mu_0 i}{\sqrt{2}\pi a}$

(B) 0

(C) $\frac{2\sqrt{2}\mu_0 i}{\pi a}$ ln 2

(D) none of these

20.32 A straight wire current element is carrying current 100 A, as shown in the figure. The magnitude of magnetic field at point P which is at perpendicular distance $(\sqrt{3}-1)$m from the current element if end A and end B of the element subtend angle 30° and 60° at point P, as shown, is :



(A) 5×10^{-6} T (B) 2.5×10^{-6} T (C) 2.5×10^{-5} T (D) 8×10^{-5} T

20.33 Axis of a solid cylinder of infinite length and radius R lies along y-axis it carries a uniformly distributed current 'i' along +y direction. Magnetic field at a point $\left(\frac{R}{2}, y, \frac{R}{2}\right)$ is :

(A) $\frac{\mu_0 I}{4\pi R}(\hat{i}-\hat{k})$ (B) $\frac{\mu_0 i}{2\pi R}(\hat{j}-\hat{k})$ (C) $\frac{\mu_0 i}{4\pi R}\hat{j}$ (D) $\frac{\mu_0 I}{4\pi R}(\hat{i}+\hat{k})$

20.34 A magnetic dipole $\vec{M} = (A\hat{i}+B\hat{j})$ J/Wb is placed in magnetic field. $\vec{B} = (Cx^2\hat{i}+Dy^2\hat{j})$ Wb in XY plane at $\vec{r} = (E\hat{i}+F\hat{j})$ m. Then force experienced by the bar magnet is :

(A) $2ACE\hat{i} + 2BDF\hat{j}$ (N)

(B) $2ACE\hat{i}$ (N)

(C) 0

(D) $ACE\hat{i} + BDF\hat{j}$ (N)

20.35 A toroid of mean radius 'a', cross section radius 'r' and total number of turns N. It carries a current 'i'. The torque experienced by the toroid if a uniform magnetic field of strength B is applied :

(A) is zero

(B) is $B i N \pi r^2$

(C) is $B i N \pi a^2$

(D) depends on the direction of magnetic field.

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

20.36 A long thick conducting cylinder of radius 'R' carries a current uniformly distributed over its cross section.

(A) The magnetic field strength is maximum on the surface

(B) The magnetic field strength is zero on the surface

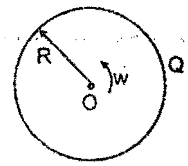
(C) The strength of the magnetic field inside the cylinder will vary as inversely proportional to r, where r is the distance from the axis.

(D) The energy density of the magnetic field outside the conductor varies as inversely proportional to $1/r^2$, where 'r' is the distance from the axis.

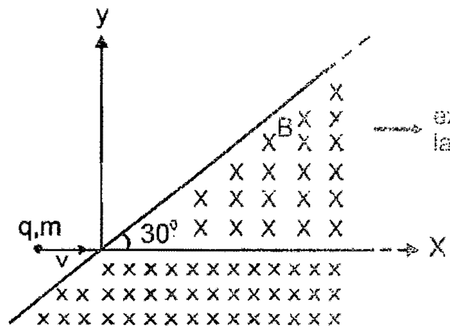
20.37 A nonconducting disc having uniform positive charge Q, is rotating about its axis in anticlock wise direction with uniform angular velocity ω. The magnetic field at the center of the disc is.

(A) directed outward (B) having magnitude $\frac{\mu_0 Q \omega}{4\pi R}$

(C) directed inwards (D) having magnitude $\frac{\mu_0 Q \omega}{2\pi R}$



20.38 A charge particle of charge q, mass m is moving with initial velocity 'v' as shown in figure in a uniform magnetic field $-B\hat{K}$. Select the correct alternative/alternatives :-



(A) Velocity of particle when it comes out from magnetic field is $\vec{v} = v\cos 60°\,\hat{i} - v\sin 60°\,\hat{j}$

(B) Time for which the particle was in magnetic field is $\frac{\pi m}{3qB}$

(C) Distance travelled in magnetic field is $\frac{\pi m V}{3qB}$

(D) The particle will never come out of magnetic field

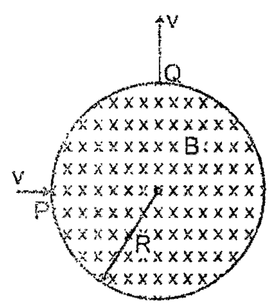
20.39 A particle of charge 'q' & mass 'm' enters normally (at point P) in a region of magnetic field with speed v. It comes out normally from Q after time T as shown in figure. The magnetic field B is present only in the region of radius R and is constant and uniform. Initial and final velocities are along radial direction and they are perpendicular to each other. For this to happen, which of the following expression(s) is/are correct :

(A) $B = \frac{mv}{qR}$

(B) $T = \frac{\pi R}{2v}$

(C) $T = \frac{\pi m}{2qB}$

(D) None of these



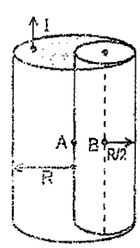
20.40 From a cylinder of radius R, a cylinder of radius R/2 is removed, as shown. Current flowing in the remaining cylinder is I. Magnetic field strength is :

(A) zero at point A

(B) zero at point B

(C) $\frac{\mu_0 I}{3\pi R}$ at point A

(D) $\frac{\mu_0 I}{3\pi R}$ at point B



SECTION - III : ASSERTION AND REASON TYPE

20.41 **Statement-1 :** A charged particle undergoes uniform circular motion in a uniform magnetic field. The only force acting on the particle is that exerted by the uniform magnetic field. If now the speed of the same particle is somehow doubled keeping its charge and external magnetic field constant, then the centripetal force on the particle becomes four times.

Statement-2 : The magnitude of centripetal force on a particle of mass m moving in a circle of radius R with uniform speed v is $\frac{mv^2}{R}$:

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

20.42 **Statement - 1 :** A current carrying closed loop remains in equilibrium in a uniform and constant magnetic field parallel to its axis. Consider forces only due to this magnetic field .

Statement - 2 : Torque on a current carrying closed loop due to a magnetic field is maximum when the plane of the coil is parallel to the direction of the magnetic field.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

20.43 **Statement-1 :** A solenoid tend to contract (along its length) when a current is passed through it.

Statement-2 : If current in two coaxial circular rings of equal radii is in same sense(as seen by an observer on axis away from both the rings), the rings attract each other. Further the given current carrying rings attract each other because parallel current attract.

(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True.

20.44 **Statement 1 :** A direct uniformly distributed current flows through a solid long metallic cylinder along its length. It produces magnetic field only outside the cylinder .

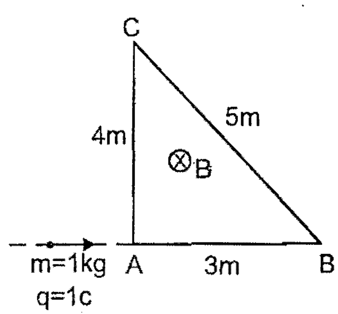
Statement 2 : A thin long cylindrical tube carrying uniformly distributed current along its circumference does not produce a magnetic field inside it. Moreover, a solid cylinder can be supposed to be made up of many thin cylindrical tubes.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

SECTION - IV : COMPREHENSION TYPE

COMPREHENSION # 1

A small particle of mass m = 1kg and charge of 1C enters perpendicularly in a triangular region of uniform magnetic field of strength 2T as shown in figure :



20.45 Calculate maximum velocity of the particle with which it should enter so that it complete a half–circle in magnetic region :

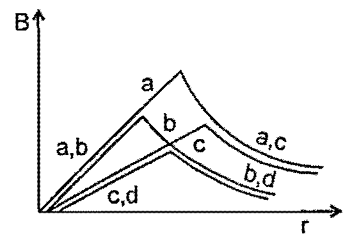
(A) 2 m/s (B) 2.5 m/s (C) 3 m/s (D) 4 m/s

20.46 In previous question, if particle enters perpendicularly with velocity 48 m/s in magnetic region. Then, how much time will it spend in magnetic region :

(A) $\frac{11\pi}{360}$ sec. (B) $\frac{7\pi}{360}$ sec. (C) $\frac{13\pi}{360}$ sec. (D) $\frac{17\pi}{360}$ sec.

COMPREHENSION #2

Curves in the graph shown give, as functions of radial distance r, the magnitude B of the magnetic field inside and outside four long wires a, b, c and d, carrying currents that are uniformly distributed across the cross sections of the wires. Overlapping portions of the plots are indicated by double labels.



20.47 Which wire has the greatest radius ?

(A) a (B) b (C) c (D) d

20.48 Which wire has the greatest magnitude of the magnetic field on the surface ?

(A) a (B) b (C) c (D) d

20.49 The current density in wire a is

(A) greater than in wire c.

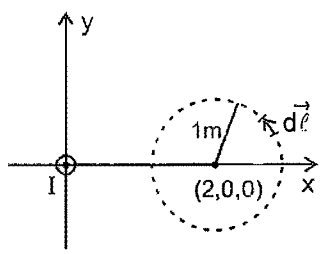
(B) less than in wire c.

(C) equl to that in wire c.

(D) not comparable to that of in wire c due to lack of information.

COMPREHENSION # 3

An infinitely long wire lying along z-axis carries a current I, flowing towards positive z-direction. There is no other current, consider a circle in x-y plane with centre at (2 meter, 0, 0) and radius 1 meter. Divide the circle in small segments and let $d\vec{\ell}$ denote the length of a small segment in anticlockwise direction, as shown.



20.50 The path integral $\oint \vec{B}\cdot d\vec{\ell}$ of the total magnetic field $\vec{B}$ along the perimeter of the given circle is,

(A) $\frac{\mu_0 I}{8}$ (B) $\frac{\mu_0 I}{2}$ (C) $\mu_0 I$ (D) 0

20.51 Consider two points A(3,0,0) and B(2,1,0) on the given circle. The path integral $\int_A^B \vec{B}\cdot d\vec{\ell}$ of the total magnetic field $\vec{B}$ along the perimeter of the given circle from A to B is,

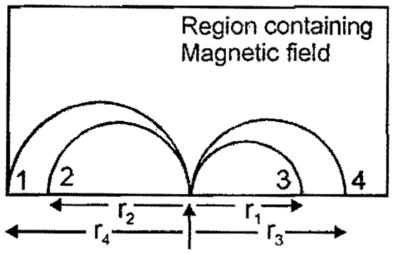
(A) $\frac{\mu_0 I}{\pi}\tan^{-1}\frac{1}{2}$ (B) $\frac{\mu_0 I}{2\pi}\tan^{-1}\frac{1}{2}$ (C) $\frac{\mu_0 I}{2\pi}\sin^{-1}\frac{1}{2}$ (D) 0

20.52 The maximum value of path integral $\int \vec{B}\cdot d\vec{\ell}$ of the total magnetic field $\vec{B}$ along the perimeter of the given circle between any two points on the circle is

(A) $\frac{\mu_0 I}{12}$ (B) $\frac{\mu_0 I}{8}$ (C) $\frac{\mu_0 I}{6}$ (D) 0

SECTION - V : MATRIX - MATCH TYPE

20.53 A beam consisting of four types of ions A, B, C and D enters a region that contains a uniform magnetic field as shown. The field is perpendicular to the plane of the paper, but its precise direction is not given. All ions in the beam travel with the same speed. The table below gives the masses and charges of the ions.



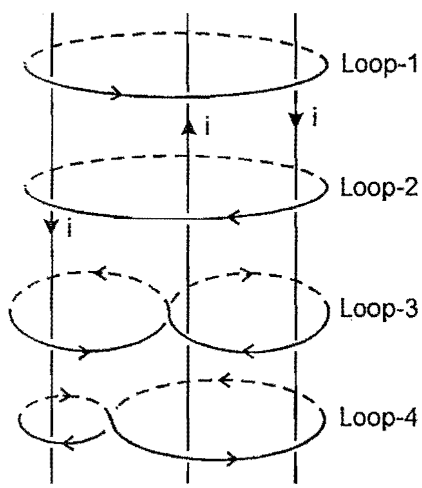
$r_4 > r_3 = r_2 > r_1$

ION	MASS	CHARGE
A	2m	+e
B	4m	– e
C	2m	– e
D	m	+ e

The ions fall at different positions 1, 2, 3 and 4, as shown. Correctly match the ions with respective falling positions.

Column-I		Column-II	
(A)	a	(p)	1
(B)	b	(q)	2
(C)	c	(r)	3
(D)	d	(s)	4

20.54 Three wires are carrying same constant current i in different directions. Four loops enclosing the wires in different manners are shown. The direction of $d\vec{\ell}$ is shown in the figure :

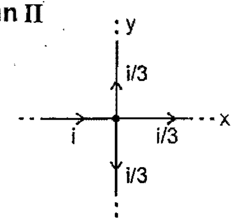


Column-I		Column-II	
(A)	Along closed Loop-1	(p)	$\oint \vec{B}.d\vec{\ell} = \mu_0 i$
(B)	Along closed Loop-2	(q)	$\oint \vec{B}.d\vec{\ell} = -\mu_0 i$
(C)	Along closed Loop-3	(r)	$\oint \vec{B}.d\vec{\ell} = 0$
(D)	Along closed Loop-4	(s)	net work done by the magnetic force to move a unit charge along the loop is zero.
		(t)	$\oint \vec{B}.d\vec{\ell} = \mu_0 (2i)$

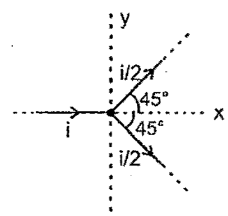
20.55 Column-II gives four situations in which three or four semi infinite current carrying wires are placed in xy-plane as shown. The magnitude and direction of current is shown in each figure. Column-I gives statements regarding the x and y components of magnetic field at a point P whose coordinates are P (0, 0, d). Match the statements in column-I with the corresponding figures in column-II

Column I / **Column II**

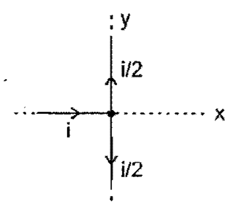
(A) The x component of magnetic field at point P is zero in

(p) 

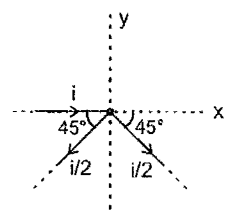
(B) The z component of magnetic field at point P is zero in

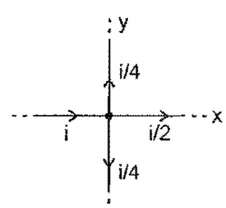
(q) 

(C) The magnitude of magnetic field at point P is $\frac{\mu_0 i}{4\pi d}$ in

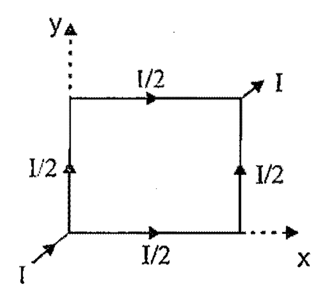
(r) 

(D) The magnitude of magnetic field at point P is less than $\frac{\mu_0 i}{2\pi d}$ in

(s) 

(t) 

20.56 A square loop of uniform conducting wire is as shown in figure. A current I (in amperes) enters the loop from one end and exits the loop from opposite end as shown in figure. The length of one side of square loop is ℓ metre. The wire has uniform cross section area and uniform linear mass density. In four situations of column I, the loop is subjected to four different magnetic field. Under the conditions of coulmn I, match the column I with corresponding results of column II (B_o in column I is a positive nonzero constant)



Column I	Column II
(A) $\vec{B} = B_o\hat{i}$ in tesla	(p) magnitude of net force on loop is $\sqrt{2}\,B_oI\,\ell$ newton
(B) $\vec{B} = B_o\hat{j}$ in tesla	(q) magnitude of net force on loop is zero
(C) $\vec{B} = B_o(\hat{i}+\hat{j})$ in tesla	(r) magnitude of net torque on loop about its centre is zero
(D) $\vec{B} = B_o\hat{k}$ in tesla	(s) magnitude of net force on loop is $B_oI\,\ell$ newton
	(t) magnitude of force on wire along y axis is $B_0I\,\ell/2$ along x axis

20.57 A particle enters a space where exists uniform magnetic field $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$ & uniform electric field $\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$ with initial velocity $\vec{u} = u_x\hat{i} + u_y\hat{j} + u_z\hat{k}$. Depending on the values of various components the particle selects a path. Match the entries of column A with the entries of column B. The components other than specified in column A in each entry are non-zero. Neglect gravity.

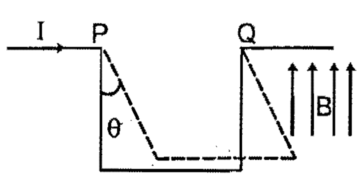
Column -I		**Column -II**	
(A)	$B_y = B_z = E_x = E_z = 0$; $u = 0$	(p)	circle
(B)	$E = 0$, $u_xB_x + u_yB_y \neq -u_zB_z$	(q)	helix with uniform pitch and constant radius
(C)	$\vec{u} \times \vec{B} = 0$, $\vec{u} \times \vec{E} = 0$	(r)	cycloid
(D)	$\vec{u} \perp \vec{B}$, $\vec{B} \parallel \vec{E}$	(s)	helix with variable pitch and constant radius
		(t)	straight line

SECTION - VI : INTEGER TYPE

20.58 A uniformly charged ring of radius 10 cm rotates at a frequency of 10^4 rps about its axis. Find the ratio of energy density of electric field to the energy density of the magnetic field at a point on the axis at distance 20 cm from the centre is $\frac{x}{10} \times 10^9$ then x is .

20.59 A neutral particle is at rest in a uniform magnetic field $\vec{B}$. At t = 0, particle decays into two particles each of mass ' m ' and one of them having charge ' q '. Both of these move off in separate paths lying in plane perpendicular to $\vec{B}$. At later time, the particles collide. if this time of collision is $x\pi m/qB$ then x is (neglecting the interaction force).

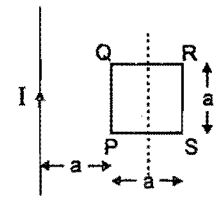
20.60 As shown in the figure, three sided frame is pivoted at P and Q and hangs vertically. Its sides are of same length and have a linear density of $\sqrt{3}$ kg/m. A current of $10\sqrt{3}$ Amp is sent through the frame, which is in a uniform magnetic field of 2T directed upwards as shown. Then angle in degree through which the frame will be deflected in equilibrium is : (Take g = 10 m/s²)



TOPIC 21 ELECTRO MAGNETIC INDUCTION (EMI)

SECTION - I : STRAIGHT OBJECTIVE TYPE

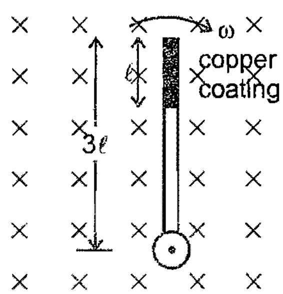
21.1 In the figure shown a square loop PQRS of side 'a' and resistance 'r' is placed near an infinitely long wire carrying a constant current I. The sides PQ and RS are parallel to the wire. The wire and the loop are in the same plane. The loop is rotated by 180° about an axis parallel to the long wire and passing through the mid points of the side QR and PS. The total amount of charge which passes through any point of the loop during rotation is :



(A) $\frac{\mu_0 Ia}{2\pi r}\ \ell n2$ (B) $\frac{\mu_0 Ia}{\pi r}\ \ell n2$

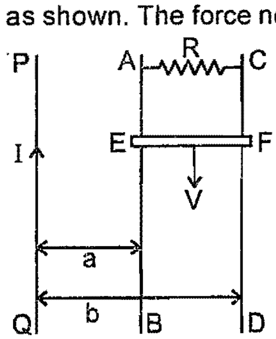
(C) $\frac{\mu_0 Ia^2}{2\pi r}$ (D) cannot be found because time of rotation not give.

21.2 A wooden stick of length 3ℓ is rotated about an end with constant angular velocity ω in a uniform magnetic field B perpendicular to the plane of motion. If the upper one third of its length is coated with copper, the potential difference across the whole length of the stick is



(A) $\frac{9B\omega\ell^2}{2}$ (B) $\frac{4B\omega\ell^2}{2}$ (C) $\frac{5B\omega\ell^2}{2}$ (D) $\frac{B\omega\ell^2}{2}$

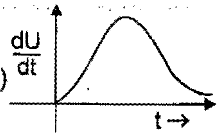
21.3 PQ is an infinite current carrying conductor. AB and CD are smooth conducting rods on which a conductor EF moves with constant velocity V as shown. The force needed to maintain constant speed of EF is.

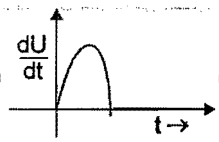


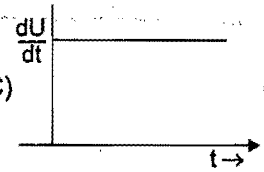
(A) $\frac{1}{VR}\left[\frac{\mu_0 IV}{2\pi}\ell n\left(\frac{b}{a}\right)\right]^2$ (B) $\left[\frac{\mu_0 IV}{2\pi}\ell n\left(\frac{b}{a}\right)\right]\frac{1}{R}$

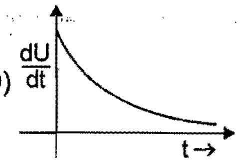
(C) $\left[\frac{\mu_0 IV}{2\pi}\ell n\left(\frac{b}{a}\right)\right]^2\frac{V}{R}$ (D) $\left[\frac{\mu_0 IV}{2\pi}\ell n\left(\frac{b}{a}\right)\right]$

21.4 Rate of increment of energy in an inductor with time in series LR circuit getting charge with battery of e.m.f. E is best represented by: [inductor has initially zero current]

(A) 

(B) 

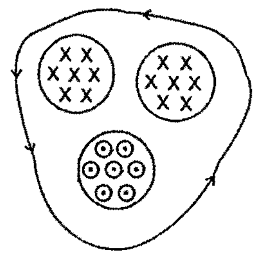
(C) 

(D) 

21.5 A wire of fixed length is wound on a solenoid of length 'ℓ' and radius 'r'. Its self inductance is found to be L. Now if same wire is wound on a solenoid of length $\frac{\ell}{2}$ and radius $\frac{r}{2}$, then the self inductance will be

(A) 2 L (B) L (C) 4 L (D) 8 L

21.6 Figure shows three regions of magnetic field, each of area A, and in each region magnitude of magnetic field decreases at a constant rate α. If $\vec{E}$ is induced electric field then value of line integral $\oint \vec{E} \cdot d\vec{r}$ along the given loop is equal to



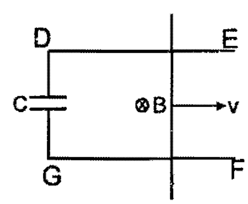
(A) αA (B) $-\alpha A$ (C) $3\alpha A$ (D) $-3\alpha A$

21.7 In an ideal transformer, the voltage and the current in the primary are 200 volt and 2 amp respectively. If the voltage in the secondary is 2000 volt. Then value of current in the secondary will be :
(A) 0.2 amp (B) 2 amp (C) 10 amp (D) 20 amp

21.8 A superconducting loop of radius R has self inductance L. A uniform & constant magnetic field B is applied perpendicular to the plane of the loop. Initially current in this loop is zero. The loop is rotated by 180°. The current in the loop after rotation is equal to

(A) zero (B) $\frac{B\pi R^2}{L}$ (C) $\frac{2B\pi R^2}{L}$ (D) $\frac{B\pi R^2}{2L}$

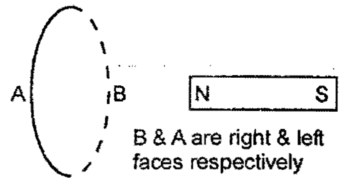
21.9 In the figure shown the section EDFG is fixed. A rod having resistance 'R' is moved with constant velocity in a uniform magnetic field B as shown in the figure. DE & FG are smooth and resistanceless. Initially capacitor is uncharged. The charge on the capacitor:



(A) remains constant (B) increases with time
(C) increases linearly with time (D) oscillates.

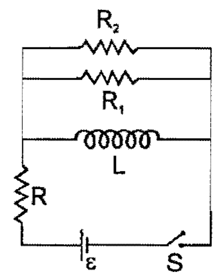
21.10 A rectangular loop of sides 'a' and 'b' is placed in xy plane. A very long wire is also placed in xy plane such that side of length 'a' of the loop is parallel to the wire. The distance between the wire and the nearest edge of the loop is 'd'. The mutual inductance of this system is proportional to :
(A) a (B) b (C) 1/d (D) current in wire I

21.11 In the figure shown, the magnet is pushed towards the fixed ring along the axis of the ring and it passes through the ring.



(A) when magnet goes towards the ring the face B becomes south pole and the face A becomes north pole
(B) when magnet goes away from the ring the face B becomes north pole and the face A becomes south pole
(C) when magnet goes away from the ring the face A becomes north pole and the face B becomes south pole
(D) the face A will always be a north pole.

21.12 Switch S is closed for a long time at t = 0. It is opened, then :



(A) total heat produced in resistor R after opening the switch is $\frac{1}{2}\frac{L\varepsilon^2}{R^2}$

(B) total heat produced in resistor R_1 after opening the switch is $\frac{1}{2}\frac{L\varepsilon^2}{R^2}\left(\frac{R_1}{R_1+R_2}\right)$

(C) heat produced in resistor R_1 after opening the switch is $\frac{1}{2}\frac{R_2 L\varepsilon^2}{(R_1+R_2)R^2}$

(D) no heat will be produced in R_1.

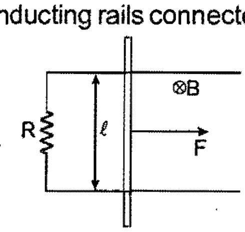
21.13 A rod of length ℓ having uniformly distributed charge Q is rotated about one end with constant frequency 'f'. Its magnetic moment is.

(A) $\pi f Q 1^2$ (B) $\frac{\pi f Q \ell^2}{3}$ (C) $\frac{2\pi f Q \ell^2}{3}$ (D) $2\pi f Q\, 1^2$

21.14 A vertical rod of length ℓ is moved with constant velocity v towards East. The vertical component of the earth's magnetic field is B and the angle of dip is θ. The induced e.m.f. in the rod is :
(A) $B\ell v \cot\theta$ (B) $B\ell v\sin\theta$ (C) $B\ell v\tan\theta$ (D) $B\ell v\cos\theta$

21.15 Two identical cycle wheels (geometrically) have different number of spokes connected from centre to rim. One is having 20 spokes and other having only 10 (the rim and the spokes are resistanceless). One resistance of value R is connected between centre and rim. The current in R will be :
(A) double in first wheel than in the second wheel
(B) four times in first wheel than in the second wheel
(C) will be double in second wheel than that of the first wheel
(D) will be equal in both these wheels.

21.16 A constant force F is being applied on a rod of length 'ℓ' kept at rest on two parallel conducting rails connected at ends by resistance R in uniform magnetic field B as shown.

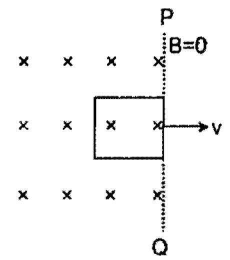


(A) the power delivered by force will be constant with time
(B) the power delivered by force will be increasing first and then will decrease
(C) the rate of power delivered by the external force will be increasing continuously
(D) the rate of power delivered by external force will be decreasing continuously.

21.17 A uniform magnetic field exists in region given by $\vec{B} = 3\hat{i} + 4\hat{j} + 5\hat{k}$. A rod of length 5 m is placed along y–axis is moved along x–axis with constant speed 1 m/sec. Then induced e.m.f. in the rod will be:
(A) zero (B) 25 volt (C) 20 volt (D) 15 volt

21.18 In a L-R growth circuit, inductance and resistance used are 1 H and 20 Ω respectively. If at t = 50 millisecond, current in the circuit is 3.165 A then applied direct current emf is :
(A) 200 V (B) 100 V
(C) 50 V (D) Data is insufficient to find out the value.

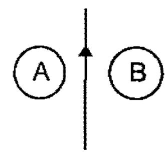
21.19 Figure shows a square loop of side 1 m and resistance 1 Ω. The magnetic field on left side of line PQ has a magnitude B = 1.0T. The work done in pulling the loop out of the field uniformly in 1 s is



(A) 1 J (B) 10 J (C) 0.1 J (D) 100 J

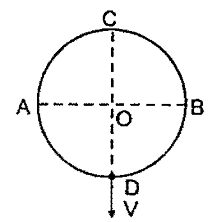
21.20 A and B are two metallic rings placed at opposite sides of an infinitely long straight conducting wire as shown. If current in the wire is slowly decreased, the direction of induced current will be :

(A) clockwise in A and anticlockwise in B
(B) anticlockwise in A and clockwise in B
(C) clockwise in both A and B
(D) anticlockwise in both A & B

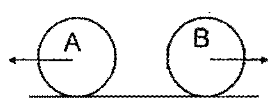


21.21 A vertical conducting ring of radius R falls vertically with a speed V in a horizontal uniform magnetic field B which is perpendicular to the plane of the ring :

(A) A and B are at same potential
(B) C and D are at same potential
(C) current flows in clockwise direction
(D) current flows in anticlockwise direction



21.22 Two identical conducting rings A & B of radius R are in pure rolling over a horizontal conducting plane with same speed (of center of mass)υ but in opposite direction. A constant magnetic field B is present pointing inside the plane of paper. Then the potential difference between the highest points of the two rings, is :

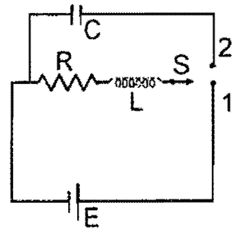


(A) zero (B) 2 Bvr (C) 4Bvr (D) none of these

21.23 An inductor L and a resistor R are connected in series with a direct current source of emf E. The maximum rate at which energy is stored in the magnetic field is :

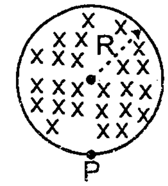
(A) $\frac{E^2}{4R}$ (B) $\frac{E^2}{R}$ (C) $\frac{4E^2}{R}$ (D) $\frac{2E^2}{R}$

21.24 In the circuit shown in figure, the switch S was initially at position 1. After sufficiently long time, the switch S was thrown from position 1 to position 2. The voltage drop across the resistor at that instant is :



(A) zero (B) E (C) $\frac{E}{R}LC$ (D) none of these

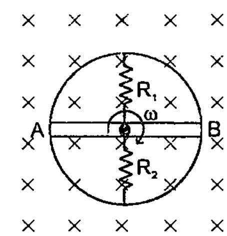
21.25 A uniform magnetic field of induction B is confined to a cylindrical region of radius R. The magnetic field is increasing at a constant rate of $\frac{dB}{dt}$ (tesla/second). An electron of charge q, placed at the point P on the periphery of the field experiences an acceleration :



(A) $\frac{1}{2}\frac{eR}{m}\frac{dB}{dt}$ towards left (B) $\frac{1}{2}\frac{eR}{m}\frac{dB}{dt}$ towards right

(C) $\frac{eR}{m}\frac{dB}{dt}$ towards left (D) zero

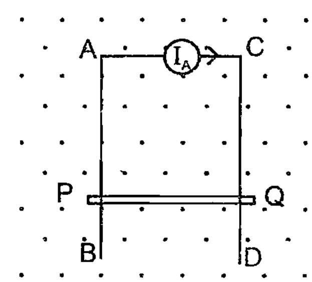
21.26 AB is a resistanceless conducting rod which forms a diameter of a conducting ring of radius r rotating in a uniform magnetic field B as shown. The resistors R_1 and R_2 do not rotate. Then current through the resistor R_1 is :



(A) $\frac{B\omega r^2}{2R_1}$ (B) $\frac{B\omega r^2}{2R_2}$

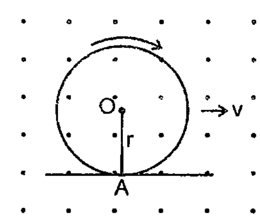
(C) $\frac{B\omega r^2}{2R_1R_2}(R_1 + R_2)$ (D) $\frac{B\omega r^2}{2(R_1+R_2)}$

21.27 AB and CD are fixed conducting smooth rails placed in a vertical plane and joined by a constant current source at its upper end. PQ is a conducting rod which is free to slide on the rails. A horizontal uniform magnetic field exists in space as shown. If the rod PQ is released from rest then,



(A) The rod PQ will move downward with constant acceleration
(B) The rod PQ will move upward with constant acceleration
(C) The rod will move downward with decreasing acceleration and finally acquire a constant velocity
(D) either A or B.

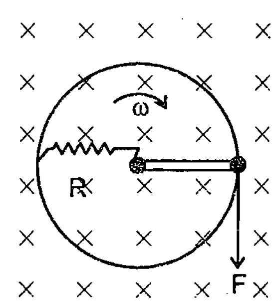
21.28 A conducting ring of radius r with a conducting spoke is in pure rolling on a horizontal surface in a region having a uniform magnetic field B as shown, v being the velocity of the centre of the ring. Then the potential difference $V_0 - V_A$ is -



(A) $\frac{Bvr}{2}$ (B) $\frac{3Bvr}{2}$

(C) $\frac{-Bvr}{2}$ (D) $\frac{3Bvr}{2}$

21.29 A metallic ring of mass m and radius r with a uniform metallic spoke of same mass m and length r is rotated about its axis with angular velocity ω in a perpendicular uniform magnetic field B as shown. If the central end of the spoke is connected to the rim of the wheel through a resistor R as shown. The resistor does not rotate, its one end is always at the center of the ring and other end is always in contact with the ring. A force F as shown is needed to maintain constant angular velocity of the wheel. F is equal to (The ring and the spoke has zero resistance)



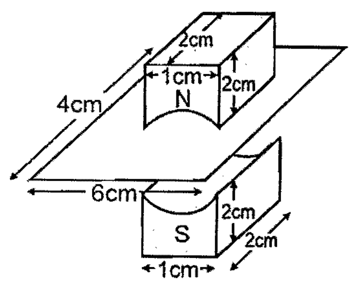
(A) $\frac{B^2\omega r^2}{8R}$ (B) $\frac{B^2\omega r^2}{2R}$

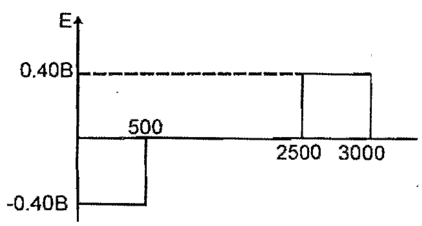
(C) $\frac{B^2\omega r^3}{2R}$ (D) $\frac{B^2\omega r^3}{4R}$

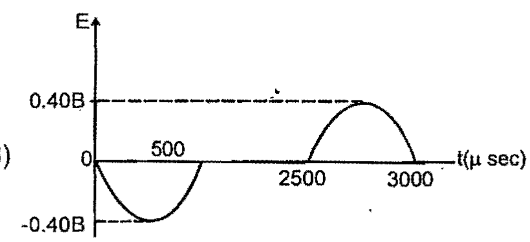
21.30 A closed circuit consists of a resistor R, inductor of inductance L and a source of emf E are connected in series. If the inductance of the coil is abruptly decreased to L/4 (by removing its magnetic core), the new current immediately after this moment is : (before decreasing the inductance the circuit is in steady state)

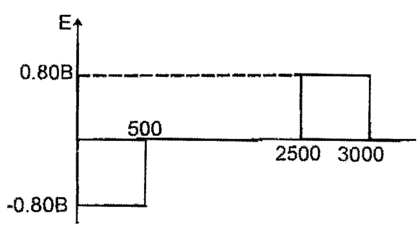
(A) zero (B) E/R (C) $4\frac{E}{R}$ (D) $\frac{E}{4R}$

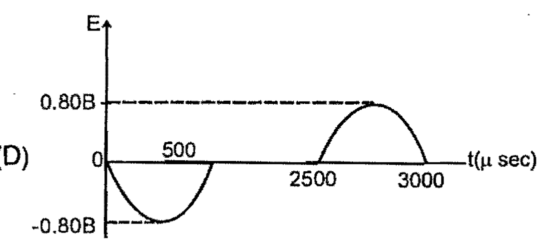
21.31 A magnetic field (B), uniform between two magnets can be determined measuring the induced voltage in the loop as it is pulled through the gap at uniform speed 20 m/sec. Size of magnet and coil is 2cm × 1cm × 2cm and 4cm × 6cm as shown in figure. The correct variation of induced emf with time is : (Assume at t = 0, the coil enters in the magnetic field) :



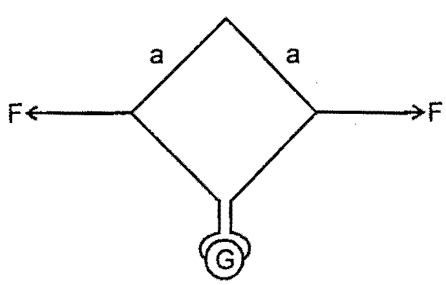
(A)


(B)


(C)


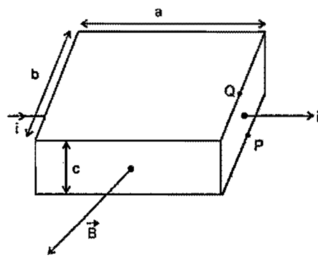
(D)


21.32 The plane of a square loop of wire with edge length a = 0.2 m is perpendicular to the earth's magnetic field B_E at a point where B_E = 15 μT. The total resistance of the loop and the wires connecting it to the galvanometer is 0.5 Ω. If the loop is suddenly collapsed(such that area of the loop becomes zero) by horizontal forces as shown, the total charge passing through the galvanometer is :



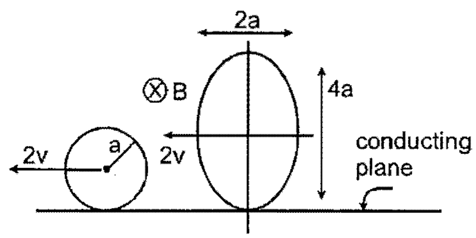
(A) 0.4 μC (B) 0.75 μC (C) 0.9 μC (D) 1.2 μC

21.33 A current flows through a rectangular conductor in the presence of uniform magnetic field B pointing out of the page as shown. Then the potential difference $V_P - V_Q$ is equal to. (assume charge carriers in the conductor to be positively charged moving with a drift velocity of v)



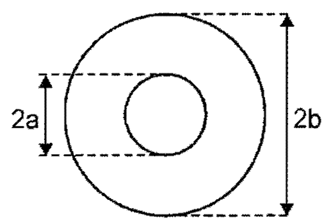
(A) Bvb (B) –Bvb (C) Bvc (D) –Bvc

21.34 A conducting circular ring and a conducting elliptical ring both are moving pure translationally in a uniform magnetic field of strength B as shown in figure on a horizontal conducting plane then potential difference between top most points of circle and ellipse is :



(A) 12 va (B) 4 vBa (C) 8 vBa (D) 2 vBa

21.35 A small circular wire loop of radius a is located at the centre of a much larger circular wire loop of radius b as shown above(b>>a). Both loops are coaxial and coplanar. The larger loop carries a time (t) varying current $I = I_0 \cos \omega t$, where I_0 and ω are constants. The magnetic field generated by the current in the large loop induces in the small loop an emf that is approximately equal to which of the following.



(A) $\frac{\pi \mu_0 I_0}{2} \frac{a^2}{b} \omega \cos \omega t$ (B) $\frac{\pi \mu_0 I_0}{2} \frac{a^2}{b} \omega \sin \omega t$

(C) $\frac{\pi \mu_0 I_0}{2} \frac{a}{b^2} \omega \sin \omega t$ (D) $\frac{\pi \mu_0 I_0}{2} \frac{a}{b^2} \omega \cos \omega t$

21.36 The number of turns, cross-sectional area and length for four solenoids are given in the following table.

Solenoid	Total Turns	Area	Length
1	2N	2A	ℓ
2	2N	A	ℓ
3	3N	3A	2ℓ
4	2N	2A	$\ell/2$

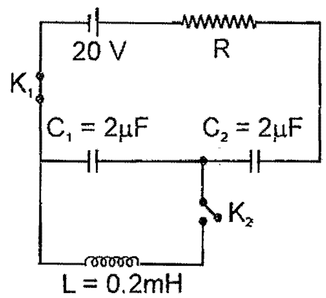
The solenoid with maximum self inductance is :

(A) 1 (B) 2 (C) 3 (D) 4

21.37 Assume Earth's surface is a conductor with a uniform surface charge density σ. It rotates about its axis with angular velocity ω. Suppose the magnetic field due to Sun at Earth at some instant is a uniform field B pointing along earth's axis. Then the emf developed between the pole and equator of earth due to this field is. (R_e = radius of earth)

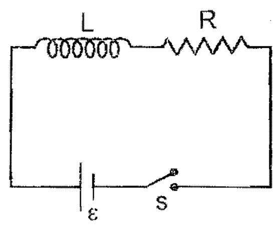
(A) $\frac{1}{2}B\omega R_e^2$ (B) $B\omega R_e^2$ (C) $\frac{3}{2}B\omega R_e^2$ (D) zero

21.38 A circuit containing capacitors C_1 and C_2 as shown in the figure are in steady state with key K_1 closed. At the instant t = 0, if K_1 is opened and K_2 is closed then the maximum current in the circuit will be :



(A) 1 A (B) $\frac{1}{2}$A (C) 2 A (D) None of these

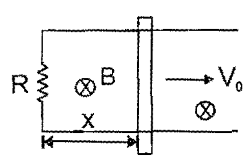
21.39 In the given circuit having an ideal inductor of inductance L, resistor of resistance R and an ideal cell of emf ε, the work done by the battery in one time constant after the switch is closed is



(A) $\frac{\varepsilon^2}{R}$ (B) $\frac{\varepsilon^2 L}{R^2}e$ (C) $\frac{\varepsilon^2 L}{R^2}$ (D) $\frac{\varepsilon^2 L}{eR^2}$

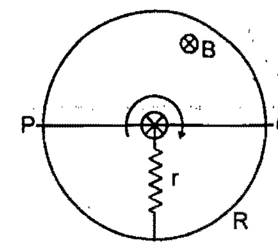
SECTION - II : MULTIPLE CORRECT ANSWER TYPE

21.40 A conducting rod of length ℓ is moved at constant velocity 'v_0' on two parallel, conducting, smooth, fixed rails, that are placed in a uniform constant magnetic field B perpendicular to the plane of the rails as shown in figure. A resistance R is connected between the two ends of the rail. Then which of the following is/are correct :



(A) The thermal power dissipated in the resistor is equal to rate of work done by external person pulling the rod.
(B) If applied external force is doubled than a part of external power increases the velocity of rod.
(C) Lenz's Law is not satisfied if the rod is accelerated by external force
(D) If resistance R is doubled then power required to maintain the constant velocity v_0 becomes half.

21.41 In the figure shown 'R' is a fixed conducting fixed ring of negligible resistance and radius 'a'. PQ is a uniform rod of resistance r. It is hinged at the centre of the ring and rotated about this point in clockwise direction with a uniform angular velocity ω. There is a uniform magnetic field of strength 'B' pointing inwards. 'r' is a stationary resistance



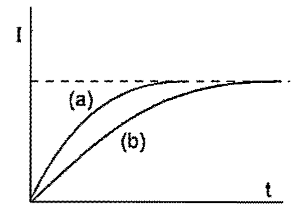
(A) Current through 'r' is zero.

(B) Current through 'r' is $\frac{2B\omega a^2}{5r}$.

(C) Direction of current in external 'r' is from centre to circumference.

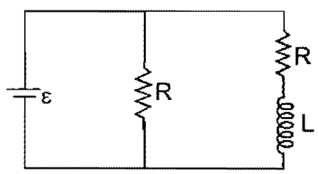
(D) Direction of current in external 'r' is from circumference to centre.

21.42 A circuit consisting of a constant e.m.f. 'E', a self induction 'L' and a resistance 'R' is closed at t = 0. The relation between the current I in the circuit and time t is as shown by curve 'a' in the figure. When one or more of parameters E, R & L are changed, the curve 'b' is obtained.The steady state current is same in both the cases. Then it is possible that:



(A) E & R are kept constant and L is increased. (B) E & R are kept constant and L is decreased
(C) E & R are both halved and L is kept constant (D) E & L are kept constant and R is decreased

21.43 In the circuit diagram shown



(A) time constant is L/R (B) time constant is 2L/R
(C) steady state current in inductor is $2\varepsilon/R$ (D) steady state current in inductor is ε/R

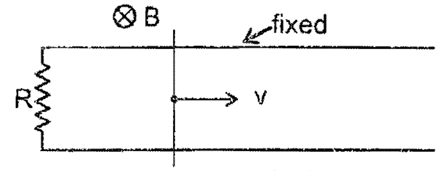
21.44 A conducting loop rotates with constant angular velocity about its fixed diameter in a uniform magnetic field. Whose direction is perpendicular to that fixed diameter.
(A) The emf will be maximum at the moment when flux is zero.
(B) The emf will be '0' at the moment when flux is maximum.
(C) The emf will be maximum at the moment when plane of the loop is parallel to the magnetic field
(D) The phase difference between the flux and the emf is $\pi/2$

21.45 An ideal inductor, (having initial current zero) a resistor and an ideal battery are connected in series at time t = 0. At any time t, the battery supplies energy at the rate P_B, the resistor dissipates energy at the rate P_R and the inductor stores energy at the rate P_L.
(A) $P_B = P_R + P_L$ for all times t. (B) $P_R < P_L$ for all times t.
(C) $P_R < P_L$ only near the starting of the circuit. (D) $P_R > P_L$ only near the starting of the circuit.

SECTION - III : ASSERTION AND REASON TYPE

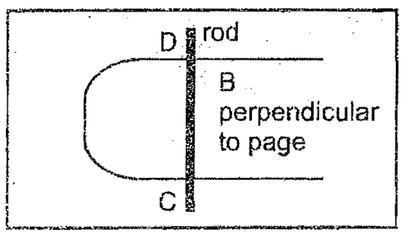
21.46 **Statement-1 :** Two coaxial conducting rings of different radii are placed in space. The mutual inductance of both the rings is maximum if the rings are also coplanar.
Statement-2 : For two coaxial conducting rings of different radii, the magnitude of magnetic flux in one ring due to current in other ring is maximum when both rings are coplanar.
(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True.

21.47 **Statement-1 :** A resistance R is connected between the two ends of the parallel smooth conducting rails. A conducting rod lies on these fixed horizontal rails and a uniform constant magnetic field B exists perpendicular to the plane of the rails as shown in the figure. If the rod is given a velocity v and released as shown in figure, it will stop after some time. The total work done by magnetic field is negative.



Statement-2 : If force acts opposite to direction of velocity its work done is negative.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

21.48 **Statement-1 :** Consider the arrangement shown below. A smooth conducting rod, CD, is lying on a smooth U-shaped conducting wire making good electrical contact with it. The U-shape conducting wire is fixed and lies in horizontal plane. There is a uniform and constant magnetic field B in vertical direction (perpendicular to plane of page in figure). If the magnetic field strength is decreased, the rod moves towards right.



Statement-2 : In the situation of statement-1, the direction in which the rod will slide is that which tends to maintain constant flux through the loop. Providing a larger loop area counteracts the decrease in magnetic flux. So the rod moves to the right independent of the fact that the direction of magnetic field is into the page or out of the page.
(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True.

21.49 **Statement-1 :** No electric current will be present within a region having uniform and constant magnetic field.

Statement-2 : Within a region of uniform and constant magnetic field $\vec{B}$, the path integral of magnetic field $\oint \vec{B}\cdot d\vec{\ell}$ along any closed path is zero. Hence from Ampere circuital law $\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I$ (where the given terms have usual meaning), no current can be present within a region having uniform and constant magnetic field.
(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True.

21.50 **Statement-1 :** Electric field produced by changing magnetic field is nonconservative.

Statement-2: For the electric field $\vec{E}$ induced by a changing magnetic field which has closed lines of force, $\oint \vec{E}.d\vec{\ell} = 0$
(A) Statement -1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

SECTION - IV : COMPREHENSION TYPE

COMPREHENSION # 1

A fan operates at 200 volt (DC) consuming 1000 W when running at full speed. It's internal wiring has resistance 1 Ω. When the fan runs at full speed, its speed becomes constant. This is because the torque due to magnetic field inside the fan is balanced by the torque due to air resistance on the blades of the fan and torque due to friction between the fixed part and the shaft of the fan. The electrical power going into the fan is spent (i) in the internal resistance as heat, call it P_1 (ii) in doing work against internal friction and air resistance producing heat, sound etc., call it P_2. When the coil of fan rotates, an emf is also induced in the coil. This opposes the external emf applied to send the current into the fan. This emf is called back-emf, call it 'e'.
Answer the following questions when the fan is running at full speed.

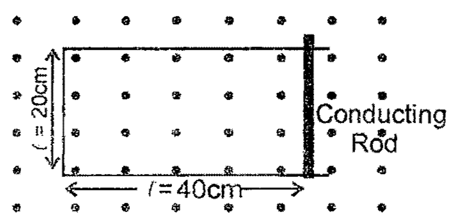
21.51 The current flowing into the fan and the value of back emf 'e' is :
(A) 200 A, 5 volt (B) 5 A, 200 volt (C) 5 A, 195 volt (D) 1 A , 0 volt

21.52 The value of power 'P_1' is
(A) 1000 W (B) 975 W (C) 25 W (D) 200 W

21.53 The value of power 'P_2 is
(A) 10000 W (B) 975 W (C) 25 W (D) 200 W

COMPREHENSION # 2

Figure shows a conducting rod of negligible resistance that can slide on smooth U-shaped rail made of wire of resistance 1Ω/m. Position of the conducting rod at t = 0 is shown. A time t dependent magnetic field B = 2t Tesla is switched on at t = 0.



21.54 The current in the loop at t = 0 due to induced emf is
(A) 0.16 A, clockwise (B) 0.08 A, clockwise
(C) 0.08 A, anticlockwise (D) zero

21.55 At t = 0, when the magnetic field is switched on, the conducting rod is moved to the left at constant speed 5 cm/s by some external means. The rod moves perpendicular to the rails. At t = 2s, induced emf has magnitude.
(A) 0.12 V (B) 0.08 V (C) 0.04 V (D) 0.02 V

21.56 Following situation of the previous question, the magnitude of the force required to move the conducting rod at constant speed 5 cm/s at the same instant t = 2s, is equal to
(A) 0.16 N (B) 0.12 N (C) 0.08 N (D) 0.06 N

COMPREHENSION # 3

A train of mass 100 tons (1 ton = 1000 kg) runs on a meter gauge track(distance between the two rails is 1 m.) The coefficient of friction between the rails and the train is 0.045. The train is powered by an electric engine of 90% efficiency. The train is moving with uniform speed of 72 Kmph at its highest speed limit. Horizontal and vertical component of earth's magnetic field are $B_H = 10^{-5}$T and $B_V = 2 \times 10^{-5}$ T. Assume the body of the train and rails to be perfectly conducting.

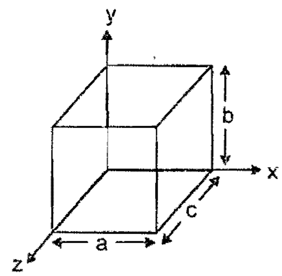
21.57 The electrical power consumed by the train is -
(A) 1.11 MW (B) 1 MW (C) 0.50 MW (D) 0.90 MW

21.58 The potential difference between the two rails is -
(A) 100 μV (B) 200 μV (C) 400 μV (D) 800 μV

21.59 If now a resistor of $10^{-3}\ \Omega$ is attached of between the two rails, the extra units of energy (electricity) consumed during a 324 km run of the train is (1 unit of power = 1 kW hour) (assume the speed of train to remain unchanged)
(A) 8×10^{-4} KW hour (B) 8×10^{-5} KW hour
(C) 8×10^{-6} KW hour (D) 8×10^{-7} KW hour

SECTION - V : MATRIX - MATCH TYPE

21.60 The figure shows a metallic solid block, placed in a way so that its faces are parallel to the coordinate axes. Edge lengths along axis x, y and z are a, b and c respectively. The block is in a region of uniform magnetic field of magnitude 30mT. One of the edge length of the block is 25 cm. The block is moved at 4 m/s parallel to each axis and in turn, the resulting potential difference V that appears across the block is measured. When the motion is parallel to the y axis, V = 24 mV; with the motion parallel to the z axis, V = 36 mV; with the motion parallel to the x axis, V = 0. Using the given information, correctly match the dimensions of the block with the values given.

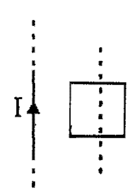


Column I		Column II	
(A)	a	(p)	20 cm
(B)	b	(q)	24 cm
(C)	c	(r)	25 cm
(D)	$\frac{bc}{a}$	(s)	30 cm
		(t)	26 cm

21.61 Column-I gives situations involving a charged particle which ***may be*** realised under the condition given in column-II. Match the situations in column-I with the condition in column-II.

Column I		Column II	
(A)	Increase in speed of a charged particle	(p)	Electric field uniform in space and constant with time
(B)	Exert a force on an electron initially at rest	(q)	Magnetic field uniform in space and constant with time.
(C)	Move a charged particle in a circle with uniform speed	(r)	Magnetic field uniform in space but varying with time.
(D)	Accelerate a moving charged particle	(s)	Magnetic field non-uniform in space but constant with time
		(t)	Electric field non–uniform in space but constant with time.

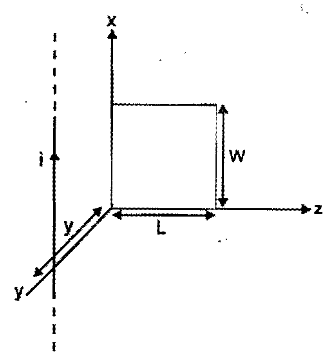
21.62 A square loop of conducting wire is placed near a long straight current carrying wire as shown. Match the statements in column-I with the corresponding results in column-II.



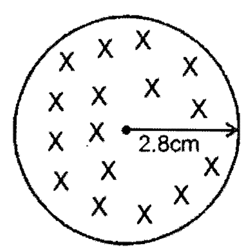
Column-I		Column-II	
(A)	If the magnitude of current I is increased	(p)	Induced current in the loop will be clock wise
(B)	If the magnitude of current I is decreased	(q)	Induced current in the loop will be anticlockwise
(C)	If the loop is moved away from the wire	(r)	wire will attract the loop
(D)	If the loop is moved towards the wire	(s)	wire will repel the loop
		(t)	Torque about centre of mass of loop is zero due to magnetie force

SECTION - VI : INTEGER TYPE

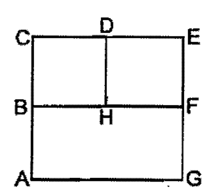
21.63 In the figure, a long thin wire carrying a varying current $i = i_0 \sin \omega t$ lies at a distance y above one edge of a rectangular wire loop of length L and width W lying in the x-z plane. If $\frac{\mu_0 I_0 W_0 \cos\theta wt}{A\pi} \ell n\left[\frac{L^B}{Y^C}+1\right]$ emf is induced in the loop. Find the value of (A+B+C)



21.64 The magnetic field of a cylindrical magnet that has a pole-face radius 2.8 cm can be varied sinusoidally between minimum value 16.8 T and maximum value 17.2 T at a frequency of $\frac{60}{\pi}$ Hz. Cross section of the magnetic field created by the magnet is shown. At a radial distance of 2cm from the axis find the amplitude of the electric field (in mN/C) induced by the magnetic field variation.



21.65 In the figure shown ABCDEFGH is a square conducting frame of side 2m and resistance 1 Ω/m. A uniform magnetic field B is applied perpendicular to the plane and pointing inwards. It increases with time at a constant rate of 10 T/s. Find the rate at which heat in watt is produced in the circuit, AB = BC = CD = BH.



21.66 A long coaxial cable consists of two thin walled conducting cylinders with inner radius 2cm and outer radius 8cm. The inner cylinder carries a steady current 1A, and the outer cylinder provides the return path for that current. The current produces a magnetic field between the two cylinders. Find the energy stored in the magnetic field for length 1 m of the cable. Express answer in nJ (use $\ell n2 = 0.7$)

TOPIC 22

ALTERNATING CURRENT

SECTION - I : STRAIGHT OBJECTIVE TYPE

22.1 An alternating EMF of angular frequency $\omega \left(=\frac{1}{\sqrt{LC}}\right)$ is applied to a series LCR circuit. For this frequency of the applied EMF,
(A) The circuit is at 'resonance' and its impedance is made up only of a reactive part
(B) The current in the circuit is in phase with the applied EMF and the voltage across R equals this applied EMF
(C) The sum of the potential differences across the inductance and capacitance equals the applied EMF which is 180° ahead of phase of the current in the circuit
(D) Impedance of the circuit is less than R

22.2 An LCR series circuit with 100 Ω resistance is connected to an AC source of 200 V and angular frequency 300 radians per second. When only the capacitance is removed, the current lags the voltage by 60°. When only the inductance is removed, the current leads the voltage by 60°. Then the current and power dissipated in LCR circuit are respectively
(A) 1A, 200 watt. (B) 1A, 400 watt. (C) 2A, 200 watt. (D) 2A, 400 watt.

22.3 A bulb is rated at 100 V, 100 W, it can be treated as a resistor. Find out the inductance of an inductor (called choke coil) that should be connected in series with the bulb to operate the bulb at its rated power with the help of an ac source of 200 V and 50 Hz.

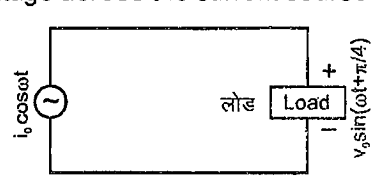
(A) $\frac{\pi}{\sqrt{3}}$ H (B) 100 H (C) $\frac{\sqrt{2}}{\pi}$ H (D) $\frac{\sqrt{3}}{\pi}$ H

22.4 An ac source of angular frequency ω is fed across a resistor R and a capacitor C in series. The current registered is I. If now the frequency of source is changed to $\omega/3$ (but maintaining the same voltage), the current in the circuit is found to be halved. Then the ratio of reactance to resistance at the original frequency ω is :

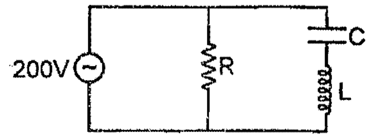
(A) $\sqrt{3/5}$ (B) $\sqrt{5/3}$ (C) $\sqrt{2/3}$ (D) $\sqrt{3/2}$

22.5 Current in an ac circuit is given by $i = 3 \sin \omega t + 4 \cos \omega t$ then :
(A) rms value of current is 5 A.
(B) mean value of this current in one half period will be $6/\pi$.
(C) if voltage applied is $V = V_m \sin \omega t$ then the circuit must be containing resistance and capacitance.
(D) if voltage applied is $V = V_m \sin \omega t$, the circuit may contain resistance and inductance.

22.6 A current source sends a current $i = i_0 \cos(\omega t)$. When connected across an unknown load gives a voltage output of, $v = v_0 \sin(\omega t + \pi/4)$ across that load. Then voltage across the current source may be brought in phase with the current through it by:
(A) connecting an inductor in series with the load
(B) connecting a capacitor in series with the load
(C) connecting an inductor in parallel with the load
(D) connecting a capacitor in parallel with the load.



22.7 In the circuit diagram shown, $X_C = 100\ \Omega$, $X_L = 200\ \Omega$ & $R = 100\ \Omega$. The effective current through the source is:



(A) 2 A (B) $2\sqrt{2}$ A (C) 0.5 A (D) $\sqrt{0.4}$ A

22.8 For a LCR series circuit with an A.C. source of angular frequency ω.

(A) circuit will be capacitive if $\omega > \frac{1}{\sqrt{LC}}$

(B) circuit will be inductive if $\omega = \frac{1}{\sqrt{LC}}$

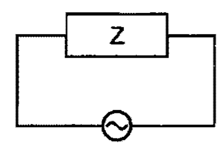
(C) power factor of circuit will by unity if capacitive reactance equals inductive reactance

(D) current will be leading voltage if $\omega > \frac{1}{\sqrt{LC}}$

22.9 The value of current in two series L C R circuits at resonance is same when connected across a sinusoidal voltage source. Then:
(A) both circuits must be having same value of capacitance and inductor
(B) in both circuits ratio of L and C will be same
(C) for both the circuits X_L/X_C must be same at that frequency
(D) both circuits must have same impedance at all frequencies.

22.10 In series L C R circuit voltage drop across resistance is 8 volt, across inductor is 6 volt and across capacitor is 12 volt. Then:
(A) voltage of the source will be leading current in the circuit
(B) voltage drop across each element will be less than the applied voltage
(C) power factor of circuit will be 4/3
(D) none of these

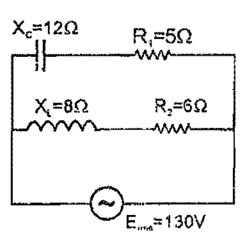
22.11 In a black box of unknown elements (L, C or R or any other combination) an AC voltage $E = E_0 \sin(\omega t + \phi)$ is applied and current in the circuit was found to be $i = i_0 \sin(\omega t + \phi + \pi/4)$. Then the unknown elements in the box may be :



(A) only capacitor
(B) inductor and resistor both
(C) either capacitor, resistor and inductor or only capacitor and resistor
(D) only resistor

22.12 A series AC circuit has resistance of 4 Ω and a reactance of 3 Ω. The impedance(Z) of the circuit is
(A) 5 Ω (B) 7 Ω (C) 12/7 Ω (D) 7/12 Ω

22.13 What is the amount of power delivered by the ac source in the circuit shown (in watts).



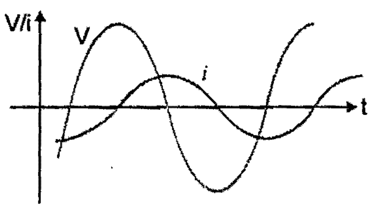
(A) 500 watt (B) 1014 watt (C) 1514 watt (D) 2013 watt

22.14 The secondary coil of an ideal step down transformer is delivering 500 watt power at 12.5 A current. If the ratio of turns in the primary to the secondary is 5 : 1, then the current flowing in the primary coil will be :

(A) 62.5 A (B) 2.5 A (C) 6 A (D) 0.4 A

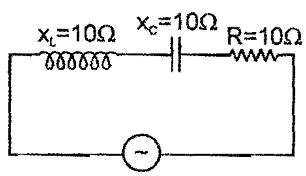
SECTION - II : MULTIPLE CORRECT ANSWER TYPE

22.15 Graph shows variation of source emf V and current i in a series RLC circuit, with time.



(A) The current leads the emf in the circuit.
(B) The circuit is more inductive than capacitive.
(C) To increase the rate at which energy is transferred to the resistive load, L should be decreased.
(D) To increase the rate at which energy is transferred to the resistive load, C should be decreased.

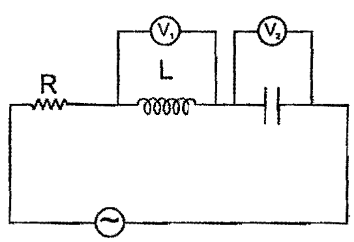
22.16 In the given AC circuit, which of the following is incorrect :



$\varepsilon=100\sin(100\pi t+\pi/2)$

(A) Voltage across resistance is lagging by 90° than the voltage across capacitor.
(B) voltage across capacitor is lagging by 180° than voltage across inductor.
(C) voltage across inductor is leading by 90° than voltage across resistance.
(D) Resistance of the circuit is equal to reactance of circuit.

22.17. In the circuit shown, ressitance R = 100 Ω, inductance L = $\frac{2}{\pi}$ H and capacitance C = $\frac{8}{\pi}\mu F$ are connected in series with an ac source of 200 volt and frequency 'f'. If the readings of the hot wire voltmeters V_1 and V_2 are same then :

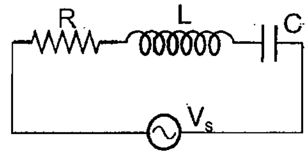


(A) f = 125 Hz (B) f = 250 π Hz
(C) current through R is 2A (D) $V_1 = V_2$ = 1000 volt

SECTION - III : ASSERTION AND REASON TYPE

22.18 **Statement-1 :** The D.C. and A.C. both can be measured by a hot wire instrument.
Statement-2 : The hot wire instrument is based on the principle of magnetic effect of current.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True.

22.19 **Statement-1 :** In a series R,L,C circuit if V_R, V_L, and V_C denote rms voltage across R, L and C repectively and V_S is the rms voltage across the source, **then** $V_S=V_R + V_L + V_C$.



Statement-2 : In AC circuits, kirchoff voltage law is correct at every instant of time.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True.

22.20 **Statement-1 :** The electrostatic energy stored in capacitor plus magnetic energy stored in inductor will always be zero in a series LCR circuit driven by ac voltage source under condition of resonance.
Statement-2 : The complete voltage of ac source appears across the resistor in a series LCR circuit driven by ac voltage source under condition of resonance.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True.

22.21 **Statement-1 :** An inductor is connected to an ac source. When the magnitude of current decreases in the circuit, energy is absorbed by the ac source.
Statement-2 : When current through an inductor decreases, the energy stored in inductor decreases.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True.

SECTION - IV : COMPREHENSION TYPE

Comprehension # 1

In a series L-R circuit, connected with a sinusoidal ac source, the maximum potential difference across L and R are respectively 3 volts and 4 volts.

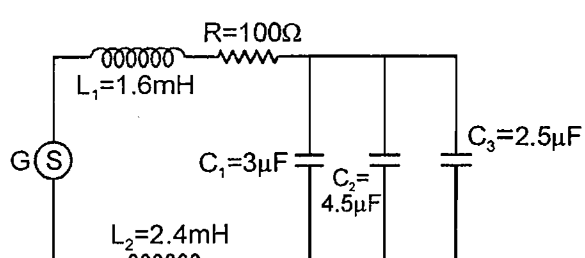
22.22 At an instant the potential difference across resistor is 2 volts. The potential difference in volt, across the inductor at the same instant will be :
(A) 3 cos 30° (B) 3 cos 60° (C) 6 cos 45° (D) 6

22.23 At the same instant, the magnitude of the potential difference in volt, across the ac source will be
(A) 3 cos 67° (B) 5 sin 67° (C) 6 cos 97° (D) 0

22.24 If the current at this instant is decreasing the magnitude of potential difference at that instant across the ac source is
(A) Increasing (B) Decreasing (C) constant (D) cannot be said

Comprehension # 2

An ac generator G with an adjustable frequency of oscillation is used in the circuit, as shown.



22.25 Current drawn from the ac source will be maximum if its angular frequency is -
(A) 10^5 rad/s (B) 10^4 rad/s (C) 5000 rad/s (D) 500 rad/s

22.26 To increase resonant frequency of the circuit, some of the changes in the circuit are carried out. Which change(s) would certainly result in the increase in resonant frequency ?
(A) R is increased.
(B) L_1 is increased and C_1 is decreased.
(C) L_2 is decreased and C_2 is increased.
(D) C_3 is removed from the circuit.

22.27 If the ac source G is of 100 V rating at resonant frequency of the circuit, then average power supplied by the source is -
(A) 50 W (B) 100 W (C) 500 W (D) 1000 W

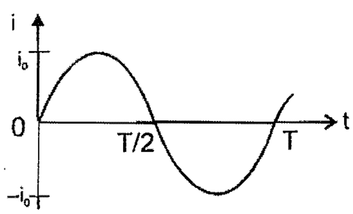
22.28 Average energy stored by the inductor L_2 (Source is at resonance frequency) is equal to
(A) zero (B) 1.2 mJ (C) 2.4 mJ (D) 4 mJ

22.29 Thermal energy produced by the resistance R in time duration 1 μs, using the source at resonant condition, is
(A) 0 J
(B) 1 μJ
(C) 100 μJ
(D) not possible to calculate from the given information

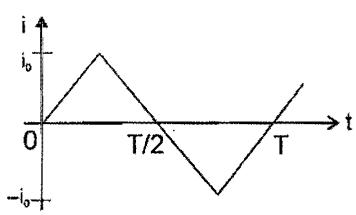
SECTION - V : MATRIX - MATCH TYPE

22.30 In Column I, variation of current i with time t is given in figures. In column II root mean square current i_{rms}, and average current is given. Match the column I with corresponding quantities given in Column II

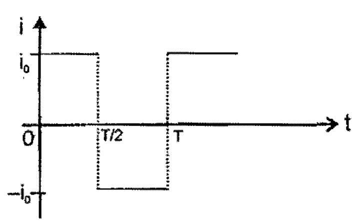
Column I / **Column II**

(A) 

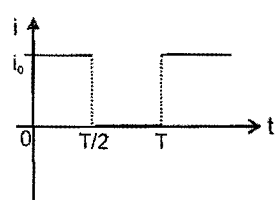
(p) $i_{rms} = \frac{i_0}{\sqrt{3}}$

(B) 

(q) Average current for positive half cycle is i_0

(C) 

(r) Average current for positive half cycle is $\frac{i_0}{2}$

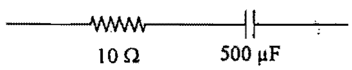
(D) 

(s) Full cycle average current is zero.

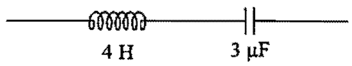
(t) Root mean square value of current for positive half cycle is i_0

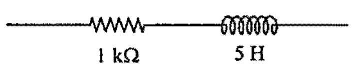
22.31 Four different circuit components are given in each situation of column-I and all the components are connected across an AC source of same angular frequency $\omega = 200$rad/sec. The information of phase difference between the current and source voltage in each situation of column-I is given in column-II. Match the circuit components in column-I with corresponding results in column-II.

Column - I

(A) 

(B) 

(C) 

(D) 

Column - II

(p) the magnitude of required phase difference is $\frac{\pi}{2}$.

(q) the magnitude of required phase difference is $\frac{\pi}{4}$.

(r) the current leads in phase to source voltage.

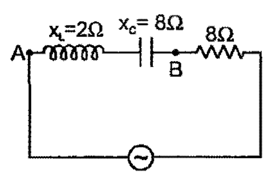
(s) the magnitude of required phase difference is zero.

(t) the current lags in phase to source voltage.

SECTION - VI : INTEGER TYPE

22.32 Find the average value of current from t = 0 to $t = \frac{2\pi}{\omega}$ if the current varies as $i = I_m \sin \omega t$.

22.33 An inductor ($x_L = 2\Omega$), a capacitor ($x_C = 8\Omega$) and a resistance (8Ω) are connected in series with an AC source. The voltage output of A.C source is given by $v = 10 \cos 100\pi t$, then the impedance of the circuit (inΩ).

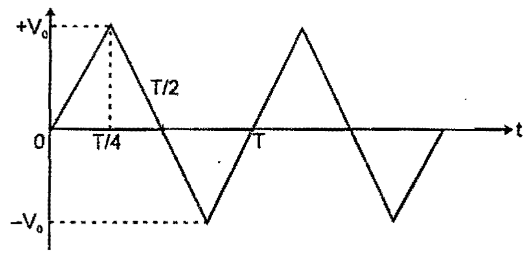


22.34 In previous question the instantaneous p.d. between A and B when it is half of the voltage output from source at that instant is $\frac{x}{5}$ volt then x is.

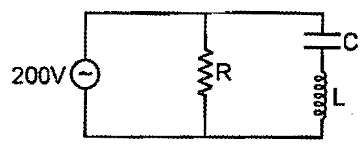
22.35 In an AC circuit the potential differences across an inductance and resistance joined in series are respectively 16 V and 20 V. The total potential difference across the circuit is $\frac{X}{5}$ then X is :

.36 The voltage time (V - t) graph for triangular wave having peak value. V_0 is as shown in figure.

The rms value of V in time interval from t = 0 to $\frac{T}{4}$ is $\frac{\sqrt{3}V}{X}$ then, X is :

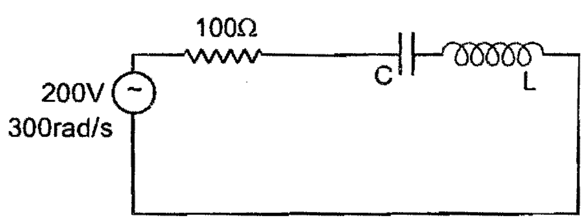


22.37 In the circuit diagram shown, $X_C = 100\ \Omega$, $X_L = 200\ \Omega$ and $R = 100\ \Omega$. The effective current through the source is $\sqrt{N}$, then N is :



22.38 An LCR series circuit with 100 Ω resistance is connected to an AC source of 200 V and angular frequency 300 radians per second. When only the capacitance is removed, the current lags behind the voltage by 60°. When only the inductance is removed, the current leads the voltage by 60°. Calculate the average power dissipated in LCR circuit in watts.

22.39 In the LCR circuit shown if only L is removed, the current leads the supply voltage by 30°. If only C is removed, the current lags the voltage by 60°. The resonant frequency is $\frac{50x}{\sqrt{3}\pi}$ Hz, then write the value of 'x'.



2.40 A series RLC, AC circuit operates at half its resonant frequency and at a power factor of 0.8. Then write the ratio of $\frac{X_C}{R}$:

TOPIC 23

MODERN PHYSICS

SECTION - I : STRAIGHT OBJECTIVE TYPE

23.1 In an α-decay the Kinetic energy of α particle is 48 MeV and Q-value of the reaction is 50 MeV. The mass number of the mother nucleus is:- (Assume that daughter nucleus is in ground state)

(A) 96 (B) 100 (C) 104 (D) none of these

23.2 A sample of radioactive material decays simultaneously by two processes A and B with half lives $\frac{1}{2}$ and $\frac{1}{4}$ hr respectively. For first half hour it decays with the process A, next one hr with the process B and for further half an hour with both A and B. If originally there were N_0 nuclei, find the number of nuclei after 2 hr of such decay.

(A) $\frac{N_0}{(2)^8}$ (B) $\frac{N_0}{(2)^4}$ (C) $\frac{N_0}{(2)^6}$ (D) $\frac{N_0}{(2)^5}$

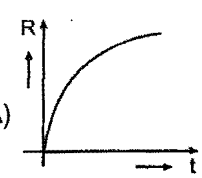
23.3 In which of the following process the number of protons in the nucleus increases .

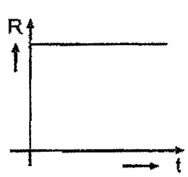
(A) α – decay (B) β^- – decay (C) β^+ – decay (D) k – capture

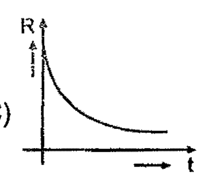
23.4 The angular momentum of an electron in first orbit of Li^{++} ion is :

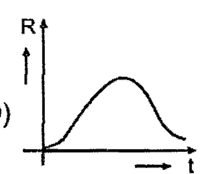
(A) $\frac{3h}{2\pi}$ (B) $\frac{9h}{2\pi}$ (C) $\frac{h}{2\pi}$ (D) $\frac{h}{6\pi}$

23.5 A radioactive nucleus 'X' decays to a stable nucleus 'Y'. Then the graph of rate of formation of 'Y' against time 't' will be :

(A)


(B)


(C)


(D)


23.6 A heavy nucleus having mass number 200 gets disintegrated into two small fragments of mass number 80 and 120. If binding energy per nucleon for parent atom is 6.5 M eV and for daughter nuclei is 7 MeV and 8 MeV respectively, then the energy released in the decay will be:

(A) 200 MeV (B) – 220 MeV (C) 220 MeV (D) 180 MeV

23.7 If first excitation potential of a hydrogen like atom is V electron volt, then the ionization energy of this atom will be:

(A) V electron volt (B) $\frac{3V}{4}$ electron volt

(C) $\frac{4V}{3}$ electron volt (D) cannot be calculated by given information.

23.8 All electrons ejected from a surface by incident light of wavelength 200 nm can be stopped before travelling 1 m in the direction of uniform electric field of 4 N/C. The work function of the surface is:
(A) 4 eV (B) 6.2 eV (C) 2 eV (D) 2.2 eV

23.9 An electron of mass 'm', when accelerated through a potential V has de–Broglie wavelength λ. The de–Broglie wavelength associated with a proton of mass M accelerated through the same potential difference will be:

(A) $\lambda\sqrt{\frac{M}{m}}$ (B) $\lambda\sqrt{\frac{m}{M}}$ (C) $\lambda\left(\frac{M}{m}\right)$ (D) $\lambda\left(\frac{m}{M}\right)$

23.10 Two hydrogen atoms are in excited state with electrons residing in n = 2. First one is moving towards left and emits a photon of energy E_1 towards right. Second one is moving towards right with same speed and emits a photon of energy E_2 towards right. Taking recoil of nucleus into account during emission process
(A) $E_1 > E_2$ (B) $E_1 < E_2$ (C) $E_1 = E_2$ (D) information insufficient

23.11 In a hydrogen atom following the Bohr's postulates the product of linear momentum and angular momentum is proportional to $(n)^x$ where 'n' is the orbit number. Then 'x' is :
(A) 0 (B) 2 (C) –2 (D) 1

23.12 The voltage applied to an X-ray tube is 18 kV. The maximum mass of photon emitted by the X-ray tube will be:
(A) 2×10^{-13} kg (B) 3.2×10^{-36} kg (C) 3.2×10^{-32} kg (D) 9.1×10^{-31} kg

23.13 The wavelengths of K_α x-rays of two metals 'A' and 'B' are $\frac{4}{1875\,R}$ and $\frac{1}{675\,R}$ respectively, where 'R' is Rydberg's constant. The number of elements lying between 'A' and 'B' according to their atomic numbers is
(A) 3 (B) 6 (C) 5 (D) 4

23.14 One of the lines in the emission spectrum of Li^{2+} has the same wavelength as that of the 2^{nd} line of Balmer series in hydrogen spectrum. The electronic transition corresponding to this line is :
(A) $n = 4 \rightarrow n = 2$ (B) $n = 8 \rightarrow n = 2$ (C) $n = 8 \rightarrow n = 4$ (D) $n = 12 \rightarrow n = 6$

23.15 If the short wavelength limit of the continuous spectrum coming out of a coolidge tube is 10 Å, then the debroglie wavelength of the electrons reaching the target metal in the coolidge tube is approximately
(A) 0.3 Å (B) 3 Å (C) 30 Å (D) 10 Å

23.16 The photon radiated from hydrogen corresponding to 2^{nd} line of Lyman series is absorbed by a hydrogen like atom 'X' in 2^{nd} excited state. As a result the hydrogen like atom 'X' makes a transition to n^{th} orbit. Then,
(A) X = He^+, n = 4 (B) X = Li^{++}, n = 6 (C) X = He^+, n = 6 (D) X = Li^{++}, n = 9

23.17 In a photoelectric experiment, with light of wavelength λ, the fastest electron has speed v. If the exciting wavelength is changed to $\frac{3\lambda}{4}$, the speed of the fastest emitted electron will become

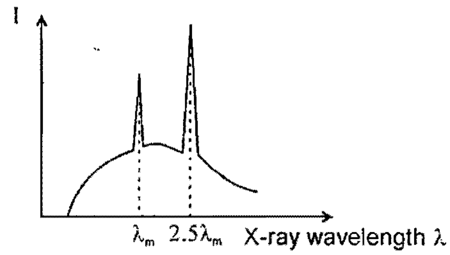
(A) $v\sqrt{\frac{3}{4}}$ (B) $v\sqrt{\frac{4}{3}}$ (C) less than $v\sqrt{\frac{3}{4}}$ (D) greater than $v\sqrt{\frac{4}{3}}$

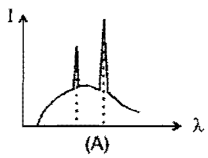
23.18 An element X decays, first by positron emission and then two α-particles are emitted in successive radioactive decay. If the product nuclei has a mass number 229 and atomic number 89, the mass number and atomic number of element X are
(A) 237, 93 (B) 237, 94 (C) 221, 84 (D) 237, 92

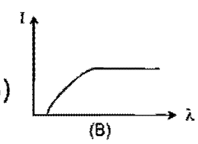
23.19 1.5 MW of 400 nm light is directed at a photoelectric cell. If 0.10% of the incident photons produce photoelectrons, the current in the cell is
(A) 0.36 μA (B) 0.48 μA (C) 0.42 mA (D) 0.32 mA

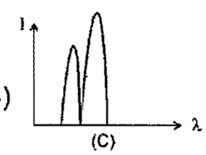
23.20 The element which has a k_α x-rays line of wavelength 1.8 Å is
($R = 1.1 \times 10^7\ m^{-1}$, b = 1 and $\sqrt{5/33} = 0.39$)
(A) Co, Z = 27 (B) Iron, Z = 26 (C) Mn, z = 25 (D) Ni, z = 28

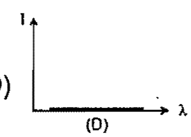
23.21 When an electron accelerated by potential difference U is bombarded on a specific metal, the emitted X-ray spectrum obtained is shown in adjoining graph. If the potential difference is reduced to U/3, the correct spectrum is



(A)


(B)


(C)


(D)


23.22 In the hydrogen atom, an electron makes a transition from n = 2 to n = 1. The magnetic field produced by the circulating electron at the nucleus
(A) decreases 16 times (B) increases 4 times
(C) decreases 4 times (D) increases 32 times

23.23 90% of a radioactive sample is left undecayed after time t has elapsed. What percentage of the initial sample will decay in a total time 2t.
(A) 20% (B) 19% (C) 40% (D) 38%

23.24 A radioactive element X converts into another stable element Y. Half life of X is 2 hrs. Initially only X is present. After time t, the ratio of atoms of X and Y is found to be 1 : 4, then t in hours is :
(A) 2 (B) 4 (C) between 4 and 6 (D) 6

23.25 An electron in a hydrogen atom makes a transition from first excited state to ground state. The equivalent current due to circulating electron
(A) increases 2 times (B) increases 4 times (C) increases 8 times (D) remains the same

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

23.26 When a hydrogen atom is excited from ground state to first excited state then
(A) its kinetic energy increases by 10.2 eV. (B) its kinetic energy decreases by 10 .2 eV.
(C) its potential energy increases by 20.4 eV. (D) its angular momentum increases by 1.05×10^{-34} J-s.

23.27 In an x–ray tube the voltage applied is 20KV. The energy required to remove an electron from L shell is 19.9 KeV. In the x–rays emitted by the tube
(A) minimum wavelength will be 62.1 pm
(B) energy of the characterstic x–rays will be equal to or less than 19.9 KeV
(C) L_α x–ray may be emitted
(D) L_α x–ray will have energy 19.9 KeV

23.28 Suppose the potential energy between electron and proton at a distance r is given by $-\frac{Ke^2}{3r^3}$. Application of Bohr's theory to hydrogen atom in this case shows that
(A) energy in the nth orbit is proportional to n^6
(B) energy is proportional to m^{-3} (m : mass of electron)
(C) energy of the nth orbit is proportional to n^{-2}
(D) energy is proportional to m^3 (m = mass of electron)

23.29 Let A_n be the area enclosed by the nth orbit in a hydrogen atom. The graph of $\ell n(A_n/A_l)$ against $\ell n(n)$
(A) will pass through origin
(B) will be a straight line with slope 4
(C) will be a monotonically increasing nonlinear curve
(D) will be a circle

SECTION - III : ASSERTION AND REASON TYPE

23.30 **Statement-1 :** Though light of a single frequency (monochromatic light) is incident on a metal, the energies of emitted photoelectrons are different.
Statement-2 : The energy of electrons just after they absorb photons incident on metal surface may be lost in collision with other atoms in the metal before the electron is ejected out of the metal.
(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True.

23.31 **Statement-1 :** The de-Broglie wavelength of a molecule (in a sample of ideal gas) varies inversely as the square root of absolute temperature.
Statement-2 : The rms velocity of a molecule (in a sample of ideal gas) depends on temperature.
(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True.

23.32 **Statement-1 :** Heavy nuclides tend to have more number of neutrons than protons.
Statement-2 : As there is coulombic repulsion between protons, so in heavy nuclei, excess of neutrons are preferable.
(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True.

23.33 **Statement–1 :** ${}_{Z}X^{A}$ undergoes 2 α decays, 2β decays (negative β) and 2 γ decays. As a result the daughter product is ${}_{Z-2}Y^{A-8}$.
Statement–2 : In α decay the mass number decreases by 4 unit and atomic number decreases by 2 unit. In β decay (negative β) the mass number remains unchanged and atomic number increases by 1 unit. In γ decay, mass number and atomic number remains unchanged.
(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True.

SECTION - IV : COMPREHENSION TYPE

Comprehension # 1

The radionuclide ^{56}Mn is being produced in a cyclotron at a constant rate P by bombarding a maganese target with deutrons. ^{56}Mn has a half life of 2.5 hours and the target contains large number of only the stable maganese isotope ^{55}Mn. The reaction that produces ^{56}Mn is :

$$^{55}Mn + d \rightarrow {}^{56}Mn + p$$

After being bombarded for a long time, the activity of ^{56}Mn becomes constant equal to $13.86 \times 10^{10}\ s^{-1}$. (Use $\ell n2 = 0.693$; Avogadro No $= 6 \times 10^{23}$; atomic weight $^{56}Mn = 56$ gm/mole)

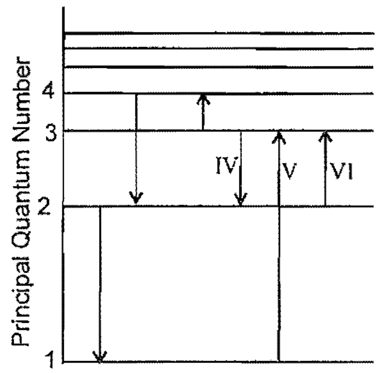
23.34 At what constant rate P, ^{56}Mn nuclei are being produced in the cyclotron during the bombardment ?
(A) 2×10^{11} nuclei/s
(B) 13.86×10^{10} nuclei/s
(C) 9.6×10^{10} nuclei/s
(D) 6.93×10^{10} nuclei/s

23.35 After the activity of ^{56}Mn becomes constant, number of ^{56}Mn nuclei present in the target, is equal to
(A) 5×10^{11}
(B) 20×10^{11}
(C) 1.2×10^{14}
(D) 1.8×10^{15}

23.36 After a long time bombardment, number of ^{56}Mn nuclei present in the target depends upon
(a) the number of ^{56}Mn nuclei present at the start of the process.
(b) half life of the ^{56}Mn
(c) the constant rate of production P.
(A) All (a), (b) and (c) are correct (B) only (a) and (b) are correct
(C) only (b) and (c) are correct (D) only (a) and (c) are correct

Comprehension # 2

Pertain to the statement and diagram below :



The figure given shows an energy level diagram for the hydrogen atom. Several transitions are marked as I, II, III, ________. The diagram is only indicative and not to scale.

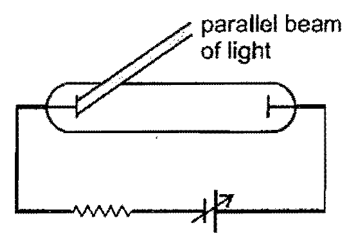
23.37 In which transition is a Balmer series photon absorbed ?
(A) II (B) III (C) IV (D) VI

23.38 The wavelength of the radiation involved in transition II is
(A) 291 nm (B) 364 nm (C) 487 nm (D) 652 nm

23.39 Which transition will occur when a hydrogen atom is irradiated with radiation of wavelength 103nm?
(A) I (B) II (C) IV (D) V

SECTION - V : MATRIX - MATCH TYPE

23.40 In the shown experimental setup to study photoelectric effect, two conducting electrodes arc enclosed in an evacuated glass-tube as shown. A parallel beam of monochromatic light, falls on photosensitive electrodes. The emf of battery shown is high enough such that all photoelectrons ejected from left electrode will reach the right electrode. Under initial conditions photoelectrons are emitted. As changes are made in each situation of column I; Match the statements in column I with results in column II.



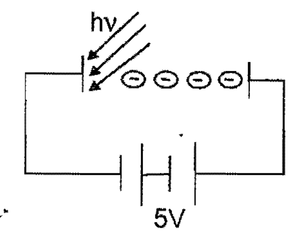
	Column-I		Column-II
(A)	If frequency of incident light is increased keeping number of photons per second constant	(p)	magnitude of stopping potential will increase
(B)	If frequency of incident light is increased and number of photons per second is decreased.	(q)	current through circuit may stop
(C)	If work function of photo sensitive electrode is increased	(r)	maximum kinetic energy of ejected photoelectrons will increase
(D)	If number of photons per second of incident light is increased keeping its frequency constant	(s)	saturation current will increase
		(t)	saturation current will decrease

23.41 In column-I, consider each process just before and just after it occurs. Initial system is isolated from all other bodies. Consider all product particles (even those having rest mass zero) in the system. Match the system in column-I with the result they produce in column-II.

	Column-I		Column-II
(A)	Spontaneous radioactive decay of an uranium nucleus initially at rest as given by reaction ${}^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He + ...$	(p)	Number of protons is increased
(B)	Fusion reaction of two hydrogen nuclei as given by reaction ${}^{1}_{1}H + {}^{1}_{1}H \rightarrow {}^{2}_{1}H + ...$	(q)	Momentum is conserved
(C)	Fission of U^{235} nucleus initiated by a thermal neutron as given by reaction ${}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{144}_{56}Ba + {}^{89}_{36}Kr + 3{}^{1}_{0}n + ...$	(r)	Mass is converted to energy or vice versa
(D)	β^- decay (negative beta decay)	(s)	Charge is conserved
		(t)	No. of protons is decreased

SECTION - VI : INTEGER TYPE

23.42 Photons of energy 5 eV are incident on cathode as shown in the figure. Electrons reaching the anode have kinetic energies varying from 6eV to 8eV. Find the work function of the metal in ev. & state whether the current in the circuit is less than or equal to saturation current.



23.43 Consider a nuclear reaction A + B → C. A nucleus 'A' moving with kinetic energy of 5 MeV collides with a nucleus 'B' moving with kinetic energy of 3 MeV and form a nucleus 'C' in excited state. The kinetic energy of nucleus 'C' just after its formation is $\frac{265}{N}$ MeV then x is and it is formed in a state with excitation energy 10 MeV. Take masses of nuclei of A, B and C as 25.0, 10.0, 34.995 amu respectively. 1 amu = 930 MeV/c^2.

23.44 A sample has two isotopes A^{150} and B having masses 50 g and 30 g respectively. A is radioactive and B is stable. A decays to A' by emitting α particles. The half life of A is 2 hrs. Find the mass of the sample after 4 hour in gm.

23.45 A radioactive source, in the form of a metallic sphere of radius 10^{-2} m emits β–particles at the rate of 5×10^{10} particles per second. The source is electrically insulated. How long in μ -sec.will it take for its potential to be raised by 2 volt, assuming that 40% of the emitted β–particles escape the source.

23.46 magnitude of Q value of the reaction is x/10 mev. then x is

$$N^{14} + \alpha \longrightarrow O^{17} + p$$

The masses of N^{14}, He^4, H^1, O^{17} are respectively 14.00307 u, 4.00260 u, 1.00783 u and 16.99913 u.

23.47 In previous question the total kinetic energy of the products if the striking α particle has the minimum kinetic energy required to initiate the reaction is x/100 mev. then x is.

23.48 A sample of hydrogen atom gas contains 100 atoms. All the atoms are excited to the same n^{th} excited state. The total energy released by all the atoms is $\frac{4800}{49}$ Rch (where Rch = 13.6 eV), as they come to the ground state through various types of transitions. Find then maximum energy of the emitted photon is x/49 Rch. then x is :

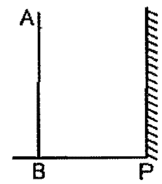
23.49 Value of 'n' in previous question.

23.50 In previous question, maximum total number of photons that can be emitted by this sample.

TOPIC 24
GEOMETRICAL OPTICS

SECTION - I : STRAIGHT OBJECTIVE TYPE

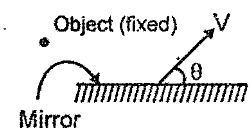
24.1 In the figure shown a person AB of height 170 cm is standing infront of a plane mirror. His eyes are at height 164 cm. At what distance from P should a hole be made in the mirror so that he cannot see the top of his head.



(A) 167 cm (B) 161 cm (C) 163 cm (D) none of these

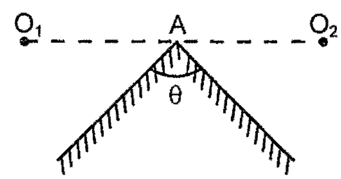
24.2 I is the image of a point object O formed by spherical mirror, then which of the following statement is incorrect :
(A) If O and I are on same side of the principal axis, then they have to be on opposite sides of the mirror.
(B) If O and I are on opposite side of the principal axis, then they have to be on same side of the mirror.
(C) If O and I are on opposite side of the principal axis, then they can be on opposite side of the mirror as well.
(D) If O is on principal axis then I has to lie on principal axis only.

24.3 An object and a plane mirror are shown in figure. Mirror is moved with velocity V as shown. The velocity of image is :

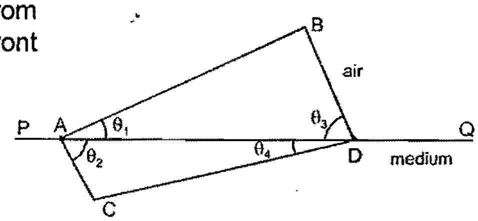


(A) 2 V sinθ (B) 2 V (C) 2V cosθ (D) none of these

24.4 Two plane mirrors are joined together as shown in the figure. Two point objects O_1 and O_2 are placed symmetrically such that $AO_1 = AO_2$. The image of the
two objects is common if :
(A) θ = 60° (B) θ = 90°
(C) θ = 30° (D) θ = 45°



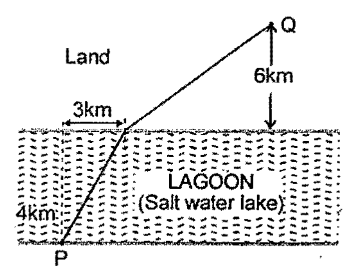
24.5 The following figure represents a wave front AB which passes from air to another transparent medium and produces a new wave front CD after refraction. The refractive index of the medium is (PQ is the boundary between air and the medium).



(A) $\frac{\cos\theta_1}{\cos\theta_4}$ (B) $\frac{\cos\theta_4}{\cos\theta_1}$

(C) $\frac{\sin\theta_1}{\sin\theta_4}$ (D) $\frac{\sin\theta_2}{\sin\theta_3}$

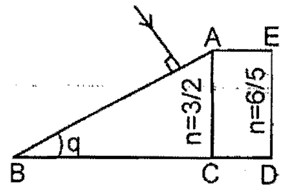
24.6 A man starting from point P crosses a 4 km wide lagoon and reaches point Q in the shortest possible time by the path shown in the figure. If the person swims at a speed of 3 km/hr and walks at a speed of 4 km/hr, then his time of journey is ($\mu_{salt\ water} = 4/3$) :
(A) 4hr, 10 min. (B) 4 hr and 30 min.
(C) 3 hr and 50 min (D) 5 hr and 10 min.



24.7 A mango tree is at the bank of a river and one of the branch of tree extends over the river. A tortoise lives in the river . A mango falls just above the tortoise. The acceleration of the mango falling from tree appearing to the tortoise is (Refractive index of water is 4/3 and the tortoise is stationary)

(A) g (B) $\frac{3g}{4}$ (C) $\frac{4g}{3}$ (D) None of these

24.8 In the figure ABC is the cross section of a right angled prism and ACDE is the cross section of a glass slab. The value of θ so that light incident normally on the face AB does not cross the face AC is (given $\sin^{-1}(3/5) = 37°$)

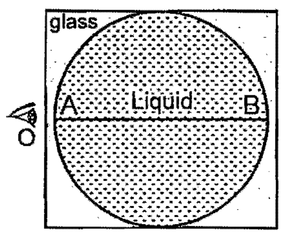


(A) $\theta \le 37°$ (B) $\theta < 37°$
(C) $\theta \le 53°$ (D) $\theta < 53°$

24.9 Refractive index of a prism is $\sqrt{\frac{7}{3}}$ and the angle of prism is 60°. The minimum angle of incidence of a ray that will be transmitted through the prism is :
(A) 30° (B) 45° (C) 15° (D) 50°

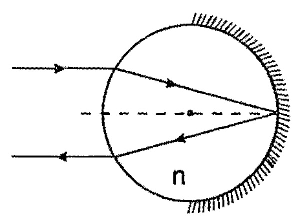
24.10 For a prism kept in air it is found that for an angle of incidence 60°, the angle of refraction 'A', angle of deviation 'δ' and angle of emergence 'e' become equal. Then the refractive index of the prism is
(A) 1.73 (B) 1.15 (C) 1.5 (D) 1.33

24.11 As shown in the figure The observer 'O' sees the distance AB as infinitely large. If refractive index of liquid is μ_1 and that of glass is μ_2, then $\frac{\mu_1}{\mu_2}$ is :



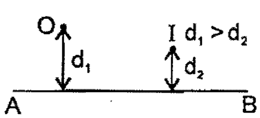
(A) 2 (B) 1/2 (C) 4 (D) None of these

24.12 A transparent cylinder has its right half polished so as to act as a mirror. A paraxial light ray is incident from left, that is parallel to principal axis, exits parallel to the incident ray as shown in the figure. The refractive index n of the material of the cylinder is :



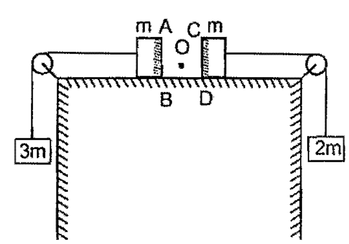
(A) 1.2 (B) 1.5
(C) 1.8 (D) 2.0

24.13 In the figure shown, the image of a real object O is formed at point I. AB is the principal axis of the mirror. The mirror must be:



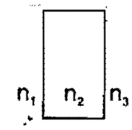
(A) concave & placed towards right of I (B) concave & placed towards left of I
(C) convex & placed towards right of I (D) convex & placed towards left of I.

24.14 Two blocks each of mass m lie on a smooth table. They are attached to two other masses as shown in the figure. The pulleys and strings are light. An object O is kept at rest on the table. The sides AB & CD of the two blocks are made reflecting. The acceleration of two images formed in those two reflecting surfaces w.r.t. each other is:



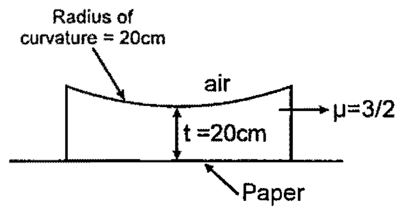
(A) 5g/6 (B) 5g/3
(C) g/3 (D) 17 g/6

24.15 The figure shows a parallel slab of refractive index n_2 which is surrounded by media of refractive indices n_1 and n_3. Light is incident on the slab at angle of incidence θ ($\neq 0$). The time taken by the ray to cross the slab is 't_1' if incidence is from 'n_1' and it is 't_2' if the incidence is from 'n_3'. Then assuming that $n_2 > n_1$, $n_2 > n_3$ and $n_3 > n_1$, then value of t_1/t_2:



(A) = 1 (B) > 1
(C) < 1 (D) cannot be decided

24.16 A planoconcave lens is placed on a paper on which a flower is drawn. How far above its actual position does the flower appear to be?

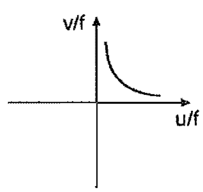


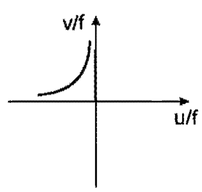
(A) 10 cm (B) 15 cm (C) 50 cm (D) none of these

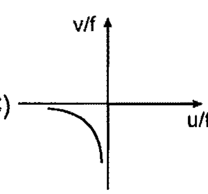
24.17 Two symmetric double convex lenses A and B have same focal length, but the radii of curvature differ so that $R_A = 0.9\, R_B$. If $n_A = 1.63$, find n_B.
(A) 1.7 (B) 1.6 (C) 1.5 (D) 4/3

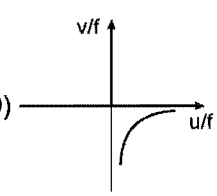
24.18 The distance between an object and the screen is 100 cm. A lens produces an image on the screen when the lens is placed at either of the positions 40 cm apart. The power of the lens is nearly:
(A) 3 diopters (B) 5 diopters (C) 2 diopters (D) 9 diopters

24.19 A virtual erect image by a **diverging lens** is represented by (u, v, f are coordinates)

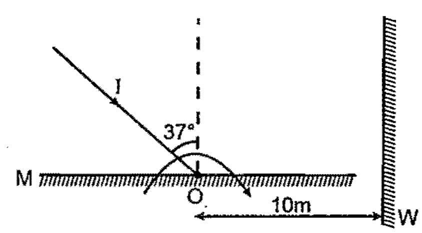
(A) 

(B) 

(C) 

(D) 

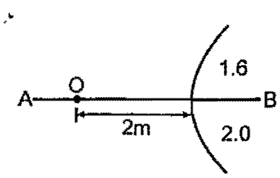
24.20 A light ray I is incident on a plane mirror M. The mirror is rotated in the direction as shown in the figure by an arrow at frequency $9/\pi$ rps. The light reflected by the mirror is received on the wall W at a distance 10 m from the axis of rotation. When the angle of incidence becomes 37° the speed of the spot (a point) on the wall is:
(A) 10 m/s (B) 1000 m/s
(C) 500 m/s (D) None of these



24.21 If a prism having refractive index $\sqrt{2}$ has angle of minimum deviation equal to the angle of refraction of the prism, then the angle of refraction of the prism is:
(A) 30° (B) 45° (C) 60° (D) 90°

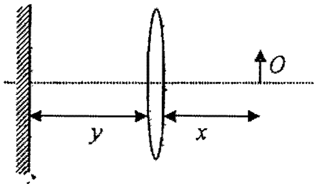
24.22 A bird is flying up at angle $\sin^{-1}(3/5)$ with the horizontal. A fish in a pond looks at that bird when it is vertically above the fish. The angle at which the bird appears to fly (to the fish) is: [$n_{water} = 4/3$]
(A) $\sin^{-1}(3/5)$ (B) $\sin^{-1}(4/5)$ (C) 45° (D) $\sin^{-1}(9/16)$

24.23 In the figure shown a point object O is placed in air. A spherical boundary separates two media. AB is principal axis. The refractive index above AB is 1.6 and below AB is 2.0. The separation between the images formed due to refraction at spherical surface is :
(A) 12 m (B) 21 m
(C) 14 m (D) 10 m



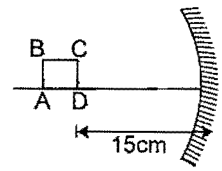
24.24 Light of wavelength 4000 Å is incident at small angle on a prism of apex angle 4°. The prism has n_v = 1.5 & n_r = 1.48. The angle of dispersion produced by the prism in this light is :
(A) 0.2°   (B) 0.08°   (C) 0.192°   (D) none of these

24.25 A convex lens of focal length f and a plane mirror are y distance apart. An object O is kept on the principal axis of the lens at a distance x from the lens. The values of x and y for the final image of O to fall exactly **(position & size)** on the object 'O' :



(A) x = f, y = f   (B) x = f, y = 2f
(C) x = 2f, y = f   (D) x = 2f, y = 2f

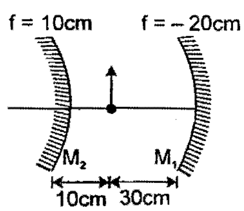
24.26 A square ABCD of side 1mm is kept at distance 15 cm infront of the concave mirror as shown in the figure. The focal length of the mirror is 10 cm. The length of the perimeter of its image will be :



(A) 8 mm   (B) 2 mm
(C) 12 mm   (D) 6 mm

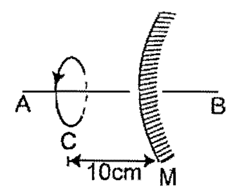
24.27 A point object is kept in front of a plane mirror. The plane mirror is doing SHM of amplitude 2 cm. The plane mirror moves along the x-axis and x- axis is normal to the mirror. The amplitude of the mirror is such that the object is always infront of the mirror. The amplitude of SHM of the image is
(A) zero   (B) 2 cm   (C) 4 cm   (D) 1 cm

24.28 In the figure shown find the total magnification after two successive reflections first on M_1 & then on M_2



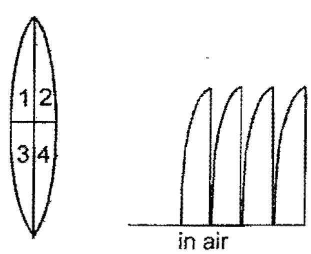
(A) + 1   (B) – 2   (C) + 2   (D) – 1

24.29 A particle revolves in clockwise direction (as seen from point A) in a circle C of radius 1 cm and completes one revolution in 2 sec. The axis of the circle and the principal axis of the mirror M coincide. Call it AB. The radius of curvature of the mirror is 21 cm. Then the direction of revolution (as seen from A) of the image of the particle and its speed is



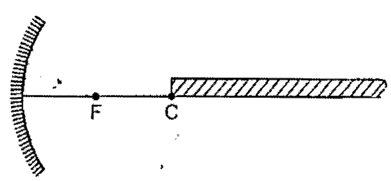
(A) Clockwise , 1.57 cm/s   (B) Clockwise , 3.14 cm/s
(C) Anticlockwise , 1.57 cm/s   (D) Anticlockwise , 3.14 cm/s

24.30 The given lens is broken into four parts and rearranged as shown in the figure. If the initial focal length is f then after rearrangement the equivalent focal length is



(A) f   (B) $\frac{f}{2}$
(C) $\frac{f}{4}$ (D) 4f

24.31 An infinitely long rectangular strip is placed on the principal axis of a concave mirror as shown in the figure. One end of the strip coincides with centre of curvature as shown. The height of rectangular strip is very small in comparison to focal length of the mirror. Then the shape of image of strip formed by concave mirror is :



(A) Rectangle   (B) Trapezium   (C) Triangle   (D) Square

24.32 Figure I given below shows a glass vessel, partially filled with water. A narrow beam of light is incident vertically down into the water and passes straight through. Figure II shows the vessel glass tilted until the angle θ, such that the light is refracted along the lower surface of the glass. If refractive indices of air, water and glass are 1, $\frac{4}{3}$ and 1.5 respectively then :

(A) $\sin\theta = \frac{3}{4}$ (B) $\cos\theta = \frac{3}{4}$

(C) $\sin\theta = \frac{8}{9}$ (D) $\cos\theta = \frac{8}{9}$

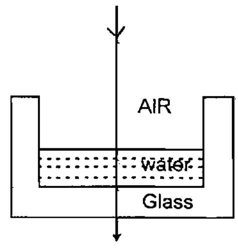


Figure 1

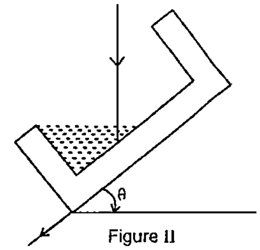


Figure II

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

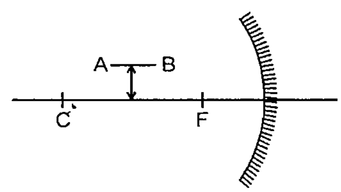
24.33 A particle is moving towards a fixed convex mirror. The image also moves. If V_i = speed of image and V_o = speed of the object, then

(A) $V_i \le V_o$ if $|u| < |F|$ (B) $V_i > V_o$ if $|u| > |F|$

(C) $V_i < V_o$ if $|u| > |F|$ (D) $V_i = V_o$ if $|u| = |F|$

24.34 An object AB is placed parallel and close to the optical axis between focus F and centre of curvature C of a converging mirror of focal length f as shown in figure.

(A) Image of A will be closer than that of B from the mirror.
(B) Image of AB will be parallel to the optical axis.
(C) Image of AB will be straight line inclined to the optical axis.
(D) Image of AB will not be straight line.



24.35 Which of the following statements is/are correct about the refraction of light from a plane surface when light ray is incident in denser medium. [C is critical angle]

(A) The maximum angle of deviation during refraction is $\frac{\pi}{2} - C$, it will be at angle of incidence C.

(B) The maximum angle of deviation for all angle of incidences is $\pi - 2C$, when angle of incidence is slightly greater than C.

(C) If angle of incidence is less than C then deviation increases if angle of incidence is also increased.

(D) If angle of incidence is greater than C then angle of deviation decreases if angle of incidence is increased.

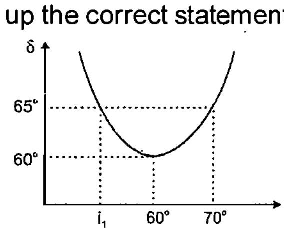
24.36 The angle of deviation (δ) vs angle of incidence (i) is plotted for a prism. Pick up the correct statements.

(A) The angle of prism is 60°

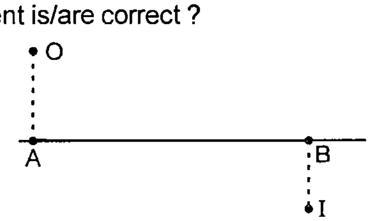
(B) The refractive index of the prism is $n = \sqrt{3}$

(C) For deviation to be 65° the angle of incidence i_1 = 55°

(D) The curve of 'δ' vs 'i' is parabolic



24.37 A luminous point object is placed at O, whose image is formed at I as shown in figure. Line AB is the optical axis. Which of the following statement is/are correct ?



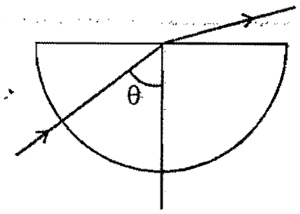
(A) If a lens is used to obtain the image, then it must be a converging lens and its optical centre will be the intersection point of line AB and OI.

(B) If a lens is used to obtain the image, then it must be a diverging lens and its optical centre will be the intersection point of line AB and OI.

(C) If a mirror is used to obtain the image then the mirror must be concave and object and image subtend equal angles at the pole of the mirror.

(D) I is a real Image.

SECTION - III : ASSERTION AND REASON TYPE



24.38 **Statement-1 :** A beam of white light enters the curved surface of a semicircular piece of glass along the normal. The incoming beam is moved clockwise (so that the angle θ increases), such that the beam always enters along the normal to the curved side. Just before the refracted beam disappears, it becomes predominantly red.

Statement-2 : The index of refraction for light at the red end of the visible spectrum is more than at the violet end.

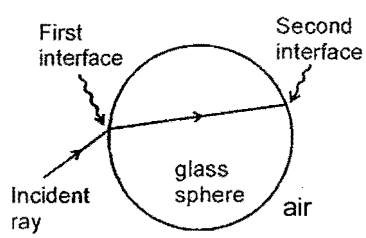
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

24.39 **Statement-1 :** There exists two angles of incidence for the same magnitude of deviation (except minimum deviation) by a prism kept in air.

Statement-2 : In a prism kept in air, a ray is incident on first surface and emerges out of second surface. Now if another ray is incident on second surface (of prism) along the previous emergent ray, then this ray emerges out of first surface along the previous incident ray. This principle is called principle of reversibility of light.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

24.40 **Statement-1 :** A ray is incident from outside on a glass sphere surrounded by air as shown. This ray may suffer total internal reflection at second interface.



Statement 2 : For a ray going from denser to rarer medium, the ray may suffer total internal reflection.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True.

24.41 **Statement-1 :** Keeping a point object fixed, if a plane mirror is moved, the image will also move.

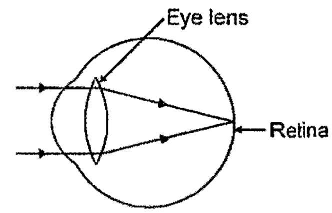
Statement-2 : In case of a plane mirror, distance of object and its image is equal from any point on the mirror.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

SECTION - IV : COMPREHENSION TYPE

Comprehension # 1

The ciliary muscles of eye control the curvature of the lens in the eye and hence can alter the effective focal length of the system. When the muscles are fully relaxed, the focal length is maximum. When the muscles are strained the curvature of lens increases (that means radius of curvature decreases) and focal length decreases. For a clear vision the image must be on retina. The image distance is therefore fixed for clear vision and it equals the distance of retina from eye-lens. It is about 2.5 cm for a grown-up person.



A person can theoretically have clear vision of objects situated at any large distance from the eye. The smallest distance at which a person can clearly see is related to minimum possible focal length. The ciliary muscles are most strained in this position. For an average grown-up person minimum distance of object should be around 25 cm.

A person suffering for eye defects uses spectacles (Eye glass). The function of lens of spectacles is to form the image of the objects within the range in which person can see clearly. The image of the spectacle-lens becomes object for eye-lens and whose image is formed on retina.

The number of spectacle-lens used for the remedy of eye defect is decided by the power of the lens required and the number of spectacle-lens is equal to the numerical value of the power of lens with sign. For example power of lens required is +3D (converging lens of focal length $\frac{100}{3}$ cm) then number of lens will be +3.

For all the calculations required you can use the lens formula and lens maker's formula. Assume that the eye lens is equiconvex lens. Neglect the distance between eye lens and the spectacle lens.

24.42 Minimum focal length of eye lens of a normal person is

(A) 25 cm (B) 2.5 cm (C) $\frac{25}{9}$ cm (D) $\frac{25}{11}$ cm

24.43 Maximum focal length of eye lens of normal person is

(A) 25 cm (B) 2.5 cm (C) $\frac{25}{9}$ cm (D) $\frac{25}{11}$ cm

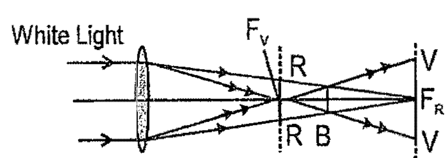
24.44 A nearsighted man can clearly see object only upto a distance of 100 cm and not beyond this. The number of the spectacles lens necessary for the remedy of this defect will be.

(A) +1 (B) – 1 (C) + 3 (D) – 3

Comprehension # 2

Chromatic Aberration

The image of a white object in white light formed by a lens is usually coloured and blurred. This defect of image is called chromatic aberration and arises due to the fact that focal length of a lens is different for different colours. As μ of lens is maximum for violet while minimum for red, violet is focused nearest to the lens while red farthest from it as shown in figure.



As a result of this, in case of convergent lens if a screen is placed at F_V centre of the image will be violet and focused while sides are red and blurred. While at F_R, reverse is the case, i.e., centre will be red and focused while sides violet and blurred. The difference between f_V and f_R is a measure of the longitudinal chromatic aberration (L.C.A), i.e.,

$$L.C.A. = f_R - f_V = -df \text{ with } df = f_V - f_R \qquad ....(1)$$

However, as for a single lens,

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \qquad ....(2)$$

$$\Rightarrow \qquad -\frac{df}{f^2} = d\mu\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \qquad ....(3)$$

Dividing Eqn. (3) by (2);

$$-\frac{df}{f} = \frac{d\mu}{(\mu-1)} = \omega \qquad \left[\omega = \frac{d\mu}{(\mu-1)}\right] = \text{dispersive power} \qquad ....(4)$$

And hence, from Eqns. (1) and (4),

$$L.C.A. = -df = \omega f$$

Now, as for a single lens neither f nor ω can be zero, we cannot have a single lens free from chromatic aberration.

Condition of Achromatism :

In case of two thin lenses in contact

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} \qquad \text{i.e.,} \qquad -\frac{dF}{F^2} = -\frac{df_1}{f_1^2} - \frac{df_2}{f_2^2}$$

The combination will be free from chromatic aberration if dF = 0

$$\text{i.e.,} \qquad \frac{df_1}{f_1^2} + \frac{df_2}{f_2^2} = 0$$

which with the help of Eqn. (4) reduces to

$$\frac{\omega_1 f_1}{f_1^2} + \frac{\omega_2 f_2}{f_2^2} = 0 \qquad \text{i.e.,} \qquad \frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0 \qquad ....(5)$$

This condition is called condition of achromatism (for two thin lenses in contact) and the lens combination which satisfies this condition is called achromatic lens, From this condition, i.e., from Eqn. (5) it is clear that in case of achromatic doublet :

(1) The two lenses must be of different materials.

Since, if $\omega_1 = \omega_2$, $\frac{1}{f_1} + \frac{1}{f_2} = 0$ i.e., $\frac{1}{F} = 0$ or $F = \infty$

i.e., combination will not behave as a lens, but as a plane glass plate.

(2) As ω_1 and ω_2 are positive quantities, for Eqn. (5) to hold, f_1 and f_2 must be of opposite nature, i.e., if one of the lenses is converging the other must be diverging.

(3) If the achromatic combination is convergent,

$$f_C < f_D$$

and as $-\frac{f_C}{f_D} = \frac{\omega_C}{\omega_D}$, $\omega_C < \omega_D$

i.e., in a convergent achromatic doublet, convex lens has lesser focal length and dispersive power than the divergent one.

24.45 Chromatic aberration in the formation of images by a lens arises because :
(A) of non-paraxial rays.
(B) the radii of curvature of the two sides are not same.
(C) of the defect in grinding.
(D) the focal length varies with wavelength.

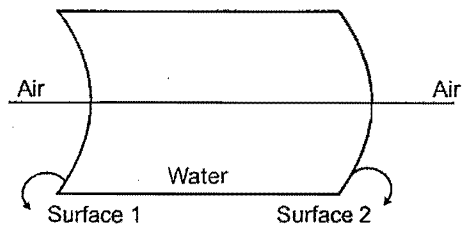
24.46 A combination is made of two lenses of focal lengths f and f' in contact ; the dispersive powers of the materials of the lenses are ω and ω'. The combination is achromatic when :
(A) $\omega = \omega_0$, $\omega' = 2\omega_0$, $f' = 2f$
(B) $\omega = \omega_0$, $\omega' = 2\omega_0$, $f' = f/2$
(C) $\omega = \omega_0$, $\omega' = 2\omega_0$, $f' = -f/2$
(D) $\omega = \omega_0$, $\omega' = 2\omega_0$, $f' = -2f$

24.47 The dispersive power of crown and flint glasses are 0.02 and 0.04 respectively. An achromatic converging lens of focal length 40 cm is made by keeping two lenses, one of crown glass and the other of flint glass, in contact with each other. The focal lengths of the two lenses are :

(A) 21 cm and 40 cm (B) 21 cm and –40 cm (C) –21cm and 40 cm (D) 10 cm and –21cm

Comprehension # 3

All objects referred to the subsequent problems lie on the principle axis.



24.48 If light is incident on surface 1 from left, the image formed after the first refraction is definitely :

(A) Real for a real object (B) Virtual for a real object
(C) Real for a virtual object (D) Virtual for a virtual object

24.49 In above question if the object is real, then the final image formed after two refractions :

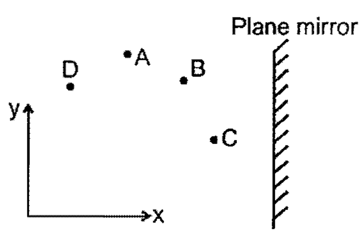
(A) may be real (B) may be virtual (C) must be virtual (D) both A and B

24.50 If light is incident on surface 2 from right then which of the following is true for image formed after a single refraction.

(A) Real object will result in a real image (B) Virtual object will result in a virtual image
(C) Real object will result in a virtual image (D) Virtual object will result in a Real image

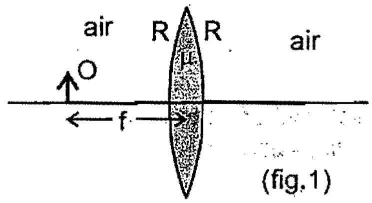
SECTION - V : MATRIX - MATCH TYPE

24.51 Four particles are moving with different velocities in front of stationary plane mirror (lying in y-z plane). At t = 0, velocity of A is $\vec{v}_A = \hat{i}$, velocity of B is $\vec{v}_B = -\hat{i} + 3\hat{j}$, velocity of C is $\vec{v}_C = 5\hat{i} + 6\hat{j}$, velocity of D is $\vec{v}_D = 3\hat{i} - \hat{j}$. Acceleration of particle A is $\vec{a}_A = 2\hat{i} + \hat{j}$ and acceleration of particle C is $\vec{a}_C = 2t\hat{j}$. The particle B and D move with uniform velocity (Assume no collision to take place till t = 2 seconds). All quantities are in S.I. Units. Relative velocity of image of object A with respect to object A is denoted by $\vec{V}_{A',A}$. Velocity of images relative to corresponding objects are given in column I and their values are given in column II at t = 2 second. Match column I with corresponding values in column II.



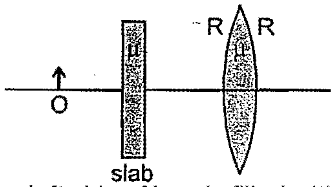
	Column-I		Column-II
(A)	$\vec{V}_{A',A}$	(p)	$2\hat{i}$
(B)	$\vec{V}_{B',B}$	(q)	$-6\hat{i}$
(C)	$\vec{V}_{C',C}$	(r)	$-12\hat{i} + 4\hat{j}$
(D)	$\vec{V}_{D',D}$	(s)	$-10\hat{i}$

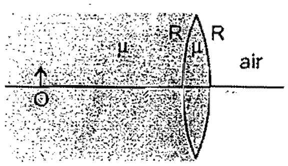
24.52 An object O (real) is placed at focus of an equi-biconvex lens as shown in figure 1. The refractive index of lens is $\mu = 1.5$ and the radius of curvature of either surface of lens is R. The lens is surrounded by air. In each statement of column-I some changes are made to situation given above and information regarding final image formed as a result is given in column-II. The distance between lens and object is unchanged in all statements of column-I. Match the statements in column-I with resulting image in column-II.



(fig.1)

	Column-I		Column-II
(A)	If the refractive index of the lens is doubled (that is, made 2μ) then	(p)	final image is real
(B)	If the radius of curvature is doubled (that is, made 2R) then	(q)	final image is virtual
(C)	If a glass slab of refractive index $\mu = 1.5$ is introduced between the object and lens as shown, then	(r)	final image becomes smaller in size in comparison to size of image before the change was made
(D)	If the left side of lens is filled with a medium of refractive index $\mu = 1.5$ as shown, then (t)	(s)	final image is of same size of object.
		(t)	final image is of larger size of object





24.53 **Match the following :**

An object O is kept perpendicular to the principal axis of a spherical mirror. Each situation (A,B,C and D) gives object coordinate u in centimeters with sign, the type of mirror, and then the distance (centimeters, without sign) between the focal point and the pole of the mirror. On the right side information regarding the image is given. Correctly match the situations on the left side with the images described on the right side.

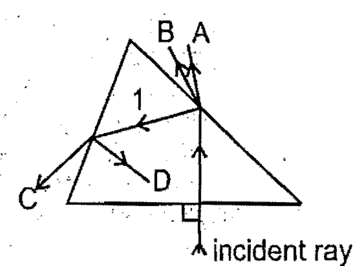
Column-I

Situation	u	Mirror
A	–18	Concave, 12
B	–12	Concave, 18
C	–8	Convex, 10
D	–10	Convex, 8

Column-II

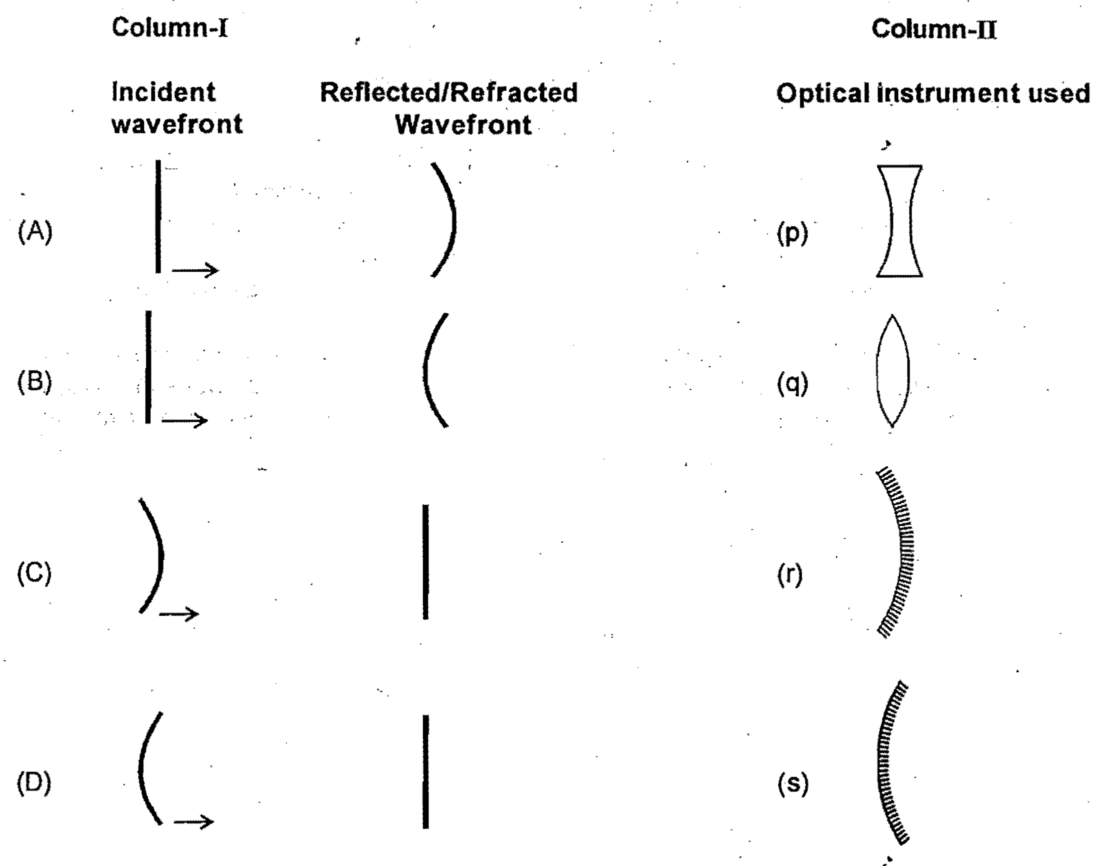
	Image
(p)	Real, Errect, Enlarged
(q)	Virtual, Errect, Diminished
(r)	Real Inverted, Enlarged
(s)	Virtual, Erect, Enlarged

24.54 A white light ray is incident on a glass prism, and it create four refracted rays A, B, C and D. Match the refracted rays with the colours given (1 & D are rays due to total internal reflection.):

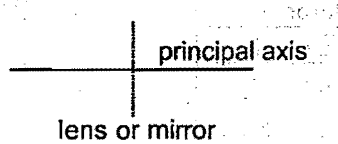


Column-I Ray		**Column-II** Colour	
(A)	A	(p)	red
(B)	B	(q)	green
(C)	C	(r)	yellow
(D)	D	(s)	blue

24.55 In each situation of column-I, an incident wavefront and its corresponding reflected or refracted wavefront is shown. In column-II the optical instrument used for reflection or refraction is given. Always take the optical instrument to the right of incident wavefront. The incident wavefront is moving towards right. Match each pair of incident and reflected/refracted wavefront in column-I with the correct optical instrument given in column-II.



24.56 Column-I gives certain situations regarding a point object and its image formed by an optical instrument. The possible optical instruments are concave and convex mirrors or lenses as given in Column-II. Same side of principal axis means both image and object should either be above the principal axis or both should be below the principal axis as shown in figure. Same side of optical instrument means both image and object should be either left of the optical instrument or both should be on right of the optical instrument as shown in figure. Match the statements in column-I with the corresponding statements in column-II.



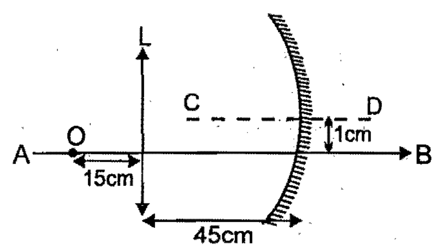
Column I	Column II
(A) If a point object and its image are on same side of principal axis and opposite sides of the optical instrument then the optical instrument is	(p) Concave mirror
(B) If a point object and its image are on opposite side of principal axis and same sides of the optical instrument then the optical instrument is	(q) Convex mirror
(C) If a point object and its image are on same side of principal axis and same sides of the optical instrument then the optical instrument is	(r) Concave lens
(D) If a point object and its image are on opposite side of principal axis and opposite sides of the optical instrument then the optical instrument is	(s) Convex lens

SECTION - VI : INTEGER TYPE

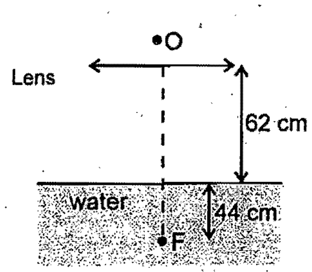
24.57 As shown in the figure, an object O is at the position (– 10, 2) with respect to the origin P. The concave mirror M_1 has radius of curvature 30 cm. A plane mirror M_2 is kept at a distance 40 cm infront of the concave mirror. Considering first reflection on the concave mirror M_1 and second on the plane mirror M_2. if x co-ordinate is $-x_0$ of the second image w.r.t. the origin P.then x_0 is

24.58 A ray of light travelling in air is incident at angle of incident 30° on one surface of a slab in which refractive index varies with y. The light travels along the curve $y = 4x^2$ (y and x are in metre) in the slab. if the refractive index of the slab at y = 1/2 m in the slab is $15/x_0$.then x_0 is :

24.59 In the figure shown L is a converging lens of focal length 10cm and M is a concave mirror of radius of curvature 21cm. A point object O is placed in front of the lens at a distance 15cm. AB and CD are optical axes of the lens and mirror respectively. if the distance of the final image formed by this system from the optical centre of the lens is $6\sqrt{x}$ cm. then x is The distance between CD & AB is 1 cm.



24.60 A stationary observer O looking at a fish F (in water of, $\mu = 4/3$) through a converging lens of focal length 90.0 cm. The lens is allowed to fall freely from a height 62.0 cm with its axis vertical. The fish and the observer are on the principal axis of the lens. The fish moves up with constant velocity 100 cm/s. Initially it was at a depth of 44.0 cm. if the velocity with which the fish appears to move to the observer at t = 0.2 sec.is x/4 cms^{-1}.then x is ($g = 10\ m/s^2$)



TOPIC 25

WAVE OPTICS

SECTION - I : STRAIGHT OBJECTIVE TYPE

25.1 Two light waves are given by, $E_1 = 2\sin(100\pi t - kx + 30^\circ)$ and $E_2 = 3\cos(200\pi t - k'x + 60^\circ)$
The ratio of intensity of first wave to that of second wave is :

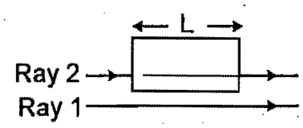
(A) $\frac{2}{3}$ (B) $\frac{4}{9}$ (C) $\frac{1}{9}$ (D) $\frac{1}{3}$

25.2 The wavefront of a light beam is given by the equation $x + 2y + 3z = c$, (where c is arbitrary constant) then the angle made by the direction of light with the y-axis is :

(A) $\cos^{-1}\frac{1}{\sqrt{14}}$ (B) $\cos^{-1}\frac{2}{\sqrt{14}}$ (C) $\sin^{-1}\frac{1}{\sqrt{14}}$ (D) $\sin^{-1}\frac{2}{\sqrt{14}}$

25.3 If the ratio of the intensity of two coherent sources is 4 then the visibility $[(I_{max} - I_{min})/(I_{max} + I_{min})]$ of the fringes is
(A) 4 (B) 4/5 (C) 3/5 (D) 9

25.4 As shown in arrangement waves with identical wavelengths and amplitudes and that are initially in phase travel through different media, Ray 1 travels through air and Ray 2 through a transparent medium for equal length L, in four different situations. In each situation the two rays reach a common point on the screen. The number of wavelengths in length L is N_2 for Ray 2 and N_1 for Ray 1. In the following table, values of N_1 and N_2 are given for all four situations, The order of the situations according to the intensity of the light at the common point in descending order is :



Situations	1	2	3	4
N_1	2.25	1.8	3	3.25
N_2	2.75	2.8	2.25	4

(A) $I_3 = I_4 > I_2 > I_1$ (B) $I_1 > I_3 = I_4 > I_2$ (C) $I_1 > I_2 > I_3 > I_4$ (D) $I_2 > I_3 = I_4 > I_1$

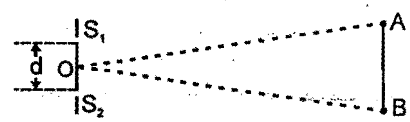
25.5 If the distance between the first maxima and fifth minima of a double slit pattern is 7mm and the slits are separated by 0.15 mm with the screen 50 cm from the slits, then wavelength of the light used is :
(A) 600 nm (B) 525 nm (C) 467 nm (D) 420 nm

25.6 In a YDSE: D = 1 m, d = 1 mm and λ = 5000 n m. The distance of 100th maxima from the central maxima is:

(A) $\frac{1}{2}$ m (B) $\frac{\sqrt{3}}{2}$ m (C) $\frac{1}{\sqrt{3}}$ m (D) does not exist

25.7 Let S_1 and S_2 be the two slits in Young's double slit experiment. If central maxima is observed at P and angle $\angle S_1PS_2 = \theta$, then the fringe width for the light of wavelength λ will be. (Assume θ to be a small angle)
(A) λ/θ (B) $\lambda\theta$ (C) $2\lambda/\theta$ (D) $\lambda/2\theta$

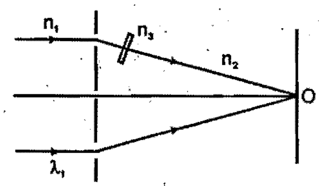
25.8 Figure shows two coherent sources S_1, S_2 vibrating in same phase. AB is an irregular wire lying at a far distance from the sources S_1 and S_2. Let $\frac{\lambda}{d} = 10^{-3}$. $\angle BOA = 0.12°$. How many bright spots will be seen on the wire, including points A and B.



(A) 2 (B) 3 (C) 4 (D) more than 4

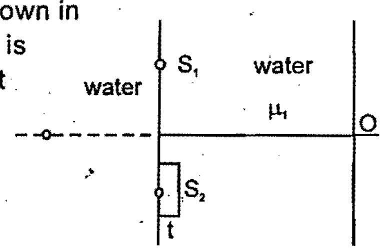
25.9 The path difference between two interfering waves at a point on the screen is λ/6. The ratio of intensity at this point and that at the central bright fringe will be : (Assume that intensity due to each slit in same)
(A) 0.853 (B) 8.53 (C) 0.75 (D) 7.5

25.10 In the figure shown in a YDSE, a parallel beam of light is incident on the slits from a medium of refractive index n_1. The wavelength of light in this medium is λ_1. A transparent slab of thickness 't' and refractive index n_3 is put infront of one slit. The medium between the screen and the plane of the slits is n_2. Find the phase difference between the light waves reaching point 'O' (symmetrical, relative to the slits)



(A) $\frac{2\pi}{n_1\lambda_1}(n_3 - n_2)t$ (B) $\frac{2\pi}{\lambda_1}(n_3 - n_2)t$ (C) $\frac{2\pi n_1}{n_2\lambda_1}\left(\frac{n_3}{n_2}-1\right)t$ (D) $\frac{2\pi n_1}{\lambda_1}(n_3 - n_2)t$

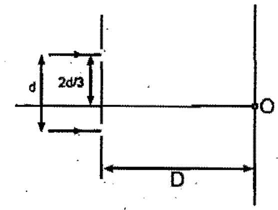
25.11 A Young's double slit experiment is conducted in water (μ_1) as shown in the figure, and a glass plate of thickness t and refractive index μ_2 is placed in the path of S_2. The magnitude of the phase difference at O is : (Assume that 'λ' is the wavelength of light in air)



(A) $\left|\left(\frac{\mu_2}{\mu_1}-1\right)t\right|\frac{2\pi}{\lambda}$ (B) $\left|\left(\frac{\mu_1}{\mu_2}-1\right)t\right|\frac{2\pi}{\lambda}$

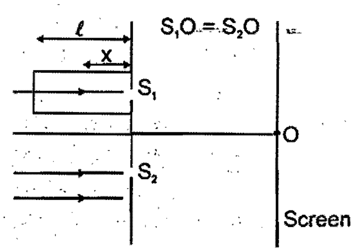
(C) $|(\mu_2 - \mu_1)t|\frac{2\pi}{\lambda}$ (D) $|(\mu_2 - 1)t|\frac{2\pi}{\lambda}$

25.12 In the figure shown if a parallel beam of white light is incident on the plane of the slits then the distance of the nearest white spot on the screen from O is: [assume d << D, λ << d]



(A) 0 (B) d/2
(C) d/3 (D) d/6

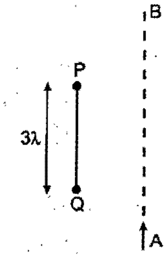
25.13 In the figure shown, a parallel beam of light is incident on the plane of the slits of a Young's double slit experiment. Light incident on the slit, S_1 passes through a medium of variable refractive index $\mu = 1 + ax$ (where 'x' is the distance from the plane of slits as shown), upto a distance 'ℓ' before falling on S_1. Rest of the space is filled with air. If at 'O' a minima is formed, then the minimum value of the positive constant a (in terms of ℓ and wavelength 'λ' in air) is :



(A) $\frac{\lambda}{\ell}$ (B) $\frac{\lambda}{\ell^2}$ (C) $\frac{\ell^2}{\lambda}$ (D) None of these

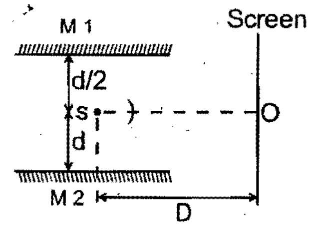
25.14 Interference fringes were produced using white light in a double slit arrangement. When a mica sheet of uniform thickness of refractive index 1.6 (relative to air) is placed in the path of light from one of the slits, the central fringe moves through some a distance. This distance is equal to the width of 30 interference bands if light of wavelength 4800 Å is used. The thickness (in 1μm) of mica is:
(A) 90 (B) 12 (C) 14 (D) 24

25.15 Two coherent light sources each of wavelength λ are separated by a distance 3λ. The total number of minima formed on line AB which runs from $-\infty$ to $+\infty$ is:



(A) 2 (B) 4 (C) 6 (D) 8

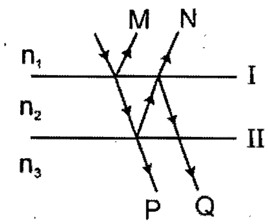
25.16 M1 and M2 are two plane mirrors which are kept parallel to each other as shown. There is a point 'O' on perpendicular screen just infront of 'S'. What should be the wavelength of light coming from monchromatic source 'S'. So that a maxima is formed at 'O' due to interference of reflected light from both the mirrors. [Consider only 1st reflection].



(A) $\frac{3d^2}{D}$ (B) $\frac{3d^2}{2D}$

(C) $\frac{d^2}{D}$ (D) $\frac{2d^2}{D}$

25.17 A ray of light is incident on a thin film. As shown in figure M,N are two reflected rays and P,Q are two transmitted rays. Rays N and Q undergo a phase change of π. Correct order of the refracting indices is:



(A) $n_2 > n_3 > n_1$

(B) $n_3 > n_2 > n_1$

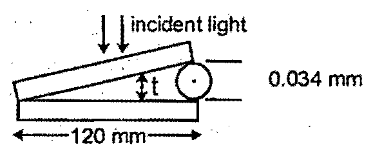
(C) $n_3 > n_1 > n_2$

(D) None of these, the specified changes can not occur

25.18 From a medium of index of refraction n_1, monochromatic light of wavelength λ is incident normally on a thin film of uniform thickness L (where $L > 0.1\lambda$) and index of refraction n_2. The light transmitted by the film travels into a medium with refractive index n_3. The value of minimum film thickness when maximum light is transmitted if ($n_1 < n_2 < n_3$) is :

(A) $\frac{n_1\lambda}{2n_2}$ (B) $\frac{n_1\lambda}{4n_2}$ (C) $\frac{\lambda}{4n_2}$ (D) $\frac{\lambda}{2n_2}$

25.19 A broad source of light ($l = 680$ nm) illuminates normally two glass plates 120 mm long that touch at one end and are separated by a wire 0.034 mm in diameter at the other end. The total number of bright fringes that appear over the 120 mm distance is :



(A) 50 (B) 100 (C) 200 (D) 400

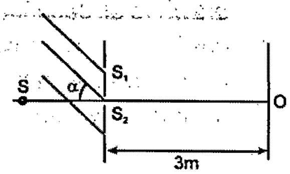
SECTION - II : MULTIPLE CORRECT ANSWER TYPE

25.20 If the first minima in a Young's slit experiment occurs directly in front of one of the slits, (distance between slit & screen D = 12 cm and distance between slits d = 5 cm) then the wavelength of the radiation used can be :

(A) 2 cm (B) 4 cm (C) $\frac{2}{3}$ cm (D) $\frac{4}{3}$ cm

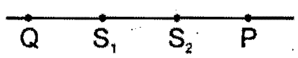
25.21 If one of the slits of a standard Young's double slit experiment is covered by a thin parallel sided glass slab so that it transmits only one half the light intensity of the other, then:
(A) The fringe pattern will get shifted towards the covered slit
(B) The fringe pattern will get shifted away from the covered slit
(C) The bright fringes will become less bright and the dark ones will become more bright
(D) The fringe width will remain unchanged

25.22 A parallel beam of light (λ =5000 Å) is incident at an angle α = 30° with the normal to the slit plane in a young's double slit experiment. Assume that the intensity due to each slit at any point on the screen is I_0. Point O is equidistant from S_1 & S_2. The distance between slits is 1mm.
(A) the intensity at O is $4I_0$
(B) the intensity at O is zero
(C) the intensity at a point on the screen 4m from O is $4I_0$
(D) the intensity at a point on the screen 4m from O is zero



SECTION - III : ASSERTION AND REASON TYPE

25.23 **Statement-1 :** Two point coherent sources of light S_1 and S_2 are placed on a line as shown. P and Q are two points on that line. If at point P maximum intensity is observed then maximum intensity should also be observed at Q.



Statement-2 : In the figure of statment 1, the distance $|S_1P - S_2P|$ is equal to distance $|S_2Q - S_1Q|$.
(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True.

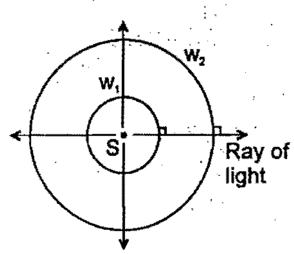
25.24 **Statement-1 :** Two coherent point sources of light having nonzero phase difference are seperated by small distance. Then on the perpendicular bisector of line segment joining both the point sources, constructive interference cannot be obtained.
Statement-2 : For two waves from coherent point sources to interfere constructively at a point, the magnitude of their phase difference at that point must be $2m\pi$ (where m is a nonnegative integer).
(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True.

SECTION - IV : COMPREHENSION TYPE

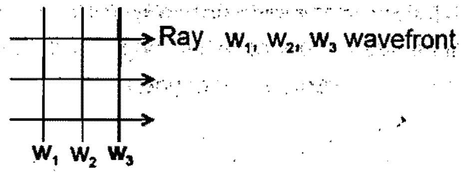
Read the following comprehensions carefully and answer the questions.

Comprehension # 1

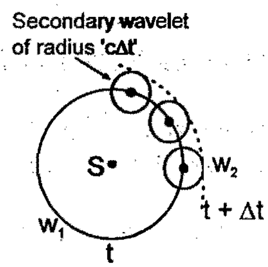
Huygen was the first scientist who proposed the idea of wave theory of light. He said that the light propagates in form of wavefronts. A wavefront is an imaginary surface at every point of which waves are in the same phase. For example the wavefronts for a point source of light is collection of concentric spheres which have centre at the origin, w_1 is a wavefront, w_2 is another wavefront.



The radius of the wavefront at time 't' is 'ct' in this case where 'c' is the speed of light. The direction of propagation of light is perpendicular to the surface of the wavefront. The wavefronts are plane wavefronts in case of a parallel beam of light.



Huygen also said that every point of the wavefront acts as the source of secondary wavelets. The tangent drawn to all secondary wavelets at a time is the new wavefront at that time. The wavelets are to be considered only in the forward direction (i.e. the direction of propagation of light) and not in the reverse direction. If a wavefront w_1 at time t is given, then to draw the wavefront at time $t + \Delta t$ take some points on the wavefront w_1 and draw spheres of radius '$c\Delta t$'. They are called secondary wavelets.



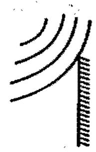
Draw a surface w_2 which is tangential to all these secondary wavelets w_2 is the wavefront at time '$t + \Delta t$'.

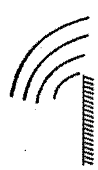
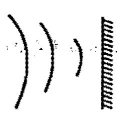
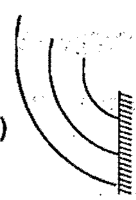
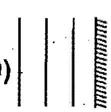
Huygen proved the laws of reflection and laws of refraction using concept of wavefronts.

25.25 A point source of light is placed at origin, in air. The equation of wave front of the wave at time t, emitted by source at t = 0, is (take refractive index of air as 1)

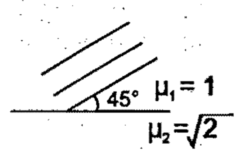
(A) $x + y + z = ct$ (B) $x^2 + y^2 + z^2 = t^2$ (C) $xy + yz + zx = c^2t^2$ (D) $x^2 + y^2 + z^2 = c^2t^2$

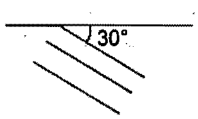
25.26 Spherical wave fronts shown in figure, strike a plane mirror. Reflected wave fronts will be as shown in

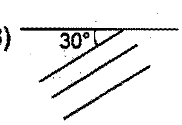


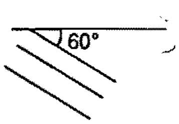
(A)  (B)  (C)  (D) 

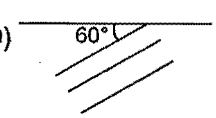
25.27 Wavefronts incident on an interface between the media are shown in the figure. The refracted wavefronts will be as shown in



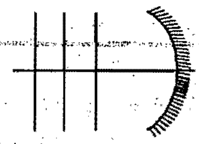
(A) 

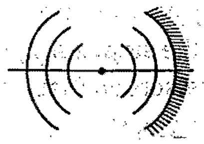
(B) 

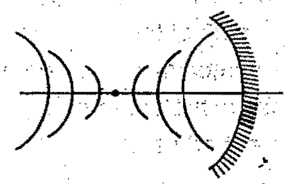
(C) 

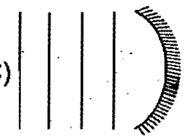
(D) 

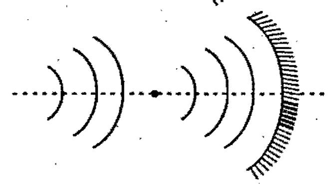
25.28 Plane wavefronts are incident on a spherical mirror as shown. The reflected wavefronts will be in the figure



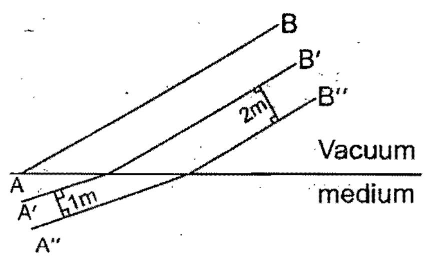
(A) 

(B) 

(C) 

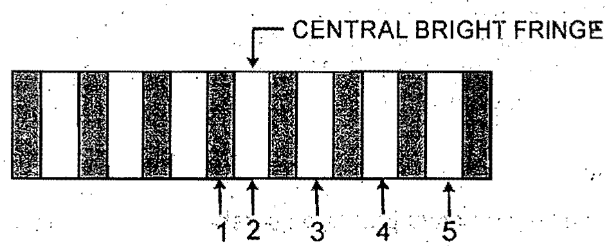
(D) 

25.29 Certain plane wavefronts are shown in figure. The refractive index of medium is



(A) 2 (B) 4 (C) 1.5 (D) Cannot be determined

Comprehension # 2



The figure shows the interference patttern obtained in a double-slit experiment using light of wavelength 600 nm. 1, 2, 3, 4 and 5 are marked on five fringes.

25.30 The third order bright fringe is

(A) 2 (B) 3
(C) 4 (D) 5

25.31 Which fringe results from a phase difference of 4π between the light waves incidenting from two slits ?

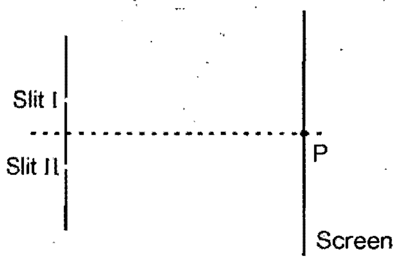
(A) 2 (B) 3
(C) 4 (D) 5

25.32 Let ΔX_A and ΔX_C represent path differences between waves interfering at 1 and 3 respectively then $(|\Delta X_C| - |\Delta X_A|)$ is equal to

(A) 0 (B) 300mn
(C) 600 nm (D) 900nm.

SECTION - V : MATRIX - MATCH TYPE

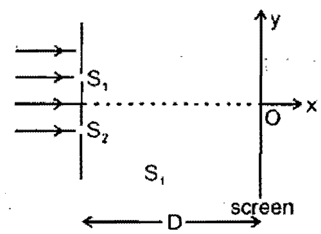
25.33 A double slit interference pattern is produced on a screen, as shown in the figure, using monochromatic light of wavelength 500 nm. Point P is the location of the central bright fringe, that is produced when light waves arrive in phase without any path difference. A choice of three strips A, B and C of transparent materials with different thicknesses and refractive indices is available, as shown in the table. These are placed over one or both of the slits, singularly or in conjunction, causing the interference pattern to be shifted across the screen from the original pattern. In the column-I, how the strips have been placed, is mentioned whereas in the column-II, order of the fringe at point P on the screen that will be produced due to the placement of the strip(s), is shown. Correctly match both the column.



Film	A	B	C
Thickness (in μm)	5	1.5	0.25
Refractive Index	1.5	2.5	2

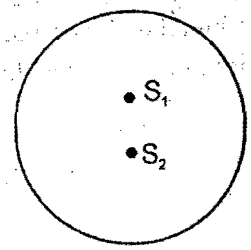
Column - I		Column - II	
(A)	Only strip B is placed over slit-I	(p)	First Bright
(B)	Strip A is placed over slit-I and strip C is placed over slit-II	(q)	Fourth Dark
(C)	Strip A is placed over the slit-I and strip B and strip C are placed over the slit-II in conjunction.	(r)	Fifth Dark
(D)	Strip A and strip C are placed over slit-I (in conjuction) and strip B is placed over slit-II.	(s)	Central Bright

25.34 A monochromatic parallel beam of light of wavelength λ is incident normally on the plane containing slits S_1 and S_2. The slits are of unequal width such that intensity only due to one slit on screen is four times that only due to the other slit. The screen is placed perpendicular to x-axis as shown. The distance between slits is d and that between screen and slit is D. Match the statements in column-I with results in column-II.



Column-I		Column-II	
(A)	The distance between two points on screen having equal intensities, such that intensity at those points is $\frac{1}{9}$th of maximum intensity.	(p)	$\frac{D\lambda}{3d}$
(B)	The distance between two points on screen having equal intensities, such that intensity at those points is $\frac{3}{9}$th of maximum intensity.	(q)	$\frac{D\lambda}{d}$
(C)	The distance between two points on screen having equal intensities, such that intensity at those points is $\frac{5}{9}$th of maximum intensity.	(r)	$\frac{2D\lambda}{d}$
(D)	The distance between two points on screen having equal intensities, such that intensity at those points is $\frac{7}{9}$th of maximum intensity.	(s)	$\frac{3D\lambda}{d}$
		(t)	$\frac{2D\lambda}{3d}$

25.35 Two coherent point sources of light having wavelength λ are seperated by a distance d. A circle is drawn in space surrounding both the point sources as shown. The plane of circle contains both the point sources. The distance d between both the sources is given in column-I and the total number of corresponding points of maximum intensity and minimum intensity on the periphery of the shown circle are given in column-II. Match each situation of column-I with the results in column-II.



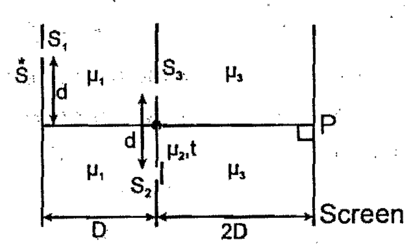
	Column-I		Column-II
(A)	$d = 99.4\ \lambda$	(p)	398 points of maximum intensity
(B)	$d = 99.6\ \lambda$	(q)	400 points of maximum intensity
(C)	$d = 100\ \lambda$	(r)	396 points of minimum intensity
(D)	$d = 100.4\ \lambda$	(s)	400 points of minimum intensity
		(t)	402 points of maximum intensity

SECTION - VI : INTEGER TYPE

25.36 An interference pattern is obtained by using a Fresnel's biprism. If the fringe width is 4 mm when air is the surrounding medium, then find the fringe width(in mm.) if water is the surrounding medium. Keeping the same source. Assume $n_{glass} = 1.5$, $n_{water} = 4/3$, $n_{air} = 1$.

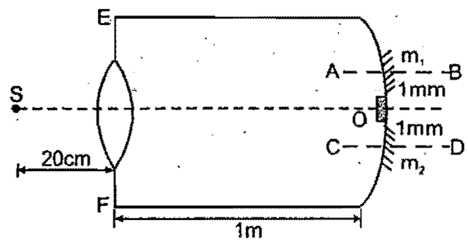
25.37 In the figure shown S_1 and S_2 are two coherent sources emitting light of wavelength 'λ' and having no initial phase difference. S_1 and S_2 oscillate simple harmonically with amplitude 'a' each and frequency 'f' each on the line PQ which is perpendicular to the x-axis. The initial position and initial direction of motion of 'S_1' and 'S_2' are shown in the figure. S_1 and S_2 are at their mean position at t = 0 sec. if the y-coordinates of 3rd maxima at time 't'.is $\frac{x\lambda D}{2a(2-\sin\omega t)}$ then x is Assume that $\lambda << a$ and $a << D$.

25.38 In the figure shown 'S' is a monochromatic source of light emitting light of wavelength λ (in air). Light falls on slit 'S_1' from 'S' and then reach the slits 'S_2' and 'S_3' through a medium of refractive index 'μ_1'. Light from slits S_2 and S_3 reach the screen through medium of refractive index μ_3. A thin transparent film of refractive index μ_2 and thickness 't' is placed infront of 'S_2'. point 'P' is symmetrical w.r.t. 'S_2' and 'S_3'. Using the values $d = 1$ mm, $D = 1$ m, $\mu_1 = 4/3$, $\mu_2 = 3/2$, $\mu_3 = 9/5$ and $t = \frac{4}{9} \times 10^{-5}$ m. Find the distance of central maxima from P.(in mm.)



25.39 In previous question if the film in front of S_2 is removed, then x/27 mm. distance and central maxima shift then x is

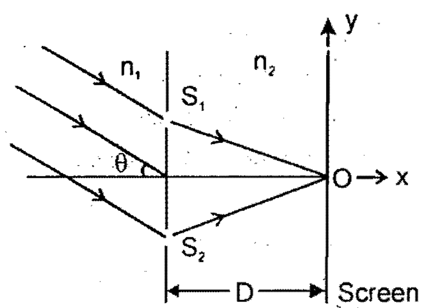
25.40 An equil convex lens of focal length 10 cm (in air) and R.I. 3/2 is put at a small opening on a tube of length 1 m fully filled with liquid of R.I. 4/3. A concave mirror of radius of curvature 20 cm is cut into two halves m_1 and m_2 and placed at the end of the tube. m_1 & m_2 are placed such that their principal axis AB and CD respectively are separated by 1 mm each from the principal axis of the lens. A slit S placed in air illuminates the lens with light of frequency 7.5×10^{14} Hz. The light reflected from m_1 and m_2 forms interference pattern on the left end EF of the tube. O is an opaque substance to cover the hole left by m_1 & m_2. Find the distance of the image formed by lens water combination in cm.



25.41 In previous the distance between the images formed by m_1 & m_2 in (mm.)

25.42 In previous width of the fringes on EF in μm.

25.43 In the figure an arrangement of young's double slit experiment is shown. A parallel beam of light of wavelength 'λ' (in medium n_1) is incident at an angle 'θ' as shown. Distance $S_1O = S_2O$. Point 'O' is the origin of the coordinate system. The medium on the left and right side of the plane of slits has refractive index n_1 and n_2 respectively. Distance between the slits is d. The distance between the screen and the plane of slits is D. Using D = 1m, d = 1mm, θ = 30°, λ = 0.3mm, $n_1 = \frac{4}{3}$, $n_2 = \frac{10}{9}$, answer the following



If y-coordinate of the point where the total phase difference between the interefering waves is zero, is $y = -x_0$ cm then x_0 is

25.44 In previous question y-coordinate of the nearest maxima above 'O' is $\frac{x}{\sqrt{154}}$ cm. then x is :

25.45 A lens of diameter 5.0 cm and focal length f = 25.0 cm was cut along the diameter into two identical halves. In the process, the layer of the lens a = 1.00 mm in thickness was lost. Then the halves were put together to form a composite lens. In it's focal plane a narrow slit was placed, emitting monochromatic light with wavelength λ = 0.60 μm. Behind the lens a screen was located at a distance b = 50 cm from it. Find number of possible maxima.

SECTION-II

PRACTICE TEST PAPERS

Section II : Rank Refiner

Pattern of PTs & FSTs

PTs Pattern : (Based on JEE(Advanced) 2014)

S.No.	Type of Que.	No. of Ques.	Marks Allotted Per Que.	Negative Marks
1	SCQ	7	3	-1
2	MCQ	7	3	0
3	PARAGRAPH TYPE	6 (3 Paragraph, 2 que. in each)	3	-1
4	MATCHING LIST	3	3	-1
5	INTEGER TYPE	7	3	0
	Total	30	90	

FST 01 (Class XI Syllabus) & FST 02 (Class XII Syllabus) Pattern

S.No.	Type of Que.	No. of Ques.	Marks Allotted Per Que.	Negative Marks
1	SCQ	5	3	-1
2	MCQ	5	3	0
3	PARAGRAPH TYPE	4 (2 Paragraph, 2 que. each)	3	-1
4	MATCHING LIST	1	3	-1
5	INTEGER	5	3	0
	Total	20	60	

FSTs 3 (Class XI+XII Syllabus) Pattern

S.No.	Type of Que.	No. of Ques.	Marks Allotted Per Que.	Negative Marks
1	SCQ	10	3	-1
2	MCQ	10	3	0
3	PARAGRAPH TYPE	8 (4 Paragraph, 2 que. each)	3	-1
4	MATCHING LIST	2	3	-1
5	INTEGER	10	3	0
	Total	40	120	

PART TEST - 1 (PT-1)

TOPIC : MECHANICS : CLASS-XI

Duration : 1 Hour **Max. Marks : 90**

GENERAL INSTRUCTIONS

1. This Question Paper contains 30 questions.
2. For each question in Section I, you will be awarded 3 Marks if you give the correct answer and Zero Mark if no answer is given. In all other cases, minus one (–1) Mark will be awarded.
3. For each question in Section II , you will be awarded 3 Marks if you give the correct answer. There is no negative marking.
4. For each question in Section III, you will be awarded 3 Marks if you give the correct answer and zero Mark if no answer is given. In all other cases, minus one (–1) Mark will be awarded.
5. For each question in Section IV, you will be awarded 3 Marks if you give the correct answer and zero Mark if no answer is given. In all other cases, minus one (–1) Mark will be awarded.
6. For each question in Section V, you will be awarded 3 Marks if you give the correct answer. There is no negative marking.

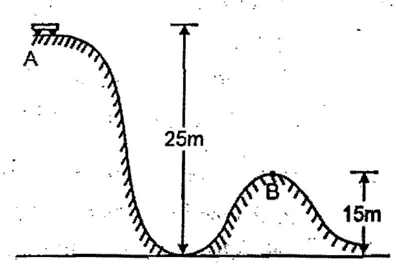
SECTION - I
Straight Objective Type

This section contains 7 Single choice questions. Each question has choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

1. An automobile enters a turn of radius R. If the road is banked at an angle of 45^0 and the coefficient of friction is 1, the minimum and maximum speed with which the automobile can negotiate the turn without skidding is :

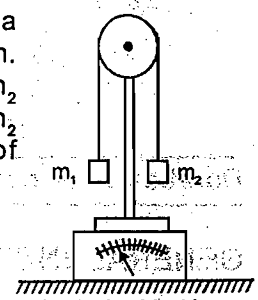
(A) $\sqrt{\frac{rg}{2}}$ and $\sqrt{rg}$ (B) $\frac{\sqrt{rg}}{2}$ and $\sqrt{rg}$ (C) $\frac{\sqrt{rg}}{2}$ and $2\sqrt{rg}$ (D) 0 and infinite

2. Figure shows the roller coaster track. Each car will start from rest at point A and will roll with negligible friction. It is important that there should be at least some small positive normal force exerted by the track on the car at all points, otherwise the car would leave the track. With the above fact, the minimum safe value for the radius of curvature at point B is ($g = 10\ m/s^2$) :



(A) 20 m
(B) 10 m
(C) 40 m
(D) 25 m

3. Two masses m_1 and m_2 which are connected with a light string, are placed over a frictionless pulley. This set up is placed over a weighing machine, as shown. Three combination of masses m_1 and m_2 are used, in first case m_1 = 6 kg and m_2 = 2 kg, in second case m_1 = 5 kg and m_2 = 3kg and in third case m_1 = 4 kg and m_2 = 4 kg. Masses are held stationary initially and then released. If the readings of the weighing machine after the release in three cases are W_1, W_2 and W_3 respectively then :



(A) $W_1 > W_2 > W_3$
(B) $W_1 < W_2 < W_3$
(C) $W_1 = W_2 = W_3$
(D) $W_1 = W_2 < W_3$

4. The work done in increasing the size of a rectangular soap film with dimensions 8 cm × 3.75 cm to 10 cm × 6 cm is 2×10^{-4} J. The surface tension of the film in N/m is :
(A) 1.65×10^{-2} (B) 3.3×10^{-2} (C) 6.6×10^{-2} (D) 8.25×10^{-2}

5. The property of surface tension is to :
(A) increase the volume (B) decrease the volume
(C) increase the surface area (D) decrease the surface area

6. A shell of mass 2 m projected with a speed 'u' at an angle θ to the horizontal explodes at the highest point of its motion into two pieces of mass 'm' each. If one piece whose initial speed is zero, falls vertically, the distance at which the other piece will fall from the gun is given by :
(A) $\frac{3u^2 \sin 2\theta}{g}$ (B) $\frac{3}{2}\frac{u^2 \sin 2\theta}{g}$ (C) $\frac{u^2 \sin 2\theta}{g}$ (D) none of these

7. A man of 80 kg attempts to jump from a small boat of mass 40 kg on to the shore. He can generate a relative velocity of 6 m/s between himself and boat. His velocity towards the shore is :
(A) 4 m/s (B) 8 m/s (C) 2 m/s (D) 3 m/s

SECTION - II
Multiple Correct Answers Type

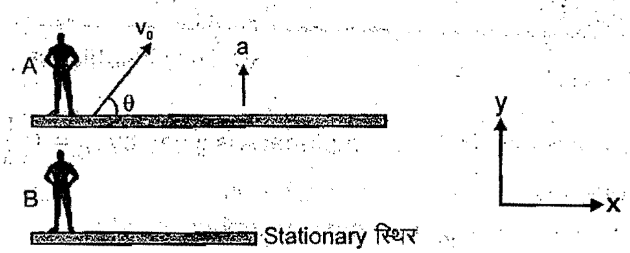
This section contains 7 Multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONE OR MORE** may be correct.

8. A ball tied to the end of a string swings in a vertical circle under the influence of gravity
(A) when the string makes an angle 90° with the vertical the tangential acceleration is zero amd radial acceleration is somewhere between maximum and minimum
(B) when the string makes an angle 90° with the vertical the tangential acceleration is maximum and radial acceleration is somewhere between maximum and minimum
(C) at no place in the circular motion, tangential acceleration is equal to radial acceleration
(D) throughout the path whenever radial acceleration has its extreme value, the tangential acceleration is zero.

9. A particle moves along the X-axis as
$x = u(t - 2) + a(t - 2)^2$
(A) the initial velocity of the particle is u (B) the acceleration of the particle is a
(C) the acceleration of the particle is 2a (D) at t =2s particle is at the origin.

10. If the resultant force on a system of particles is non-zero, then :
(A) The linear momentum of the system must increase.
(B) The velocity of the centre of mass of the system must change.
(C) The distance of the centre of mass may remain constant from a fixed point.
(D) kinetic energy of all particles must either increase simultaneuosly or decrease simultaneously.

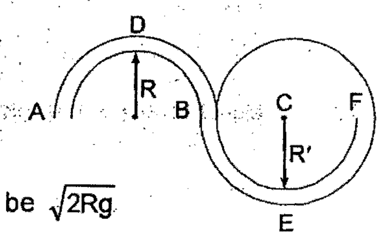
11. A projectile has initial velocity v_0 relative to the large plate which is moving upwrads with a constant upwards acceleration 'a'. Which of the following remains equal for the observes A and B :



(A) maximum height (B) distance travelled (C) time of flight (D) none of these

12. Two particle P and Q are in motion under gravity. Then :
(A) their relative acceleration is constant but not zero
(B) their relative velocity is constant
(C) their centre of mass has constant velocity
(D) their centre of mass has constant acceleration.

13. In the figure shown ADB & BEF are two fixed circular paths. A block of mass m enters in the tube ADB through point A with minimum velocity to reach point B. From there it moves on another circular path of radius R'. There it is just able to complete the circle.



(A) velocity at A must be $\sqrt{4Rg}$ (B) velocity at A must be $\sqrt{2Rg}$

(C) $\frac{R'}{R} = \frac{2}{3}$ (D) the normal reaction at point E is 6 m g

14. The displacement of a body from a reference point is given by, $\sqrt{x} = 2t - 3$, where 'x' is in metres and t in seconds. This shows that the body :
(A) is at rest at t = 3/2 (B) is accelerated
(C) is decelerated (D) is in uniform motion

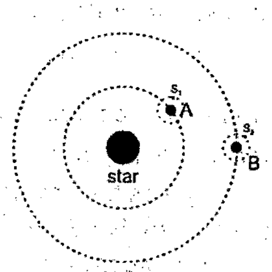
SECTION - III

Comprehension Type

This section contains 3 Paragraphs. Based upon each paragraph, **2 Multiple choice questions** have to be answered. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 15 to 16

Consider a star and two planet system. The star has mass M. The planets A and B have the same mass m, radius a and they revolve around the star in circular orbits of radius r and 4r respectively (M > > m, r > > a). Planet A has intelligent life and the people of this planet have achieved a very high degree of technological advance. They wish to shift a geostationary satellite of their own planet to a geostationary orbit of planet B. They achieve this through a series of high-precision maneuvers in which the satellite never has to apply brakes and not a single joule of energy is wasted. S_1 is a geostationary satellite of planet A and S_2 is a geostationary satellite of planet B. Neglect interaction between A and B, S_1 and S_2, S_1 and B & S_2 and A.



15. If the time period of the satellite in geostationary orbit of planet A is T, then its time period in geostationary orbit of planet B is :

(A) T (B) 4T
(C) 8 T (D) Data insufficient

16. If the radius of the geostationary orbit in planet A is given by $r_G = r\left(\frac{m}{M}\right)^{1/3}$, then the time in which the geostationary satellite will complete 1 revolution is

I. 1 planet year = time in which planet revolves around the star
II. 1 planet day = time in which planet revolves about its axis.

(A) I (B) II
(C) both I and II (D) neither I nor II

Paragraph for Question Nos. 17 to 18

A particle moves in xy-plane according to the equations $x = \alpha t$ and $y = \alpha t(1 - \beta t)$ where α and β are positive constants and t is time.

17. Equation of the trajectory (path) of the particle is :

(A) $y = x\left(1-\frac{\alpha}{\beta}x\right)$ (B) $y = x\left(1-\frac{\beta}{\alpha}x\right)$ (C) $y = \alpha x(1 - \beta x)$ (D) $y = \beta x(1 - \alpha x)$

18. Speed of the particle at time $t = \frac{1}{4\beta}$ is :

(A) $\frac{\sqrt{3}}{2}\alpha$ (B) $\frac{\sqrt{3}}{2}\beta$ (C) $\frac{\sqrt{5}}{2}\alpha$ (D) $\frac{\sqrt{5}}{2}\beta$

Paragraph for Question Nos. 19 to 20

Experiment 1 : The student pushes horizontally (rightward) on the crate of mass 100 kg and gradually increases the strength of this push force. The crate does not begin to move until the push force reaches 400 N.

Experiment 2 : The student applies a constant horizontal (rightward) push force for 1.0 s and measures how far the crate moves during that time interval. In each trial the crate starts at rest, and the student stops pushing after the 1.0 s interval. The following table summarizes the results.

Trial	Push force (N)	Distance (m)
1	500	1.5
2	600	2
3	700	2.5

19. The coefficient of static friction between the crate and the floor is approximately :

(A) 0.25 (B) 0.40 (C) 2.5 (D) 4.0

20. In experiment 1, when the rightward push force was 50 N the crate didn't move. Why didn't it move ?

(A) The push force was weaker than the frictional force on the crate
(B) The push force had the same strength as the gravitational force on the crate
(C) The push force was stronger than the frictional force on the crate
(D) The push force had the same strength as the frictional force on the crate

SECTION – IV
Matching List Type (Only One Option Correct)

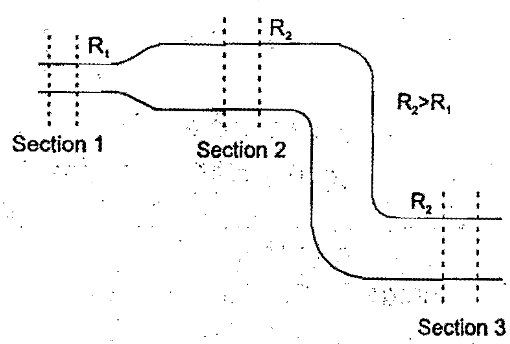
This section contains 3 questions, each having two matching lists. Choices for the correct combination of elements from List-I and List-II are given as options (A), (B), (C) and (D) out of which **ONE** is correct.

21. A particle is taken to a distance r (> R) from centre of the earth. R is radius of the earth. It is given velocity V which is perpendicular to $\vec{r}$. With the given values of V in column I you have to match the values of total energy and path of particle in column II. Here 'G' is the gravitational constant and 'M' is the mass of the earth.

List I (Velocity)	List II
P. $V = \sqrt{GM/r}$	1. Total energy Negative
Q. $V = \sqrt{2GM/r}$	2. Total energy Positive
R. $V > \sqrt{2GM/r}$	3. Total energy Zero
S. $\sqrt{GM/r} < V < \sqrt{2GM/r}$	4. Path is circular

(A) P-1, Q-3, R-4, S-1
(B) P-3, Q-1, R-2, S-3
(C) P-2, Q-2, R-1, S-4
(D) P-4, Q-3, R-2,S-1

22. An arrangement of the pipes is shown in the figure. The flow of water (incompressible and nonviscous) through the pipes is steady in nature. Three sections of the pipe are marked in which section 1 and section 2 are at same horizontal level, while being at a greater height than section 3. Correctly match order of the different physical parameter with the options given.

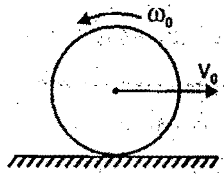


List I	List II
P. volume flow rate	1. same everywhere
Q. kinetic energy per unit mass	2. same at 2 and 3
R. pressure in the sections.	3. maximum at 1
S. flow speed in sections	4. minimum at 1

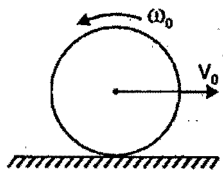
(A) P-1, Q-1, R-4,S-1
(B) P-2, Q-3, R-4,S-2
(C) P-4, Q-2, R-3,S-4
(D) P-3, Q-3, R-3,S-2

23. A uniform disc of mass M and radius R lies on a fixed rough horizontal surface at time t = 0. Initial angular velocity ω_0 of each disc (magnitude and sense of rotation) and horizontal velocity v_0 of centre of mass is shown for each situation of column-I. Match each situation in column-I with the results given in column-II.

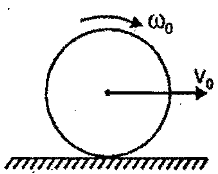
List-I

P. 

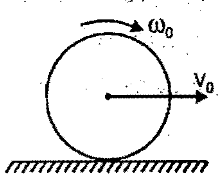
It is given that $v_0 = 2R\omega_0$

Q. 

It is given that $2v_0 = R\omega_0$

R. 

It is given that $v_0 = 2R\omega_0$

S. 

It is given that $2v_0 = R\omega_0$

List-II

1. The magnitude of angular speed keeps on decreasing till the disc starts rolling without slipping.

2. After the disc starts rolling without slipping, the angular velocity is nonzero and in clockwise sense.

3. After the disc starts rolling without slipping, the velocity of centre of disc is towards right.

4. After the disc starts rolling without slipping, the kinetic energy of disc is less than its initial value.

(A) P-1, Q-1, R-1, S-3
(B) P-2, Q-2, R-2, S-3
(C) P-1, Q-3, R-4, S-2
(D) P-2, Q-4, R-3, S-3

SECTION - V
Integer value correct Type

This section contains 7 **questions**. The answer to each question is a **integer type.**

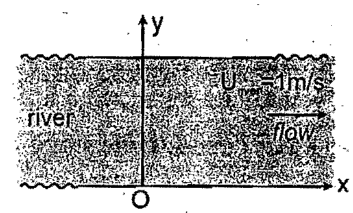
24. The ratio of work done by the internal forces of a car in order to change its speed from 0 to V and from V to 2V is $\frac{X}{3}$ then X is (Assume that the car moves on a horizontal road) -

25. A weightless rod of length 2ℓ carries two equal masses 'm', one tied at lower end A and the other at the middle of the rod at B. The rod can rotate in vertical plane about a fixed horizontal axis passing through C. The rod is released from rest in horizontal position. The speed of the Smass B at the instant rod, become vertical is $\sqrt{\frac{X}{5}g\ell}$ then X is.

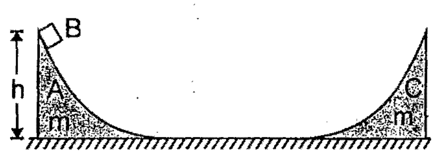
26. A body is thrown horizontally with a velocity $\sqrt{2gh}$ from the top of a tower of height h. It strikes the level ground through the foot of the tower at a distance 'xh' from the tower. The value of x is....

27. Two particles are projected simultaneously with the same speed v in the same vertical plane with angles of elevation θ and 2θ, where $\theta < 45°$. Their velocity becomes parallel at $t = \frac{v}{g} f\left(\frac{\theta}{2}\right)$ what will be value of $\sqrt{\frac{t}{\cos 15°}}$ for $v = 20\sqrt{2}$ m/s and $\theta = 30°$ take $g = 10 m/s^2$.

28. A man can swim in still water with a speed of 3 m/s. x and y axis are drawn along and normal to the bank of river flowing to right with a speed of 1 m/s. The man starts swimming from origin O at t = 0 second. Assume size of man to be negligible.locus of all the possible points where man can reach at t = 1 sec. is $(x-a)^2 + y^2 = c^2$ Find value of ac^2



29. In the figure shown a small block 'B' of mass 'm' is released from the top of a smooth movable wedge 'A' of the same mass 'm'. 'B' ascends another movable smooth wedge 'C' of the same mass. Neglecting friction any where the maximum height attained by 'B' on 'C' is $\frac{h}{2x}$ Find the value of x.



30. A cylinder rotating at an angular speed of 50 rev/s is brought in contact with an identical stationary cylinder. Because of the kinetic friction, torques act on the two cylinders, accelerating the stationary one and decelerating the moving one. If the common magnitude of the acceleration and deceleration be one revolution per second square. If it take before the two cylinders have equal angular speed is 5 x second then x is.

PART TEST - 2 (PT-2)

TOPIC : HEAT & THERMODYNAMICS : CLASS-XI

Duration : 1 Hour | **Max. Marks : 90**

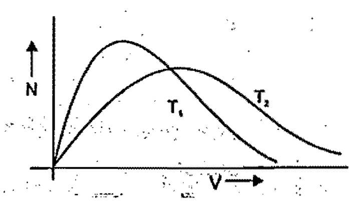
GENERAL INSTRUCTIONS

1. This Question Paper contains **30** questions.
2. For each question in Section I, you will be awarded **3 Marks** if you give the correct answer and **Zero Mark** if no answer is given. In all other cases, minus one (–1) Mark will be awarded.
3. For each question in Section II , you will be awarded 3 Marks if you give the correct answer. There is no negative marking.
4. For each question in Section III, you will be awarded 3 Marks if you give the correct answer and zero Mark if no answer is given. In all other cases, minus one (–1) Mark will be awarded.
5. For each question in Section IV, you will be awarded 3 Marks if you give the correct answer and zero Mark if no answer is given. In all other cases, minus one (–1) Mark will be awarded.
6. For each question in Section V, you will be awarded **3** Marks if you give the correct answer. There is no negative marking.

SECTION - I
Straight Objective Type

This section contains 7 Single choice questions. Each question has choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

1. Maxwell's velocity distribution curve is given for two different temperatures. For the given curves.



(A) $T_1 > T_2$
(B) $T_1 < T_2$
(C) $T_1 \leq T_2$
(D) $T_1 = T_2$

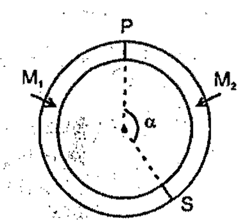
2 12 gm He and 4 gm H_2 is filled in a container of volume 20 litre maintained at temperature 300 K. The pressure of the mixture is nearly :

(A) 3 atm (B) 5 atm (C) 6.25 atm (D) 12.5 atm

3. Which of the following will have maximum total kinetic energy at temperature 300 K.

(A) 1 kg H_2 (B) 1 kg He (C) $\frac{1}{2}$ kg H_2 + $\frac{1}{2}$ kg He (D) $\frac{1}{4}$ kg H_2 + $\frac{3}{4}$ kg He

4. A ring shaped tube contains two ideal gases with equal masses and atomic mass numbers $M_1 = 32$ and $M_2 = 28$. The gases are separated by one fixed partition P and another movable conducting partition S which can move freely without friction inside the ring. The angle α as shown in the figure in equilibrium is:



(A) $\frac{7\pi}{8}$ (B) $\frac{8\pi}{7}$

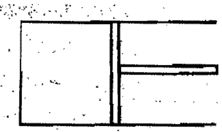
(C) $\frac{15\pi}{16}$ (D) $\frac{16\pi}{15}$

5. In an experiment the speeds of any five molecules of an ideal gas are recorded. The experiment is repeated N times where N is very large. The average of recorded values, is :

(A) $\sqrt{\frac{2RT}{M}}$ (B) $\sqrt{\frac{8RT}{\pi M}}$ (C) $\sqrt{\frac{3RT}{M}}$ (D) $\sqrt{\frac{RT}{M}}$

6. Temperature at which Fahrenheit and Kelvin pair of scales give the same reading will be:
(A) $\theta = -40$ (B) $\theta = 40$ (C) $\theta = 574.25$ (D) 512.45

7. n moles of a gas filled in a container at temperature T is in thermodynamic equilibrium initially. If the gas is compressed slowly and isothermally to half its initial volume the work done by the atmosphere on the piston is:



(A) $\frac{nRT}{2}$ (B) $-\frac{nRT}{2}$ (C) $nRT\left(\ell n\,2-\frac{1}{2}\right)$ (D) $-nRT\,\ell n\,2$

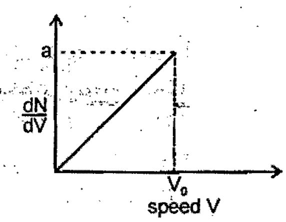
SECTION - II
Multiple Correct Answers Type

This section contains 7 Multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONE OR MORE** may be correct.

8. During an experiment, an ideal gas is found to obzey a condition $\frac{P^2}{\rho}$ = constant [ρ = density of the gas]. The gas is initially at temperature T, pressure P and density ρ. The gas expands such that density changes to $\frac{\rho}{2}$
(A) The pressure of the gas changes to $\sqrt{2}$ P.
(B) The temperature of the gas changes to $\sqrt{2}$ T.
(C) The graph of the above process on the P-T diagram is parabola.
(D) The graph of the above process on the P-T diagram is hyperbola.

9. When the temperature of a copper coin is raised by 80 °C, its diameter increases by 0.2%,
(A) percentage rise in the area of a face is 0.4%
(B) percentage rise in the thickness is 0.4%
(C) percentage rise in the volume is 0.6%
(D) coefficient of linear expansion of copper is 0.25×10^{-4} /°C.

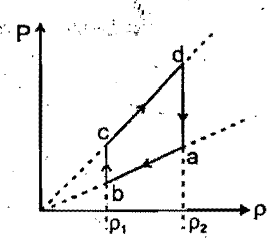
10. Graph shows a hypothetical speed distribution for a sample of N gas particles (for $V > V_0$; $\frac{dN}{dV} = 0$)



(A) The value of aV_0 is 2N.
(B) The ratio V_{avg}/V_0 is equal to 2/3.
(C) The ratio V_{rms}/V_0 is equal to $1/\sqrt{2}$.
(D) Three fourth of the total particle has a speed between 0.5 V_0 and V_0.

11. Pick the correct statement(s) :
(A) The rms translational speed for all ideal-gas molecules at the same temperature is not the same but it depends on the mass.
(B) Each particle in a gas has average translational kinetic energy and the equation $\frac{1}{2}mv^2_{rms} = \frac{3}{2}kT$ establishes the relationship between the average translational kinetic energy per particle and temperature of an ideal gas. It can be concluded that single particle has a temperature.
(C) Temperature of an ideal gas is doubled from 100°C to 200°C. The average kinetic energy of each particle is also doubled.
(D) It is possible for both the pressure and volume of a monoatomic ideal gas to change simultaneously without causing the internal energy of the gas to change.

12. An ideal gas undergoes a cyclic process abcda which is shown by pressure- density curve.



(A) Work done by the gas in the process 'bc' is zero
(B) Work done by the gas in the process 'cd' is negative
(C) Internal energy of the gas at point 'a' is greater than at state 'c'
(D) Net work done by the gas in the cycle is negative.

13. The emissive power of a black body at T = 300 K is 100 Watt/m². Consider a body B of area A = 10 m² coefficient of reflectivity r = 0.3 and coefficient of transmission t = 0.5 and at temperature 300 K. Then which of the following is correct :
(A) The emissive power of B is 20 W/m²
(B) The emissive power of B is 200 W/m²
(C) The power emitted by B is 200 Watts
(D) The emissivity of B is = 0.2

14. The ends of a rod of uniform thermal conductivity are maintained at different (constant) temperatures. After the steady state is achieved :
(A) heat flows in the rod from high temperature to low temperature even if the rod has nonuniform cross sectional area.
(B) temperature gradient along length is same even if the rod has non uniform cross sectional area.
(C) heat current is same even if the rod has non-uniform cross sectional area.
(D) if the rod has uniform cross sectional area the temperature is same at all points of the rod.

SECTION - III

Comprehension Type

This section contains 3 Paragraphs. Based upon each paragraph, 2 Multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 15 to 16

The atmospheric lapse rate

For small volumes of gas, according to KINETIC THEORY OF GASES, all parts of the gas are at the same temperature. But for huge volumes of gas like atmosphere, assumption of a uniform temperature throughout the gas is not valid. Different parts of the atmosphere are at different temperatures. Apart from the surface of the earth, variations also occur in temperature at different heights in the atmosphere. The decrease in temperature with height called the ATMOSPHERIC LAPSE RATE is similar at various locations across the surface of the Earth. By analyzing the data collected at various locations, it is found that average global lapse rate is – 6.7 °C/Km.

The linear decrease with temperature only occurs in the lower part of the atmosphere called the TROPOSPHERE. This is the part of the atmosphere in which weather occurs and our planes fly. Above the troposphere is the stratosphere, with an imaginary boundary separating the two layers. In the stratosphere, temperature tends to be relatively constant.

Absorption of sunlight at the Earth's surface warms the troposphere from below, so vertical convection currents are continually mixing in the air. As a parcel of air rises, its pressure drops and it expands. The parcel does work on its surrounding, so that its internal energy and therefore, its temperature drops. Assume that the vertical mixing is so rapid as to be adiabatic and the quantity $TP^{(1-\gamma)/\gamma}$ has a uniform value through the layers of troposphere.

(M is molecular mass of the air, R is universal gas constant, g is gravitational acc., P and T are pressure and temperature respectively at the point under consideration and y is height,)

15. If behaviour of the mixing of parcels of air is approximately assumed to be adiabatic then lapse rate can be expressed as :

(A) $-\frac{P}{T}(\gamma-1)\frac{dp}{dy}$ (B) $\frac{T}{P}\left(\frac{1-\gamma}{\gamma}\right)\frac{dp}{dy}$ (C) $-\frac{\gamma-1}{\gamma}\frac{gM}{R}$ (D) $\frac{T}{P}\left(1-\frac{1}{\gamma}\right)\frac{dp}{dy}$

16. Mechanical equilibrium of the atmosphere requires that the pressure decreases with altitude according to $\frac{dP}{dy} = -\rho g$. Assuming free fall acceleration to be uniform, then lapse rate is given by

(A) $-\left(1-\frac{1}{\gamma}\right)\frac{Mg}{R}$ (B) $\left(1-\frac{1}{\gamma}\right)\frac{Mg}{R}$ (C) $-(\gamma-1)\frac{Mg}{R}$ (D) $-\gamma\frac{Mg}{R}$

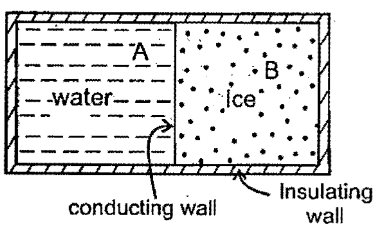
Paragraph for Question Nos. 17 to 18

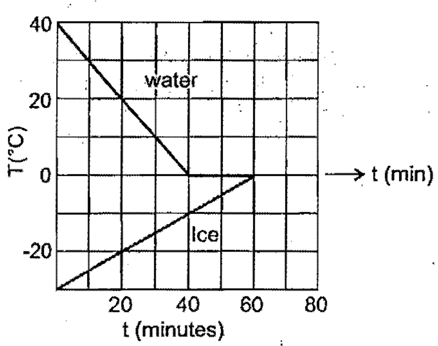
A 0.60 kg sample of water and a sample of ice are placed in two compartments A and B that are separated by a conducting wall, in a thermally insulated container. The rate of heat transfer from the water to the ice through the conducting wall is constant P, until thermal equilibrium is reached. The temperature T of the liquid water and the ice are given in graph as functions of time t. Temperature of the each compartment remain homogeneous during whole heat transfer process.

Given specific heat of ice = 2100 J/kg-K

Given specific heat of water = 4200 J/kg-K

Latent heat of fusion of ice = 3.3×10^5 J/kg





17 The value of rate P is

(A) 42.0 W (B) 36.0 W (C) 21.0 W (D) 18.0 W

18. The initial mass of the ice in the container is equal to

(A) 0.36 kg (B) 1.2 kg (C) 2.4 kg (D) 3.6 kg

Paragraph for Question Nos. 19 to 20

A quantity of an ideal monoatomic gas consists of n moles initially at temperature T_1. The pressure and volume both are then slowly doubled in such a manner so as to trace out a straight line on a P-V diagram.

19. For this process, the ratio $\frac{W}{nRT_1}$ is equal to (where W is work done by the gas) :

(A) 1.5 (B) 3 (C) 4.5 (D) 6

20. For the same process, the ratio $\frac{Q}{nRT_1}$ is equal to (where Q is heat supplied to the gas) :

(A) 1.5 (B) 3 (C) 4.5 (D) 6

SECTION – IV

Matching List Type (Only One Option Correct)

This section contains 3 questions, each having two matching lists. Choices for the correct combination of elements from List-I and List-II are given as options (A), (B), (C) and (D) out of which **ONE** is correct.

21. The figures given below show different processes (relating pressure P and volume V) for a given amount of an ideal gas. ΔW is work done by the gas and ΔQ is heat absorbed by the gas.

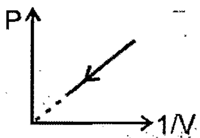


fig (i)

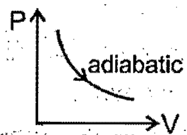


fig (ii)

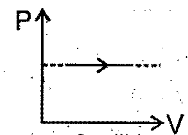


fig (iii)

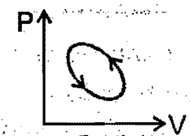


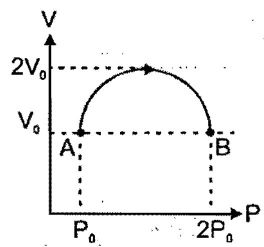
fig (iv)

List-I	Lsist-II
P. In Figure (i)	1. $\Delta Q > 0$.
Q. In Figure (ii)	2. $\Delta W < 0$.
R. In Figure (iii)	3. $\Delta Q < 0$.
S. In Figure (iv) (for complete cycle)	4. $\Delta W > 0$.

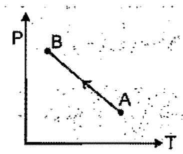
(A) P-1, Q-4, R-4, S-3 (B) P-2, Q-1, R-2, S-3
(C) P-2, Q-2, R-3, S-4 (D) P-4, Q-3, R-1, S-3

22. A sample of gas goes from state A to state B in four different manners, as shown by the graphs. Let W be the work done by the gas and ΔU be change in internal energy along the path AB. Correctly match the graphs with the statements provided.

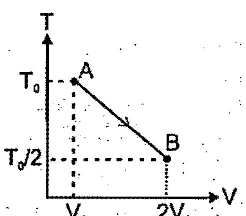
List I — List II

P. 

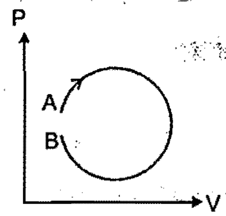
1. W is positive

Q. 

2. ΔU is negative

R. 

3. W is negative

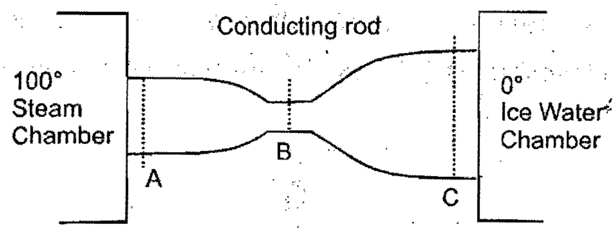
S. 

4. ΔU is positive

States A and B are very close

(A) P-1, Q-1, R-1, S-3 (B) P-2, Q-2, R-2, S-3
(C) P-3, Q-2, R-1, S-1 (D) P-4, Q-2, R-3, S-1

23. A copper rod (initially at room temperature 20°C) of non-uniform cross section is placed between a steam chamber at 100°C and ice-water chamber at 0°C.



List I	List II
P. Initially rate of heat flow $\left(\frac{dQ}{dt}\right)$ will be	1. maximum at section A
Q. At steady state rate of heat flow $\left(\frac{dQ}{dt}\right)$ will be	2. maximum at section B
R. At steady state temperature gradient $\left|\left(\frac{dT}{dx}\right)\right|$ will be	3. minimum at section C
S. At steady state rate of change of temperature $\left(\frac{dT}{dt}\right)$ at a certain point will be	4. same for all section

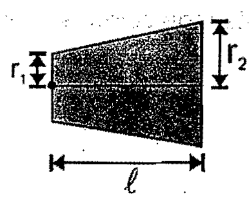
(A) P-1, Q-1, R-1, S-3
(B) P-1, Q-4, R-3, S-4
(C) P-2, Q-2, R-2, S-3
(D) P-2, Q-2, R-3, S-3

SECTION - V

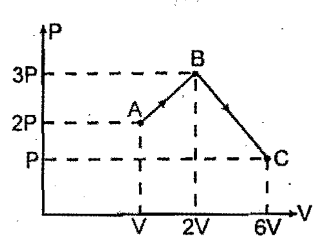
Integer value correct Type

This section contains 7 questions. The answer to each question is a integer type.

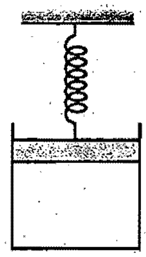
24. The thermal conductance (reciprocal of the thermal resistance) for axial flow of a truncated cone of length ℓ, the radius of the two ends are r_1 and r_2 assuming that thermal conductivity of the material is K is $\frac{\pi k(r_1 r_2)}{\ell} n^2$. Find the value of n^2.



25. One mole of monoatomic ideal gas undergoes a process ABC as shown in figure. The maximum temperature of the gas during the process ABC is in the form $\frac{XPV}{R}$. Find X.



26. One mole of an ideal gas is kept enclosed under a light piston (area=10^{-2} m^2) connected by a compressed spring (spring constant 200 N/m). The volume of gas is 0.83 m^3 and its temperature is 100K. The gas is heated so that it compresses the spring further by 0.1 m. Find the work done by the gas in the process in joules ? (Take R = 8.3 J/K-mole and suppose there is no atmosphere).



27. A piston can freely move inside a horizontal cylinder closed from both ends. Initially, the piston separates the inside space of the cylinder into two equal parts each of volume V_0, in which an ideal gas is contained under the same pressure p_0 and at the same temperature. Work has to be performed in order to increase isothermally the volume of one part of gas 2 times compared to that of the other by slowly moving the piston is $P_0V_0V_0 \ell n \frac{x}{8}$ then x is

28. Two rods of same dimensions, but made of different materials are joined end to end with their free ends being maintained at 100°C and 0°C respectively. The temperature of the junction is 70°C. Then the temperature of the junction if the rods are interchanged will be equal to 6x °C then x is :

29. Assume a sample of an ideal gas in a vessel. Where velocity of molecules are between 2 m/sec to 5 m/sec and velocity of molecules (v) and number of molecules (n) are related as $n = 7v - v^2 - 10$. The most probable velocity in sample is $\frac{x}{2}$ m,5' then x is :

30 The molecules of an ideal gas have 6 degrees of freedom. The temperature of the gas is T. The average translational kinetic energy of its molecules is $\frac{x}{2}$ kT then x is :

PART TEST - 3 (PT-3)

TOPIC : SHM & WAVES : CLASS-XI

Duration : 1 Hour — **Max. Marks : 90**

GENERAL INSTRUCTIONS

1. This Question Paper contains **30 questions**.
2. For each question in **Section I**, you will be awarded **3 Marks** if you give the correct answer and **Zero Mark** if no answer is given. In all other cases, minus one (–1) Mark will be awarded.
3. For each question in **Section II**, you will be awarded **3 Marks** if you give the correct answer. There is no negative marking.
4. For each question in **Section III**, you will be awarded **3 Marks** if you give the correct answer and zero Mark if no answer is given. In all other cases, minus one (–1) Mark will be awarded.
5. For each question in **Section IV**, you will be awarded **3 Marks** if you give the correct answer and zero Mark if no answer is given. In all other cases, minus one (–1) Mark will be awarded.
6. For each question in **Section V**, you will be awarded **3 Marks** if you give the correct answer. There is no negative marking.

SECTION - I
Straight Objective Type

This section contains **7 Single choice questions**. Each question has choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

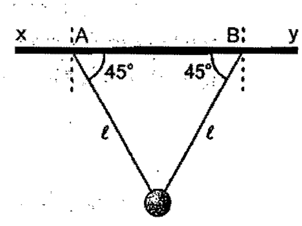
1. For a particle in S.H.M., if the amplitude of displacement is 'a' and the amplitude of velocity is 'v' the amplitude of acceleration is :

(A) va (B) $\frac{v^2}{a}$ (C) $\frac{v^2}{2a}$ (D) $\frac{v}{a}$

2. A system is shown in the figure.
The time period for small oscillations of the two blocks will be :

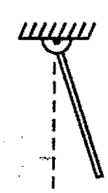
(A) $2p\sqrt{\frac{3m}{k}}$ (B) $2p\sqrt{\frac{3m}{2k}}$ (C) $2p\sqrt{\frac{3m}{4k}}$ (D) $2p\sqrt{\frac{3m}{8k}}$

3. Two light strings, each of length ℓ, are fixed at points A and B on a fixed horizontal rod xy. A small bob is tied by both strings and in equilibrium, the strings are making angle 45° with the rod. If the bob is slightly displaced normal to the plane of the strings and released then period of the resulting small oscillation will be :



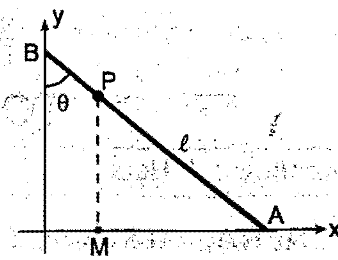
(A) $2\pi\sqrt{\frac{2\sqrt{2}\ell}{g}}$ (B) $2\pi\sqrt{\frac{\sqrt{2}\ell}{g}}$ (C) $2\pi\sqrt{\frac{\ell}{g}}$ (D) $2\pi\sqrt{\frac{\ell}{\sqrt{2}g}}$

4. A metre stick swinging in vertical plane about a fixed horizontal axis passing through its one end undergoes small oscillation of frequency f_0 as shown in figure. If the bottom half of the stick were cut



(A) f_0 (B) $\sqrt{2}\, f_0$ (C) $2f_0$ (D) $2\sqrt{2}\, f_0$

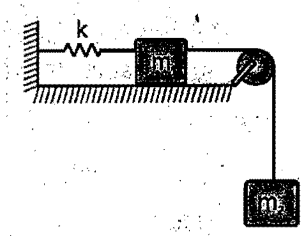
5. A rod of length ℓ is in motion such that its ends A and B are moving along x-axis and y-axis respectively. It is given that $\frac{d\theta}{dt} = 2$ rad/s always. P is a fixed point on the rod as shown in figure. Let M be the projection of P on x-axis. For the time interval in which θ changes from 0 to $\frac{\pi}{2}$, choose the correct statement :



(A) The acceleration of M is always directed towards right
(B) M executes SHM
(C) M moves with constant speed
(D) M moves with constant acceleration

6. Which of the following is greatest in SHM ? (assuming potential energy = 0 at mean position)
(A) Average kinetic energy with respect to position
(B) Average potential energy with respect to position
(C) Average kinetic energy with respect to time
(D) Average potential energy with respect to time

7. m_1 & m_2 are connected with a light inextensible string with m_1 lying on smooth table and m_2 hanging as shown in figure. m_1 is also connected to a light spring which is initially unstretched and the system is released from rest :



(A) system performs SHM with angular frequency given by $\sqrt{\frac{k(m_1+m_2)}{m_1m_2}}$

(B) system performs SHM with angular frequency given by $\sqrt{\frac{k}{m_1+m_2}}$.

(C) tension in string will be zero when the system is released.

(D) maximum displacement of m_1 will be $\frac{m_2g}{k}$.

SECTION - II
Multiple Correct Answers Type

This section contains 7 Multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONE OR MORE** may be correct.

8. A particle is executing SHM between points $-X_m$ and X_m, as shown in figure-I. The velocity V(t) of the particle is partially graphed and shown in figure-II. Two points A and B corresponding to time t_1 and time t_2 respectively are marked on the V(t) curve :

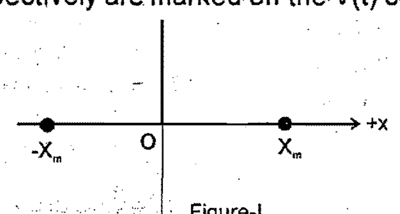


Figure-I

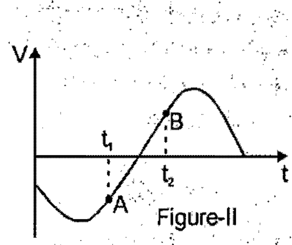
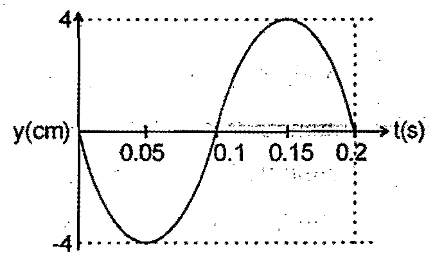


Figure-II

(A) At time t_1 , it is going towards X_m.
(B) At time t_1, its speed is decreasing.
(C) At time t_2, its position lies in between $-X_m$ and O.
(D) The phase difference $\Delta\phi$ between points A and B must be expressed as $90° < \Delta\phi < 180°$.

9. In a standing wave on a string rigidly fixed at both ends.
(A) In one time period all the particles are simultaneously at rest twice.
(B) All the particles must be at their positive extremes simultaneously once in one time period
(C) All the particles may be at their positive extremes simultaneously once in one time period.
(D) All the particles are never at rest simultaneously.

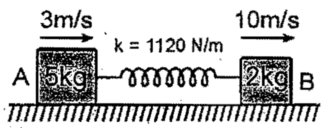
10. For a certain transverse standing wave on a long string, an antinode is formed at x = 0 and next to it, a node is formed at x = 0.10 m. the displacement y(t) of the string particle at x = 0 is shown in figure.



(A) Transverse displacement of the particle at x = 0.05m and t = 0.05 s is $-2\sqrt{2}$ cm.
(B) Transverse displacement of the particle at x = 0.04 m and t = 0.025 s is $-2\sqrt{2}$ cm.
(C) Speed of the traveling waves that interfere to produce this standing wave is 2 m/s.
(D) The transverse velocity of the string particle at x = $\frac{1}{15}$ m and t = 0.1 s is 20 π cm/s

11. A wave pulse moving in the positive x-direction along the x-axis is represented by the wave function $y(x, t) = \frac{2.0}{(x-3.0t)^2+1}$, where x and y are in centimeters and t is in seconds. Then
(A) The speed of particle at time t = 1 sec. and x = 3cm is zero.
(B) The speed of particle at time t = 1 sec. and x = 3cm is 2 cm/s.
(C) The speed of the pulse is 3.0 cm/s
(D) The speed of the pulse is 0.33 cm/s

12. Two blocks A (5kg) and B(2kg) attached to the ends of a spring constant 1120N/m are placed on a smooth horizontal plane with the spring undeformed. Simultaneously velocities of 3m/s and 10m/s along the line of the spring in the same direction are imparted to A and B then



(A) when the extension of the spring is maximum the velocities of A and B are zero.
(B) the maximum extension of the spring is 25cm.
(C) the first maximum compression occurs $3\pi/56$ seconds after start.
(D) maximum extension and maximum compression occur alternately.

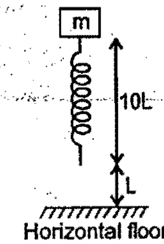
13 A mass of 0.2kg is attached to the lower end of a massless spring of force-constant 200 N/m, the upper end of which is fixed to a rigid support. Which of the following statements is/are true?
(A) In equilibrium, the spring will be stretched by 1cm.
(B) If the mass is raised till the spring is unstretched state and then released, it will go down by 2cm before moving upwards.
(C) The frequency of oscillation will be nearly 5 Hz.
(D) If the system is taken to the moon, the frequency of oscillation will be the same as on the earth.

14. The apparent frequency of a sound wave as heard by an observer is 10% more than actual frequency. If the velocity of sound in air is 330 m/s.
(A) the sound may be moving towards the stationary observer with a velocity of 30 m/s
(B) the source may be moving towards the stationary observer with a velocity of 33 m/s
(C) the observer may be moving towards the stationary source with velocity of 30 m/s
(D) the observer may be moving towards the stationary source with velocity of 33 m/s

SECTION - III
Comprehension Type

This section contains **3 Paragraphs**. Based upon each paragraph, **2 Multiple choice questions** have to be answered. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 15 to 16



A small block of mass m is fixed at upper end of a massless vertical spring of spring constant $K = \frac{4mg}{L}$ and natural length '10L'. The lower end of spring is free and is at a height L from fixed horizontal floor as shown. The spring is initially unstressed and the spring-block system is released from rest in the shown position.

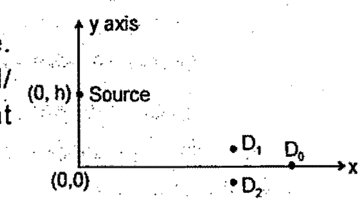
15. At the instant speed of block is maximum, the magnitude of force exerted by spring on the block is :

(A) $\frac{mg}{2}$ (B) mg (C) Zero (D) None of these

16. As the block is coming down, the maximum speed attained by the block is :

(A) $\sqrt{gL}$ (B) $\sqrt{3gL}$ (C) $\frac{3}{2}\sqrt{gL}$ (D) $\sqrt{\frac{3}{2}gL}$

Paragraph for Question Nos. 17 to 18



There is a point source of sound placed at (0, h) as shown in figure. Two detectors D_1 and D_2 are placed at positions (D,d/2) and (D,– d/2) respectively. Take h < < D. The source emitted a sound pulse at a certain time. Assuming velocity of sound in the surrounding medium is v.

17. The time gap between the recordings made by the detectors will approximately be :

(A) $\frac{1}{2}\frac{dh}{Dv}$ (B) $2\frac{dh}{Dv}$ (C) $\frac{hd}{Dv}$ (D) None of these

18. If the source emits continuous waves, and the pressures recorded by the two detectors are superposed at every instant in detector D_0 (which is equidistant from D_1 & D_2), the resultant pressure amplitude will be maximum if the minimum frequency of the source is :

(A) $\frac{1}{2}\frac{vD}{d.h}$ (B) $2\frac{vD}{d.h}$ (C) $\frac{3}{2}\frac{vD}{d.h}$ (D) $\frac{vD}{d.h}$

Paragraph for Question Nos. 19 to 20

A sinusoidal wave is propagating in negative x–direction in a string stretched along x-axis. A particle of string at x = 2m is found at its mean position and it is moving in positive y direction at t = 1 sec. The amplitude of the wave, the wavelength and the angular frequency of the wave are 0.1meter, $\pi/4$ meter and 4π rad/sec respectively.

19. The equation of the wave is :

(A) $y = 0.1 \sin [4\pi(t-1) + 8(x-2)]$ (B) $y = 0.1 \sin [(t-1) - (x-2)]$
(C) $y = 0.1 \sin [4\pi(t-1) - 8(x-2)]$ (D) none of these

20. The speed of particle at x = 2 m and t = 1sec is :

(A) 0.2π m/s (B) 0.6π m/s (C) 0.4π m/s (D) 0

SECTION – IV
Matching List Type (Only One Option Correct)

This section contains 3 questions, each having two matching lists. Choices for the correct combination of elements from List-I and List-II are given as options (A), (B), (C) and (D) out of which **ONE** is correct.

In the column-I, a system is described in each option and corresponding time period is given in the column-II. Suitably match them.

	List-I		List-II
P.	A simple pendulum of length 'ℓ' oscillating with small amplitude in a lift moving down with retardation g/2.	1.	$T = 2\pi\sqrt{\frac{2\ell}{3g}}$
Q.	A block attached to an end of a vertical spring, whose other end is fixed to the ceiling of a lift, stretches the spring by length 'ℓ' in equilibrium. It's time period when lift moves up with an acceleration g/2 is	2.	$T = 2\pi\sqrt{\frac{\ell}{g}}$
R.	The time period of small oscillation of a uniform rod of length 'ℓ' smoothly hinged at one end. The rod oscillates in vertical plane.	3.	$T = 2\pi\sqrt{\frac{2\ell}{g}}$
S.	A cubical block of edge 'ℓ' and specific density $\rho/2$ is in equilibrium with some volume inside water filled in a large fixed container. Neglect viscous forces and surface tension. The time period of small oscillations of the block in vertical direction is	4.	$T = 2\pi\sqrt{\frac{\ell}{2g}}$

(A) P-1, Q-2, R-1, S-4 (B) P-2, Q-2, R-2, S-3
(C) P-2, Q-2, R-2, S-4 (D) P-2, Q-2, R-3, S-3

22 Match the statements in column-I with the statements in column-II.

	List-I		List-II
P.	A tight string is fixed at both ends and sustaining standing wave	1.	At the middle, antinode is formed in odd harmonic
Q.	A tight string is fixed at one end and free at the other end	3.	At the middle, node is formed in even harmonic
R.	Standing wave is formed in an open organ pipe. End correction is not negligible.	3.	At the middle, neither node nor antinode is formed
S.	Standing wave is formed in a closed any organ pipe. End correction is not negligible.	4.	Phase difference between SHMs of

(A) P-1, Q-1, R-1, S-3 (B) P-2, Q-2, R-2, S-3
(C) P-2, Q-2, R-2, S-4 (D) P-2, Q-2, R-3, S-3

23. For a particle executing SHM along a straight line, match the statements in column-I with statement in column-II. (Note that displacement given in column-I is to be measured from mean position).

List-I		List-II
P. Velocity-displacement graph may be	1.	straight line
Q. Acceleration-velocity graph may be	2.	circle
R. Acceleration-displacement graph will be	3.	ellipse
S. Acceleration-time graph will be	4.	sine curve

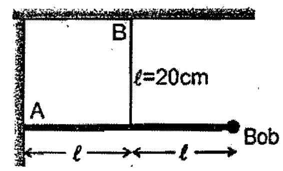
(A) P-1, Q-1, R-1, S-3 (B) P-2, Q-2, R-2, S-3
(C) P-2, Q-3, R-1, S-4 (D) P-2, Q-2, R-3, S-3

SECTION - V
Integer value correct Type

This section contains 7 questions. The answer to each question is a integer type.

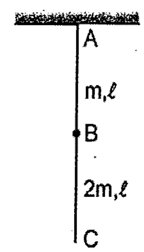
27. Two particles P_1 and P_2 are performing SHM along the same line about the same mean position. Initially they are at their positive extreme position. If the time period of each particle is 12 sec and the difference of their amplitudes is 12 cm then find the minimum time in seconds after which the separation between the particles become 6 cm.

25. A weightless rigid rod with a small iron bob at the end is hinged at point A to the wall so that it can rotate in all directions. The rod is kept in the horizontal position by a vertical inextensible string of length 20 cm, fixed at its mid point. The bob is displaced slightly, perpendicular to the plane of the rod and string. Find period of small oscillations of the system in the form $\frac{\pi X}{10}$ second. and fill value of X. (g = 10 m/s²).

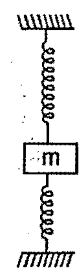


26. A straight line source of sound of length L = 10m, emitts a pulse of sound that travels radially outward from the source. If '4N' mwatt sound energy is intercepted by an acoustic cylindrical detector of surface area 2.4cm², located at a perpendicular distance 7m from the source then value of N is. The waves reach perpendicularly at the surface of the detector. The total power emitted by the source in the form of sound is 2.2×10^4W. and Use $\pi = 22/7$)

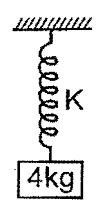
27. In the figure shown strings AB and BC have masses m and 2m respectively. Both are of same length ℓ. Mass of each string is uniformly distributed on its length. The string is suspended vertically from the ceiling of a room. A small jerk wave pulse is given at the end 'C'. It goes up to upper end 'A' in time 't'. If m = 2 kg, $\ell = \frac{9610}{1681}$ m, g = 10 m/s², $\sqrt{2} = 1.4$, $\sqrt{3} = 1.7$ Then find the value of 't' in seconds .



28. One end of a spring is fixed to the ceiling and other end is attached to a block. The block is released when spring is relaxed. The product of time period and amplitude is 8 S.I. units. If spring is cut in two equal parts and the two springs are attached to the block as shown in figure. The block is released when both springs are relaxed. Now find the product of time period and amplitude in S.I. units.



29. A body of mass 4 kg is suspended from a spring of spring constant 400 N/m. Another body of mass 4 kg moving vertically upward with 2 m/s hits it & gets embedded in it. If amptitude is $\frac{1}{x}\sqrt{\frac{3}{4}}$ m. find 'x'.



30. A point source of sound emiting sound of frequency 700 Hz and observer starts moving from a point along mutually perpendicular directions with velocity 20 m/s and 15 m/s respectively. If change in observed frequency by observer is 10x Hz then calculate 'x'. [speed of sound in 334 m/sec]

PART TEST - 4 (PT-4)

TOPIC: OPTICS : CLASS-XII

Duration : 1 Hour — Max. Marks : 90

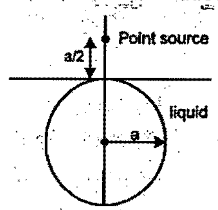
GENERAL INSTRUCTIONS

1. This Question Paper contains 30 questions.
2. For each question in Section I , you will be awarded 3 Marks if you give the correct answer and zero Mark if no answer is given. In all other cases, minus one (–1) Mark will be awarded.
3. For each question in Section II , you will be awarded 3 Marks if you give the correct answer. There is no negative marking.
4. For each question in Section III, you will be awarded 3 Marks if you give the correct answer and zero Mark if no answer is given. In all other cases, minus one (–1) Mark will be awarded.
5. For each question in Section IV, you will be awarded 3 Marks if you give the correct answer and zero Mark if no answer is given. In all other cases, minus one (–1) Mark will be awarded.
6. For each question in Section V, you will be awarded 3 Marks if you give the correct answer. There is no negative marking.

SECTION - I
Straight Objective Type

This section contains 7 Single choice questions. Each question has choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

1. An opaque sphere of radius a is just immersed in a transparent liquid as shown in figure. A point source is placed on the vertical diameter of the sphere at a distance a/2 from the top of the sphere. One ray originating from the point source after refraction from the air liquid interface forms tangent to the sphere. The angle of refraction for that particular ray is 30°. The refractive index of the liquid is -



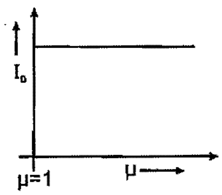
(A) $\frac{2}{\sqrt{3}}$ (B) $\frac{3}{\sqrt{5}}$

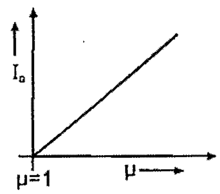
(C) $\frac{4}{\sqrt{5}}$ (D) $\frac{4}{\sqrt{7}}$

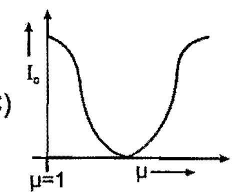
2. A thin oil film of refractive index 1.2 floats on the surface of water ($\mu = \frac{4}{3}$). When a light of wavelength $\lambda = 9.6 \times 10^{-7}$ m falls normally on the film from air, then it appears dark when seen normally. The minimum change in its thickness for which it will appear bright in normally reflected light by the same light is:

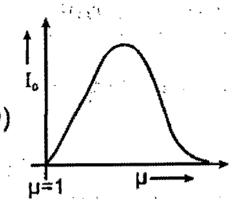
(A) 10^{-7} m (B) 2×10^{-7} m (C) 3×10^{-7} m (D) 5×10^{-7} m

3. In a YDSE experiment if a slab whose refractive index can be varied is placed in front of one of the slits then the variation of resultant intensity at mid-point of screen with 'μ' will be best represented by (μ ≥ 1).[Assume slits of equal width and there is no absorption by slab; mid point of screen is the point where waves interfere with zero phase difference in absence of slab]

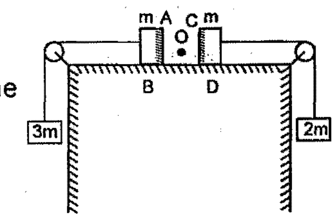
(A) 

(B) 

(C) 

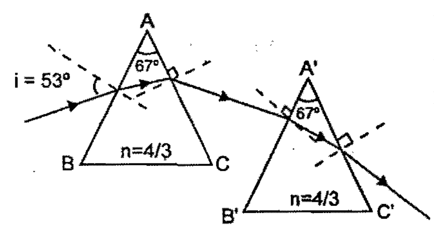
(D) 

4. Two blocks each of mass m lie on a smooth table. They are attached to two other masses as shown in the figure. The pulleys and strings are light. An object O is kept at rest on the table. The sides AB & CD of the two blocks are made reflecting. The acceleration of two images formed in those two reflecting surfaces w.r.t. each other is:



(A) 5g/6 (B) 5g/3
(C) g/3 (D) 17 g/6

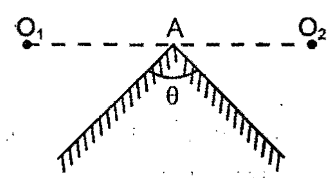
5. A ray is incident on the first prism at an angle of incidence 53° as shown in the figure. The angle between side CA and B'A' for the net deviation by both the prisms to be double of the deviation produced by the first prism, will be



(A) $2\sin^{-1}\frac{2}{3}$ (B) $\sin^{-1}\frac{2}{3}$
(C) $\cos^{-1}\frac{2}{3}$ (D) $\sin^{-1}\frac{1}{3}$

6. Light of wavelength 4000 Å is incident at small angle on a prism of apex angle 4°. The prism has n_v = 1.5 & n_r = 1.48. The angle of dispersion produced by the prism in this light is :
(A) 0.2° (B) 0.08° (C) 0.192° (D) none of these

7. Two plane mirrors are joined together as shown in figure. Two point objects O_1 and O_2 are placed symmetrically such that $AO_1 = AO_2$. The image of the two objects is common if :



(A) θ = 60° (B) θ = 90°
(C) θ = 30° (D) θ = 45°

SECTION - II
Multiple Correct Answers Type

This section contains 7 Multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONE OR MORE** may be correct.

8. A particle is moving towards a fixed convex mirror. The image also moves. If V_i = speed of image and V_o = speed of the object, then
(A) Vi £ VO if |u| < |F| (B) Vi > VO if |u| > |F|
(C) Vi < VO if |u| > |F| (D) Vi = VO if |u| = |F|

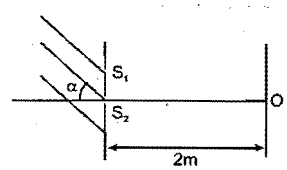
9. The speed of light in the material of a plano convex lens is 2 × 108 m/s and its greatest thickness is 3 mm. If the aperture diameter of the lens in 6.0 cm then :
(A) focal length of the lens is 6.0 cm.
(B) focal length of the lens is 30.4 cm.
(C) radius of curvature of the lens is 15.2 cm.
(D) parallel rays after falling on the lens converge at a distance of 6.0 cm on the other side of the lens.

10. In displacement method, the distance between object and screen is 96 cm. The ratio of length of two images formed by a convex lens placed between them is 4.84.
(A) Ratio of the length of object to the length of shorter image is 11/5.
(B) Distance between the two positions of the lens is 36 cm.
(C) Focal length of the lens is 22.5 cm.
(D) Distance of the lens from the shorter image is 30 cm.

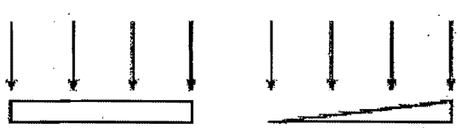
11. Which of the following statements is/are correct about the refraction of light from a plane surface when light ray is incident in denser medium. [C is critical angle]
(A) The maximum angle of deviation during refraction is $\frac{\pi}{2} - C$, it will be at angle of incidence C.
(B) The maximum angle of deviation for all angle of incidences is $\pi - 2C$, when angle of incidence is slightly greater than C.
(C) If angle of incidence is less than C then deviation increases if angle of incidence is also increased.
(D) If angle of incidence is greater than C then angle of deviation decreases if angle of incidence is increased.

12. A small air bubble is trapped inside a transparent cube of size 12 cm. When viewed from one of the vertical faces, the bubble appears to be at 5 cm from it. When viewed from opposite face, it appears at 3 cm from it.
(A) The distance of the air bubble from the first face is 7.5 cm.
(B) The distance of the air bubble from the first face is 9 cm.
(C) Refractive index of the material of the prism is 2.0.
(D) Refractive index of the material of the prism is 1.5.

13. A parallel beam of light ($\lambda = 5000$ Å) is incident at an angle $\alpha = 30°$ with the normal to the slit plane in a young's double slit experiment. Assume that the intensity due to each slit at any point on the screen is I_0. Point O is equidistant from S_1 & S_2. The distance between slits is 1mm.
(A) the intensity at O is $4I_0$
(B) the intensity at O is zero
(C) the intensity at a point on the screen 4m from O is $4I_0$
(D) the intensity at a point on the screen 4m from O is zero



14. Figure shows two thin slabs of glass, one is rectangular and the other is triangular wedge shaped. A monochromatic light incident nearly normally on the slabs as shown in figure.



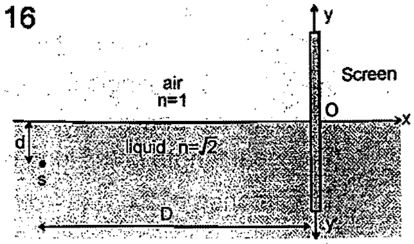
(A) In situation (X) fringes are thinner as compared to fringes obtained in situation (Y)
(B) In case (X) no fringes are obtained and in case (Y) fringes are obtained
(C) The shape of the fringe obtained is circular.
(D) The shape of the fringe obtained is straight line.

SECTION - III

Comprehension Type

This section contains 3 Paragraphs. Based upon each paragraph, 2 **Multiple choice questions** have to be answered. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

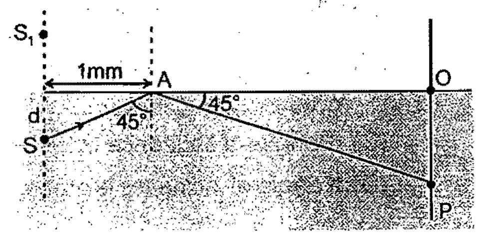
Paragraph for Question Nos. 15 to 16



A monochromatic point source S of wavelength $\lambda = 5000\sqrt{2}$ Å (in air) is placed at a distance d = 1 mm below the surface of transparent liquid as shown in figure. A very large screen is placed along y-axis at horizontal distance D = 1 metre from point source. The refractive index of liquid is $\sqrt{2}$. (Neglect partial reflection of rays from source S at liquid-air interface.)

$$\sin c = \frac{\mu_{air}}{\mu_{liquid}} = \frac{1}{\sqrt{2}}$$

The critical angle is C = 45°



Hence all rays from S which are incident on interface at an angle of incidence greater than 45° are reflected back (and appear to come from S_1) and fall on screen between OP. Light also directly falls on OP from S.

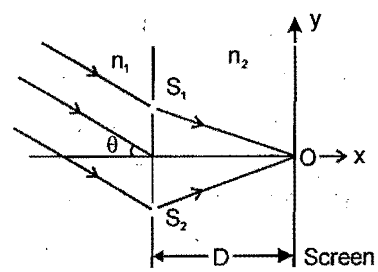
∴ Interference pattern is formed in region OP.

Since waves from S do not under go phase change during reflection, We can take S and S_1 to be coherent sources in same phase. Hence zero order bright is formed at y = 0.

15. The y-coordinates of second order bright formed on the screen is
(A) 250 μm (B) 500 μm (C) – 250 μm (D) – 500 μm

16. The region on screen where interference pattern is formed lies in
(A) $-\infty < y < +\alpha$ (B) $-\infty < y < 0$ (C) $1(m) < y \le 0$ (D) $-1\ (m) < y \le 0$

Paragraph for Question Nos. 17 to 18



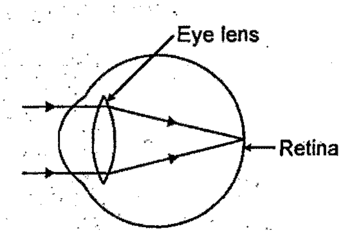
In the figure an arrangement of young's double slit experiment is shown. A parallel beam of light of wavelength 'λ' (in medium n_1) is incident at an angle 'θ' as shown. Distance $S_1O = S_2O$. Point 'O' is the origin of the coordinate system. The medium on the left and right side of the plane of slits has refractive index n_1 and n_2 respectively. Distance between the slits is d. The distance between the screen and the plane of slits is D. Using

$D = 1\text{m}, d = 1\text{mm}, \theta = 30°, \lambda = 0.3\text{mm}, n_1 = \frac{4}{3}, n_2 = \frac{10}{9}$, answer the following

17. The y-coordinate of the point where the total phase difference between the interefering waves is zero, is
(A) y = 0 (B) $y = +\frac{3}{4}$ m (C) $y = -\frac{3}{4}$ m (D) $-\frac{1}{\sqrt{3}}$ m

18. If the intensity due to each light wave at point 'O' is I_0 then the resultant intensity at point 'O' will be -
(A) Zero (B) $2I_0\left(1+\cos\frac{40\pi}{9}\right)$ (C) $3I_0$ (D) I_0

Paragraph for Question Nos. 19 to 20



The ciliary muscles of eye control the curvature of the lens in the eye and hence can alter the effective focal length of the system. When the muscles are fully relaxed, the focal length is maximum. When the muscles are strained the curvature of lens increases (that means radius of curvature decreases) and focal length decreases. For a clear vision the image must be on retina. The image distance is therefore fixed for clear vision and it equals the distance of retina from eye-lens. It is about 2.5 cm for a grown-up person.

A person can theoretically have clear vision of objects situated at any large distance from the eye. The smallest distance at which a person can clearly see is related to minimum possible focal length. The ciliary muscles are most strained in this position. For an average grown-up person minimum distance of object should be around 25 cm.
A person suffering from eye defects uses spectacles (Eye glass). The function of the lens of spectacles is to form the image of the objects within the range in which person can see clearly. The image of the spectacle-lens becomes object for eye-lens and whose image is formed on retina.
The number of spectacle-lens used for the remedy of eye defect is decided by the power of the lens required and the number of spectacle-lens is equal to the numerical value of the power of lens with sign.

For example power of lens required is +3D (converging lens of focal length $\frac{100}{3}$ cm) then number of lens will be +3.
For all the calculations required you can use the lens formula and lens maker's formula. Assume that the eye lens is equiconvex lens. Neglect the distance between eye lens and the spectacle lens.

19. Minimum focal length of eye lens of a normal person is

(A) 25 cm (B) 2.5 cm (C) $\frac{25}{9}$ cm (D) $\frac{25}{11}$ cm

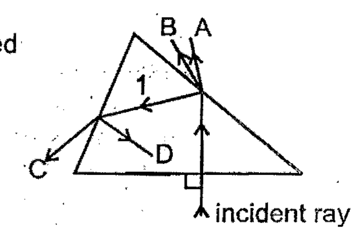
20. Maximum focal length of eye lens of normal person is

(A) 25 cm (B) 2.5 cm (C) $\frac{25}{9}$ cm (D) $\frac{25}{11}$ cm

SECTION – IV
Matching List Type (Only One Option Correct)

This section contains 3 questions, each having two matching lists. Choices for the correct combination of elements from List-I and List-II are given as options (A), (B), (C) and (D) out of which **ONE** is correct.

21. A white light ray is incident on a glass prism, and it create four refracted rays A, B, C and D. Match the refracted rays with the colours given (1 & D are rays due to total internal reflection.):



	List-I (Ray)		List-II (Colour)
P.	A	1.	red
Q.	B	2.	green
R.	C	3.	yellow
S.	D	4.	blue

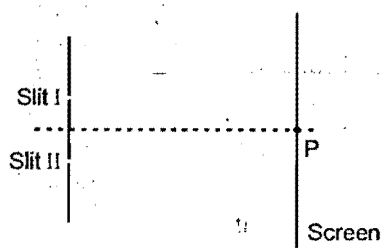
(A) P-1, Q-4, R-1,S-3 (B) P-1, Q-3, R-2,S-4
(C) P-2, Q-1, R-3,S-4 (D) P-2, Q-4, R-3,S-2

22. Match the following :

	List-I		List-II
P.	Object is between optic center and 1[st] principle focus in a diverging lens	1.	Image is inverted
Q.	Object is between optic center and 1[st] principle focus of a converging lens	2.	Image is Erect
R.	Object is between optic center and 2[nd] principle focus of a diverging lens	3.	Image is of greater size than the object
S.	Object is between optic center and 2[nd] principle focus of a converging lens	4.	Image is of smaller size than the object

(A) P-2, Q-3, R-3,S-4 (B) P-3, Q-2, R-2,S-4
(C) P-4, Q-3, R-2,S-2 (D) P-1, Q-2, R-4,S-2

23. A double slit interference pattern is produced on a screen, as shown in the figure, using monochromatic light of wavelength 500 nm. Point P is the location of the central bright fringe, that is produced when light waves arrive in phase without any path difference. A choice of three strips A, B and C of transparent materials with different thicknesses and refractive indices is available, as shown in the table. These are placed over one or both of the slits, singularly or in conjunction, causing the interference pattern to be shifted across the screen from the original pattern. In the column-I, how the strips have been placed, is mentioned whereas in the column-II, order of the fringe at point P on the screen that will be produced due to the placement of the strip(s), is shown. Correctly match both the columns.



Film	A	B	C
Thickness (in μm)	5	1.5	0.25
Refractive Index	1.5	2.5	2

List - I		List - II	
P.	Only strip B is placed over slit-I	1.	First Bright
Q.	Strip A is placed over slit-I and strip C is placed over slit-II	2.	Fourth Dark
R.	Strip A is placed over the slit-I and strip B and strip C are placed over the slit-II in conjunction.	3.	Fifth Dark
S.	Strip A and strip C are placed over slit-I (in conjuction) and strip B is placed over slit-II.	4.	Central Bright

(A) P-1, Q-3, R-1, S-1 (B) P-2, Q-3, R-2, S-3

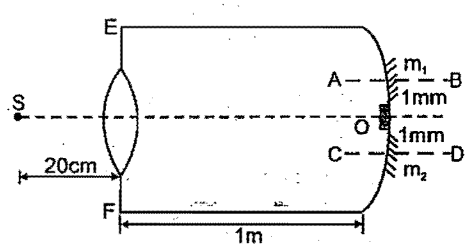
(C) P-2, Q-2, R-2, S-4 (D) P-3, Q-4, R-1, S-1

SECTION - V

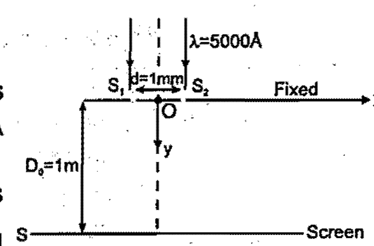
Integer value correct Type

This section contains 7 questions. The answer to each question is a integer type.

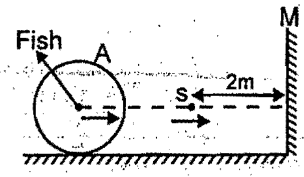
24. An equiconvex lens of focal length 10 cm (in air) and R.I. 3/2 is put at a small opening on a tube of length 1 m fully filled with liquid of R.I. 4/3. A concave mirror of radius of curvature 20 cm is cut into two halves m_1 and m_2 and placed at the end of the tube. m_1 & m_2 are placed such that their principal axes AB and CD respectively are separated by 1 mm each from the principal axes of the lens. A slit S placed in air illuminates the lens with light of frequency 7.5×10^{14} Hz. The light reflected from m_1 and m_2 forms interference pattern on the left end EF of the tube. O is an opaque substance to cover the gap left by m_1 & m_2. The position of the image formed by lens water combination is $a \times 10^{-2}$m from lens and the distance between the images formed by m_1 & m_2 is $b \times 10^{-3}$ m then find $\frac{ab}{40}$.



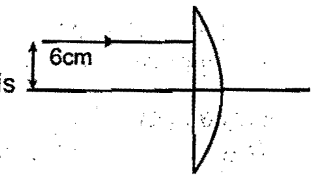
25. In the figure shown light of wave length $\lambda = 5000$ Å is incident on the slits (in a horizontal fixed plane) S_1 and S_2 separated by distance d = 1 mm. A horizontal screen 'S' is released from rest from initial distance $D_0 = 1$ m from the plane of the slits. Taking origin at O and positive x and y axis as shown, at t = 2 seconds; (Use g = 10 m/s²) velocity is $10n\hat{j}$ m/s and acceleration is $m\hat{j}$ m/s² of central maxima, find the value of $\left[\frac{mn}{25}\right]$



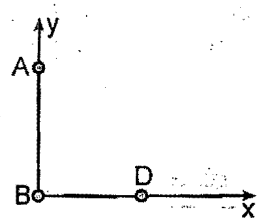
26. A is a thin walled sphere at rest made up of glass. The radius of the sphere is 1m and it is filled with a transparent liquid of refractive index μ. S is the luminous source moving directly towards a plane vertical mirror M. A fish in the liquid is moving towards S. The eye of the fish and S are collinear & perpendicular to mirror M. At the moment S is 3m away from the centre of the sphere, fish observes that image of S due to reflection is moving with speed of 13m/s. If speed of the fish relative to the sphere is 10m/s and μ = 1.5 then find the speed of the source at that instant. The system is placed in air.



27. A light ray parallel to the principal axis is incident (as shown in the figure) on a thin planoconvex lens with radius of curvature of its curved part equal to 10 cm. Assuming that the refractive index of the material of the lens is 4/3 and medium on both sides of the lens is air, find the distance of the point from the lens where this ray meets the principal axis. Find your answer in the form $\frac{26X}{7}$ cm and fill value of X.

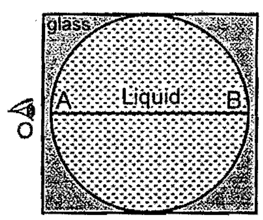


28. An interference is observed due to two coherent sources 'A' & 'B' having phase constant zero, separated by a distance 4 l along the y - axis where l is the wavelength of the sources. A detector D is moved on the positive x - axis. The total number of points on the x - axis excluding the points, x = 0 & x = ¥ at which maximum will be observed



29. The observer 'O' sees the distance AB as infinitely large.

If refractive index of liquid is μ_1 and that of glass is μ_2, then $\frac{\mu_1}{\mu_2}$ is :



30. An object of height 1 cm is kept perpendicular to the principal axis of a convex mirror of radius of curvature 20 cm. distance of the object from the mirror is 20 cm. If the distance between heads of the image and the object will be $\sqrt{\frac{1601 \times x}{y}}$ then what is the value of y - x.

PART TEST - 5 (PT-5)

TOPIC: ELECTRODYNAMICS : CLASS-XII

Duration : 1 Hour **Max. Marks : 90**

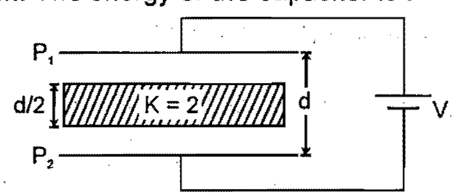
GENERAL INSTRUCTIONS

1. This Question Paper contains 30 questions.
2. For each question in Section I , you will be awarded 3 Marks if you give the correct answer and zero Mark if no answer is given. In all other cases, minus one (–1) Mark will be awarded.
3. For each question in Section II , you will be awarded 3 Marks if you give the correct answer. There is no negative marking.
4. For each question in Section III, you will be awarded 3 Marks if you give the correct answer and zero Mark if no answer is given. In all other cases, minus one (–1) Mark will be awarded.
5. For each question in Section IV, you will be awarded 3 Marks if you give the correct answer and zero Mark if no answer is given. In all other cases, minus one (–1) Mark will be awarded.
6. For each question in Section V, you will be awarded 3 Marks if you give the correct answer. There is no negative marking.

SECTION - I
Straight Objective Type

This section contains 7 Single choice questions. Each question has choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

1. In the figure shown a parallel plate capacitor has a dielectric of width d/2 and dielectric constant K = 2. The other dimensions of the dielectric are same as that of the plates. The plates P_1 and P_2 of the capacitor have area 'A' each. The energy of the capacitor is :



(A) $\frac{\epsilon_0 AV^2}{3d}$ (B) $\frac{2\epsilon_0 AV^2}{d}$ (C) $\frac{3}{2}\frac{\epsilon_0 AV^2}{d}$ (D) $\frac{2\epsilon_0 AV^2}{3d}$

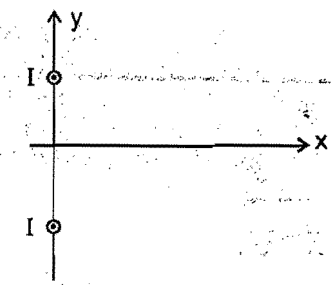
2. A rod of length ℓ having uniformly distributed charge Q is rotated about one end with constant frequency ' f '. Its magnetic moment.

(A) πfQl^2 (B) $\frac{\pi fQ\ell^2}{3}$ (C) $\frac{2\pi fQ\ell^2}{3}$ (D) $2\pi fQ l^2$

3. Two identical spheres of same mass and specific gravity (which is the ratio of density of a substance and density of water) 2.4 have different charges of Q and – 3Q. They are suspended from two strings of same length ℓ fixed to points at the same horizontal level, but distant ℓ from each other. When the entire set up is transferred inside a liquid of specific gravity 0.8, it is observed that the inclination of each string in equilibrium remains unchanged. Then the dielectric constant of the liquid is

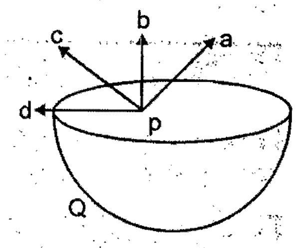
(A) 2 (B) 3 (C) 1.5 (D) None of these

4. Two infinitely long parallel wires are a distance d apart and carry equal parallel currents I in the same direction as shown in the figure. If the wires are located on y axis (normal to x-y plane) at $y = \frac{d}{2}$ and $y = -\frac{d}{2}$, then the magnitude of x-coordinate of the point on x-axis where the magnitude of magnetic field is maximum is (Consider points on x-axis only)



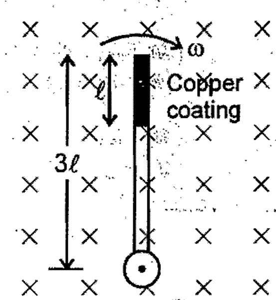
(A) $\sqrt{2}$ d (B) d/$\sqrt{2}$
(C) 2d (D) d/2

5. Figure shows a uniformly charged hemispherical shell. The direction of electric field at point p, that is off-centre (but in the plane of the largest circle of the hemisphere), will be along



(A) pa (B) pb
(C) pc (D) pd

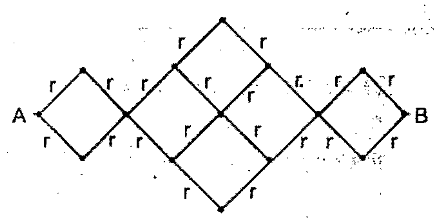
6. A wooden stick of length 3ℓ is rotated about an end with constant angular velocity ω in a uniform magnetic field B perpendicular to the plane of motion. If the upper one third of its length in coated with copper, the potential difference across the copper coating of the stick is



(A) $\frac{9B\omega\ell^2}{2}$ (B) $\frac{4B\omega\ell^2}{2}$

(C) $\frac{5B\omega\ell^2}{2}$ (D) $\frac{B\omega\ell^2}{2}$

7. The resistance of each straight section is r. Find the equivalent resistance between A and B.

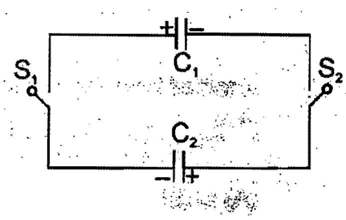


(A) 3 r (B) 3.5 r
(C) 4 r (D) 4.5 r

SECTION - II
Multiple Correct Answers Type

This section contains 7 Multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONE OR MORE may be correct.

8. Two capacitors C_1 & C_2 are charged to same potential V, but with opposite polarity as shown in fig. The switch S_1 & S_2 are then closed.



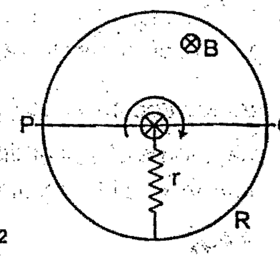
(A) P.d. across two capacitors are same & is given by $\frac{(C_1-C_2)V}{(C_1+C_2)}$

(B) P.d. across two capacitors are same & is given by $\frac{C_1V}{(C_1+C_2)}$

(C) Ratio of final energy to initial energy of the system is $\left(\frac{(C_1-C_2)}{(C_1+C_2)}\right)^2$

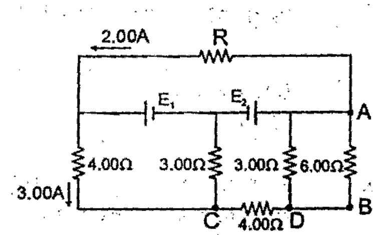
(D) Ratio of final energy to initial energy of the system is $\left(\frac{(C_1)}{(C_1+C_2)^2}\right)$

9. In the figure shown 'R' is a fixed conducting ring of negligible resistance and radius 'a'. PQ is a uniform rod of resistance r. It is hinged at the centre of the ring and rotated about this point in clockwise direction with a uniform angular velocity ω. There is a uniform magnetic field of strength 'B' pointing inwards. 'r' is a stationary resistance, then choose correct statements



(A) Current through 'r' is zero.
(B) Current through 'r' is $\frac{2B\omega a^2}{5r}$.
(C) Direction of current in external 'r' is from centre to circumference.
(D) Direction of current in external 'r' is from circumference to centre.

10. In the circuit shown in figure, E_1 and E_2 are two ideal sources of unknown emfs. Some currents are shown. Potential difference appearing across 6Ω resistance is $V_A - V_B = 10V$.

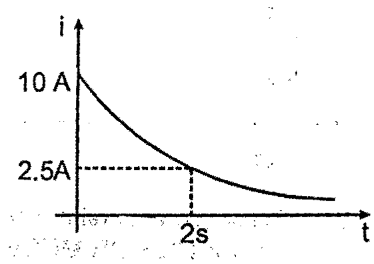


(A) The current in the 4.00 Ω resistor in between C and D is 5A.
(B) The unknown emf E_1 is 36 V.
(C) The unknown emf E_2 is 54 V.
(D) The resistance R is equal to 9 Ω.

11. A proton of charge 'e' and mass 'm' enters a uniform and constant magnetic field $\vec{B} = B\hat{i}$ with an initial velocity $\vec{V} = V_{OX}\hat{i} + V_{OY}\hat{j}$. Which of the following will be correct at any later time 't' :

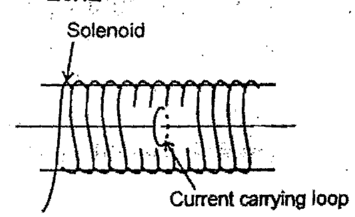
(A) x - component of velocity of the proton will be $V_{ox}\cos\left(\frac{qBt}{m}\right)$
(B) y - component of velocity will be $V_{oy}\cos\left(\frac{qBt}{m}\right)$
(C) z-component of velocity will be $V_{oy}\sin\left(\frac{qBt}{m}\right)$ along $(+\hat{k})$ direction.
(D) x-component of velocity will remain unchanged.

12. The figure shows, a graph of the current in a discharging circuit of a capacitor through a resistor of resistance 10 Ω.



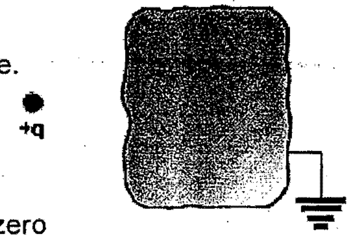
(A) The initial potential difference across the capacitor is 100 volt.
(B) The capacitance of the capacitor is $\frac{1}{10\ell n2}$ F.
(C) The total heat produced in the circuit will be $\frac{500}{\ell n2}$ joules.
(D) The thermal power in the resistor will decrease with a time constant $\frac{1}{2\ell n2}$ second.

13. A single circular loop of wire with radius 0.02 m carries a current of 8.0 A. It is placed at the centre of a solenoid that has length 0.65 m, radius 0.080 m and 1300 turns.



(A) The value of the current in the solenoid so that the magnetic field at the centre of the loop becomes zero, is equal to 44 mA.
(B) The value of the current in the solenoid so that the magnetic field at the centre of the loop becomes zero, is equal to 100 mA.
(C) The magnitude of the total magnetic field at the centre of the loop (due to both the loop and the solenoid) if the current in the loop is reversed in direction from that needed to make the total field equal to zero tesla, is $8\pi \times 10^{-5}$ T.
(D) The magnitude of the total magnetic field at the centre of the loop (due to both the loop and the solenoid) if the current in the loop is reversed in direction from that needed to make the total field equal to zero tesla, is $16\pi \times 10^{-5}$ T.

14. In front of an earthed conductor a point charge + q is placed as shown in figure :



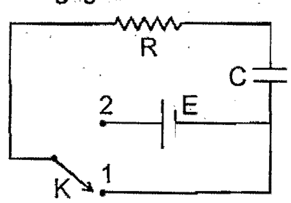
(A) On the surface of conductor the net charge is always negative.
(B) On the surface of conductor at some points charges are negative and at some points charges may be positive distributed non uniformly
(C) Inside the conductor electric field due to point charge is non zero
(D) None of these

SECTION - III
Comprehension Type

This section contains 3 Paragraphs. Based upon each paragraph, **2 Multiple choice questions** have to be answered. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 15 to 16

In the shown circuit involving a resistor of resistance R Ω, capacitor of capacitance C farad and an ideal cell of emf E volts, the capacitor is initially uncharged and the key is in position 1. At t = 0 second the key is pushed to position 2 for t_0 = RC seconds and then key is pushed back to position 1 for t_0 = RC seconds. This process is repeated again and again. Assume the time taken to push key from position 1 to 2 and vice versa to be negligible.



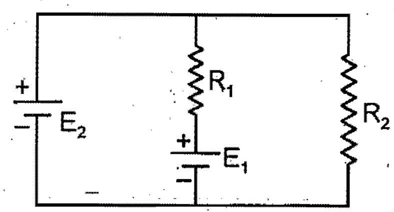
15. The charge on capacitor at t = 2RC second is

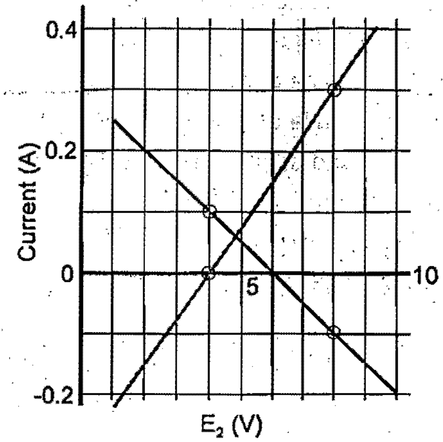
(A) CE (B) $CE\left(1-\frac{1}{e}\right)$ (C) $CE\left(\frac{1}{e}-\frac{1}{e^2}\right)$ (D) $CE\left(1-\frac{1}{e}+\frac{1}{e^2}\right)$

16. Then the variation of charge on capacitor with time is best represented by

Paragraph for Question Nos. 17 to 18

In the circuit given below, both batteries are ideal. EMF E_1 of battery 1 has a fixed value, but emf E_2 of battery 2 can be varied between 1.0 V and 10.0 V. The graph gives the currents through the two batteries as a function of E_2, but are not marked as which plot corresponds to which battery. But for both plots, current is assumed to be negative when the direction of the current through the battery is opposite the direction of that battery's emf. (Direction of emf is from negative to positive)



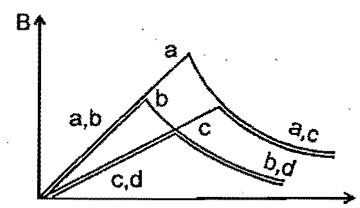


17. The value of emf E_1 is
(A) 8 V (B) 6 V (C) 4 V (D) 2V

18. The resistance R_1 has value
(A) 10 Ω (B) 20 Ω (C) 30 Ω (D) 40 Ω

Paragraph for Question Nos. 19 to 20

Curves in the graph shown give, as functions of radial distance r (from the axis), the magnitude B of the magnetic field (due to individual wire) inside and outside four long wires a, b, c and d, carrying currents that are uniformly distributed across the cross sections of the wires. Overlapping portions of the plots are indicated by double labels. All curves start from the origin.



19. Which wire has the greatest radius ?
(A) a (B) b (C) c (D) d

20. Which wire has the greatest magnitude of the magnetic field on the surface ?
(A) a (B) b (C) c (D) d

SECTION – IV
Matching List Type (Only One Option Correct)

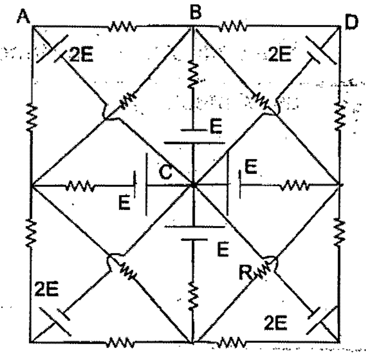
This section contains 3 questions, each having two matching lists. Choices for the correct combination of elements from List-I and List-II are given as options (A), (B), (C) and (D) out of which ONE is correct.

21. A charged particle having non zero velocity is subjected to certain conditions given in Column I. Column II gives possible trajectories of the particle. Match the conditions in column I with the results in Column II

List-I		List-II	
P.	In only uniform electric field	1.	the path of the charged particle may be a straight line
Q.	In only uniform magnetic field	2.	the path of the charged particle may be a parabola
R.	In uniform magnetic and uniform electric field such that both are parallel	3.	the path of the charged particle may be a circle
S.	Subjected to a net force of constant magnitude	4.	the path of the charged particle may be a helix with uniform pitch

(A) P-1, Q-1, R-1, S-3 (B) P-2, Q-2, R-2, S-3
(C) P-2, Q-2, R-2, S-4 (D) P-2, Q-2, R-3, S-3

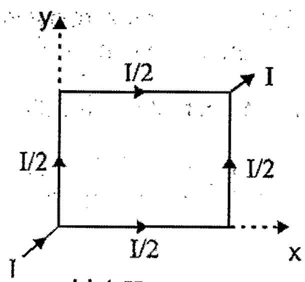
22. Consider the circuit shown in the figure.
Given E = 4 Volts and R = 1Ω . Take V_C = 0 Volt.



List-I	List-II	
P. Magnitude of current in AB in Amp	1.	1
Q. Magnitude of current in BC in Amp	2.	16
R. Potential at A in volt	3.	4
S. Potential at B in volt	4.	8

(A) P-3, Q-4, R-4,S-3
(B) P-4, Q-2, R-1,S-3
(C) P-3, Q-3, R-2,S-4
(D) P-3, Q-1, R-3,S-3

23. A square loop of uniform conducting wire is as shown in figure. A current-I (in amperes) enters the loop from one end and exits the loop from opposite end as shown in figure. The length of one side of square loop is ℓ metre. The wire has uniform cross section area and uniform linear mass density. In four situations of column-I, the loop is subjected to four different uniform and constant magnetic field. Under the conditions of column-I, match the column-I with corresponding results of column-II (B_0 in column I is a positive nonzero constant)



List-I	List-II
P. $\vec{B} = B_0\hat{i}$ in tesla	1. magnitude of net force on loop is $\sqrt{2}\,B_0 I \ell$ newton
Q. $\vec{B} = B_0\hat{j}$ in tesla	2. magnitude of net force on loop is zero
R. $\vec{B} = B_0(\hat{i}+\hat{j})$ in tesla	3. magnitude of net torque on loop about its centre is zero
S. $\vec{B} = B_0\hat{k}$ in tesla	4. magnitude of net force on loop is $B_0 I \ell$ newton

(A) P-1, Q-4, R-3,S-3
(B) P-4, Q-3, R-2,S-1
(C) P-3, Q-2, R-2,S-1
(D) P-4, Q-3, R-2,S-4

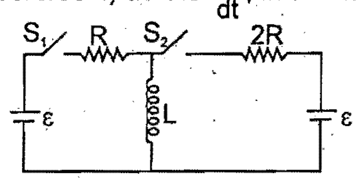
SECTION - V

Integer value correct Type

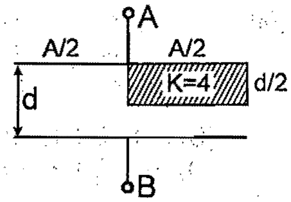
This section contains 7 questions. The answer to each question is a **integer type.**

24. A uniformly charged ring of radius 0.1 m rotates at a frequency of 10^4 rps about its axis. The ratio of energy density of electric field to the energy density of the magnetic field at a point on the axis at distance 0.2 m from the centre is in form X × 10^9. Find the value of X. (Use speed of light c = 3 × 10^8 m/s, π^2 = 10)

25. In the circuit shown S_1 and S_2 are switches. S_2 remains closed for a long time and S_1 open. Now S_1 is also closed. It is given that R = 10 Ω, L = 1 mH and ε = 3V. Just after S_1 is closed, the magnitude of rate of change of current (in ampere/sec.) that is $\frac{di}{dt}$, in the inductor L is x × 10^3 A/s find x

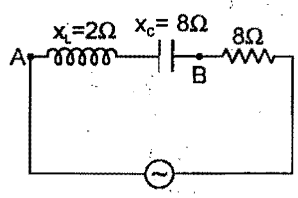


26. The equivalent capacitance between terminals 'A' and 'B' is $\frac{x}{10}\left(\frac{A\epsilon_0}{d}\right)$ Find x-5. The letters have their usual meaning.

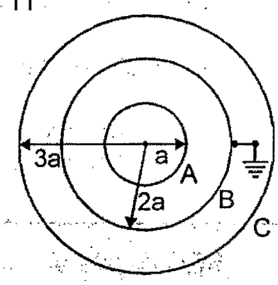


27. The current density $\vec{J}$ inside a long, solid, cylindrical wire of radius a = 12 mm is in the direction of the central axis and its magnitude varies linearly with radial distance r from the axis according to $J = \frac{J_0 r}{a}$,

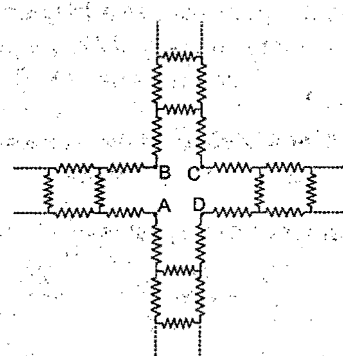
28. An inductor ($x_L = 2\Omega$) a capacitor ($x_C = 8\Omega$) and a resistance (8Ω) is connected in series with an A.C. source. The voltage output of A.C. source is given by v = 10 cos 100πt. If the instantaneous potential difference between A and B is $\frac{4x}{7}$ when it is half of the voltage output from source at that instant than what is the value of x :



29. Figure shows a system of three concentric metal shells A, B and C with radii a, 2a and 3a respectively. Shell B is earthed and shell C is given a charge Q. Now if shell C is connected to shell A, then the final charge on the shell B $\frac{-Qx}{11}$, is equal to :



30. Four infinite ladder network containing identical resistances of R Ω each, are combined as shown in figure. The equivalent resistance between A and B is R_{AB} and between A and C is R_{AC}. If the value of $\frac{R_{AB}}{R_{AC}}$ is $\frac{x}{4}$ than what is the value of x :



PART TEST - 6 (PT-6)

TOPIC : MODERN PHYSICS : CLASS-XII

Duration : 1 Hour **Max. Marks : 90**

GENERAL INSTRUCTIONS

1. This Question Paper contains 30 questions.
2. For each question in **Section I**, you will be awarded **3 Marks** if you give the correct answer and zero Mark if no answer is given. In all other cases, minus one (–1) Mark will be awarded.
3. For each question in Section II, you will be awarded 3 Marks if you give the correct answer. There is no negative marking.
4. For each question in **Section III**, you will be awarded **3 Marks** if you give the correct answer and zero Mark if no answer is given. In all other cases, minus one (–1) Mark will be awarded.
5. For each question in **Section IV**, you will be awarded 3 Marks if you give the correct answer and zero Mark if no answer is given. In all other cases, minus one (–1) Mark will be awarded.
6. For each question in **Section V**, you will be awarded 3 Marks if you give the correct answer. There is no negative marking.

SECTION - I
Straight Objective Type

This section contains 7 Single choice questions. Each question has choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

1. A radioactive nuclide A decays to nuclide B which further decays to C. Their disintegration constant are λ_1 and λ_2 respectively. At t = 0 only nuclei A are present. Number of nuclei A at t = 0 is N_0.

$$A \xrightarrow{\lambda_1} B \xrightarrow{\lambda_2} C$$

	A	B	C
t = 0	N_0	0	0
t	N_1	N_2	N_3

In the above radioactive decay C is stable nucleus. Then:

(A) rate of decay of A will first increase and then decrease
(B) number of nuclei of B will first increase and then decrease
(C) if $\lambda_2 > \lambda_1$, then activity of B will always be higher than activity of A
(D) if $\lambda_1 >> \lambda_2$, then number of nucleus of C will always be less than number of nucleus of B.

2. An enclosure filled with helium is heated to a temperature of 400 K. Helium atom emerges out of the enclosure. The mean debroglie wavelength of the helium atoms is :

(A) 0.44 Å (B) 0.63 Å (C) 0.77 Å (D) none of these

3. When a metallic surface is illuminated with monochromatic light of wavelength λ, the stopping potential is $5V_0$. When the same surface is illuminated with light of wavelength 3λ, the stopping potential is V_0. Then the work function of the metallic surface is :

(A) $\frac{hc}{6\lambda}$ (B) $\frac{hc}{5\lambda}$ (C) $\frac{hc}{4\lambda}$ (D) $\frac{2hc}{4\lambda}$

4. No. of identical photons incident on a perfectly black body of mass m kept at rest on smooth horizontal surface. Then the acceleration of the body if n number of photons incident per second is (Assume wavelength of photon to be λ) :

(A) $\frac{nh}{2\pi\lambda m}$ (B) $\frac{nh}{\lambda m}$ (C) $\frac{2\pi nh}{\lambda m}$ (D) $\frac{\lambda m}{nh}$

5. Two radioactive sources A and B initially contain equal number of radioactive atoms. Source A has a half-life of 1 hour and source B has a half-life of 2 hours. At the end of 2 hours, the ratio of activity of source A to that of B is :
(A) 1 : 2 (B) 2 : 1 (C) 1 : 1 (D) 1 : 4

6. The radionuclide ^{238}U decays by emitting an alpha particle.
$^{238}U \rightarrow {}^{234}Th + {}^{4}He$
The atomic masses of the three isotopes are.
^{238}U 238.05079 amu
^{234}U 234.04363 amu
^{4}He 4.00260 amu
What is the maximum kinetic energy of the emitted alpha particle? Express your answer in Joule.
(1 amu = 1.67×10^{-27} kg)
(A) 6.8×10^{-14} J (B) 6.8×10^{-13} J (C) 4.3×10^{-14} J (D) 4.3×10^{-13} J

7. Choose the correct statement.
(A) The nuclear force between two nucleons depends upon charge on each nucleon.
(B) The nuclear force is not a central force
(C) The nuclear force between the two nucleons increases rapidly as size of nucleus increases.
(D) Nuclear force is a conservative force.

SECTION - II
Multiple Correct Answers Type

This section contains 7 Multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONE OR MORE may be correct.

8. The decay constant of a radio active substance is 0.173 $(years)^{-1}$. Therefore:
(A) Nearly 63% of the radioactive substance will decay in (1/0.173) year.
(B) half life of the radio active substance is (1/0.173) year.
(C) one -forth of the radioactive substance will be left after nearly 8 years.
(D) average life of the substance is nearly 5.8 years.

9. In an x–ray tube the voltage applied is 20 kV. The energy required to remove an electron from L shell is 19.9 keV. In the x–rays emitted by the tube (Use hc = 12420 eVÅ)
(A) minimum wavelength will be 62.1 pm
(B) energy of the characterstic x-rays will be equal to or less than 19.9 KeV
(C) L_α x-ray may be emitted
(D) L_α x-ray will have energy 19.9 keV

10. When a hydrogen atom is excited from ground state to first excited state then
(A) its kinetic energy increases by 10.2 eV. (B) its kinetic energy decreases by 10 .2 eV.
(C) its potential energy increases by 20.4 eV. (D) its angular momentum increases by 1.05×10^{-34} J-s.

11. At t = 0, a sample of radionuclide A has the same decay rate as a sample of radionuclide B has at t = 60 min. The disintegration constants of A and B are λ_A and λ_B respectively, with $\lambda_A < \lambda_B$.
(A) The half life of radionuclide A is greater than that of B.
(B) At t = 60 min, number of atoms in sample of material A is greater than that of sample B.
(C) The two samples will never have the same decay rate simultaneously.
(D) After some time, the two samples will have the same decay rate simultaneously for an instant.

12. Ionization energy of a hydrogen-like ion B is less than that of hydrogen like ion A. Let r, u, E and L represent the radius of the orbit, speed of the electron, energy of the atom and orbital angular momentum of the electron respectively. In ground state

(A) $r_A > r_B$ (B) $u_A > u_B$ (C) $E_A > E_B$ (D) $L_A > L_B$

13. Mark the correct options.

(A) An atom with a vacancy has smaller energy than a neutral atom
(B) K X-ray is emitted when a hole makes a jump from the K shell to some other shell
(C) The wavelength of K X-ray is smaller than the wavelength of L X-ray of the same material
(D) The wavelength of K_α X-ray is smaller than the wavelength of K_β X-ray of the same material

14. An electron makes a transition from n = 2 to n = 1 state in a hydrogen like atom.

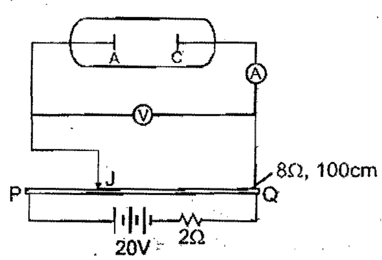
(A) magnetic field at the site of nucleus is decreased by 16 times.
(B) magnetic field at the site of nucleus is increased by 32 times
(C) angular momentum of electron is decreased
(D) angular momentum of electron is increased

SECTION - III
Comprehension Type

This section contains 3 Paragraphs. Based upon each paragraph, 2 **Multiple choice questions have** to be answered. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 15 to 16

An experimental setup of verification of photoelectric effect is shown in the diagram. The voltage across the electrodes is measured with the help of an ideal voltmeter and which can be varied by moving jockey 'J' on the potentiometer wire. The battery used in potentiometer circuit is of 20 V and its internal resistance is 2Ω. The resistance of 100 cm long potentiometer wire is 8Ω.



The photocurrent is measured with the help of an ideal ammeter. Two plates of potassium oxide of area 50 cm^2 at separation 0.5 mm are used in the vacuum tube. Photo current in the circuit is very small so we can treat potentiometer circuit an independent circuit.

The wavelengths of various colours is as follows :

Light	1 Violet	2 Blue	3 Green	4 Yellow	5 Orange	6 Red
λ in Å →	4000-4500	4500-5000	5000-5500	5500-6000	6000-6500	6500-7000

15. The number of electrons appeared on the surface of the cathode plate, when the jockey is connected at the end 'P' of the potentiometer wire. Assume that no radiation is falling on the plates.

(A) 8.85×10^6 (B) 11.06×10^9 (C) 8.85×10^9 (D) 0

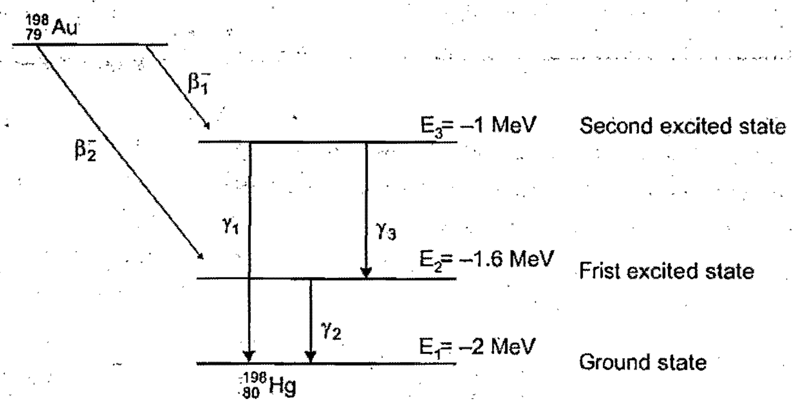
16. It is found that ammeter current remains unchanged (2μA) even when the jockey is moved from the end 'P' to the middle point of the potentiometer wire. Assuming all the incident photons eject electron and the power of the light incident is 4×10^{-6} W. Then the colour of the incident light is

(A) Green (B) Violet (C) Red (D) Orange

Paragraph for Question Nos. 17 to 18

Gold Nucleus ($_{79}Au^{198}$) can decay into mercury nucleus ($_{80}Hg^{198}$) by two decay schemes shown in figure. (i) It can emit a β particle (β_1) and come to ground state by either emitting one γ ray(γ_1) or emitting two γ rays (γ_3 & γ_2)

(ii) It can emit one β particle (β_2) and come to ground state by emitting γ_2 ray. Atomic masses : ^{198}Au = 197.9682 amu, ^{198}Hg = 197.9662 amu, 1 amu = 930 MeV/c^2. The energy levels of the nucleus are shown in figure.



17. What is the maximum kinetic energy of emitted β_2 particles is

(A) 1.44 MeV (B) 0.59 MeV (C) 1.86 MeV (D) 1.46 MeV

18. What is the maximum kinetic energy of emitted β_1 particles is -

(A) 1.28 MeV (B) 0.77 MeV (C) 1.86 MeV (D) 0.86 MeV

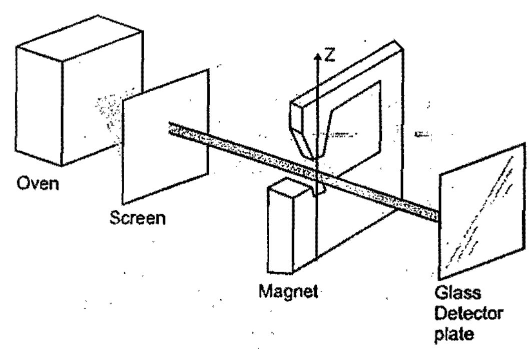
Paragraph for Question Nos. 19 to 20

Atomic Magneism

The orbital and spin angular momentum of the atom influence its magnetic structure and these properties are most directly studied by placing the atom in a magnetic field. Also, a magnetic field can affect the wavelengths of the emitted photons.

The angular momentum vector associated with an atomic state can take up only certain specified directions in space. This concept of space quantization was shown by Otto Stern and Walthor Gerlach in their experiment.

In the experiment, silver is vapourized in an electric oven and silver atoms spray into the evacuated apparatus through a small hole in the oven wall. The atoms which are electrically neutral but have a magnetic moment, are formed into a narrow beam as they pass through a slit in a screen. The beam, thus collimated, then passes between the poles of an electromagnet and finally, deposits its silver atoms on a glass plate that serves as a detector. The pole faces of the magnet are shaped to make the magnetic field as nonuniform as possible.



In a non-uniform magnetic field, there is a net force on a magnetic dipole. Its magnitude and direction depends on the orientation of the dipole. Thus the silver atoms in the beam are deflected up or down, depending on the orientation of their magnetic dipole moments with respect to the z–direction.

The potential energy of a magnetic dipole in a magnetic field $\vec{B}$ is given by $U = -\vec{\mu}.\vec{B}$

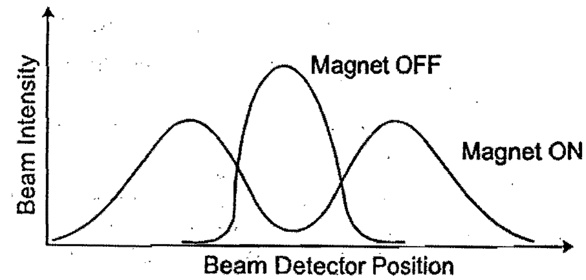
where μ is magnetic dipole moment of the atom. From symmetry, the magnetic field at the beam position has no x or y components i.e. $B = B_z$ then $U = -\mu_z B_z$.

The net force F_z on the dipole is $\left(-\frac{dU}{dz}\right)$ or $F_z = \mu_z \frac{dB_z}{dz}$.

Thus, the net force depends, not on the magnitude of the field itself, but on its spatial derivative or gradient.

The Results

If space quantization did not exist, then μ could take on any value from $+\mu$ to $-\mu$, the result would be a spreading out of the beam when the magnet was turned ON. However, the beam was split cleanly into two subbeams, each subbeam corresponding to one of the two permitted orientations of the magnetic moment of the silver atom, as shown.



In a silver atom, all the spin and orbital magnetic moments of the electrons cancel, except for those of the atom's single valance electron. For this electron the orbital magnetic moment is zero because orbital angular momentum is zero (because for electrons of s–orbit, L = 0), leaving only the spin magnetic moment. This can take up only two orientations in a magnetic field, corresponding to $m_s = +1/2$ and $m_s = -1/2$. Hence there are two subbeams – and not some other number.

19. A hydrogen atom in ground state passes through a magnetic field that has a gradient of 16mT/m in the vertical direction. If vertical component magnetic moment of the atom is 9.3×10^{-24} J/T, then force on it due to the magnetic moment of the electron is :

(A) 5.8×10^{-22} N (B) 1.5×10^{-25} N (C) 5.8×10^{-24} N (D) 1.5×10^{-24} N

20. In the Stern–Gerlach experiment conducted with silver atoms, the magnetic field $\vec{B}$ has a magnitude of 620 mT. If magnetic moment of a silver atom is 5.8×10^{-5} eV/T, then the energy difference between the orientation of the silver atoms in the two subbeams is :

(A) 3.6×10^{-5} eV (B) 3.6×10^{-2} eV (C) 7.2×10^{-5} eV (D) 7.2×10^{-2} eV

SECTION – IV

Matching List Type (Only One Option Correct)

This section contains 3 questions, each having two matching lists. Choices for the correct combination of elements from List-I and List-II are given as options (A), (B), (C) and (D) out of which **ONE** is correct.

21. Using Bohr's model for H–like atom, match the following. Here n = orbit number, Z = nuclear charge, m = mass of electron

List-I	List-II
P. Due to revolving electron, the magnetic field produced at its centre is proportional to	**1.** n^{-5}
Q. Magnetic moment of revolving electron is proportional to	**2.** n
R. De-Broglie wave length of revolving electron is proportional to	**3.** Z^3
S. Areal velocity of revolving electron about nucleus is proportional to	**4.** independent of Z

(A) P-1, Q-4, R-2, S-4 (B) P-2, Q-4, R-3, S-2
(C) P-3, Q-2, R-1, S-4 (D) P-1, Q-4, R-2, S-2

22. Match the statements given in column-I with their corresponding possible results in column-II.

List-I		List-II	
P.	If photons of ultraviolet light of energy 12 eV are incident on a metal surface of work function of 4 eV, then the stopping potential (in eV) will be	1.	8
Q.	The ratio of wavelengths of K_α lines of two elements is $\left(\frac{85}{81}\right)^2$ Number of elements having atomic numbers between these elements will be	2.	3
R.	If 20 gm of a radioactive substance due to radioactive decay reduces to 10 gm in 4 minutes, then in what time (in minutes) 80 gm of the same substance will reduce to 20 gm	3.	1
S.	The mass defect for the nucleus of helium is 0.0302a.m.u. The binding energy per nucleon for helium in MeV is approximately (1amu = 930 MeV/c^2)	4.	7

(A) P-1, Q-2, R-2,S-4
(B) P-4, Q-3, R-2,S-1
(C) P-2, Q-1, R-2,S-3
(D) P-1, Q-2, R-1,S-4

23. In column-I, consider each process just before and just after it occurs. Initial system is isolated from all other bodies. Consider all product particles (even those having rest mass zero) in the system. Match the system in column-I with the result they produce in column-II.

List-I	List-II
P. Spontaneous radioactive decay of an uranium nucleus initially at rest as given by reaction $^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He + ...$	1. Number of protons is increased
Q. Fusion reaction of two hydrogen nuclei as given by reaction $^{1}_{1}H + {}^{1}_{1}H \rightarrow {}^{2}_{1}H + ...$	2. Momentum is conserved
R. Fission of U^{235} nucleus initiated by a thermal neutron as given by reaction $^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{144}_{56}Ba + {}^{89}_{36}Kr + 3{}^{1}_{0}n + ...$	3. Mass is converted to energy or vice versa
S. β^- decay (negative beta decay)	4. Charge is conserved

(A) P-1, Q-2, R-3,S-1
(B) P-2, Q-1, R-4,S-2
(C) P-3, Q-4, R-2,S-3
(D) P-4, Q-1, R-1,S-4

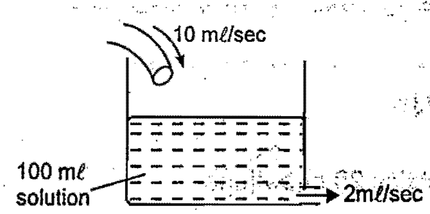
SECTION - V

Integer value correct Type

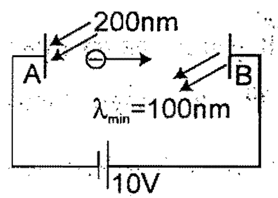
This section contains 7 **questions.** The answer to each question is a **integer type.**

24. A radioactive sample has 12.0×10^{18} active nuclei at a certain instant. Number of nuclei still in the same active state after two half-lives is $n \times 10^{18}$. Find n.

25. A 100 $m\ell$ solution having activity 50 dps is kept in a beaker. It is now constantly diluted by adding water at a constant rate of 10 $m\ell$/sec and 2 $m\ell$/sec of solution is constantly being taken out. The activity of 10 $m\ell$ solution which is taken out, assuming half life to be effectively very large is $A_0\left[1-\left(\frac{5}{a}\right)^{1/b}\right]$ where A_0 = 50 dps find a×b



26. In the figure shown electromagnetic radiations of wavelength 200nm are incident on the metallic plate A. The photo electrons are accelerated by a potential difference 10V. These electrons strike another metal plate B from which electromagnetic radiations are emitted. The minimum wavelength of the emitted photons is 100nm. The work function of the metal 'A' is x eV then find x + 2.2 use hc = 12400 eVÅ, use Rch = 13.6 eV.



27. An X-ray tube is working at potential of 20 kV. The potential difference is decreased to 10 kV. It is found that the difference of the wavelength of K_α X-ray and the most energetic continuous X-ray becomes 4 times the difference before the change of voltage. Find the atomic number of the target element. Take b = 1 and $\frac{1}{\sqrt{3.4}}$ = 0.54.

28. The wavelengths of K_α x-rays of two metals 'A' and 'B' are $\frac{4}{1875\,R}$ and $\frac{1}{675\,R}$ respectively, where 'R' is Rydberg constant. The number of elements lying between 'A' and 'B' according to their atomic numbers is

29. A cobalt (atomic no. = 27) target is bombarded with electrons, and the wavelengths of its characteristic x-ray spectrum are measured. A second weak characteristic spectrum is also found, due to an impurity in the target. The wavelengths of the K_α lines are 225.0 pm (cobalt) and 100.0 pm (impurity). If atomic number of the impurity is 5N find the value of N (take b = 1)

30. The radii of nuclei of two atoms are in ratio $\frac{3}{2}$. Assuming them to be Hydrogen like atom, the ratio of their orbital radius for K shell is $\frac{N}{27}$, find the value of N (assume no. of proton = No. of neutron for each atom)

FULL SYLLABUS TEST - 1 (FST-1)

CLASS : XI

Duration : 1 Hour **Max. Marks : 60**

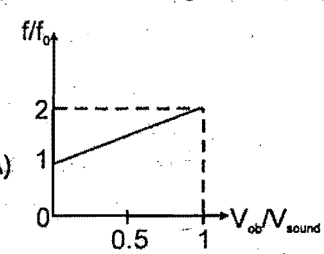
GENERAL INSTRUCTIONS :

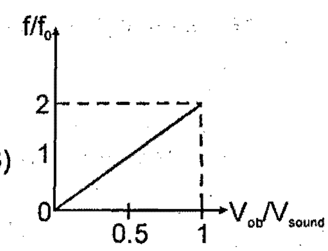
1. This Question Paper contains **20 questions.**
2. For each question in **Section I**, you will be **awarded 3 Marks** if you give the correct answer and zero **Mark** if no answer is given. In all other cases, **minus one (–1) Mark** will be awarded.
3. For each question in **Section II**, you will be **awarded 3 Marks** if you give the correct answer. There is **no negative marking.**
4. For each question in **Section III**, you will be **awarded 3 Marks** if you give the correct answer and zero **Mark** if no answer is given. In all other cases, **minus one (–1) Mark** will be awarded.
5. For each question in **Section IV**, you will be **awarded 3 Marks** if you give the correct answer and zero **Mark** if no answer is given. In all other cases, **minus one (–1) Mark** will be awarded.
6. For each question in **Section V**, you will be **awarded 3 Marks** if you give the correct answer. There is **no negative marking.**

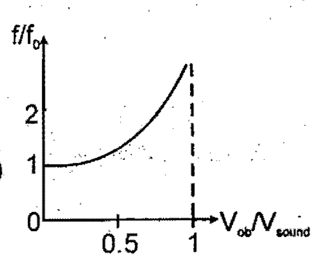
SECTION - I
Straight Objective Type

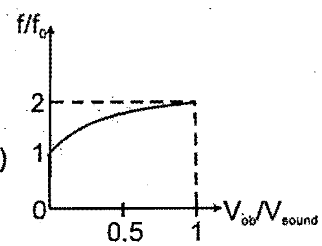
This section contains **5 Single choice questions.** Each question has choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

1. A curve is plotted to represent the dependence of the ratio of the received frequency f to the frequency f_0 emitted by the source on the ratio of the speed of observer V_{ob} to the speed of sound V_{sound} in a situation in which an observer is moving towards a stationary sound source. The curve is best represented by :

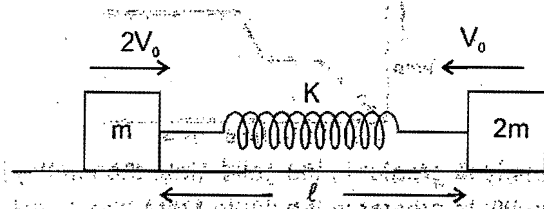
(A)


(B)


(C)


(D)


2. Two blocks A & B of masses m and 2m respectively, attached at opposite ends of a spring of spring constant K, placed on smooth horizontal surface. Spring is initially at its natural length ℓ. A is given a velocity $2V_0$ and B given velocity V_0 as shown.



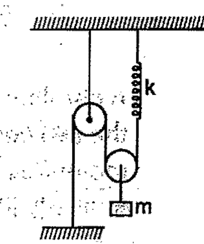
maximum seperation between m and centre of mass of the system will be :

(A) $\frac{\ell}{3}+\sqrt{\frac{8mV_0^2}{3K}}$ (B) $\frac{\ell}{3}+\sqrt{\frac{2mV_0^2}{3K}}$ (C) $\frac{2\ell}{3}+\sqrt{\frac{2mV_0^2}{3K}}$ (D) $\frac{2\ell}{3}+\sqrt{\frac{8mV_0^2}{3K}}$

3. A particle is executing SHM according to the equation $x = A\cos\omega t$. Average speed of the particle during the interval $0 \le t \le \frac{\pi}{6\omega}$.

(A) $\frac{\sqrt{3}A\omega}{2}$ (B) $\frac{\sqrt{3}A\omega}{4}$ (C) $\frac{3A\omega}{\pi}$ (D) $\frac{3A\omega}{\pi}\left(2-\sqrt{3}\right)$

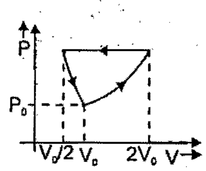
4. Mass m shown in figure is in equilibrium. If it is displaced further by x and released find its acceleration just after it is released. Take pulleys to be light & smooth and strings light.

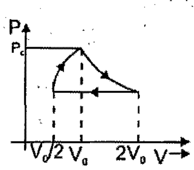


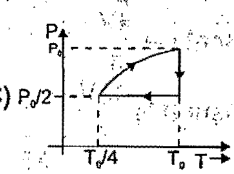
(A) $\frac{4kx}{5m}$ (B) $\frac{2kx}{5m}$

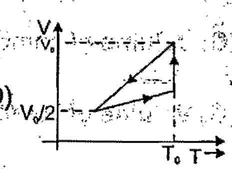
(C) $\frac{4kx}{m}$ (D) none of these

5. One mole of an ideal gas at pressure P_0 and temperature T_0 is expanded isothermally to twice its volume and then compressed at constant pressure to $(V_0/2)$ and the gas is brought back to original state by a process in which P α V (Pressure is directly proportional to volume). The correct representation of process is -

(A) 

(B) 

(C) 

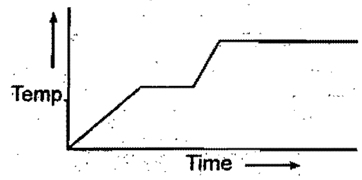
(D) 

SECTION - II

Multiple Correct Answers Type

This section contains 5 Multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONE OR MORE may be correct.

6. Heat is supplied to a certain homogeneous sample of matter at a uniform rate. Its temperature is plotted against time as shown in the figure. Which of the following conclusions can be drawn?



(A) its specific heat capacity is greater in the solid state than in the liquid state.
(B) its specific heat capacity is greater in the liquid state than in the solid state.
(C) its latent heat of vaporization is greater than its latent heat of fusion.
(D) its latent heat of vaporization is smaller than its latent heat of fusion.

7. The vibrations of a string of length 600 cm fixed at both ends are represented by the equation

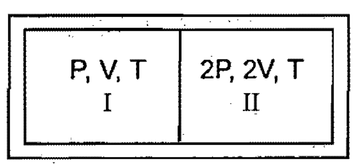
$$y = 4 \sin\left(\pi \frac{x}{15}\right) \cos(96\pi t)$$

where x and y are in cm and t in seconds.

(A) The maximum displacement of a particle at x = 5 cm is $2\sqrt{3}$ cm.
(B) The nodes located along the string are 15n where integer n varies from 0 to 40.
(C) The velocity of the particle at x = 7.5 cm at t = 0.25 sec is zero
(D) The equations of the component waves whose superposition gives the above wave are

$$2 \sin 2\pi\left(\frac{x}{30} + 48t\right),\ 2 \sin 2\pi\left(\frac{x}{30} - 48t\right).$$

8. A partition divides a container having insulated walls into two compartments I and II. The same gas fills the two compartments whose initial parameters are given. The partition is a conducting wall which can move freely without friction. Which of the following statements is/are correct, with reference to the final equilibrium position?



(A) The Pressure in the two compartments are equal.
(B) Volume of compartment I is $\frac{3V}{5}$
(C) Volume of compartment II is $\frac{12V}{5}$
(D) Final pressure in compartment I is $\frac{5P}{3}$

9. In a resonance tube experiment, a closed organ pipe of length 120 cm resonates when tuned with a tuning fork of frequency 340 Hz. If water is poured in the pipe then (given vair = 340 m/sec.) :
(A) minimum length of water column to have the resonance is 45 cm.
(B) the distance between two succesive nodes is 50 cm.
(C) the maximum length of water column to create the resonance is 95 cm.
(D) none of these.

10. A wire of length 'ℓ' having tension T and radius 'r' vibrates with fundamental frequency 'f'. Another wire of the same metal with length 2ℓ having tension 2T and radius 2r will vibrate with fundamental frequency:

(A) f (B) 2f (C) $\frac{f}{2\sqrt{2}}$ (D) $\frac{f}{2}\sqrt{2}$

SECTION - III
Comprehension Type

This section contains 2 Paragraphs. Based upon each paragraph, 2 Multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 11 to 12

A quantity of an ideal monoatomic gas consists of n moles initially at temperature T_1. The pressure and volume are then slowly doubled in such a manner so as to trace out a straight line on a P-V diagram.

11. For this process, the ratio $\frac{W}{nRT_1}$ is equal to (where W is work done by the gas) :

(A) 1.5 (B) 3 (C) 4.5 (D) 6

12. If C is defined as the average molar specific heat for the process then $\frac{C}{R}$ has value

(A) 1.5 (B) 2 (C) 3 (D) 6

Paragraph for Question Nos. 13 to 14

Figure shows an electrical calorimeter to determine specific heat capacity of an unknown liquid. First of all, the mass of empty calorimeter (a copper container) is measured and suppose it is 'm_1'. Then the unknown liquid is poured in it. Now the combined mass of calorimeter + liquid system is measured and let it be 'm_2'. So the mass of liquid is $(m_2 - m_1)$. Initially both were at room temperature (θ_0).

Now a heater is immersed in it for time interval 't'. The voltage drop across the heater is 'V' and current passing through it is 'I'. Due to heat supplied, the temperature of both the liquid and calorimeter will rise simultaneously. After t sec; heater was switched off, and final temperature is θ_f. If there is no heat loss to surroundings

Heat supplied by the heater = Heat absorbed by the liquid + heat absorbed by the calorimeter

$(VI)t = (m_2 - m_1) S_\ell (\theta_f - \theta_0) + m_1 S_c (\theta_f - \theta_0)$

The specific heat of the liquid $S_\ell = \dfrac{\dfrac{(VI)t}{\theta_f - \theta_0} - m_1 S_C}{(m_2 - m_1)}$

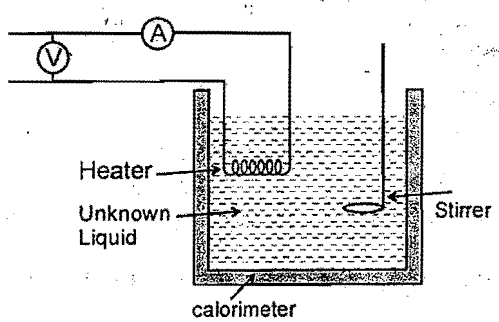


Figure 1

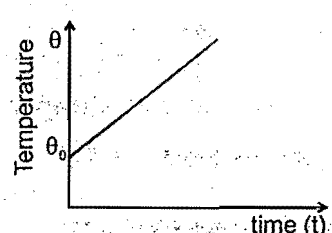


Figure 2
Temperature vs time graph assuming no heat losses to surrounding.

Radiation correction : There can be heat loss to environment. To compensate this loss, a correction is introduced.

Let the heater was on for t sec, and then it is switched off. Now the temperature of the mixture falls due to heat loss to environment. The temperature of the mixture is measured at t/2 sec. after switching off. Let the fall in temperature during this time is ε

Now the corrected final temperature is taken as

$$\theta'_f = \theta_f + \varepsilon$$

13. In this experiment voltage across the heater is 100.0 V and current is 10.0A, and heater was switched on for t = 700.0 sec. Initially all elements were at room temperature θ_0 = 10.0°C and final temperature was measured as θ_f = 73.0°C. Mass of empty calorimeter was 1.0 kg and the combined mass of calorimeter + liquid is 3.0 kg . The specific heat capacity of the calorimeter S_c = 3.0 × 10^3 J/kg°C. The fall in temperature 350 second after switching off the heater was 7.0°C. Find the specific heat capacity of the unknown liquid in proper significant figures.

(A) 3.5 × 10^3 J/kg°C (B) 3.50 × 10^3 J/kg°C (C) 4.0 × 10^3 J/kg°C (D) 3.500 × 10^3 J/kg°C

14. If mass and specific heat capacity of calorimeter is negligible, what would be maximum permissible error in S_ℓ. Use the data mentioned below.

$m_1 \to 0$, $S_c \to 0$, m_2 = 1.00 kg, V = 10.0 V, I = 10.0 A, t = 1.00 × 10^2 sec., θ_0 = 15°C, Corrected θ_f = 65°C

(A) 4% (B) 5% (C) 8% (D) 12%

SECTION – IV

Matching List Type (Only One Option Correct)

This section contains 1 questions, each having two matching lists. Choices for the correct combination of elements from List-I and List-II are given as options (A),(B),(C) and (D) out of which ONE is correct.

15. Consider a system of particles (it may be rigid or non rigid). In the column-I some condition on force and torque is given. Column-II contains the effects on the system. (Letters have usual meaning)

List-I	List-II
P. $\vec{F}_{res} = 0$	1. $\vec{P}_{system}$ will be constant
Q. $\vec{\tau}_{res} = 0$	2. $\vec{L}_{system}$ will be constant
R. External force is absent	3. total work done by all forces will be zero
S. No nonconservative force acts.	4. total mechanical energy will be constant.

(A) P-1, Q-2, R-3,S-4 (B) P-1, Q-2, R-2,S-4

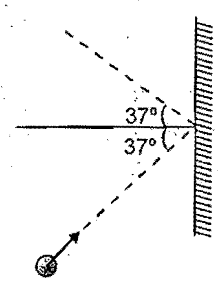
(C) P-4, Q-2, R-1,S-3 (D) P-1, Q-3, R-3,S-4

SECTION - V

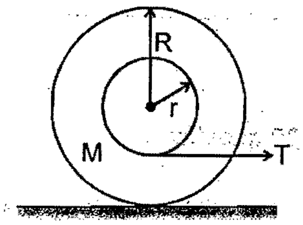
Integer value correct Type

This section contains 5 questions. The answer to each question is a integer type.

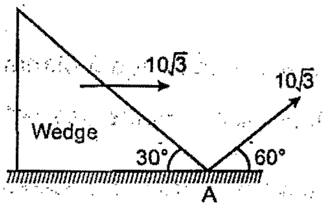
16. A particle moving on a smooth horizontal surface strikes a stationary wall. The angle of strike is equal to the angle of rebound & is equal to 37° and the coefficient of restitution with wall is e = $\frac{1}{5}$. Find the friction coefficient between wall and the particle in the form $\frac{X}{10}$ and fill value of X.



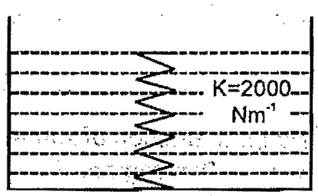
17. A spool of mass M = 3 kg and radius R = 20 cm has an axle of radius r = 10 cm around which a string is wrapped. The moment of inertia about an axis perpendicular to the plane of the spool and passing through the centre is $\frac{MR^2}{2}$. Coefficient of friction between the surface and the spool is 0.4. If the maximum value of the tension is T (in N) that can be applied so that the spool rolls without slipping, then find T/2 [Take $g = 10\ m/s^2$.]



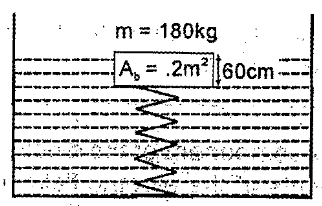
18. A particle is projected at an angle 60° with speed $10\sqrt{3}$, from the point 'A' as shown in the fig. At the same time the wedge is made to move with speed $10\sqrt{3}$ towards right as shown in the figure. Then the time (in seconds) after which particle will strike the wedge is :



19. In a tank of horizontal cross-sectional area $1 m^2$, a spring with force constant 2000 Nm^{-1} is fixed in vertical position upto the height of the water as shown in figure 1. A block of mass 180 kg is gently placed over the spring and it attains the equilibrium position as shown in figure 2. If base area of the block is $0.2 m^2$ and height 60 cm, then compression in the spring is 5X (in cm) in equilibrium position. Then find X (take $g = 10\ m/s^2$; $\rho_w = 1000\ kg/m^3$)



(Fig.1)



(Fig. 2)

20. A planet revolves about the sun in elliptical orbit of semimajor axis 2×10^{12} m. The areal velocity of the planet when it is nearest to the sun is $4.4 \times 10^{16}\ m^2/s$. The least distance between planet and the sun is 1.8×10^{12} m. The minimum speed of the planet is $a \times 10^1$ (in km/sec) then find a.

FULL SYLLABUS TEST - 2 (FST-2)

CLASS : XII

Duration : 1 Hour **Max. Marks : 60**

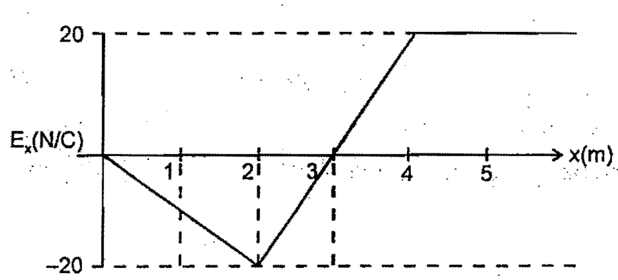
GENERAL INSTRUCTIONS :

1. This Question Paper contains 20 questions.
2. For each question in Section I, you will be **awarded 3 Marks** if you give the correct answer and zero **Mark** if no answer is given. In all other cases, **minus one** (–1) Mark will be awarded.
3. For each question in **Section II**, you will be **awarded 3 Marks** if you give the correct answer. There is no **negative** marking.
4. For each question in **Section III**, you will be **awarded 3 Marks** if you give the correct answer and zero **Mark** if no answer is given. In all other cases, **minus one (–1)** Mark will be awarded.
5. For each question in **Section IV**, you will be **awarded** 3 **Marks** if you give the correct answer and zero **Mark** if no answer is given. In all other cases, **minus** one (–1) Mark will be awarded.
6. For each question in **Section V**, you will be **awarded** 3 **Marks** if you give the correct answer. There is no **negative** marking.

SECTION - I
Straight Objective Type

This section contains 5 **Single choice questions**. Each question has choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

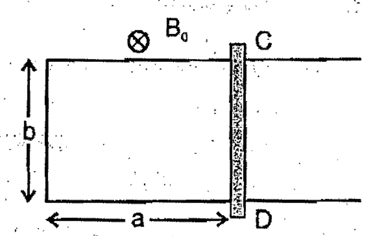
1. A graph of the x component of the electric field as a function of x in a region of space is shown. The Y and Z components of the electric field are zero in this region. If the electric potential is 10 V at the origin, then potential at x = 2.0 m is :



(A) 10 V (B) 40 V (C) – 10 V (D) 30 V

2. A metallic charged ring is placed in a uniform magnetic field with its plane perpendicular to the field. If the magnitude of field starts increasing with time, then :
(A) the ring starts translating
(B) the ring starts rotating about its axis
(C) the ring starts rotating about a diameter
(D) current starts flowing in the ring

3. A U-shaped conducting frame is fixed in space. A conducting rod CD lies at rest on the smooth frame as shown. The frame is in uniform magnetic field B_0, which is perpendicular to the plane of frame. At time t = 0, the magnitude of magnetic field begins to change with time t as, $B = \frac{B_0}{1+kt}$, where k is a positive constant. For no current to be ever induced in frame, the speed with which rod should be pulled starting from time t = 0 is (the rod CD should be moved such that its velocity must lie in the plane of frame and perpendicular to rod CD)

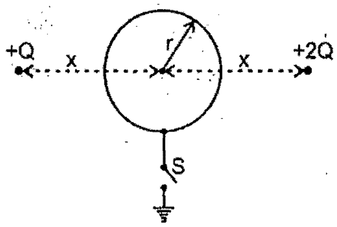


Frame in magnetic field before time t = 0

(A) ak
(B) bk
(C) a(1 + kt)
(D) b(1 + kt)

4. Refractive index of a prism is $\sqrt{\frac{7}{3}}$ and the angle of prism is 60°. The minimum angle of incidence of a ray that will be transmitted through the prism is :
(A) 30°
(B) 45°
(C) 15°
(D) 50°

5. Two particles having positive charges +Q and +2Q are fixed at equal distance x from centre of a conducting sphere having zero net charge and radius r as shown. Initially the switch S is open. After the switch S is closed, the net charge flowing out of sphere is

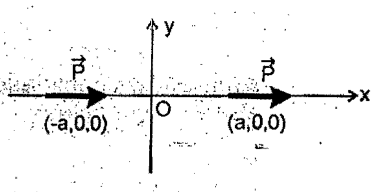


(A) $\frac{Qr}{x}$
(B) $-\frac{Qr}{x}$
(C) $\frac{3Qr}{x}$
(D) $-\frac{3Qr}{x}$

SECTION - II
Multiple Correct Answers Type

This section contains 5 Multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONE OR MORE may be correct.

6. Two identical dipoles of dipole moment $\vec{P} = p_0\hat{i}$ (p_0 is a positive constant) are placed on x-axis at points A(a, 0, 0) and B(−a, 0, 0) as shown. Then pick up the correct statements :

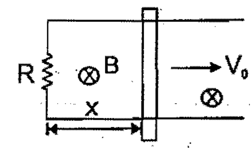


(A) The electric field at each point on y-z plane (except at infinity) must be perpendicular to y-z plane.
(B) If electric field exists at a point on y-z plane, it must be perpendicular to y-z plane.
(C) Potential at each point on y-z plane is zero.
(D) There is a circle of finite radius on y-z plane with centre at origin such that both electric field and potential are zero at each point on its periphery.

7. The plates of a parallel plate capacitor with no dielectric are connected to a voltage source. Now a dielectric of dielectric constant K is inserted to fill the whole space between the plates with voltage source remaining connected to the capacitor.
(A) the energy stored in the capacitor will become K-times
(B) the electric field inside the capacitor will decrease K-times
(C) the force of attraction between the plates will become K^2 -times
(D) the charge on the capacitor will become K-times.

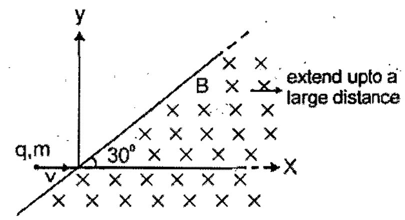
8. At distance of 5cm and 10cm outwards from the surface of a uniformly charged solid sphere, the potentials are 100V and 75V respectively. Then
(A) potential at its surface is 150V.
(B) the charge on the sphere is (5/3) × 10^{-10}C.
(C) the electric field on the surface is 1500 V/m.
(D) the electric potential at its centre is 225V.

9. A conducting rod of length ℓ is moved at constant velocity 'v_0' on two parallel, conducting, smooth, fixed rails, that are placed in a uniform constant magnetic field B perpendicular to the plane of the rails as shown in figure. A resistance R is connected between the two ends of the rail. Then which of the following is are correct :



(A) The thermal power dissipated in the resistor is equal to rate of work done by external person pulling the rod.
(B) If applied external force is doubled than a part of external power increases the velocity of rod.
(C) Lenz's Law is not satisfied if the rod is accelerated by external force
(D) If resistance R is doubled then power required to maintain the constant velocity v_0 becomes half.

10. A charged particle of charge +q, mass m is moving with initial velocity 'v' as shown in figure in a uniform magnetic field $B(-\hat{k})$. Select the correct alternative/alternatives :-



(A) Velocity of particle when it comes out from magnetic field is $\vec{v} = v\cos 30^0\hat{i} - v\sin 30^0\hat{j}$
(B) Time for which the particle was in magnetic field is $\frac{\pi m}{3qB}$
(C) Distance travelled in magnetic field is $\frac{\pi m v}{3qB}$
(D) The particle will never come out of magnetic field

SECTION - III
Comprehension Type

This section contains 2 Paragraphs. Based upon each paragraph, 2 Multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 11 to 12

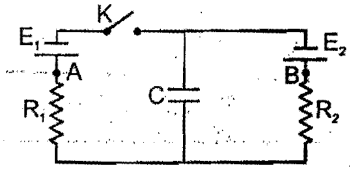
Rutherford's calculations used the inverse-square law of repulsive force between an α-particle (Z = 2) and a gold nucleus (Z = 79) ignoring multiple scattering. The scattering angle θ of the α-particle is related to the impact parameter b through the relation $b = \frac{Ze^2\cot(\theta/2)}{4\pi\varepsilon_0 E}$.

Where E is the kinetic energy of the incident α-particle. The impact parameter b is the perpendicular distance of the initial velocity vector of the α-particle at different angles agreed with Rutherford's calculations based on the nuclear model of an atom. $b = \frac{Ze^2\cot(\theta/2)}{4\pi\varepsilon_0 E}$

11. For impact parameter (b = 0) scattering angle will be :
(A) $\theta = 0^0$ (B) $\theta = \pi$ (C) $\theta = \pi/2$ (D) None of these

12. If for the β^+(positron) emission from a nucleus, there is another competing process known as electron capture, then which of the following is correct :
(A) if β^+ emission is energetically allowed then electron capture is necessarily allowed
(B) if electron capture is energetically allowed then β^+ emission is necessarily allowed
(C) both processes are independent from each other
(D) none of these

Paragraph for Question Nos. 13 to 14



In the circuit shown, switch K is initially open. Both the cells are ideal and C = 30 μF, E_1 = 1V, R_1 = 2Ω, E_2 = 3V, R_2 = 4Ω. At t = 0 second, the switch K is closed. A and B are two points on circuit as shown.

13. Just after the switch K is closed, the magnitude of current in amperes through resistance R_1 is -

(A) $\frac{1}{3}$ (B) $\frac{4}{3}$ (C) 1 (D) 2

14. Long after the switch K is closed, the magnitude of charge on the capacitor in steady state is -

(A) 10 μC (B) 30 μC (C) 50 μC (D) 90 μC

SECTION – IV

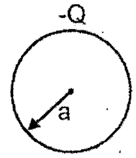
Matching List Type (Only One Option Correct)

This section contains 1 questions, each having two matching lists. Choices for the correct combination of elements from List-I and List-II are given as options (A),(B),(C) and (D) out of which ONE is correct.

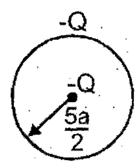
15. In each situation of List-I, some charge distributions are given with all details explained. In List-II the electrostatic potential energy, electric field and its nature is given situation in List-II. Then match situation in List-I with the corresponding results in List-II and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the OMR.

List-I

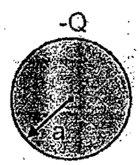
(p) A thin shell of radius a and having a charge – Q uniformly distributed over its surface as shown



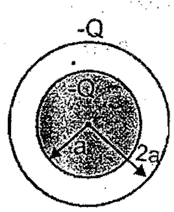
(q) A thin shell of radius $\frac{5a}{2}$ and having a charge – Q uniformly distributed over its surface and a point charge – Q placed at its centre as shown.



(r) A solid sphere of radius a and having a charge – Q uniformly distributed throughout its volume as shown.



(s) A solid sphere of radius a and having a charge – Q uniformly distributed throughout its volume. The solid sphere is surrounded by a concentric thin uniformly charged spherical shell of radius 2a and carrying charge –Q as shown



List-II

(1) Electric potential energy of the system is $\frac{1}{8\pi\epsilon_0}\frac{Q^2}{a}$ in magnitude

(2) Electric potential energy of the system is $\frac{3}{20\pi\epsilon_0}\frac{Q^2}{a}$ in magnitude

(3) Electric potential energy of the system is $\frac{2}{5\pi\epsilon_0}\frac{Q^2}{a}$ in magnitude

(4) Electric potential energy of the system is positive in sign

Codes :

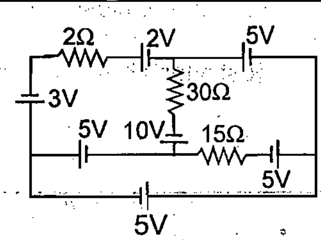
	p	q	r	s
(A)	1	2	3	4
(B)	1	3	3	4
(C)	4	3	2	1
(D)	1	2	2	4

SECTION - V

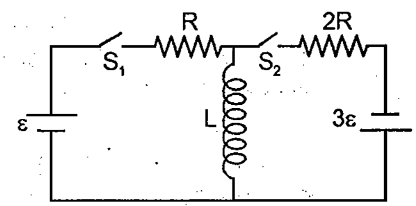
Integer value correct Type

This section contains 5 questions. The answer to each question is a **integer type.**

16. In the circuit shown, current through the resistance 2Ω is i_1 and current through the resistance 30Ω is i_2. Find the ratio $\frac{i_1}{i_2}$.

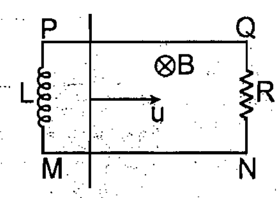


17. In the figure shown the switch S_1 remains connected for long time and the switch S_2 remains opened. Now the switch S_2 is closed. Assuming that ε = 10 volt and L = 1H, the magnitude of rate of change of current is 2X (in Amp/sec.) in the inductor just after the switch S_2 is closed then find X.



18. A point source of radiation power P is placed on the axis of completely absorbing disc. The distance between the source and the disc is 2 times the radius of the disc. The force that light exerts on the disc is $\frac{P}{4ac}$ then find a (c = speed of light).

19. In the figure, a conducting rod of length ℓ = 1 meter and mass m = 1 kg moves with initial velocity u = 5 m/s. on a fixed horizontal frame containing inductor L = 2 H and resistance R = 1 Ω. PQ and MN are smooth, conducting wires. There is a uniform magnetic field of strength B = 1T. Initially there is no current in the inductor. Find the total charge in coulomb, flown through the inductor by the time velocity of rod becomes v_f = 1 m/s and the rod has travelled a distance x= 3 meter.



20. In a YDSE arrangement composite lights of different wavelengths λ_1 = 560 nm and λ_2 = 400 nm are used. If D = 1m, d = 0.1 mm. If the distance between two nearest completely dark regions is 7X mm then X is :

FULL SYLLABUS TEST - 3 (FST-3)

CLASS : XI & XII

Duration : 1 Hour | Max. Marks : 120

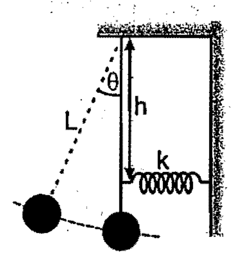
GENERAL INSTRUCTIONS :

1. This Question Paper contains 40 questions.
2. For each question in **Section I**, you will be **awarded 3 Marks** if you give the correct answer and zero Mark if no answer is given. In all other cases, **minus one** (–1) Mark will be awarded.
3. For each question in **Section II**, you will be **awarded 3 Marks** if you give the correct answer. There is no negative marking.
4. For each question in **Section III**, you will be **awarded 3 Marks** if you give the correct answer and zero Mark if no answer is given. In all other cases, **minus one** (–1) Mark will be awarded.
5. For each question in **Section IV**, you will be **awarded 3 Marks** if you give the correct answer and zero Mark if no answer is given. In all other cases, **minus one** (–1) Mark will be awarded.
6. For each question in **Section V**, you will be **awarded 3 Marks** if you give the correct answer. There is no negative marking.

SECTION - I
Straight Objective Type

This section contains **10 Single** choice questions. Each question has choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

1. A pendulum of length L and bob of mass M has a spring of force constant k connected horizontally to it at a distance h below its point of suspension. The rod is in equilibrium in vertical position. The rod of length L used for vertical suspension is rigid and massless. The frequency of vibration of the system for small values of θ is :



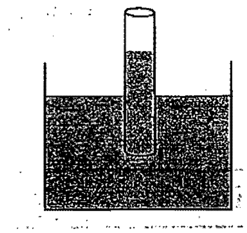
(A) $\frac{1}{2\pi L}\sqrt{gL+\frac{kh}{m}}$

(B) $\frac{1}{2\pi L}\sqrt{\frac{mgL+k}{m}}$

(C) $2\pi\sqrt{\frac{mL^2}{mgL+kh}}$

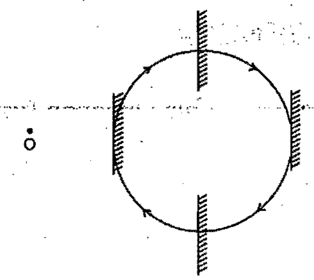
(D) $\frac{1}{2\pi L}\sqrt{gL+\left(\frac{kh^2}{m}\right)}$

2. A long capillary tube of mass 'π' gm, radius 2mm and negligible thickness, is partially immersed in a liquid of surface tension 0.1 N/m. Take angle of contact zero and neglect buoyant force of liquid. The force required to hold the tube vertically, will be - (g = 10 m/s²)



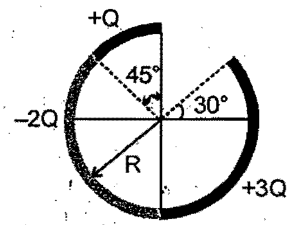
(A) 10.4 π mN (B) 10.8 π mN
(C) 0.8 π mN (D) 4.8 π mN

3. A plane mirror is moving in a circle in front of a stationary point object O as shown in fig. Then path of image of object is.



(A) circle
(B) straight line
(C) ellipse
(D) parabolic

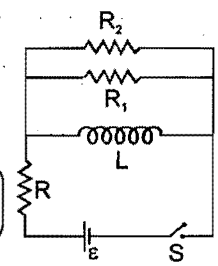
4. Figure shows three circular arcs, each of radius R and total charge as indicated. The net elecric potential at the centre of curvature is :



(A) $\frac{Q}{2\pi\varepsilon_0 R}$ (B) $\frac{Q}{4\pi\varepsilon_0 R}$

(C) $\frac{2Q}{\pi\varepsilon_0 R}$ (D) $\frac{Q}{\pi\varepsilon_0 R}$

5. Switch S is closed for a long time at t = 0. It is opened, then :



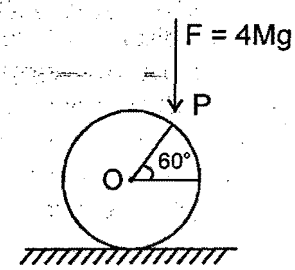
(A) total heat produced in resistor R after opening the switch is $\frac{1}{2}\frac{L\varepsilon^2}{R^2}$

(B) total heat produced in resistor R_1 after opening the switch is $\frac{1}{2}\frac{L\varepsilon^2}{R^2}\left(\frac{R_1}{R_1+R_2}\right)$

(C) heat produced in resistor R_1 after opening the switch is $\frac{1}{2}\frac{R_2 L \varepsilon^2}{(R_1+R_2)R^2}$

(D) no heat will be produced in R_1.

6. A uniform solid sphere of mass M and radius R is lying on rough horizontal plane. A constant force F = 4 Mg acts vertically downwards at point P such that OP makes 60° with horizontal as shown in figure. Find the minimum value of coefficient of friction μ so that sphere starts pure rolling.



(A) $\frac{3}{7}$ (B) $\frac{4}{7}$

(C) $\frac{2}{7}$ (D) $\frac{2}{5}$

7. In a meter bridge experiment the resistance of resistance box is 16Ω, which is inserted in right gap. The null point is obtained at 36 cm from the left end. The least count of meter scale is 1mm. The value of unknown resistance is –

(A) $9 \pm \frac{5}{128}\Omega$ (B) $9 \pm \frac{5}{256}\Omega$ (C) $9 \pm \frac{5}{512}\Omega$ (D) $9 \pm \frac{1}{2560}\Omega$

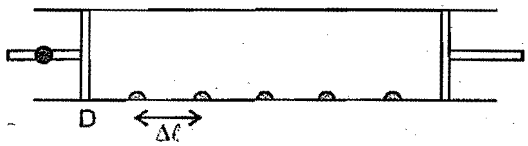
8. There exist uniform electric field is space. Electric field is parallel to xy plane. The potential of points A (2,2), B(-2, 2) and C(2, 4) are 4V, 16V, 12V, respectively. The electric field is

(A) $(4\hat{i}+5\hat{j})V/m$ (B) $(3\hat{i}+4\hat{j})V/m$ (C) $-(3\hat{i}+4\hat{j})V/m$ (D) $(3\hat{i}-4\hat{j})V/m$

9. Plane surface of a thin plano–convex lens is silvered. If a point object is placed on principal axis at a distance 60 cm from the lens and final image is formed at a distance 30 cm from the lens on same side. What will be the distance of final image from the lens if the plane surface is not silvered :

(A) 120 cm (B) 180 cm (C) 30 cm (D) 90 cm

10. In a Kundt's tube distance between two consecutive heap is $\Delta\ell$ for air, while it is increased by 50% for a gas in the same tube with same resonite. If speed of sound in air is $\frac{1000}{3}m/s$ then speed of sound in gas at same temperature is .



(A) 1500 m/s
(B) 500 m/s
(C) To calculate speed in gas degree of freedom and molecular mass of the gas is required
(D) None of these

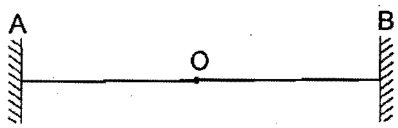
SECTION - II
Multiple Correct Answers Type

This section contains 10 Multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONE OR MORE may be correct.

11. A closed vessel contains a mixture of two diatomic gases A and B. Molar mass of A is 16 times that of B and mass of gas A, contained in the vessel is 2 times that of B. :
(A) Average kinetic energy per molecule of gas A is equal to that of gas B.
(B) Root mean square value of translational velocity of gas B is four times that of A.
(C) Pressure exerted by gas B is eight times of that exerted by gas A.
(D) Number of molecules of gas B in the cylinder is eight times that of gas A.

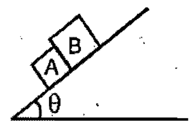
12. In a resonance tube experiment, a closed organ pipe of length 120 cm resonates when tuned with a tuning fork of frequency 340 Hz. If water is poured in the pipe then (given vair = 340 m/sec.) :
(A) minimum length of water column to have the resonance is 45 cm.
(B) the distance between two succesive nodes is 50 cm.
(C) the maximum length of water column to create the resonance is 95 cm.
(D) none of these.

13. A wire, under tension between two fixed points *A* and *B*, executes transverse vibrations in 2nd harmonium. Then :



(A) All points of wire between *A* and *B* are in the same phase
(B) All points between *A* and *O* are in the same phase
(C) A point between *A* and *O* and a point between *O* and *B* may have a phase difference of $\pi/2$
(D) A point between *A* and *O* and a point between *O* and *B* may have a phase difference of π

14. The two blocks A and B of equal mass are initially in contact when released from rest on the inclined plane. The coefficients of friction between the inclined plane and A and B are μ_1 and μ_2 respectively :
(Assume $\tan\theta > \mu_1$ and μ_2)

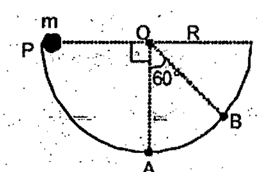


(A) If $\mu_1 > \mu_2$, the blocks will always remains in contact.
(B) If $\mu_1 < \mu_2$, the blocks will slide down with different accelerations
(C) If $\mu_1 > \mu_2$, the blocks will have a common acceleration $\frac{1}{2}(\mu_1+\mu_2)g\sin\theta$
(D) If $\mu_1 < \mu_2$, the blocks will have a common acceleration $\frac{\mu_1\mu_2}{\mu_1+\mu_2}g\sin\theta$

15. A parallel plate capacitor of capacitance 10 μF is connected to a cell of emf 10 Volt and is fully charged. Now a dielectric slab (k = 3) of thickness equal to the gap between the plates, is very slowly inserted to completely fill in the gap, keeping the cell connected. During the filling process :
(A) the increase in charge on the capacitor is 200 μC.
(B) the heat produced is zero.
(C) energy supplied by the cell = increase in stored potential energy + work done on the person who is filling the dielectric slab.
(D) energy supplied by the cell = increase in stored potential energy + work done on the person who is filling the dielectric slab + heat produced.

16. A ball tied to the end of the string swings in a vertical circle under the influence of gravity.
(A) When the string makes an angle 90° with the vertical, the tangential acceleration is zero and radial acceleration is somewhere between minimum and maximum
(B) When the string makes an angle 90° with the vertical, the tangential acceleration is maximum and radial acceleration is somewhere between maximum and minimum
(C) At no place in circular motion, tangential acceleration is equal to radial acceleration (in magnitude)
(D) When radial acceleration has its maximum value, the tangential acceleration is zero

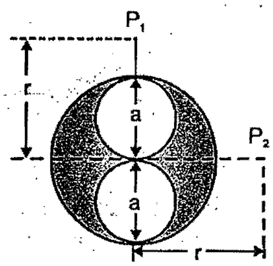
17. A small block of mass m is released from rest from point P in a smooth fixed hemispherical bowl of radius R, as shown. Choose the correct alternative(s) :



(A) The speed of block at A is $\sqrt{2gR}$
(B) The speed of block at B is $\sqrt{gR}$
(C) The normal reaction at B is $\frac{3mg}{2}$
(D) The net force on the block at B is $\frac{3mg}{2}$

18. The potential energy of a particle of mass 1 kg in a conservative field is given as $U = (3x^2y^2 + 6x)$ J, where x and y are measured in meter. Initially particle is at (1,1) & at rest then :
(A) Initial acceleration of particle is $6\sqrt{5}$ ms²
(B) Work done to slowly bring the particle to origin is 9 J
(C) Work done to slowly bring the particle to origin is – 9 J
(D) If particle is left free it moves in straight line

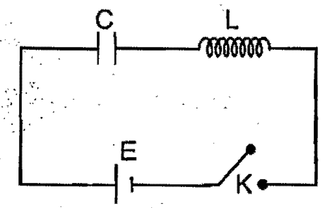
19. A long cylindrical conductor of radius a has two cylindrical cavities of diameter a through its entire length as shown in cross–section in figure. A current I is directed out of the page and is uniform throughout the cross–section of the conductor. Then the magnetic field in terms of m_0, I,r and a is :



(A) at the point P_1 $\frac{\mu_0 I}{\pi r}\left(\frac{2r^2 - a^2}{4r^2 - a^2}\right)$ (B) at the point P_1 $\frac{\mu_0 I}{\pi r}\left(\frac{2r^2 - a^2}{4r^2 + a^2}\right)$

(C) at the point P_2 $\frac{\mu_0 I}{\pi r}\left(\frac{2r^2 + a^2}{4r^2 + a^2}\right)$ (D) at the point P_2 $\frac{\mu_0 I}{\pi r}\left(\frac{2r^2 + a^2}{4r^2 - a^2}\right)$

20. A capacitor of capacitance C and an ideal inductor of inductance L are connected in series with an ideal battery of emf E. The resistance of circuit is negligible. If initially capacitor is uncharged and key is closed at t = 0 then select the correct statements :



(A) Maximum charge on capacitor is CE
(B) Maximum charge on capacitor is 2CE
(C) Maximum potential difference across inductor is E
(D) Maximum potential difference across inductor is 2E

SECTION - III
Comprehension Type

This section contains 4 Paragraphs. Based upon each paragraph, 2 Multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 21 to 22

Resistive force proportional to object velocity

At low speeds, the resistive force acting on an object that is moving through a viscous medium is effectively modeled as being proportional to the object's velocity. The mathematical representation of the resistive force can be expressed as

$$R = -bv$$

Where v is the velocity of the object and b is a positive constant that depends on the properties of the medium and on the shape and dimensions of the object. The negative sign represents the fact that the resistance force is opposite to the velocity.

Consider a sphere of mass m released from rest in a liquid. Assuming that the only forces acting on the sphere are the resistive force R and the weight mg, we can describe its motion using Newton's second law. Though the buoyant force is also acting on the submerged object, the force is constant and effect of this force can be modeled by changing the apparent weight of the sphere by a constant force, so we can ignore it here.

Thus $mg - bv = m\frac{dv}{dt} \Rightarrow \frac{dv}{dt} = g - \frac{b}{m}v$

Solving the equation.

$$v = \frac{mg}{b}(1 - e^{-bt/m})$$

where e = 2.71 is the base of the natural logarithm.

The acceleration becomes zero when the increasing resistive force eventually balances the weight. At this point, the object reaches its terminal speed v_T and then on it continues to move with zero acceleration.

$mg - bv_T = 0$

$\Rightarrow \quad v_T = \frac{mg}{b}$

Hence $\quad v = v_T\left(1 - e^{-\frac{bt}{m}}\right)$

In an experimental set–up, four objects I, II, III, IV were released in same liquid. Using the data collected for the subsequent motions, values of constant b were calculated. Respective data are shown in table.

Object	I	II	III	IV
Mass (in kg.)	1	2	3	4
Constnat b in (N-s)/m	3.7	1.4	1.4	2.8

21. Which object has greatest terminal speed in the liquid ?
(A) I (B) II (C) III (D) IV

22. A small sphere of mass 2.00 g is released from rest in a large vessel filled with oil. The sphere approaches a terminal speed of 10.00 cm/s. Time required to achieve speed 6.32 cm/s from start of the motion is (Take g = 10.00 m/s²) :
(A) 5.00×10^{-3} s (B) 1.00×10^{-2} s (C) 2.5×10^{-3} s (D) 1.00×10^{-3} s

Paragraph for Question Nos. 23 to 24

In a series L-R circuit, connected with a sinusoidal ac source, the maximum potential difference across L and R are respectively 3 volts and 4 volts.

23. At an instant the potential difference across resistor is 2 volts. The potential difference in volt, across the inductor at the same instant will be :
(A) 3 cos 30° (B) 3 cos 60° (C) 6 cos 45° (D) 6

24. At the same instant, the magnitude of the potential difference in volt, across the ac source will be -
(A) 3 cos 67° (B) 5 cos 83° (C) 6 cos 97° (D) 0

Paragraph for Question Nos. 25 to 26

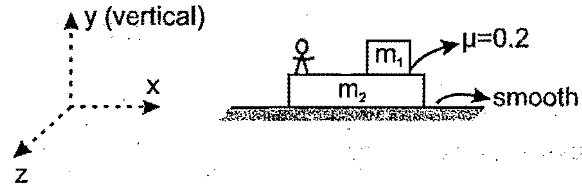
A wave represented by equation $y = 2(\text{mm}) \sin[4\pi(\text{sec}^{-1})t - 2\pi(\text{m}^{-1})x]$ is superimposed with another wave $y = 2(\text{mm}) \sin[4\pi(\text{sec}^{-1})t + 2\pi(\text{m}^{-1})x + \pi/3]$ on a tight string (Neglecting dissipative losses answer the following:

25. Phase difference between two particles which are located at $x_1 = \frac{1}{7}$ and $x_2 = \frac{5}{12}$ is :
(A) 0 (B) $\frac{5\pi}{6}$ rad (C) π rad (D) $\frac{5\pi}{3}$ rad

26. Which of the following is not a location of antinode :
(A) $\frac{5}{12}$ m (B) $\frac{11}{12}$ m (C) $\frac{2}{3}$ m (D) $\frac{17}{12}$ m

Paragraph for Question Nos. 27 to 28

The system shown in figure is initially at origin and is moving with velocity $5\hat{k}$ (m/s) on a smooth horizontal xz plane. A force F = (120 t) acts on mass m_2, [where F in newton, t in sec]. The man throws a light ball (at the instant when m_1 starts slipping on m_2) with a velocity 10 m/s vertically up with respect to himself. Taking the mass of block and man as 60 kg each and assuming that man never slips on m_2. (neglect the dimensions of system, and g = 10 m/s²)



27. Projection velocity of ball with respect to ground is :

(A) $10\hat{i}+5\hat{k}$ (B) $1.5\hat{i}+10\hat{j}+5\hat{k}$ (C) $3\hat{i}+10\hat{j}+5\hat{k}$ (D) None of these

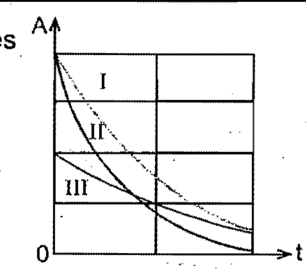
28. The time of flight of the ball is

(A) 2s (B) 4s (C) 6s (D) 8s

SECTION – IV
Matching List Type (Only One Option Correct)

This section contains 2 questions, each having two matching lists. Choices for the correct combination of elements from List-I and List-II are given as options (A), (B), (C) and (D) out of which ONE is correct.

29 Figure shows activities A of three different radioactive material's samples (labelled as I, II and III) versus time. Using the given information, correctly match the requisite parameter in the List-I with the options given in List-II. Consider only their natural decay as the cause of rate of change of no. of parent nuclei

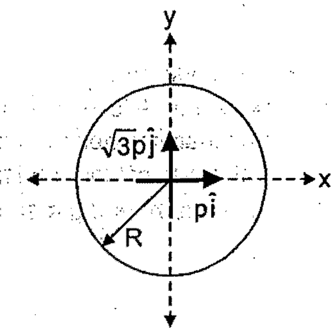


	List-I		List-II
(P)	Disintegration constant (λ) has maximum value for the material of sample ...	(1)	I
(Q)	Half life is maximum for the material of the sample	(2)	II
(R)	Initially if samples of all three materials have same number of atoms then number of parent atoms will be maximum at a later time in the sample	(3)	III
(S)	Suppose all the materials decay by emitting α-particles of same energy and initially all three samples contain same amount (in gm) of the materials. Till the end of time span equal to their respective half lives, maximum energy is radiated by the sample	(4)	it is not possible to compare with the help of data available

Codes :

	P	Q	R	S
(A)	1	2	3	4
(B)	2	3	4	1
(C)	2	3	3	4
(D)	2	4	4	3

30. List-I gives a situation in which two dipoles of dipole moment $p\hat{i}$ and are placed at origin. A circle of radius R with centre at origin is drawn as shown in figure. List-II gives coordinates of certain positions on the circle. Match the statements in List-I with the statements in List-II and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the OMR.



List-I	List-II
(P) The coordinate(s) of point on circle where potential is maximum	(1) $\left(\frac{R}{2}, \frac{\sqrt{3}R}{2}\right)$
(Q) The coordinate(s) of point on circle where potential is zero	(2) $\left(-\frac{R}{2}, -\frac{\sqrt{3}R}{2}\right)$
(R) The coordinate(s) of point on circle where magnitude of electric field intensity is $\frac{1}{4\pi\varepsilon_0}\frac{4p}{R^3}$	(3) $\left(-\frac{\sqrt{3}R}{2}, \frac{R}{2}\right)$
(S) The coordinate(s) of point on circle where magnitude of electric field intensity is $\frac{1}{4\pi\varepsilon_0}\frac{2p}{R^3}$	(4) $\left(\frac{\sqrt{3}R}{2}, -\frac{R}{2}\right)$

Codes :

	P	Q	R	S
(A)	1	4	3	4
(B)	1	4	1	3
(C)	2	3	4	1
(D)	3	4	2	3

SECTION - V
Integer value correct Type

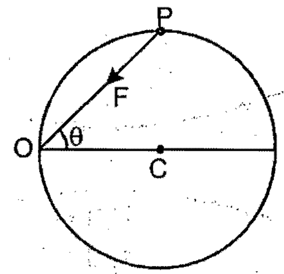
This section contains 10 **questions**. The answer to each question is a **integer type.**

31. The amplitude of the magnetic field part of a harmonic electromagnetic wave in vacuum is B_0 = 510 nT. If the amplitude of the electric field part of the wave is $X\frac{N}{C}$. Find the value of 'X'?

32. 2 kg ice at – 20 °C is mixed with 5 kg water at 20 °C . Then final amount of water (in kg) in the mixture will be: [Specific heat of ice = 0.5 cal/gm °C, Specific heat of water = 1 cal/gm °C, Latent heat of fusion of ice = 80 cal/gm]

33. The work done in increasing the size of a soap film from 10 cm × 6 cm to 10 cm × 11 cm is 3×10^{-4} J. If surface tension (in Nm^{-1}) of the film is N x 10^{-2}. Then calculate N.

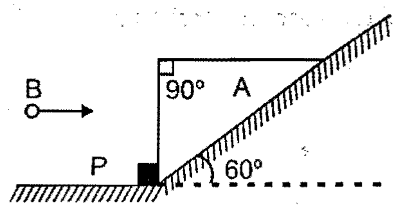
34. A particle P is moving on a circle under the action of only one force, which always acts towards a fixed point O lying on the circumference. Find ratio of $\frac{d^2\theta}{dt^2}$ to $\left(\frac{d\theta}{dt}\right)^2$ at the moment when $\theta = 45^\circ$. (C is centre of circle)



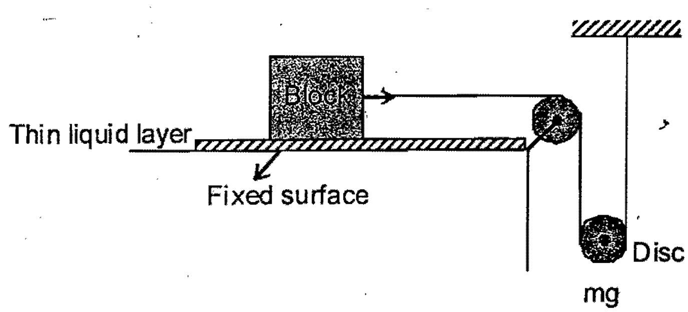
35. A fiber of length 10 km is illuminated with light from an light emitting diode (LED) which is turned on and off repeatedely for equal amount of time. The speed of the pulses of light are 2.00×10^8 m/s and 2.1×10^8 m/s in fiber. Maximum frequency of LED so that pulse arrive without overlapping is 60X (KHz). Calculate X.

36. A particle starts from point A, moves along a straight line path with an acceleration given by a = 2 (4 – x) where x is distance from point A. The particle stops at point B for a moment. Find the distance AB (in m). (All values are in S.I. units)

37. A wedge A of mass 1kg is held at rest on the smooth incline plane of inclination 60° from the horizontal by an stopper P. A bullet B of mass 500 grams, travelling horizontally with 90 m/s strikes the wedge as shown. Assuming all the impacts are perfectly inelastic and duration of impact is negligible. Find the velocity in m/s with which the wedge moves up the incline just after collision.

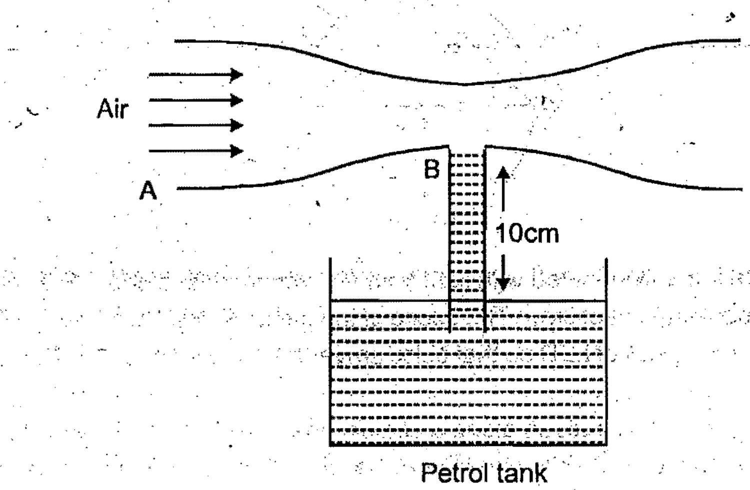


38. A cubical block of side $10\sqrt{10}$ cm is connected to a smooth and uniform disc of mass 20 g through an ideal string as shown. The block is placed on a thin liquid layer of thickness 0.2 mm. If center of the disc move downward with constant speed of 2 cm/sec after the system is released. If coefficient of viscosity of liquid is $\frac{x}{1000}$ Pa-sec. then find x (Take g = 10 m/s²)

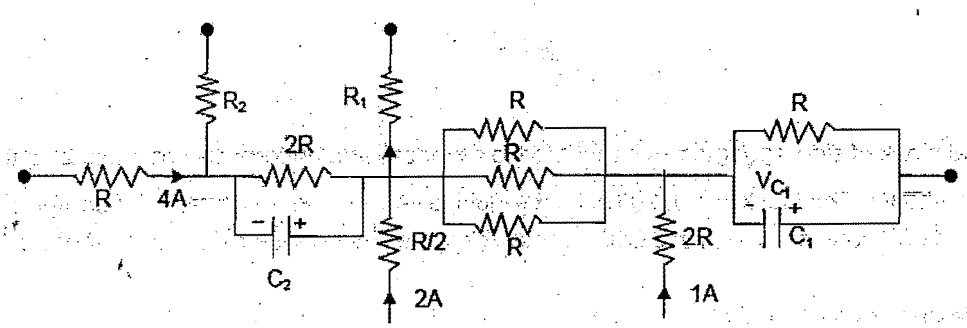


39. In a carburator of an engine, air is drawn from atmosphere through section A with a velocity of 10 m/s. The narrow section B is connected to a petrol tank as shown. The minimum ratio of area of cross section A to area of cross section B, so that the petrol can just enter the carburator tube is $\sqrt{\frac{66}{x}}$. Then the value of x is:

(density of petrol = 1000 kg/m³, density of air = 2 kg/m³, g = 10 m/s², atmospheric pressure = 10^5 Pa and assume density of air remain constant)



40. Here shown a diagram which is part of a network in steady state. Here R = 1 Ω, Potential difference across capacitor C_1 is V_{C1} = 2 volt and Potential difference across capacitor C_2 is V_{C2} = 4 volt. If currents in the resistance R_1 and R_2 are I_1 and I_2 respectively then find $\frac{I_2}{I_1}$.



SECTION-I
TOPIC WISE PROBLEMS

SOLUTIONS

Answers & Solutions

SECTION-I (TOPIC WISE PROBLEMS)

1. UNIT & DIMENSION & ERROR ANALYSIS

1.1 (B)

1.2 (B)

$$\left[\frac{E^2}{\mu_0}\right] = \left[\frac{\varepsilon_0 E^2}{\varepsilon_0 \mu_0}\right] = \left[\frac{\text{energy / volume}}{(1/\text{speed of light})^2}\right] = \left[\frac{\text{energy(speed)}^2}{\text{volume}}\right] = \left[\frac{ML^2T^{-2}L^2T^{-2}}{L^2}\right] = [MLT^{-4}]$$

1.3 (C)

The quantity $\frac{t}{a} - 1$ is dimensionless i. e.,

$[a] = [t] \quad \Rightarrow \quad \therefore \quad [\sqrt{2at - t^2}] = [t]$

or $\left[\frac{dt}{\sqrt{2at - t^2}}\right] = \left[\frac{t}{t}\right] = [\text{ m0L0T0 }]$

i.e., ax should also be dimensionless.

or $x = 0$

1.4 (C)

$[hc] = [hc] = [E\lambda] = [ML^2T^{-2}L] = [ML^3T^{-2}]$

1.5 (A)

$U(x) = K|x|^3$

$\therefore \quad [K] = \frac{[U]}{[x]^3} = \frac{[ML^2T^{-2}]}{[L^3]} = [ML^{-1}T^{-2}]$

Now time period may depend on

$T \propto (\text{mass})^x (\text{amplitude})^y (K)^z$

or $[M^0L^0T] = [M]^x [L]^y [ML^{-1}T^{-2}]^z$

$= [M^{x+z} L^{y-z} T^{-2z}]$

Equating the powers, we get

$-2z = 1$ or $z = -1/2$

$y - z = 0$ or $y = z = -1/2$

$\therefore \quad T \propto (\text{amplitude})^{-1/2}$

or $T \propto (a)^{-1/2}$

or $T \propto \frac{1}{\sqrt{a}}$

1.6 (B)

$$[Y] = \left[\frac{X}{Z^2}\right] = \left[\frac{\text{Capacitance}}{(\text{Magnetic induction})^2}\right] = \left[\frac{M^{-1}L^{-2}Q^2T^2}{M^2Q^{-2}T^{-2}}\right] = [M^{-3}L^{-2}Q^4T^4]$$

1.7 (B)

$\frac{1}{2}\varepsilon_0 E^2$ is the expression of energy density (Energy per unit volume) $= \left[\frac{1}{2}\varepsilon_0 E^2\right] = \left[\frac{ML^2T^{-2}}{L^3}\right] = [ML^{-1}T^{-2}]$

1.8 (A)

$\left[\frac{\alpha z}{k\theta}\right] = [M^0L^0T^0] \quad \Rightarrow \quad [\alpha] = \left[\frac{k\theta}{Z}\right]$

Further $[P] = \left[\frac{\alpha}{\beta}\right] \quad \therefore \quad [\beta] = \left[\frac{\alpha}{p}\right] = \left[\frac{k\theta}{ZP}\right]$

Dimensions of K θ is that to energy. Hence, $[\beta] = \left[\frac{ML^2T^{-2}}{LML^{-1}T^{-2}}\right] = [M^0L^2T^0]$

1.9 (D)

1.10 (C)

Zero error $= 5 \times \frac{0.5}{50} = 0.05\text{mm}$

Actual measurement $= 2 \times 0.5\text{mm} + 25 \times \frac{0.5}{50} - 0.05 \text{ mm} = 1 \text{ mm} + 0.25 \text{ mm} - 0.05 \text{ mm} = 1.20 \text{ mm}.$

1.11 (C)

$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1q_2}{r^2} \quad \Rightarrow \quad [\varepsilon_0] = \frac{[q_1][q_2]}{[F][r^2]} = \frac{[IT]^2}{[MLT^{-2}][L^2]} = [M^{-1}L^{-3}T^4I^2]$

Speed of light $c = \frac{1}{\sqrt{\varepsilon_0\mu_0}} \quad \therefore \quad [\mu_0] = \frac{1}{[\varepsilon_0][c]^2} = \frac{1}{[M^{-1}L^{-3}T^4I^2][LT^{-1}]^2}$

1.12 (A)

(a) $L = \frac{\phi}{i}$ or henry $= \frac{\text{weber}}{\text{ampere}}$ (b) $e = -L\left(\frac{di}{dt}\right)$

$\therefore \quad L = -\frac{e}{(di/dt)}$ or henry $= \frac{\text{volt} - \text{second}}{\text{ampere}}$

(c) $U = \frac{1}{2}Li^2 \quad \therefore \quad L = \frac{2U}{i^2} = \frac{\text{joule}}{(\text{ampere})^2}$

(d) $U = \frac{1}{2}Li^2 = i^2Rt$

$\therefore$ L = Rt or henry = ohm-second

1.13 (D)

Torque and energy have same dimensional formula but have different units.

1.14 (D)

Same physical quantities can be added or subtracted and same dimensional formula does not ensure same physical quantities.

1.15 (B)

equivalent length of pendulum

L = 23.2 cm + 1.32 cm = 24.5 cm

time period $T = \frac{10.0}{10} = 1.00$ sec (Three significant figures)

$g = 4\pi^2\frac{L}{T^2} = 4 \times 10 \frac{24.5 \times 10^{-2}\text{m}}{(10.0/10)^2 \text{sec}^2} = 9.80 \text{ m/s}^2$

1.16 (C)

Since $\ell = \left(\frac{g}{4\pi^2}\right)T^2$ so, slope of curve L v/s T^2 is $\left(\frac{g}{4\pi^2}\right)$

slope = $\frac{0.49}{2} = \frac{g}{4\pi^2}$ $\Rightarrow$ $g = 9.8\ m/s^2$.

1.17 (D)
$\ell_1 = 24.0$ cm
$\ell_2 = 74.0$ cm
$v = 2f_0(\ell_2 - \ell_1) = 2(330)(0.740 - 0.240) = (2)(330)(0.500) = 330$ m/sec.

1.18 (D)

$v \propto \sqrt{T} \Rightarrow \frac{V_{27^0}}{V_{0^0}} = \sqrt{\frac{300}{273}} \Rightarrow V_{0^o} = V_{27^o}\sqrt{\frac{273}{300}} = 330\sqrt{\frac{273}{300}} = 314$ m/sec.

1.19 (C)

$\ell_1 + \varepsilon = \frac{V}{4f_0}$ $\ell_2 + \varepsilon = \frac{3V}{4f_0}$ solve both equations and get $\varepsilon = 1$cm

for third resonance $\ell_3 + \varepsilon = \frac{5V}{4f_0}$ get $\ell_3 = 124$cm

1.20 (B)

$\varepsilon = 1$ cm $= 0.3$ d $d = \frac{1cm}{0.3} = 3.3$ cm

1.21. (A) → (p),(q); (B) → (r), (s); (C) → (r), (s); (D) → (r), (s)

(A) $\frac{GM_eM_s}{R_e^2}$ = Force

$[GM_eM_s] = [Force][R_e^2]$
$= MLT^{-2}L^2 = ML^3T^{-2}$
Hence SI unit of GM_eM_s, will be (kilogram) (meter^3)(sec^{-2})
ie same as (volt) (coulomb) (metre)

(B) $\sqrt{\frac{3RT}{M}} = V_{R.M.S.}$

$\left[\frac{3RT}{M_0}\right] = [V_{R.M.S.}]^2 = L^2T^{-2}$

Hence SI unit will be (metre)^2 (second)$^{-2}$ ie same as (farad) (volt)^2 (kg)$^{-1}$

(C) $\frac{[F^2]}{[q^2B^2]} = \frac{[q^2v^2B^2]}{[q^2B^2]} = [V^2] = L^2T^{-2}$

Hence SI unit (metre)^2 (second)$^{-2}$ i.e. same as (farad) (volt)^2 (kg)$^{-1}$

(D) $\left[\frac{GM_e}{R_e}\right] = \frac{[Force][R_e]}{[Mass]} = \frac{MLT^{-2}L}{M} = L^2T^{-2}$

Hence SI unit will be (meter)$^{-2}$ (second)$^{-2}$ i.e. same as (farad) (volt)^2 (kg)$^{-1}$

1.22 14

S.N.	Value of g	Absolute error $\Delta g = \lvert g_i - \bar{g} \rvert$
1	9.81	0.01
2	9.80	0.00
3	9.82	0.02
4	9.79	0.01
5	9.78	0.02
6	9.84	0.04
7	9.79	0.01
8	9.78	0.02
9	9.79	0.01
10	9.80	0.00
	$g_{mean} = 9.80$	$\Delta g_{mean} = \frac{\sum \Delta g_i}{10} = \frac{0.14}{10} = 0.014$

percentage error $= \frac{\Delta g_{mean}}{g_{mean}} \times 100 = \frac{0.014}{9.80} \times 100\ \% = 0.14\ \%$

1.23 50

Due to error in ℓ $\left(\frac{\Delta\rho}{\rho}\right)_{max} = \frac{\Delta\ell}{\ell} + \frac{\Delta\ell}{100-\ell} = \frac{\Delta\ell(100)}{\ell(100-\ell)}$

When $\ell(100-\ell)$ is maximum then $\left(\frac{\Delta\rho}{\rho}\right)_{max}$ will be minimum, that means $\ell = 50$ cm

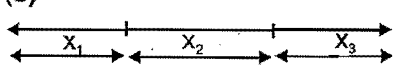
1.24 12

$$\frac{\Delta x}{x} = \pm\left[2\frac{\Delta a}{a} + 3\frac{\Delta b}{b} + \frac{\Delta c}{c} + \frac{1}{2}\frac{\Delta d}{d}\right]$$

$\therefore \quad \frac{\Delta x}{x} \times 100 = \pm\ (2 \times 2\% + 3 \times 1\% + 3\% + ½ \times 4\%) = \pm\ (4\% + 3\% + 3\% + 2\%) = \pm\ 12\%$

2. RECTILINEAR MOTION

2.1 (C)



Starting from rest $x_1 = \frac{1}{2}\,a\,(10)^2$ ....(1)

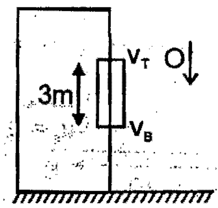
$x_1 + x_2 = \frac{1}{2}a(20)^2$ ....(2)

$x_1 + x_2 + x_3 = \frac{1}{2}a(30)^2$ ....(3)

From (2) – (1) $\quad x_2 = \frac{1}{2}\,a\,(300)$

From (3) – (2) $\quad x_3 = \frac{1}{2}\,a\,(500) \Rightarrow \quad x_1 : x_2 : x_3 :: 1 : 3 : 5$ **Ans.**

2.2 (A)



$s = \frac{(u+v)}{2}t$

$3 = \frac{(v_T + v_B)}{2} \times 0.5$

$v_T + v_B = 12$ m/s

Also, $v_B = v_T + (9.8)(0.5)$ .... (2)
$v_B - v_T = 4.9$ m/s

2.3 (C)

The retardation is given by $\frac{dv}{dt} = -av^2$

integrating between proper limits $\Rightarrow$ $-\int_u^v \frac{dv}{v^2} = \int_0^t a\,dt$ or $\frac{1}{v} = at + \frac{1}{u}$

$\Rightarrow \frac{dt}{dx} = at + \frac{1}{u}$ $\Rightarrow dx = \frac{u\ dt}{1+aut}$

integrating between proper limits $\Rightarrow$ $\int_0^s dx = \int_0^t \frac{u\,dt}{1+aut}$ $\Rightarrow$ $S = \frac{1}{a}\ell n\,(1+aut)$

2.4 (B)
Let a be the retardation produced by resistive force, t_a and t_d be the time of ascent and descent respectively. If the particle rises upto a height h

then $h = \frac{1}{2}(g+a)t_a^2$ and $h = \frac{1}{2}(g-a)t_d^2$

$\therefore \frac{t_a}{t_d} = \sqrt{\frac{g-a}{g+a}} = \sqrt{\frac{10-2}{10+2}} = \sqrt{\frac{2}{3}}$ Ans.

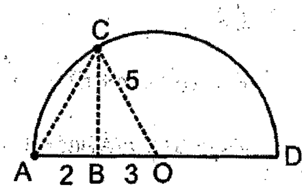
2.5 (A)
$x_A = x_B$

$10.5 + 10t = \frac{1}{2}at^2$ $a = \tan 45° = 1$

$t^2 - 20t - 21 = 0$
$t^2 - 21t + t - 21 = 0$
$t(t-21) + 1(t-21) = 0$
$\Rightarrow t = 21, -1$
rejecting negative value
$t = 21$ sec.

2.6 (D)

From triangle BCO $\Rightarrow$ BC = 4
From triangle BCA $\Rightarrow$ $AC = \sqrt{2^2+4^2} = 2\sqrt{5}$



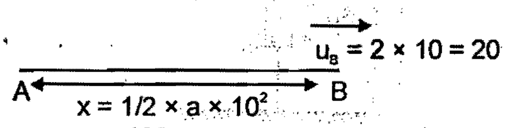
$AC = u_1 t,\quad BC = u_2 t$

$\therefore \frac{u_1}{u_2} = \frac{AC}{BC} = \frac{2\sqrt{5}}{4} = \frac{\sqrt{5}}{\sqrt{4}}$

2.7 (D)

After 10 sec

10 sec



= 100

Now $x_A = (40\,t)$

$$x_B = 100 + (ut) + \frac{1}{2}(2)\,t^2 = 100 + 20\,t + t^2$$

A will be ahead of B when

$x_B < x_A$

$\Rightarrow \quad 100 + 20\,t + t^2 < 40\,t$

$\Rightarrow \quad t^2 - 20\,t + 100 < 0$

$t^2 - 10t - 10\,t + 100 < 0$

$t(t - 10) - 10\,(t - 10) < 0$

$(t - 10)^2 < 0$

which is not possible

2.8 (B)

From given graphs : a_x is +ve & a_y is –ve, as v_x is increasing in +ve direction and v_y in –ve direction. (Checked from slope)

2.9 (A)

Distance travelled from time 't–1' sec to 't' sec is

$S = u + \frac{a}{2}(2t - 1)$ ....(1)

from given condition S = t .... (2)

(1) & (2) $\Rightarrow t = u + \frac{a}{2}(2t - 1) \Rightarrow u = \frac{a}{2} + t\,(1 - a)$.

Since u and a are arbitrary constants, and they must be constant for every time.

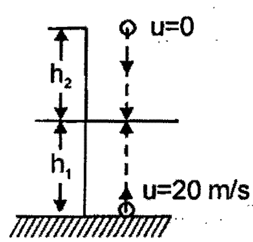
$\Rightarrow$ coefficient of t must be equal to zero.

$\Rightarrow \quad 1 - a = 0 \Rightarrow a = 1$ for $a = 1$, $u = \frac{1}{2}$ unit

Initial speed is $\frac{1}{2}$ unit. Ans.

2.10 (B)

Height of the building



$H = h_1 + h_2$

$= \frac{1}{2}gt^2 + ut - \frac{1}{2}gt^2$

$=$ ut = 60 m.

2.11 (A)

$\vec{r} = (t^2 - 4t + 6)\,\hat{i} + t^2\,\hat{j}\ ;\ \vec{v} = \frac{d\vec{r}}{dt} = (2t - 4)\,\hat{i} + 2t\,\hat{j}\ ,\ \vec{a} = \frac{d\vec{v}}{dt} = 2\,\hat{i} + 2\,\hat{j}$

if $\vec{a}$ and $\vec{v}$ are perpendicular

$\vec{a}\,.\,\vec{v} = 0 \quad \Rightarrow \quad (2\,\hat{i} + 2\,\hat{j}).\,((2t - 4)\,\hat{i} + 2t\,\hat{j}) = 0$

$8t - 8 = 0 \quad \Rightarrow \quad t = 1$ sec. Ans.

2.12 (A)

At t = 0

$\frac{dx}{dt} = 0$ for particles 1,2 and 3 and $\left|\frac{d^2x}{dt^2}\right| > 0$ for t > 0

and $\frac{dx}{dt} = -3.4$ m/s for particle 4 and $\frac{d^2x}{dt^2}$ is negative for t > 0

Therefore for t > 0 ; $\left|\frac{dx}{dt}\right|$ is increasing in all.

2.13 (BCD)

|Displacement| ≤ Distance.

So, average speed of a particle in a given time $\left(\text{i.e. } \frac{\Delta(\text{distance})}{\Delta t}\right)$ is never less than magnitude of average velocity $\left(\text{i.e } \left|\frac{\Delta}{\Delta t}(\text{displacement})\right|\right)$

It is possible to have a situation in which $\left|\frac{d\bar{v}}{dt}\right| \neq 0$ (i.e., |acceleration| ≠ 0) but $\frac{d|\bar{v}|}{dt} = 0$ (i.e., $\frac{d}{dt}$ (speed) = 0). A particle moving in a circle with constant speed follow the upper statement.

A partcile revolving in a circle has zero average velocity every time it reaches the starting point.

2.14 (AC)

2.15 (ACD)

$$\upsilon = \sqrt{x} \Rightarrow \frac{dx}{dt} = \sqrt{x} \Rightarrow \frac{dx}{x^{1/2}} = dt \Rightarrow 2\sqrt{x} = t + C$$

but given at t = 0 ; x = 4 ⇒ c = 4

$$x = \frac{(t+4)^2}{4} \Rightarrow x = \frac{(6)^2}{4} = \frac{36}{4} = 9 \text{ m} \qquad \text{[Putting t = 2 sec.]}$$

$$a = v\frac{dv}{dx} = \sqrt{x} \times \frac{1}{2\sqrt{x}} = \frac{1}{2} \text{ m/s}^2$$

2.16 (ACD)

Slope of displacement-time curve gives velocity.

(A) During OA slope is +ve but decreasing hence velocity is positive and acceleration is negative.

(C) During BC slope is – ve and going to zero hence velocity is – ve but acceleration is +ve.

(D) During DE slope is +ve and increasing hence vel. is +ve and increasing ∴ +ve acceleration

2.17 (ACD)

time		distance left
t = 0	→	x_0
t = T	→	$x_0/2$
t = 2T	→	$x_0/2^2$

$$t = nT \rightarrow \frac{x_0}{(2)^n} = \frac{x_0}{(2)^{t/T}} = x_0(2)^{-t/T} = x_0(2)^{-t} \qquad (\because T = 1s)$$

∴ distance travelled in time t = x = $x_0 - x_0(2)^{-t} = x_0(1 - 2^{-t})$

$$v = \frac{dx}{dt} = x_0 2^{-t} \times \ell n(2) = \frac{x_0 \ell n2}{2^t} \quad (\therefore \text{ slope of x-t curve is positive and decreasing with time})$$

$$a = \frac{dv}{dt} = -x_0 2^{-t} \times (\ell n2)^2$$

$|a| = x_0 2^{-t} \times (\ell n2)^2$

2.18 **(AB)**

(i) $V\frac{dv}{dx} = -\beta V$

$\therefore$ $dv = -\beta dx$

$\int_{v_0}^{0} dv = -\beta \int_{0}^{x} dx$

$-v_0 = -\beta x$

$x = \frac{v_0}{\beta}$ [when V = 0, accelaration = 0,

so x is total direction

(ii) $a = -\beta V$

$\frac{dv}{dt} = -\beta V$

$\int_{v_0}^{v} \frac{dv}{v} = -\beta \int_{0}^{t} dt$

$\ell n\left(\frac{V}{V_0}\right) = -\beta t$

$V = V_0 e^{-\beta t}$

$V = \frac{V_0}{e^{\beta t}}$ at $t \to \infty$ $V = 0$.

$\therefore$ A & B are correct answer

2.19 **(D)**

Average velocity $= \frac{\text{displacement}}{\text{Time}}$, and average speed $= \frac{\text{distance}}{\text{time}} \Rightarrow$ |Displacement| $\leq$ Distance.

2.20 **(D)**

$a = \frac{dv}{dt}$ $\Rightarrow$ If $a = 0$ $\Rightarrow$ v may or may not be zero.

2.21 **(A)**

2.22 **(B)**

2.23 **(B)**

(2.21 to 2.23)

$a = \sin\pi t$

$\therefore \int dv = \int 2\sin\pi t\, dt$ or $v = -\frac{2}{\pi}\cos\pi t + C$

at t = 0 v = 0 $\therefore C = \frac{2}{\pi}$ or $v = \frac{2}{\pi}(1-\cos\pi t)$

Note : Velocity is always non-negative as $\cos\theta \leq 1$ hence particle always moves along positive x-direction

$\therefore$ Distance from time t = 0 to t is

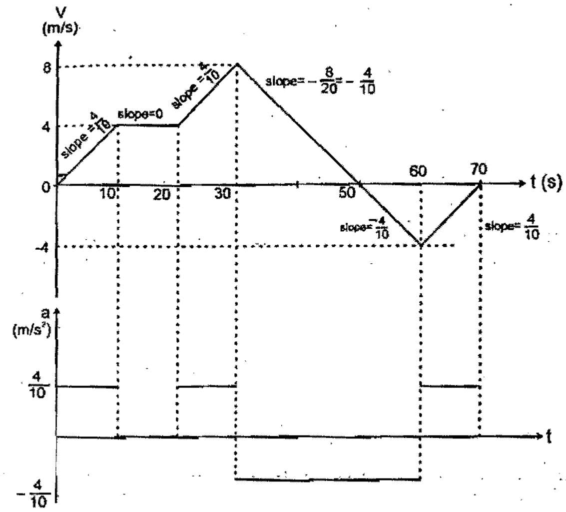
$$S = \int_0^t \frac{2}{\pi}(1-\cos\pi t)dt = \frac{2}{\pi}\left(t - \frac{1}{\pi}\sin\pi t\right]_0^t = \frac{2}{\pi}t - \frac{2}{\pi^2}\sin\pi t$$

also displacement from time t = 0 to t = $\frac{2t}{\pi} - \frac{2}{\pi^2}\sin\pi t$

Distance from time t = 0 to t = 1 s = $\frac{2}{\pi}$ meters

2.24 (D)

a = slope of v–t curve so

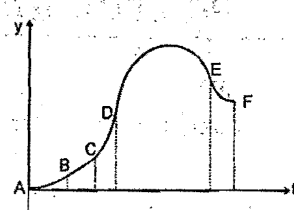


2.25 (B)

AB = Slope increasing, BC = Slope constant, CD = Slope increasing

DE = Slope decreasing, EF = Slope increasing

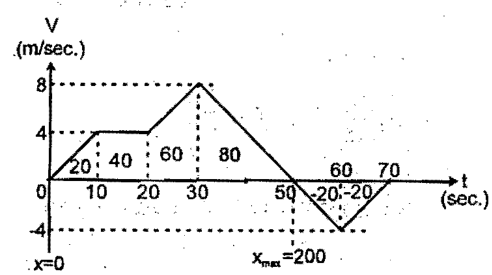
F = Slope is 0



2.26 (A)

Positive increase in area of v-t curve shows positive increase in displacement. So displacement is increasing till t = 50 s.

$\therefore$ max displacement = positive v-t area.



2.27 (A)→ r,t (B)→ r,t (C)→ p,q,r (D)→p,q,r

In case A and B acceleration is constant but speed first decreases and then increases.

In case C and D, the slope does not change sign hence direction of acceleration is constant. Speed and magnitude of acceleration decreases with time.

2.28 (A)→p, q (B)→p, q (C)→p, r (D)→r, t

(A) $v = 6t + 2$ m/s $v(t = 1) = 8$ m/s

$a = 6$ m/s^2

$v > 0$

(B) $v(t = 1) = 8$ m/s

$a = 8$ m/s^2

$v > 0$

(C) a is variable and +ve

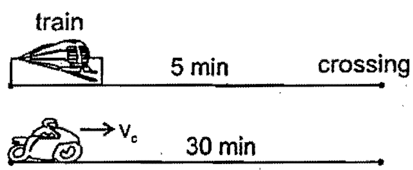
$v = \int a\,dt = 8t^2$, $v(t = 1s) = 8$ m/s

(D) $v(t = 1s) = 3$ m/s

$a = 6 - 6t$, variable

$v < 0$ for $t > 2$ s.

2.29 4



$$t_{cycle} = \frac{10km}{20kmh^{-1}} = \frac{1}{2}\ h = 30\ min$$

$$\Delta t = 5\ min = \frac{5}{60}\ hr$$

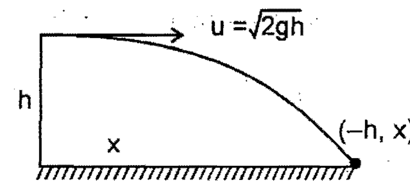
Train running as per shedule

So $\quad V_{train} = \frac{10}{(5/60)} = \frac{10 \times 60}{5} = 120\ kmh^{-1}$

3. PROJECTILE MOTION

3.1 (C)

Using equation of trajectory



$$-h = x\tan(0°) - \frac{gx^2}{2(2gh)(\cos^2 0°)} \Rightarrow x = 2h \qquad \text{Ans.}$$

Method II

time of flight $T = \sqrt{\frac{2h}{g}}$

horizontal distance covered during time of flight is $x = u_x t = \sqrt{\frac{2h}{g}} \times \sqrt{2hg} = 2h$

3.2 (D)

Ranges for complementary angles are same

$\therefore$ Required angle $= \frac{\pi}{2} - \frac{5\pi}{36} = \frac{13\pi}{36}$ Ans.

3.3 (D)

Use $\alpha = \beta = 45°$ in the formula for Range down the incline plane.

3.4 (D)

Time of flight $T = \frac{2u_y}{g}$

$$T = \frac{2 \times 20 \sin 37°}{10} = 4 \times \frac{3}{5} = \frac{12}{5}\ \text{sec.}$$

Range $R = u_x \times T = \frac{12}{5} \times (20\cos 37° + 10)$

$$R = \frac{12}{5} \times \left(20 \times \frac{4}{5} + 10\right) \qquad = 26 \times \frac{12}{5} = 62.4\ m$$

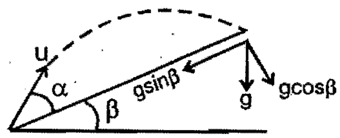
3.5 (C)

Use the given data in the formulae for projection up the inclined plane.

Let the inclination of the inclined plane = β

$u \cos\alpha = 10$ .... (1)

Time of flight $\frac{2u\sin\alpha}{g\cos\beta} = 2$ .... (2)



maximum height $\frac{u^2\sin^2\alpha}{g\cos\beta} = 5$ .... (3)

$u \sin\alpha = 5$

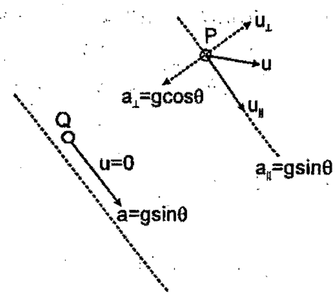
$\therefore \quad u = 5\sqrt{5}$

3.6 (D)

Path will not be straight line but parabolic hence neither stone will hit any person. Condition of collision will depend upon direction as well as magnitude of velocities of projection which are not given.

3.7 (B)

It can be observed from figure that P and Q shall collide if the initial component of velocity of P on inclined plane i.e along incline. $u_{\parallel} = 0$ that is particle is projected perpendicular to incline.



$\therefore \quad$ Time of flight $T = \frac{2u_{\perp}}{g\cos\theta} = \frac{2u}{g\cos\theta}$

$\therefore \quad u = \frac{gT\cos\theta}{2} = 10$ m/s.

3.8 (C)

$\tan\theta = \frac{9-1}{4-0} = 2$ Where θ is the angle of projection

Displacement in y-direction $s_y = u_y t + \frac{1}{2} a_y t^2$

now, $-1 = u\sin\theta\,(1) - \frac{1}{2} g\,(1)^2$

$u\sin\theta = 4$ and from triangle $\sin\theta = \frac{2}{\sqrt{5}} \Rightarrow u = 2\sqrt{5}$ m/s

Displacement in x-direction $s_x = u_x t + \frac{1}{2} a_x t^2$

now, $x = u\cos\theta\,(1) = (2\sqrt{5}) \times \frac{1}{\sqrt{5}} = 2$m

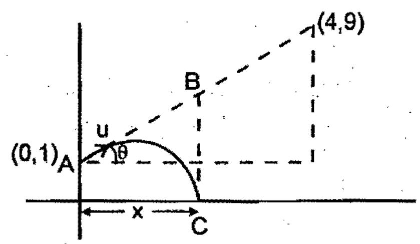
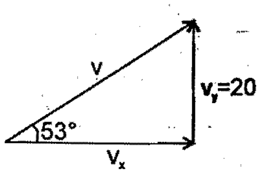


fig. (A)

3.9 (B)

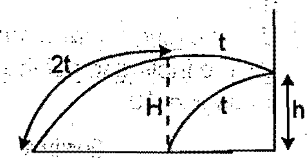
Two second before maximum height $v_y = g \times 2 = 20$ m/s



$\tan 53° = \frac{20}{v_x} \quad v_x = 15$ m/s

velocity at maximum height $v = v_x = 15$ m/s

3.10 (C)
For B to C



$H = \frac{1}{2}g(2t)^2 = 2gt^2$ ....(1)

$h' = \frac{1}{2} g t^2$ .... (2)

$h = H - h'$ $\Rightarrow$ $h = H - \frac{1}{2} gt^2$ ....(3)

By (1) & (2)

$h = H - \frac{H}{4} = \frac{3H}{4}$

3.11 (A)

velocity component $u_x = 400/3\ \hat{i}$, $u_y = 100\ \hat{j}$

Applying equation is y direction

$-1500 = -100\,t - \frac{1}{2} \times 10\,t^2 \Rightarrow \frac{t^2}{2} + 10\,t - 150 = 0$

$t = \frac{-20 \pm 40}{2}$ $\Rightarrow$ So t = 10 sec

i.e. horizontal distance $u_x \times t = \frac{500}{3} \times \frac{4}{5} \times 10 = \frac{4000}{3}$ m.

3.12 (B)
For minimum number of jumps, range must be maximum.

maximum range $= \frac{u^2}{g} = \frac{(\sqrt{10})^2}{10} = 1$ meter.

Total distance to be covered = 10 meter
So minimum number of jumps = 10

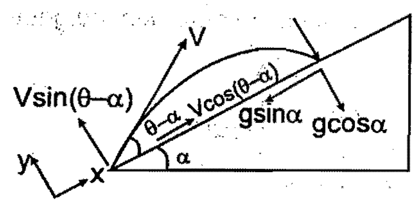
3.13 (D)
$y = bx^2$

$\frac{dy}{dt} = 2bx.\frac{dx}{dt} \Rightarrow \frac{d^2y}{dt} = 2b\left(\frac{dx}{dt}\right)^2 + 2bx\frac{d^2x}{dt^2} \Rightarrow a = 2bv^2 + 0 \Rightarrow v = \sqrt{\frac{a}{2b}}$

3.14 (D)
Applying equation of motion perpendicular to the incline for y = 0.

$y = u_y t + \frac{1}{2} a_y t^2$ $\Rightarrow t = 0\ \&\ \frac{2V\sin(\theta-\alpha)}{g\cos\alpha}$

$\Rightarrow$ $0 = V\sin(\theta)t + \frac{1}{2}(-g\cos\alpha)\,t^2$



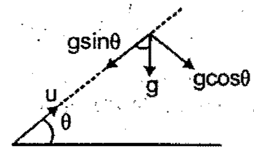
At the moment of striking the plane, as velocity is perpendicular to the inclined plane hence component of velocity along incline must be zero.

$V_x = u_x + a_x t$ $\Rightarrow 0 = v\cos(\theta-\alpha) + (-g\sin\alpha).\frac{2V\sin(\theta-\alpha)}{g\cos\alpha} \Rightarrow v\cos(\theta-\alpha) = \tan\alpha.\ 2V\sin(\theta-\alpha)$

$\Rightarrow$ $\cot(\theta-\alpha) = 2\tan\alpha$

3.15 (ABCD)

$0 = u - g\sin\theta . t$



(a) $t = \dfrac{2u\sin\theta}{g} = \dfrac{2.(10)\sin 30^\circ}{10} = 1$ sec.

(b) $t = \dfrac{2.10\sqrt{3}.\sqrt{3}}{10.2} = 3$ sec.

(c) $t = \dfrac{u}{g\sin\theta} = \dfrac{10\sqrt{3}}{10(\sqrt{3}/2)} = 2$ sec. t is less than time of flight

(d) $t = \dfrac{u}{g\sin\theta} = \dfrac{10}{10.\frac{1}{2}} = 2$ sec. But it's time of flight is 1sec.

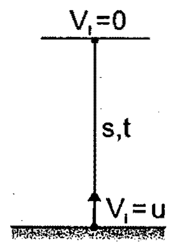
3.16 (ABC)

(A) Total displacement is zero hence its average velocity is zero.

(B) Displacement is zero.

(C) Total distance travelled is 2s and total time taken is 2t.

$< speed > = \dfrac{\text{total distance travelled}}{\text{total time taken}}$



$0^2 = u^2 - 2gs$

$\therefore \quad s = \dfrac{u^2}{2g} \Rightarrow 2s = u^2/g$

also $0 = u - g\,t \quad \Rightarrow t = u/g$

$\therefore \quad 2t = \dfrac{2u}{g}$

$<speed> = \dfrac{u^2}{g} \Big/ \dfrac{2u}{g} = \dfrac{u}{2}$

3.17 (ABC)

On the curve

$y = x^2$

at $x = 1/2 \quad \Rightarrow \quad y = \dfrac{1}{4}$

Hence the coordinate $\left(\dfrac{1}{2}, \dfrac{1}{4}\right)$

Differentiating $\quad y = x^2$

$\Rightarrow \quad v_y = 2xv_x$

$v_y = 2\left(\dfrac{1}{2}\right)(4) = 4$ m/s

Which satisfies the line

$4x - 4y - 1 = 0$ (tangent to the curve)

& magnitude of velocity :

$|\vec{v}| = \sqrt{v_x^2 + v_y^2} = 4\sqrt{2}$ m/s

As the line $4x - 4y = 1$ does not pass through the origin, therefore (D) is not correct.

3.18 (D)

Let u_x and u_y be horizontal and vertical components of velocity respectively at t = 0. Then,

$v_y = u_y - gt$

Hence, v_y – t graph is straight line.

$x = v_x t$

Hence, x - t graph is straight line passing through origin.

The relation between y and t is $y = u_y t - ½ gt^2$

Hence y-t graph is parabolic.

v_x = constant

Hence, v_x-t graph is a straight line.

3.19 (D)

[here T_1 & T_2 are times of flight in the two cases respectively]

$R_1 = \frac{2u^2 \sin\alpha \cos(\alpha+\theta)}{g\cos^2\theta}$ and $h_1 = \frac{u^2 \sin^2\alpha}{2g\cos\beta}$

$R_2 = \frac{2u^2 \sin\alpha \cos(\alpha-\theta)}{g\cos^2\theta}$ and $h_2 = \frac{u^2 \sin^2\alpha}{2g\cos\beta}$ hence $h_1 = h_2$

$R_2 - R_1 = g \sin\theta \, T_2^2$

$R_2 - R_1 = g \sin\theta \, T_1^2$

3.20 (C)

Total time taken by the ball to reach at bottom = $\sqrt{\frac{2H}{g}} = \sqrt{\frac{2x80}{10}}$ = 4 sec.

Let time taken in one collision is t

Then t x 10 = 7

t = 0.7 sec.

No. of collisions = $\frac{40}{7} = 5\frac{5}{7}$ (5th collisions from wall B)

Horizontal distance travelled in between 2 successive collisions = 7 m

∴ Horizontal distance travelled in 5/7 part of collisions = $\frac{5}{7}$ x 7 = 5 m

Distance from A is 2 m. **Ans.**

3.21 (D)

Both the stones cannot meet (collide) because their horizontal component of velocities are different. Hence statement I is false.

3.22 (B)

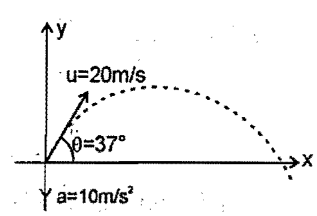
If particle moves with constant acceleration $\vec{a}$, then change in velocity in every one second is numerically equal to $\vec{a}$ by definition. Hence statement-2 is true and correct explanation of statement-1.

3.23 (B)

The question can be reframed as shown in figure.

The path of particle is parabolic.

∴ $\vec{a} \perp \vec{v}$ at maximum height, that is at half time of flight



Hence $t_0 = \frac{u\sin\theta}{a} = \frac{20 \times 3/5}{10}$ = 1.2 sec.

3.24 (B)

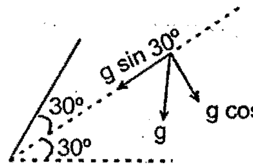
Speed is least at maximum height, that is at instant t_0=1.2 sec.

3.25 (C)

acceleration and displacement are mutually perpendicular at instant $2t_0$ = 2.4 sec.

3.26 (A) r (B) p (C) s (D) q

(A) $R = \frac{u^2 \sin 2\theta}{g} = \frac{100\sqrt{3}}{2(10)} = 5\sqrt{3}m$

(B) 

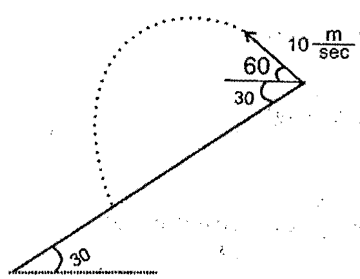
$11.25 = -10\sin 60^\circ t + \frac{1}{2}(10)t^2 \quad \Rightarrow \quad 5t^2 - 5\sqrt{3}\,t - 11.25 = 0$

$t = \frac{5\sqrt{3} \pm \sqrt{25(3)+4(5)(11.25)}}{10} = \frac{5\sqrt{3} \pm \sqrt{3}(10)}{10} = \frac{15}{10}\sqrt{3} = \frac{3}{2}\sqrt{3}$

$R = (10\cos 60)\left(\frac{3}{2}\sqrt{3}\right) = 7.5\sqrt{3}$ m

(C) $t = \frac{2u\sin 30^\circ}{g\cos 30^\circ} = \frac{2(10)\left(\frac{1}{2}\right)}{10\left(\frac{\sqrt{3}}{2}\right)} = \frac{2}{\sqrt{3}}$ sec.

$R = 10\cos 30^\circ t - \frac{1}{2} g \sin 30^\circ t^2 = \frac{10\sqrt{3}}{2}\left(\frac{2}{\sqrt{3}}\right) - \frac{1}{2}(10)\left(\frac{1}{2}\right)\frac{4}{3} = 10 - \frac{10}{3} = \frac{20}{3}$ m

(D) 

$T = \frac{2(10)}{g\cos 30} = \frac{2(10)}{10\left(\frac{\sqrt{3}}{2}\right)} = \frac{4}{\sqrt{3}}$ sec.

$R = \frac{1}{2} g \sin 30^\circ t^2 = \frac{1}{2}(10)\left(\frac{1}{2}\right)\frac{16}{3} = \frac{40}{3}$ m

3.27 (A) p, q, r (B) p (C) q, t (D) r, s

Range of the ball in absence of the wall = $\frac{u^2 \sin 2\theta}{g}$

When d < 20 m, ball will hit the wall. when d = 25 m, ball will fall 5m short of the wall.
When d < 20 m, the ball will hit the ground , at a distance, x = 20 m – d in front of the wall.

3.28 3

(a) Taking motion in vertical direction

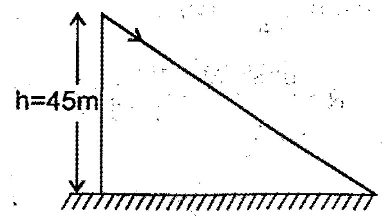
$u = 0,\ g = 10\ m/s^2, \qquad h = 45\ m$

$h = ut + ½\ gt^2$

$\Rightarrow \quad h = 0 + ½\ gt^2$

$\Rightarrow \quad t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 45}{10}}$

$\Rightarrow \quad t = 3$ sec.



3.29 25

Let t be the time after which projectile reaches the ground. Taking motion in horizontal direction

$400 = (v_0 \cos 37°)\ t$

$400 = v_0\ (4/5)\ t$

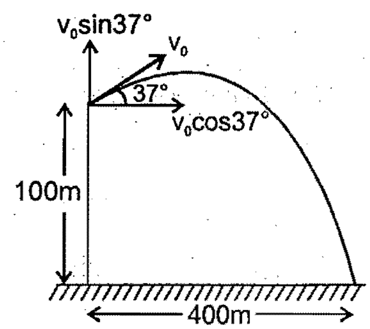
$\Rightarrow \quad v_0\ t = 500 \qquad \text{.... (1)}$

Taking motion in vertical direction

$h = ut + ½\ gt^2$

$\Rightarrow \quad 100 = (-\ v_0 \sin 37°)\ t\ + ½\ (10)\ t^2$

$\Rightarrow \quad 100 = -\frac{3}{5}(v_0\ t) + 5\ t^2$



Putting $v_0t = 500$ from equation (1) ;

$\Rightarrow \quad 100 = -\frac{3}{5}(500) + 5\ t^2$

$\Rightarrow \quad 5t^2 = 400 \qquad \Rightarrow\ t = \frac{20}{\sqrt{5}}$sec

From eqn (1) ; $\quad v_0 = 500 \times \frac{\sqrt{5}}{20}$

$v_0 = 25\sqrt{5}$ m/s

4. RELATIVE MOTION

4.1 (C)

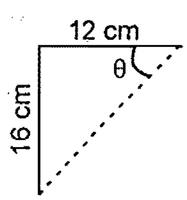
Relative to the person in the train, ' component of acceleration of the stone is 'g' downward and , a (acceleration of train) backwards.

$x = \frac{1}{2}at^2\ , \quad y = \frac{1}{2}gt^2 \quad \Rightarrow \quad \frac{x}{y} = \frac{a}{g} \Rightarrow y = \frac{g}{a}\ x \Rightarrow$ straight line.

4.2 (A)

$V_{R/G(x)} = 0\ ,\ V_{R/G(y)} = 10$ m/s

Let, velocity of man = v



$\tan\theta = \frac{16}{12} = \frac{4}{3}$

then, $v_{R/man}$ = v (opposite to man)

For the required condition :

$\tan\theta = \frac{V_{R/M(y)}}{V_{R/M(x)}} = \frac{10}{v} = \frac{4}{3} \Rightarrow V = \frac{10 \times 3}{4} = 7.5$ **Ans.**

4.3 (A)

$v = at = 2t$

Velocity of car at t = 3 $v_1 = 6$ m/s

at t = 4 $v_2 = 8$ m/s

Car moves 7m $\left(=\frac{6+8}{2}\times 1\right)$ in Li+e na Postion where first coin will hit hte ground = $\frac{1}{2}\times 2\times 3^2\times 6\times 1$ x = 15

Car moves 7m $\left(=\frac{6+8}{2}\times 1\right)$ in Li+e na Postion where second will hit hte ground $x_2 = \frac{1}{2}\times 2\times 4^2\times 8\times 1$ x = 24

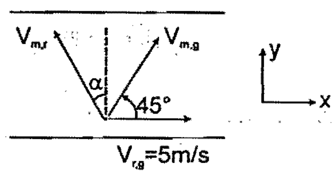
$\Rightarrow \quad x_2 - x_1 = 9$m

4.4 (C)

$\vec{V}_{m,g} = \vec{V}_{m,r} + \vec{V}_{r,g}$

Let the swimner tryer to & wim at an after $\propto$ from perpendicular to riwer flow direction.

As resulting velocity $\vec{V}_{m,g}$ is at 45° with river flow



i.e. $V_{r,g} - V_{m,r}\sin\alpha = V_{m,r}\cos\alpha$ ....(1)

and $\frac{60\text{m}}{V_{mr}\cos\alpha} = 6$ sec. ....(2)

Solving (1) & (2)

$V_{m,r} = 5\sqrt{5}$ m/s

4.5 (D)

Relative to balloon = 5 m/s

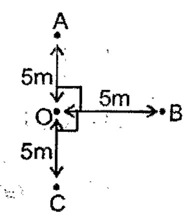
= +10–(–5) m/s² = 15

$\therefore \quad v = u + at = 5 + 15 \times 2 = 35$ m/s

$\therefore$ relative velocity after t = 2 second is 35 m/s

4.6 (C)

Let the stones be projected at t=0 sec with a speed u from point O. Then an observer ,at rest at t= 0 and having constant acceleration equal to acceleration due to gravity, shall observe the three stones move with constant velocity as shown. In the given time each ball shall travel a distance 5 metre as seen by this observer.

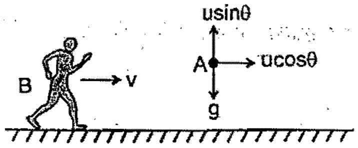


Hence the required distance between A and B will be = $\sqrt{5^2+5^2} = 5\sqrt{2}$ metre

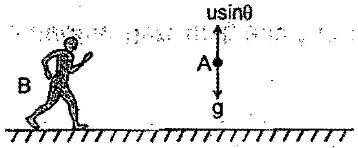
4.7 (A)

The horizontal and vertical components of initial velocity of projectile are as shown in figure. Since the observer moving with uniform velocity v sees the projectile moving in straight line

Hence $v = u\cos\theta$



velocity of A and B given from frame of ground



velocity of A given from frame of B

The time of flight as measured by observer B is T

Hence horizontal range of projectile on ground is

$R = (u\cos\theta)T = vT$

4.8 (B)

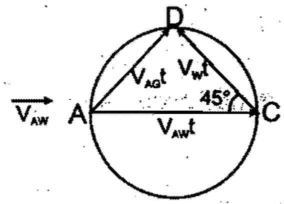
Without wind A reaches to C and with wind it reaches to D in same time so wind must deflect from C to D so wind blow in the direction of CD

$$\vec{V}_{AG} = \vec{V}_{AW} + \vec{V}_{WG}$$

$$\Rightarrow \quad \vec{V}_{AG}\,t = \vec{V}_{AW}\,t + \vec{V}_{WG}\,t$$

$$AC = \vec{V}_{AW}\,t$$

$$CD = \vec{V}_{WG}\,t$$



4.9 (B)

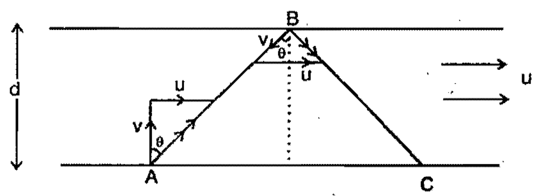
With respect to lift initial speed = v_0 $T = \dfrac{2u\sin\theta}{g} = \dfrac{2v_0}{g}$

acc = – 2g

displacement = 0

$\therefore\ S = ut + \dfrac{1}{2}at^2 \quad \Rightarrow \quad 0 = v_0T' - \dfrac{1}{2} \times 2g \times T'^2 \quad \therefore\ T' = \dfrac{v_0}{g} = \dfrac{1}{2} \times \dfrac{2v_0}{g} = \dfrac{1}{2}\,T$

4.10 (C)



V = velocity of man w.r.t . river

u = velocity of river

$A \xrightarrow{t} B = \dfrac{d}{v} \Rightarrow 10 = \dfrac{d}{v} \Rightarrow d = 10\,V \qquad \longrightarrow (1)$

$B \xrightarrow{t} C = \dfrac{d}{v\cos\theta} \Rightarrow 15 = \dfrac{d}{v\cos\theta} \Rightarrow d = 15\,v\cos\theta \qquad \longrightarrow (2)$

(1) & (2) $\Rightarrow \cos\theta = 2/3 \Rightarrow \sec\theta = 3/2$

$\because \quad \tan\theta = \dfrac{u}{v} \quad \therefore \quad \sqrt{\sec^2\theta - 1} = \dfrac{u}{v}$

$\Rightarrow \quad \dfrac{u}{v} = \sqrt{9/4 - 1} = \dfrac{\sqrt{5}}{2} \quad \Rightarrow \dfrac{v}{u} = \dfrac{2}{\sqrt{5}}$

4.11 (C)

No. of taxi = $\dfrac{240}{10}$ = 24 but when 24th start motion it reach the destination so it will meet 23 only.

4.12 (A)

$$v_{rel} = 2v\sin\frac{\theta}{2}\ ; \quad \langle v_{rel} \rangle = \frac{\int_0^{2\pi} 2v\sin\frac{\theta}{2}\,d\theta}{\int_0^{2\pi} d\theta} = \frac{4v}{\pi}$$

4.13 (B)

Let velocity of the aeroplane be $\vec{v}_P = u\cos30^0\hat{i} + u\sin30^0\hat{j}$ and velocity of the wind be v, then

$$u\frac{\sqrt{3}}{2}t\hat{i} + \left(\frac{u}{2}t - 5t^2\right)\hat{j} + vt\hat{k} = 400\sqrt{3}\hat{i} + 80\hat{j} + 200\hat{k}$$

$$\Rightarrow u\frac{\sqrt{3}}{2}t = 400\sqrt{3},\ \frac{u}{2}t - 5t^2 = 80,\ vt = 200 \Rightarrow ut = 800 \text{ and } \frac{u}{2}t - 5t^2 = 80$$

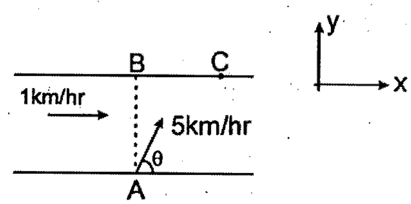
$$\Rightarrow 400 - 5t^2 = 80 \quad \Rightarrow \quad t^2 = 64 \quad \Rightarrow \quad t = 8 \text{ sec.}$$

4.14 (AB)

AB = BC = 400 m = 0.4 km

$v_x = 5\cos\theta + 1$

$v_y = 5\sin\theta$



time taken (t) = $\frac{AB}{v_y} = \frac{BC}{v_x}$

$\Rightarrow v_y = v_x \quad \Rightarrow \quad 5\sin\theta = 5\cos\theta + 1$

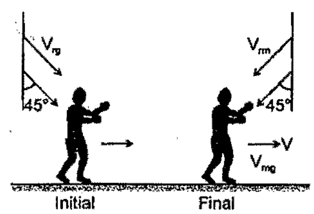
$\Rightarrow \theta = 53°$

and $t = \frac{0.4}{5\sin(53°)} = 0.1$ hr = 6 min.

4.15 (AD)

He can only reach the opposite point if he can cancel up the velocity of river by his component of velocity.

4.16 (CD)



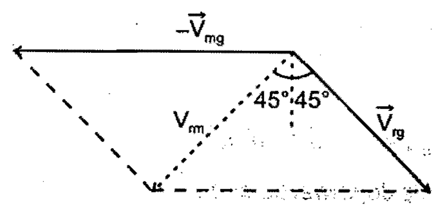
$\vec{V}_{rg} = \vec{V}_{rm} + \vec{V}_{mg}$

$\vec{V}_{rm} = \vec{V}_{rg} - \vec{V}_{mg}$

$V_{rm}\cos45° = V_{rg}\cos45°$

$V_{rm} = 2\sqrt{2}$ m/s $= V_{rg}$

$V_{rm}\cos45° = V_{mg} - V_{rg}\cos45°$



$$V_{mg} = 2\sqrt{2}\frac{1}{\sqrt{2}} + 2\sqrt{2}\frac{1}{\sqrt{2}} = 4 \text{ m/s}$$

using $v^2 = u^2 + 2as$ for the motion of man,

s = 16 m.

4.17 (AD)

While both the stones are in flight, $a_1 = g$ and $a_2 = g$

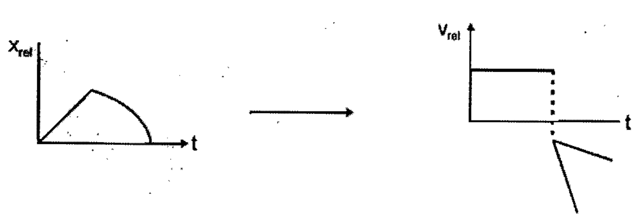
So $a_{rel} = 0 \quad \Rightarrow V_{rel}$ = constant

$\Rightarrow x_{rel}$ = (const) t

$\Rightarrow$ Curve of x_{rel} v/s t will be straight line.

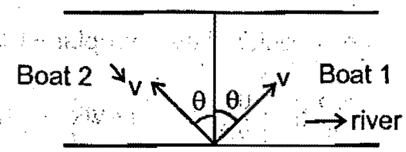
After the first particle drops on ground, the seperation (x_{rel}) will decrease parabolically (due to gravitational acceleration), and finally becomes zero.

and V_{rel} = slope of x_{rel} w.r.r time

So 

4.18 (A)

If component of velocities of boat relative to river is same normal to river flow (as shown in figure) both boats reach other bank simultaneously.



4.19 (A)

Acceleration of each of the projectile = $\vec{g}$. Relative acceleration $\vec{a}_r = \vec{g} - \vec{g} = 0$.

4.20 (A)

Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

In air their relative acceleration is zero. Hence they can,t approch the vertical distance between.

4.21 (C)

$$t_1 = \frac{d}{v_{sw}} = \frac{d}{v} \; ; \; t_2 = \frac{d}{\sqrt{v^2 - u^2}} \qquad \therefore \; \frac{t_1}{t_2} = \frac{d/v}{d/\sqrt{v^2 - u^2}} = \left(\frac{\sqrt{v^2 - u^2}}{v}\right) = \sqrt{1 - \frac{u^2}{v^2}}.$$

4.22 (A)

$$t_1' = \frac{d}{v} \; ; \; t_2' = \frac{d}{v} \qquad \therefore \; \frac{t_1'}{t_2'} = 1.$$

4.23 (B)

$$T_1 = \frac{d}{\sqrt{v^2 - u^2}} \text{ and } T_2 = \frac{d}{(v - u)} \qquad \text{so, } \frac{T_2}{T_1} = \frac{\sqrt{v^2 - u^2}}{v - u} = \sqrt{\frac{v + u}{v - u}} = \sqrt{\frac{1 + u/v}{1 - u/v}}$$

4.24 (D)

4.25 (A)

4.26 (B)

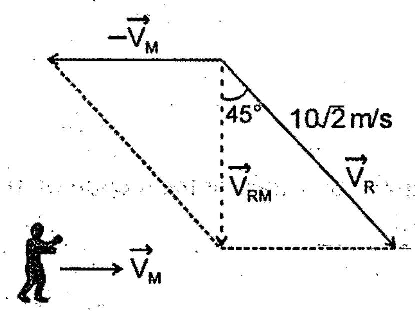
Solution. (4.24 to 4.26)

In the first case :

From the figure it is clear that

$\vec{V}_{RM}$ is 10 m/s downwards and

$\vec{V}_M$ is 10 m/s towards right.



In the second case :

Velocity of rain as observed by man becomes $\sqrt{3}$ times in magnitude.

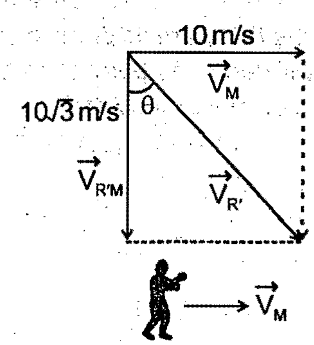
∴ New velocity of rain

$\vec{V}_{R'} = \vec{V}_{R'M} + \vec{V}_M$

∴ The angle rain makes with vertical is

$$\tan\theta = \frac{10}{10\sqrt{3}} \qquad \text{or } \theta = 30°$$

∴ Change in angle of rain = 45 – 30 = 15°.



4.27 (A) s (B) p (C) r (D) q

The initial velocity of A relative to B is $\vec{u}_{AB} = \vec{u}_A - \vec{u}_B = (8\hat{i} - 8\hat{j})$ m/s $\therefore u_{AB} = 8\sqrt{2}$ m/s

Acceleration of A relative to B is -

$\vec{a}_{AB} = \vec{a}_A - \vec{a}_B = (-2\hat{i} + 2\hat{j})$ m/s^2 $\therefore a_{AB} = 2\sqrt{2}$ m/s^2

since B observes initial velocity and constant acceleration of A in opposite directions, Hence B observes A moving along a straight line.

From frame of B

Hence time when $v_{AB} = 0$ is $t = \frac{u_{AB}}{a_{AB}} = 4$ sec.

The distance between A & B when $v_{AB} = 0$ is $S = \frac{u_{AB}^2}{2a_{AB}} = 16\sqrt{2}$ m

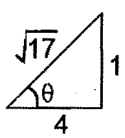
The time when both are at same position is -

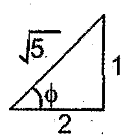
$T = \frac{2u_{AB}}{a_{AB}} = 8$ sec.

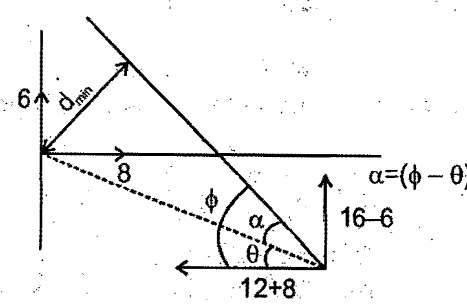
Magnitude of relative velocity when they are at same position is $u_{AB} = 8\sqrt{2}$ m/s.

4.28 (A) r, (B) t; (C) p, s; (D) p

$\tan\phi = \frac{5}{20} = \frac{1}{4}$ $\tan\phi = \frac{10}{20} = \frac{1}{2}$







$d_{min} = \sqrt{425}\sin\alpha = \sqrt{425}\sin(\phi - \theta)$

$= 5\sqrt{17}[\sin\phi\cos\theta - \cos\phi\sin\theta]$

$= 5\sqrt{17}\left[\frac{1}{\sqrt{5}} \times \frac{4}{\sqrt{17}} - \frac{1}{\sqrt{5}} \times \frac{1}{\sqrt{17}}\right] = \frac{5}{\sqrt{5}}[2] = 2\sqrt{5}$

$t = \frac{\sqrt{425}\cos\alpha}{\sqrt{500}} = \frac{5\sqrt{17}}{10\sqrt{5}}\left[\frac{2}{\sqrt{5}} \times \frac{4}{\sqrt{17}} + \frac{1}{\sqrt{5}} \times \frac{1}{\sqrt{17}}\right]$

$t = \frac{1}{10}[8 + 1] = \frac{9}{10}$ sec

$\vec{v}_1 = (8\hat{i} + 6\hat{j}) - (11t)\hat{j}$

$\vec{v}_2 = (-12\hat{i} + 16\hat{j}) - (11t)\hat{j}$

$\vec{v}_1 = 8\hat{i} + (6 - 11t)\hat{j}$

$\vec{v}_2 = -12\hat{i} + (16 - 11t)\hat{j}$

$\vec{v}_1.\vec{v}_2 = -96 + 96 - 66\,t - 176\,t + 121\,t^2$

$0 = -242\,t + 121\,t^2 = 0$

$t = 0$ and $t = \frac{242}{121} = 2$

4.29 5

Let velocity of bodies be v_1 and v_2.
in first case
$u_1 = v_1 + v_2$ .... (i)
in second case
$u_2 = v_1 - v_2$ .... (i)

$\therefore\ v_1 = \frac{u_1 + u_2}{2}$ and $v_2 = \frac{u_1 - u_2}{2}$

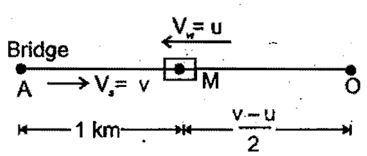
Here $u_1 = \frac{16}{10}$ m/s and $u_2 = \frac{3}{5}$

After solving we have $v_1 = \frac{11}{10}$ m/s and $v_2 = \frac{1}{2}$ m/s.

4.30 1

Let $V_w = u$ & $v_{sw} = v$

Time taken by swimmer to go from M to O and O to B = time taken by float to reach B from M



$\Rightarrow\ \frac{1}{2} + \frac{1 + \frac{v-u}{2}}{v+u} = \frac{1}{u}$ $\Rightarrow\ \frac{1}{2} + \frac{2 + v - u}{2(v+u)} = \frac{1}{u}$ $\Rightarrow\ \frac{v + u + 2 + v - u}{2(v+u)} = \frac{1}{u}$

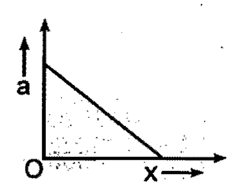
$\Rightarrow\ (2v + 2)\,u = 2\,(v + u)$

$\Rightarrow\ 2vu + 2u = 2v + 2u$ $\Rightarrow$ $u = 1$ km/h Ans.

5. NEWTON'S LAW OF MOTION

5.1 (C)

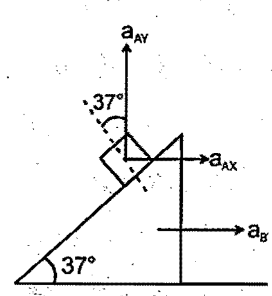
Let the initial compression of spring be ℓ.
Then the acceleration after the block travels a distance x is



$a = \frac{k}{m}(\ell - x)$

$\therefore$ The graph of a vs x is

5.2 (D)



From wedge constraint

$(a_A)_\perp = (a_B)_\perp$

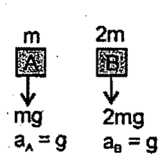
$a_{AX} \cos 53° - a_{AY} \cos 37° = a_B \cos 53°$

$a_B = -5$ m/s

$\vec{a}_B = -5\hat{i}$

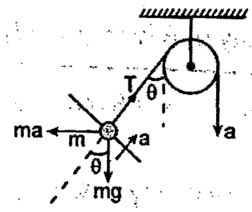
5.3 (A)

In this case spring force is zero initially
F.B.D. of A and B



5.4 (C)

(Force diagram in the frame of the car)



Applying Newton's law perpendicular to string

mg sinθ = ma cos θ

$\tan\theta = \frac{a}{g}$

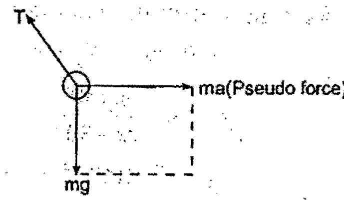
Applying Newton's law along string

$\Rightarrow \quad T - m\sqrt{g^2+a^2} = ma \quad T = m\sqrt{g^2+a^2} + ma$ Ans.

5.5 (B)

Acceleration of box = 10 m/s²

Inside the box forces acting on bob are shown in the figure

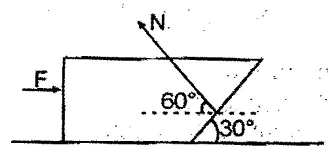


$T = \sqrt{(mg)^2+(ma)^2} = 10\sqrt{2}$ N

5.6 (D)

Acceleration of two mass system is $a = \frac{F}{2m}$ leftward

FBD of block A

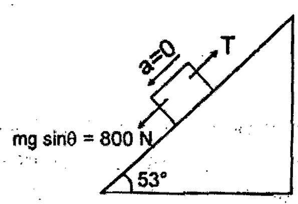


$N\cos 60° - F = ma = \frac{mF}{2m}$ solving, N = 3 F

5.7 (C)

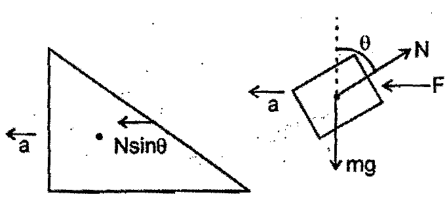
For block to be stationary T = 800 N

If man moves up by acc. 'a'



T – mg = ma ⇒ 800 – 500 = 50 a ⇒ a = 6m/s².

5.8 (C)



For no relative motion between wedge and block, let the acceleration of both block and wedge be 'a' towards left.

From FBD of block

N cosθ = mg .... (1)

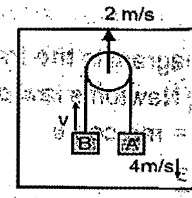
and F– N sinθ = ma .... (2)

From FBD of wedge

N sinθ = Ma .... (3)

from equation (1), (2) and (3) solving we get $F = \frac{m}{M}(M+m)\, g \tan\theta$

5.9. (D)



V = (velcoity of B w.r.t ground)

$\frac{V-4}{2} = 2 \qquad V = 8$ m/s (velcoity of B w.r.t ground)

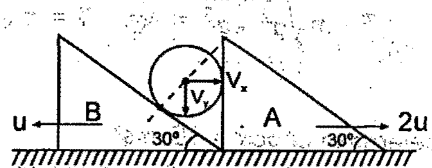
V' = 6 m/s (velcoity of B w.r.t lift)

5.10 (D)

Method - I

As cylinder will remain in contact with wedge A

$V_x = 2u$



As it also remain in contact with wedge B

$u \sin 30° = V_y \cos 30° - V_x \sin 30°$

$V_y = V_x \frac{\sin 30°}{\cos 30°} + \frac{u \sin 30°}{\cos 30°}$

$V_y = V_x \tan 30° + u \tan 30°$

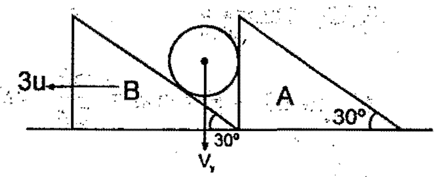
$V_y = 3u \tan 30° = \sqrt{3}\, u$

$V = \sqrt{V_x^2 + V_y^2} = \sqrt{7}\, u$ **Ans.**

Method - II

In the frame of A

$3u \sin 30° = V_y \cos 30°$

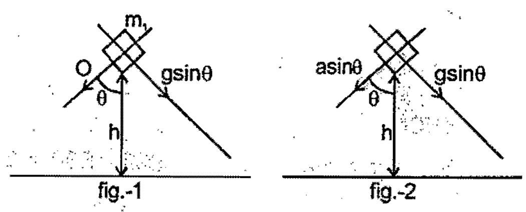


$\Rightarrow \qquad V_y = 3u \tan 30° = \sqrt{3}\, u$

and $\qquad V_x = 2u \qquad \Rightarrow \qquad V = \sqrt{V_x^2 + V_y^2} = \sqrt{7}\, u$ **Ans.**

5.11 (A)

We draw axes for each block along the incline and normal to incline. The component of acceleration for each block are as shown, where a is acceleration of wedge is figure2.

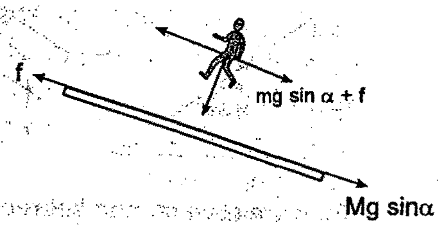


It is obvious that vertical component of acceleration is larger for block in figure 2.

$\therefore \qquad T_1 > T_2$

5.12 (B)

F.B.D. of man and plank are



For plank to be at rest, applying Newton's second law to plank along the incline

$Mg \sin \alpha = f$ ....(1)

and applying Newton's second law to man along the incline.

$mg \sin \alpha + f = ma$ ....(2)

$a = g \sin \alpha \left(1 + \frac{M}{m}\right)$ down the incline

Alternate Solution (II) :

If the friction force is taken up the incline on man, then application of Newton's second law to man and plank along incline yields.

$f + Mg \sin \alpha = 0$ ....(1)

$mg \sin \alpha - f = ma$ ....(2)

Solving (1) and (2)

$a = g \sin \alpha \left(1 + \frac{M}{m}\right)$ down the incline

Alternate Solution (III)

Application of Newton's seconds law to system of man + plank along the incline yields

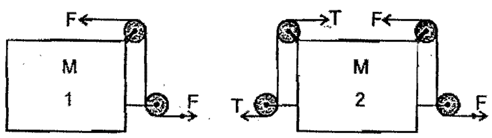
mg sin α + Mg sin α = ma

$a = g \sin \alpha \left(1 + \frac{M}{m}\right)$ down the incline

Ans. $a = g \sin \alpha \left(1 + \frac{M}{m}\right)$; downwards

5.13 (B)

The free body diagram for large blocks of figure 1 and figure 2



From FBD it is obvious net force on each block is zero in horizontal direction.

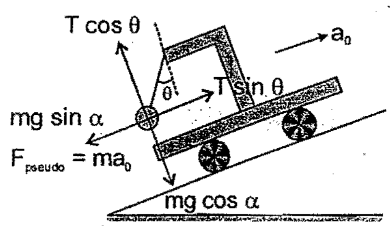
$\therefore \quad a_1 = a_2 = 0$

5.14 (D)

$T \sin \theta = ma_0 + mg \sin \alpha$

$T \cos \theta = mg \cos \alpha$

$\tan \theta = \frac{a_0 + g\sin\alpha}{g\cos\alpha}$



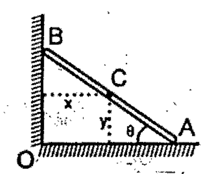
5.15 (B)

At any instant of time the rod makes an angle θ with horizontal,
the x & y coordinates of centre of rod are

$x = \ell \cos \theta \qquad y = \ell \sin \theta \qquad \therefore \quad x^2 + y^2 = \ell^2$

Hence the centre C moves along a circle of radius ℓ with centre at O.

∴ velocity vector of C is always directed along the tangent drawn at C to the circle of radius ℓ whose centre lies at O.



5.16 (A)

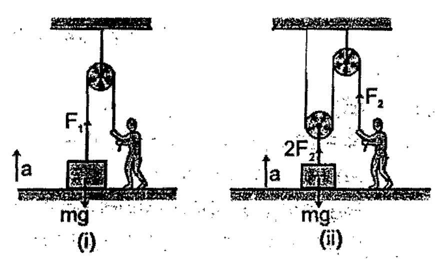
Since, $h = \frac{1}{2}at^2 \Rightarrow a$ should be same in both cases,

because h and t are same in both cases as given.

In (i) $F_1 - mg = ma. \quad \Rightarrow \quad F_1 = mg + ma.$

In (ii) $2F_2 - mg = ma \quad \Rightarrow \quad F_2 = \frac{mg + ma}{2}$

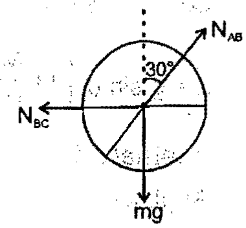
$\therefore \quad F_1 > F_2.$



5.17 (C)

The free body diagram of cylinder is as shown.

Since net acceleration of cylinder is horizontal,



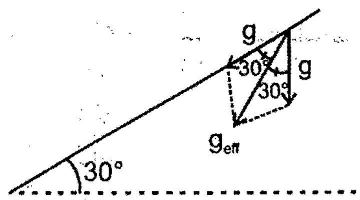
$N_{AB} \cos 30° = mg$ or $N_{AB} = \frac{2}{\sqrt{3}} mg$ .... (1)

and $N_{BC} - N_{AB} \sin 30° = ma$ or $N_{BC} = ma + N_{AB} \sin 30°$ .... (2)

Hence N_{AB} remains constant and N_{BC} increases with increase in a.

5.18 (B)

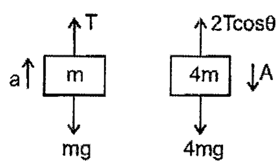
From frame of car, the effective acceleration (g_{eff}) due to gravity shall be measured as shown in figure. Hence g_{eff} makes an angle 30° with vertical direction (down wards). Since the string aligns with direction of g_{eff} in equilibrium, the required angle is θ = 30°.



5.19 (D)

The FBD of blocks is as shown

From Newton's second law



$4mg - 2T \cos\theta = 4 mA$ .... (1)

and $T - mg = ma$ .... (2)

$\cos\theta = \frac{4}{5}$ and from constraint we get $a = A \cos\theta$ ....(3)

Solving equation (1), (2) and (3)

we get acceleration of block of mass 4m, $a = \frac{5g}{11}$ downwards.

5.20 (C)

When all are pulling

$\vec{F}_{net} = 100 \times 3\hat{i}$ ....(1)

when 'A' stops

$\vec{F}_{net} - \vec{F}_A = 100 \times 1(-\hat{i})$ ....(2)

when 'B' stops

$\vec{F}_{net} - \vec{F}_B = 100 \times 24\hat{j}$ ....(3)

from these three get

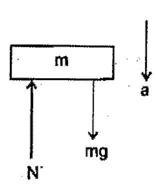
$\vec{F}_A + \vec{F}_B$ & solve

5.21 (B)

$N = m(g - a)$, $N < mg$ if a (↓)

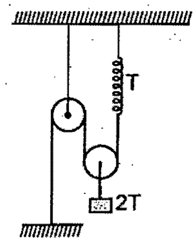
and $N > mg$ if a (↑)

Reading of spring balance is less than m if a (↓) and reading of spring balance is greater than m if a (↑)



5.22 (C)

Initially the block is at rest under action of force 2T upward and mg downwards. When the block is pulled downwards by x, the spring extends by 2x. Hence tension T increases by 2kx. Thus the net unbalanced force on block of mass m is 4kx.

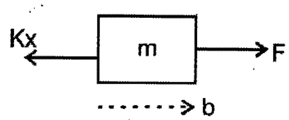


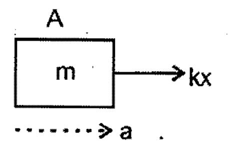
$\therefore$ acceleration of the block is = $\frac{4kx}{m}$

5.23 (C)

F - Kx = mb

and   kx = ma





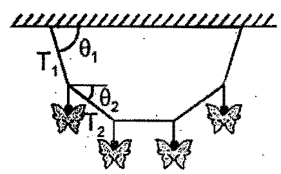
Hence

m (b – a) = F – 2kx

5.24 (C)

$T_1 \sin\theta_1 = 2\,mg \;\Rightarrow T_2 \sin\theta_2 = mg \Rightarrow T_1 \cos\theta_1 = T_2 \cos\theta_2$

$\Rightarrow\; 2mg \cot\theta_1 = mg \cot\theta_2 \Rightarrow \tan\theta_1 = 2 \tan\theta_2$



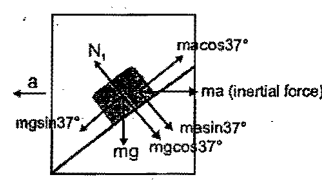
5.25 (AC)

(1) Balancing forces perpendicular to incline $N_1 = mg \cos 37° + ma \sin 37°$

$N_1 = \frac{4}{5} mg + \frac{3}{5} ma$

and along incline $mg \sin 37° - ma \cos 37° = mb_1$

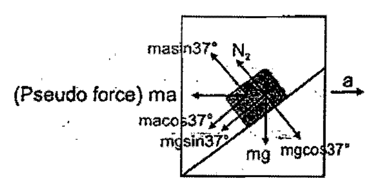
$b_1 = \frac{3}{5} g - \frac{4}{5} a$



(2) Similarly for this case get $N_2 = \frac{4}{5} mg - \frac{3}{5} ma$

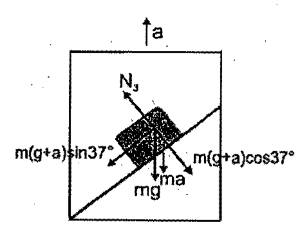
and $b_2 = \frac{3}{5} g + \frac{4}{5} a$

$N_2 = \frac{4}{5} mg - \frac{3}{5} ma$



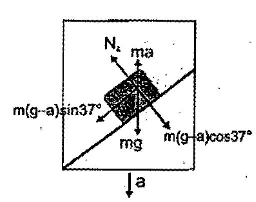
(3) Similarly for this case get $N_3 = \frac{4}{5} mg + \frac{4}{5} ma$

and $b_3 = \frac{3}{5} g + \frac{3}{5} a$



(4) Similarly for this case get $N_4 = \frac{4}{5} mg - \frac{4}{5} ma$

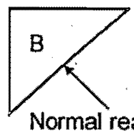
and $b_4 = \frac{3}{5} g - \frac{3}{5} a$



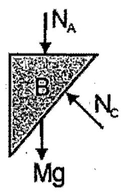
5.26 (ABCD)

There is no horizontal force on block A, therefore it does not move in x-direction, whereas there is net downward force (mg – N) is acting on it, making its acceleration along negative y-direction.

Block B moves downward as well as in negative x-direction. Downward acceleration of A and B will be equal due to constrain, thus w.r.t. B, A moves in positive x-direction.



Due to the component of normal exterted by C on B, it moves in negative x-direction.



The force acting vertically downward on block B are mg and N_A(normal reaction due to block A). Hence the component of net force on block B along the inclined surface of B is greater than mg sinθ. Therefore the acceleration of 'B' relative to ground directed along the inclined surface of 'C' is greater than g sin θ.

5.27 (AC)

For painter ;

$R + T - mg = ma$

$R + T = m(g + a)$ ....(1)

For the system ;

$2T - (m + M)g = (m + M)a$

$2T = (m + M)(g + a)$ ....(2)

where ; m = 100 kg

M = 50 kg

a = 5 m/sec^2

$\therefore \quad T = \dfrac{150 \times 15}{2} = 1125$ N

and ; R = 375 N

5.28 (BD)

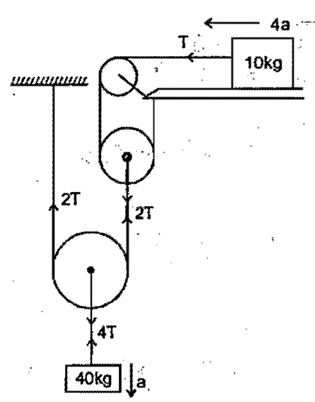
Applying NLM on 40 kg block

$400 - 4T = 40a$

For 10 kg block T = 10.4 a

Solving a = 2m/s^2

T = 80 N



5.29 (BC)

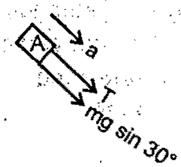
For equilibrium $N_A \cos 60° + N_B \cos 30° = Mg$

and $N_a \sin 60° = N_B \sin 30°$

On solving $N_B = \sqrt{3}\ N_A ; N_A = \dfrac{Mg}{2}$

5.30 (BD)

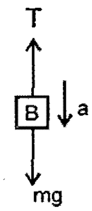
Let a be acceleration of system and T be tension in, the string.



F.B.D of block A

mg sin 30° + T = ma

$$\frac{mg}{2} + T = ma \quad ..... (i)$$



F.B.D of block B

$mg - T = ma \quad ..... (ii)$

Adding equation (i) & (ii); we get

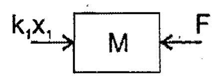
$$2ma = \frac{3mg}{2} \quad \Rightarrow \quad a = \frac{3}{4}g$$

from equation (i); $T = \frac{mg}{4}$

5.31 (ACD)

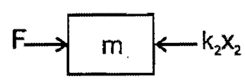
Let F be the force exerted by mass m on mass M.

FBD of mass M



$F = k_1x_1 = 2 \times 3$
$= 6$ N

FBD of mass m



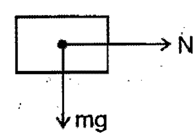
$k_2x_2 = F = 6$ N to the left

Hence the force exerted on block of mass m be the right spring (k_2x_2) is 6 N to the left. From FBD, the normal reaction (F) between blocks is 6 N.

As system is at rest, net force on block of mass m is zero.

5.32 (D)

The FBD of block A is



The force exerted by B on A is N (normal reaction).

The forces acting on A are N (horizontal) and mg (weight downwards).

Hence statementt I is false.

5.33 (D)

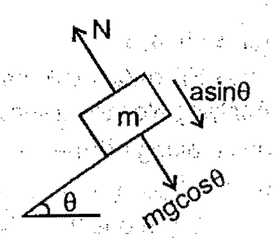
If the lift is retarding while it moves upward, the man shall feel lesser weight as compared to when lift was at rest. Hence statement1 is false and statement 2 is true.

5.34 (D)

Newton's third law of motion is valid in all reference frames. Hence statement-1 is incorrect.

5.35 (B)

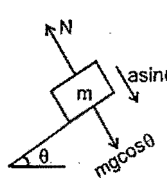
From wedge constraint, if acceleration of wedge is a then component of acceleration of block normal to inclined surface is a sinθ.



Applying Newton's law to block in direction normal to the inclined surface.

$mg\cos\theta - N = ma\sin\theta$

for N to be zero, $mg\cos\theta = ma\sin\theta$ or $a = g\cot\theta$



$mg\cos\theta - N = ma\sin\theta$

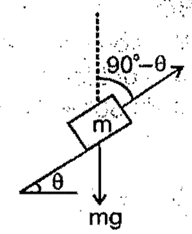
for N to be zero, $mg\cos\theta = ma\sin\theta$ or $a = g\cot\theta$

5.36 (B)

Since normal reaction is zero and the acceleration of block is horizontal, The net vertical force on block is zero.

Therefore T $\sin\theta = mg$,

or T = $mg \operatorname{cosec}\theta$



5.37 (A)

$F_{net} = Ma = Mg \cot\theta$

5.38 (C)

For equilibrium of block (A)

$F = N \sin\theta$

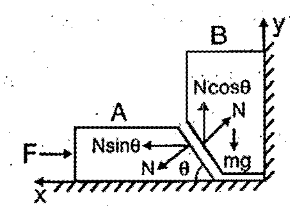
$N = F/\sin\theta$

To lift block B from ground

$N \cos\theta \geq mg$

$$\frac{F}{\sin\theta}\cos\theta \geq mg$$

$$F \geq mg\tan\theta = mg\left(\frac{3}{4}\right) \Rightarrow \text{So } F_{min} = \frac{3}{4}mg$$



5.39 (C)

If both the blocks are stationary,

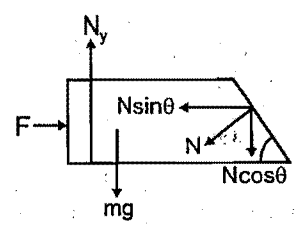
Balancing forces along x-direction

$F = N \sin\theta \Rightarrow N = F/\sin\theta$

Balancing forces along y-direction

$$N_y = mg + N\cos\theta = mg + \left(\frac{F}{\sin\theta}\right)\cos\theta = mg + F\cot\theta$$

$$N_{y1} = mg + \frac{4F}{3}$$

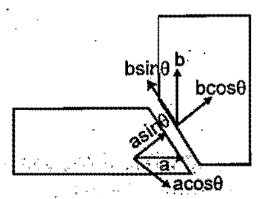


5.40 (A)

To keep regular contact

$a\sin\theta = b\cos\theta$

$$b = a\tan\theta = \frac{3}{4}a$$



5.41 (A)

5.42 (D)

5.43 (B)

(Q. 41 to 43)

The FBD of A and B are

Applying Newton's second law to block A and B along normal to inclined surface

$N_B - mg\cos 53° = ma\sin 53°$

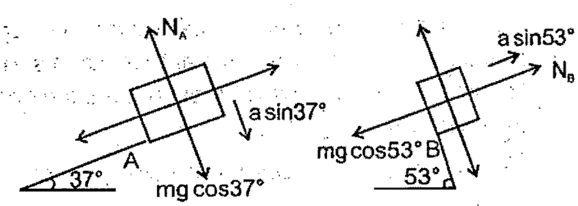
$mg\cos 37° - N_A = ma\sin 37°$

Solving $N_A = \frac{m}{5}(4g - 3a)$ and $N_B = \frac{m}{5}(3g + 4a)$

For N_A to be non zero

$4g - 3a \geq 0$

or $a \leq \frac{4g}{3}$

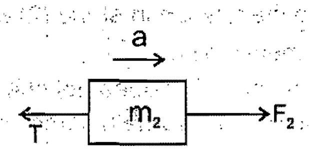


5.44. (A) t (B) r (C) t (D) r

Let a be acceleration of two block system towards right

$\therefore a = \dfrac{F_2 - F_1}{m_1 + m_2}$

The F.B.D. of m_2 is



$\therefore F_2 - T = m_2 a$

Solving $T = \dfrac{m_1 m_2}{m_1 + m_2}\left(\dfrac{F_2}{m_2} + \dfrac{F_1}{m_1}\right)$

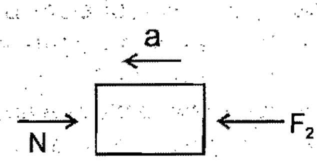
(B) Replace F_1 by $-F_1$ is result of A

$\therefore \quad T = \dfrac{m_1 m_2}{m_1 + m_2}\left(\dfrac{F_2}{m_2} - \dfrac{F_1}{m_1}\right)$

(C) Let a be acceleration of two block system towards left

$\therefore \quad a = \dfrac{F_2 - F_1}{m_1 + m_2}$

The FBD of m_2 is



$\therefore \quad F_2 - N_2 = m_2 a$

Solving $N = \dfrac{m_1 m_2}{m_1 + m_2}\left(\dfrac{F_1}{m_1} + \dfrac{F_2}{m_2}\right)$

(D) Replace F_1 by $-F_1$ in result of C

$N = \dfrac{m_1 m_2}{m_1 + m_2}\left(\dfrac{F_2}{m_2} - \dfrac{F_1}{m_1}\right)$

5.45 (A) t (B) q (C) s (D) q

(A), (B) ⇒ After spring 2 is cut, tension in string AB will not change.

$(T_{CD})_i = 4\text{ mg}$, $(T_{CD})_f = m_D g + m_D \cdot \dfrac{m_A + m_B - m_C - m_D}{m_A + m_B + m_C + m_D} \cdot g = 2\text{ mg}\left(1 + \dfrac{1}{5}\right) = 2.4\text{ mg}$

Hence T_{CD} decreases.

(C), (D) ⇒ After string between C and pulley is cut tension in string AB will become zero.

$(T_{CD})_i = (m_D + m_E)g = 4\text{ mg}$

Acceleration of C and D blocks is

$(m_C + m_D)\,g + m_E g = (m_C + m_D).a \Rightarrow a = \dfrac{6\text{mg}}{4\text{mg}} = \dfrac{3}{2}g$

$(T_{CD})_f + m_C g = m_C.a$

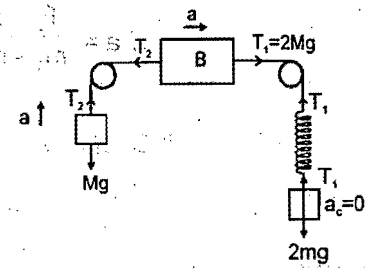
$(T_{CD})_f = 2\text{ m}\,\dfrac{3}{2}g - 2\text{mg} = \text{mg}$

The tension decreases

5.46 0

Before block A was released, the system was at rest, and all blocks were in equilibrium. hence, tension in both the strings is equal to 2Mg.

When block A is released, it will have an unbalanced force on it and hence the tension in string (2) will change to say T_2. Now the arrangement will be.



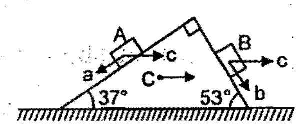
Since, tension in spring does not change instantaneously, hence, tension in string 1 will remain same i.e. 2Mg, Thus, Block C will remain at rest and $a_c = 0$.

Newton's law along the string (2), $2Mg - Mg = Ma + Ma \Rightarrow a = \frac{g}{2}$

Hence acceleration of A $= \frac{g}{2}\uparrow$, B $= \frac{g}{2}\rightarrow$, & C = 0

5.47 0

Let c be acceleration of wedge C.
a be acceleration of block A w.r.t. wedge C.
b be acceleration of block B w.r.t. wedge C.



Applying Newtons law in horizontal direction to system of A + B + C.

$mc + m(c - a\cos 37°) + m(c + b\cos 53°) = 0$ .... (1)

Applying Newton's law to block A and B along the incline gives.

In case of A : $mg\sin 37° = m(a - c\cos 37°)$ ....(2)

In case of B : $mg\sin 53° = m(b + c\cos 53°)$ ....(3)

solving (1), (2), & (3) we get c = 0   Ans. $a_c = 0$

5.48 $3 m/s^2$

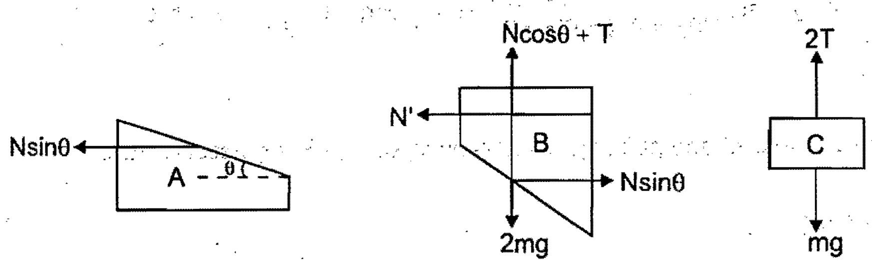
Let the acceleration of B downwards be $a_B = a$

From constraint ; acceleration of A and C are

$a_A = a\cot\theta = \frac{4a}{3}$ towards left

$a_C = \frac{a}{2}$ upwards

free body diagram of A, B and C are



$N\sin\theta = \frac{9m}{64}(a\cot\theta)$ ....(1)

$2mg - T - N\cos\theta = 2ma$ ....(2)

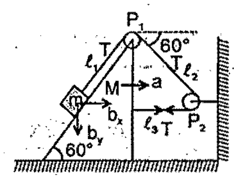
$2T - mg = m\frac{a}{2}$ ....(3)

solving we get

$a_c = \frac{a}{2} = 3 m/s^2$

Ans. $3 m/s^2$

5.49 from length constraint



$\ell_1 + \ell_2 + \ell_3 = C$

$\ell_1'' + \ell_2'' + \ell_3'' = C$

$-b_x \cos 60° + b_y \cos 30° + a \cos 60° - a \cos 60° - a = 0$

$2a = \sqrt{3}\, b_y - b_x$ ...(i)

from wedge constraint

$a \cos 30° = b_x \cos 30° + b_y \cos 60°$

$\sqrt{3}\, a = \sqrt{3}\, b_x + b_y$ ...(ii)

Applying Newton's law (wedge + block)
along horizontal direction

$T + T \cos 60° = Ma + mb_x$

$\frac{3T}{2} = 8a + 2b_x$ ...(iii)

Applying Newton's law on block along the incline plane

$T - mg \sin 60° = m (b_x \cos 60° - b_y \cos 30°)$

$T - \sqrt{3}\, g = b_x - \sqrt{3}\, b_y$ ....(iv)

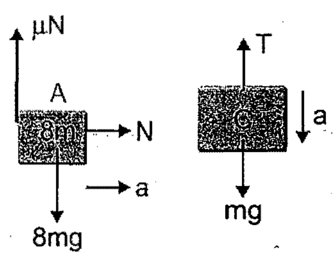
Solving equation (i); (ii); (iii) and (iv)

we get $a = \frac{3\sqrt{3}g}{23}$ m/s²

6. FRICTION

6.1 (D)

FBD of A and C



If acceleration of 'C' is a

For block 'A' $N = 8\,ma$ .... (1)

for block A to remain stationay with respect to block B,

$8\,mg = \mu N$ (Limiting condition) .... (2)

$8mg = \mu\, 8ma;$

$a = g/\mu$

and acceleration a can be written by the equation of system (A + B + C)

$m_c g = (10\,m + m_c) a$ .... (3)

$m_c g = (10\,m + m_c) g/\mu$

$\Rightarrow \quad m_c = \frac{10m}{\mu - 1}$ Ans.

6.2 (D)

Let the value of 'a' be increased from zero. As long as $a \le \mu g$, there shall be no relative motion between m_1 or m_2 and platform, that is, m_1 and m_2 shall move with acceleration a.

As $a > \mu g$ the acceleration of m_1 and m_2 shall become μg each.

Hence at all instants the velocity of m_1 and m_2 shall be same

∴ The spring shall always remain in natural length.

6.3 (C)

For the sliding not to occur when

$\tan\theta \le \mu$

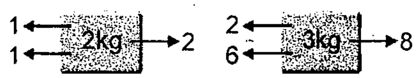
$$\tan\theta = \frac{dy}{dx} = \frac{2x}{a} = \frac{2\sqrt{2y}}{a} = 2\sqrt{\frac{y}{a}}$$

$$\therefore 2\sqrt{\frac{y}{a}} \le \mu \quad \text{or} \quad y \le \frac{a\mu^2}{4}$$

6.4 (C)

FBD

Net force without friction on system is '7N' in right side so first maximum friction will come on 3 kg block.

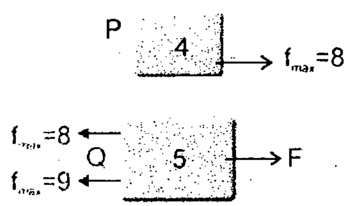


So f_1 = 1 N, f_2 = 6 N, T = 2N

6.5 (C)

So block 'Q' is moving due to force while block 'P' due to friction.

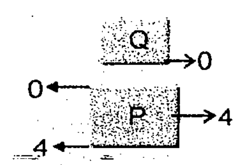
Friction direction on both P + Q blocks as shown.



First block 'Q' will move and P will move with 'Q' so by FBD taking 'P' and 'Q' as system

F – 9 = 0 $\Rightarrow$ F = 9 N

When applied force is 4 N then FBD



i.e. $a_P = a_q = 0$

4 kg block is moving due to friction and maximum friction force is 8 N.

So acceleration = $\frac{8}{4}$ = 2 m/s² = a_{max}.

Slipping will start at when Q has +ve acceleration equal to maximum acceleration of P i.e. 2 m/s².

F – 17 = 5 × 2 $\Rightarrow$ F = 27 N.

6.6 (C)

limiting condition for m to not slip in vertical downward direction, mg = μN

$$\Rightarrow \quad N = \frac{mg}{\mu} = \frac{100}{.5} = 200 \text{ Newton}$$

Same normal force would accelerate M, thus $a_M = \frac{200}{50}$ = 4 m/s²

Taking m + M as system

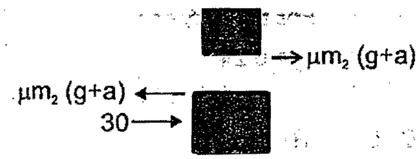
F = (m + M). 4 = 240 N

6.7 (C)

Let m_A and m_B be the mass of blocks A and B respectively.
As the force F increases from 0 to $\mu_s m_A g$, the frictional force f on block A is such that f = F. When F = $\mu_s m_A g$, the frictional force f attains maximum value f = $\mu_s$mg.
As F is further increased to $\mu_s(m_A+m_B)g$, the block A does not move. In this duration frictional force on block A remains constant at $\mu_s m_A g$.
As F further increased, system will start moving and kinetic friction ($\mu_k m_A g$) will start acting on A ($\mu_s > \mu_k$).
Hence C is correct choice.

6.8 (C)

FBD in reference frame of the lift



$$a_2 = \frac{1}{5}\left(g+\frac{g}{4}\right) = \frac{g}{4} = 2.5 \text{ m/s}^2$$

$$a_1 = \frac{30-\left[\mu m_2\left(g+\frac{g}{4}\right)\right]}{8} = \frac{30-\left[\frac{1}{5}\times 2\times\frac{50}{4}\right]}{8} = \frac{25}{8} \text{ m/s}^2.$$

6.9 (A)

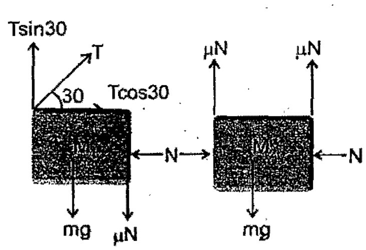
maximum friction = μmg = 0.6 × 10 ×1 = 6N
Psuedo force = ma = 5N

Applied the Pseudo force = $\frac{1\times 5}{1}$ = 5 m/s²

so required friction force is only 5N although maximum friction force available is 6 N.

6.10 (A)

Since block M_1 is attached to the string so it will remain stationary, M_2 has tendency to move downward.
So, friction on M_2 will act in upward direction.
N is the force by which the both block are pressed i.e. N = T cos30°.



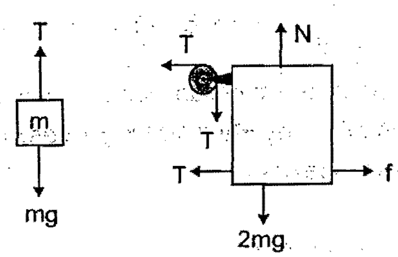
for block M_1
$M_1g + \mu(T\cos30°) - T\sin30° = M_1(0)$ .... (1)
(a = 0 for M_1 since attach to string)
for block M_2
$M_2g - (\mu N + \mu N) = M_2 a$ .... (2)

6.11 (A)

For chain to move with constant speed P needs to be equal to frictional force on the chain. As the length of chain on the rough surface increases. Hence the friction force $f_k = \mu_k N$ increases.

6.12 (C)

In equilibrium
T = mg
N = 3 mg
& f = 2T = 2mg
in limiting case f = f_{max}.
⇒ 2mg = μ N
⇒ 2mg = 3μ mg
⇒ μ = 2/3 Ans.



6.13 (D)

Limiting friction between A & B = 60 N

Limiting friction between B & C = 90 N

Limiting friction between C & ground = 50 N

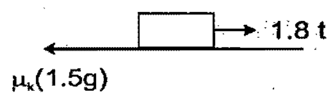
Since limiting friction is least between C and ground, slipping will occure at first between C and ground. This will occure when F = 50 N.

6.14 (B)

$1.8\,t - \mu_k\, 15 = 1.5\,(1.2\,t - 2.4)$

For t = 2.85 sec.

$\mu_k = 0.24$



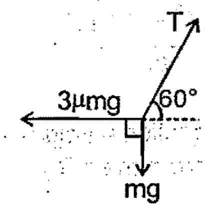
6.15 (B)

At the instant 3m is about to slip, tension in all the strings are as shown

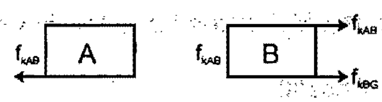
$\therefore \quad 3\,\mu mg = T\cos 60° \quad .... (1)$

and $mg = T\sin 60° \quad .... (2)$

$\therefore \quad \mu = \frac{1}{3\sqrt{3}}$

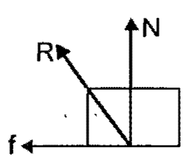


6.16 (A)



Direction of friction is such that it opposes the relative velocity.

6.17 (C)



Acceleration of train will be from right to left.

⇒ Pseudo force will act on the box from left to right therefore friction will act from right to left.

6.18 (BC)

For system remain in equilibrium, value of m can be decided in two limiting cases:

Case-I: m can take a maximum value such that 100kg block has tendency to move upward.

$mg = 100 \times g \times \sin 37° + \mu \times 100 \times g \times \cos 37°$

$m = 100 \times \frac{3}{5} + \frac{3}{10} \times 100 \times \frac{4}{5} = 60 + 24 = 84$

Case-II: m can take a minimum value such that 100kg block has tendency to move downward.

$100 \times g \times \sin 37° = mg + \mu \times 100 \times g \times \cos 37° \quad \Rightarrow m = 36$

so we got the range of m

$36 < m < 84$

In this range 37 and 83 lie.

6.19 (AD)

Where ϕ = angle of friction

θ = angle of inclined plane

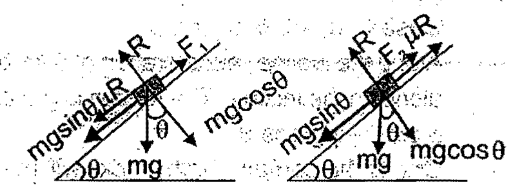
w = weight of the body

$F_1 = mg\sin\theta + \mu\, mg\cos\theta$.

$F_2 = mg\sin\theta - \mu\, mg\cos\theta$.

But $mg = w$

$\mu = \tan\phi$



$F_1 = w(\sin\theta + \frac{\sin\phi}{\cos\phi}\cos\theta) \Rightarrow F_1 = w\sin(\theta+\phi)\sec\phi$

$F_2 = w(\sin\theta - \frac{\sin\phi}{\cos\phi}\cos\theta) \Rightarrow F_2 = w\sin(\theta-\phi)\sec\phi$

Now $F_1 = 2F_2$

$mg\sin\theta + \mu\, mg\cos\theta = 2(mg\sin\theta - \mu\, mg\cos\theta)$

$\sin\theta + \mu\cos\theta = 2\sin\theta - 2\mu\cos\theta \Rightarrow 3\mu\cos\theta = \sin\theta$

$= \tan\theta = 3\mu$

$\tan\theta = 3\tan\phi$.

6.20 (BD)

Case I :

Since, no relative motion :

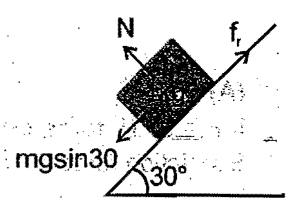
$a = \frac{F_1 - F_f}{5} = \frac{F_f}{3} \Rightarrow F_{1\,(max)} = \frac{8}{3}F_f$

Case II :

$a = \frac{F_f}{5} = \frac{F_2 - F_f}{3} \Rightarrow F_{2\,(max)} = \frac{8}{5}F_f$

Clearly ; $F_{1\,(max)} > F_{2\,(max)}$ and $\frac{F_{1(max)}}{F_{2(max)}} = \frac{5}{3}$

6.21 (AC)



Block is moving upwards due to friction

$f_r - mg\sin 30 = ma$

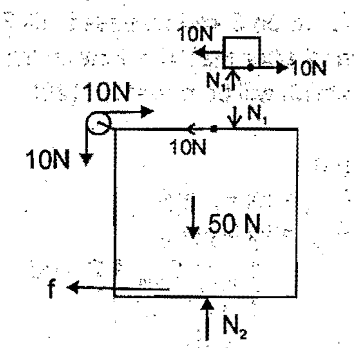
$\Rightarrow f_r - 1 \times 10 \times \frac{1}{2} = 1 \times 1 \Rightarrow f_r = 6$ N

Contact force is the resultant of N and f_r

$= \sqrt{N^2 + f_r^2} = \sqrt{(mg\cos 30)^2 + (6)^2} = 10.5$ N

6.22 (AD)

The frictional force on block A is



$\Rightarrow \mu N_1 = 10 \Rightarrow N_1 = \frac{10}{0.2} = 50$ N

The net force on block B in vertical direction is zero

$\therefore N_2 = 50 + N_1 + 10 = 110$ N

$\Rightarrow$ Normal reaction exerted by ground on block B is 110N.

The net force on block B in horizontal direction is zero

$\therefore f + 10 - 10 = 0$

$\Rightarrow$ frictional force exerted by ground on block B is zero

6.23 (AD)

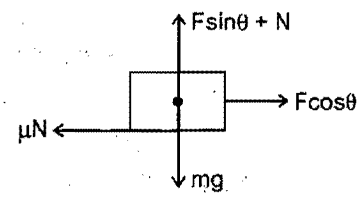
Case-I : $\mu_1 = 0.5$, $\mu_2 = 0.3$

Along the incline, acceleration of 5 kg block will be less than acceleration of 3 kg block provided they move alone on the incline. The reason is greater friction coefficient of 5 kg block, as acceleration along the incline is $g \sin\theta - \mu g \cos\theta$

One to the contain, both blocks will move together. In this case FBDs of both are shown.

6.24 (D)

The FBD of block is as shown.



Minimum force at an angle θ to pull the block horizontally,

$F\cos\theta = \mu N$ .... (1)

where $N = mg - F\sin\theta$ .... (2)

$$F = \frac{\mu mg}{\cos\theta + \mu\sin\theta}$$

which can be less than μmg

Putting $\mu = \frac{1}{3}$, $\theta = 37°$

$F_{min} = \mu mg$

6.25 (D)

Due to pseudo force, the person observes the block to move back. Also the accelerating person does not observe any relative motion between body and the rough surface.

6.26 (D)

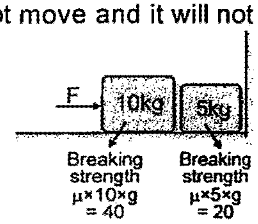
The block cannot move along x-axis by a force applied along x-axis.

For block to move along x-axis, the component of force along y-axis should be equal to mg sinθ, So that net force along y-axis is zero.

Hence statement-1 is false.

6.27 (D)

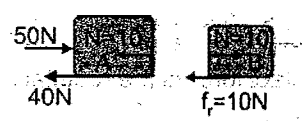
If F = 20 N, 10 kg block will not move and it will not press 5 kg block So N = 0.



6.28 (A)

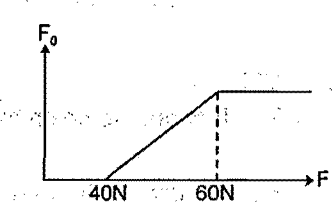
If F = 50 N, force on 5 kg block = 10 N

So friction force = 10 N



6.29 (B)

Until the 10 kg block is sticked with ground (... F = 40 N),
No force will be felt by 5 kg block. After F = **40 N**, the friction force on 5 kg increases, till F = 60 N,
and after that, the kinetic friction start acting on 5 kg block, which will be constant (20N)



6.30 (A)

$F_{max} = kx + \mu mg$

$F_{min.} = kx - \mu mg$

$\therefore \quad F_{max} - F_{min.} = 2\mu mg$

or $\quad 2 = 2\mu\, 10$

$\therefore \quad \mu = 0.1$

6.31 (A)

$F_{max} + F_{min.} = 2\,kx$ .... (1)

from graph $F_{max} + F_{min.} = 5$

and $x = 0.1$

Putting in equation (1)

$5 = 2\,k(0.1)$

$k = 25$ N/m.

6.32 (A)

When $x = 0.03$

$kx = 25 \times 0.03 = 0.75$ N, which is less than $\mu\, mg = 0.1 \times 10 = 1$ N

$\therefore$ The block will be at rest, without applying force F.

6.33 (A) p,q,r,s,t (B) p,q,r,s,t (C) p,q,r,s,t (D) p,q,r,s,t

(A) For $\mu > \tan\theta$, the magnitude of acceleration of both blocks is zero. Hence acceleration of both blocks is same.

For $\mu < \tan\theta$, the acceleration of both blocks is same and equal to $(g\sin\theta + g\cos\theta)$

Hence whatever be the value of μ, the acceleration of both blocks shall be same.

(B) For $\mu > \tan\theta$, both blocks are at rest and their binding with inclined surface is not broken. Hence the blocks cannot exert force on each other. Therefore normal reaction between both blocks is zero.

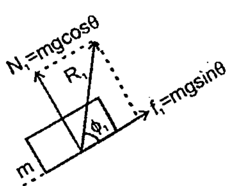
For $\mu < \tan\theta$, both blocks will move down the incline with same acceleration when they are not in contact. Hence they have no tendency to approach.

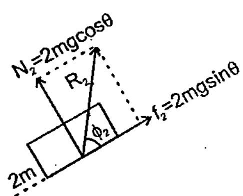
Hence when both blocks are in contact, they will not exert normal reaction no tendency to approach.

Hence whatever be the value of μ, normal reaction between both blocks is zero.

(C & D) For $\mu > \tan\theta$, both blocks are at rest.

The normal reaction (N), friction (f) and net reaction on each blocks by inclined surface are as shown.

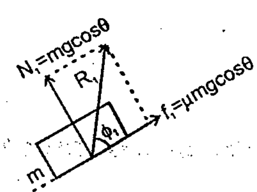


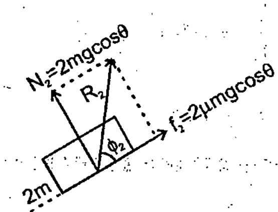


It is obvious $\phi_1 = \phi_2$ and $R_2 = 2R_1$.

For $\mu < \tan\theta$, both blocks move down the incline.

The normal reaction (N), friction (f) and net reaction on each blocks by inclined surface are as shown.





Again it can be seen that $\phi_1 = \phi_2$ and $R_2 = 2R_1$.

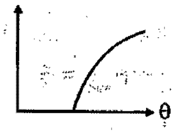
Hence whatever be the value of μ, $R_2 = 2R_1$ and $\phi_2 = \phi_1$.

6.34 (A) q , (B) s, (C) r, (D) p

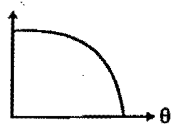
(i) Till $\theta = \tan^{-1}\mu$, T = 0

After $\theta = \tan^{-1}\mu$, $T = mg\sin\theta - \mu mg\cos\theta$

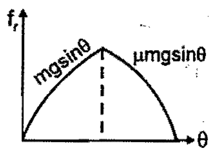
So curve will be



(ii) $N = mg\cos\theta$



(iii) Till $\theta = \tan^{-1}\mu$

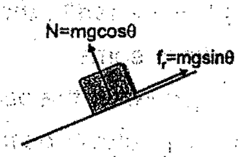


f_r will be static = mg sinθ

after $\theta = \tan^{-1}\mu$

f_r will be kinetic = $\mu\, mg\cos\theta$

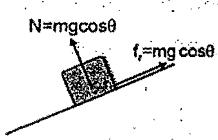
(iv) Net interaction force between the block and incline's



for $\theta = < \tan^{-1}\mu$

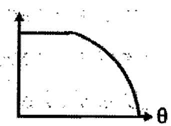
Net reation = $\sqrt{(mg\cos\theta)^2 + (mg\sin\theta)^2} = mg$

for $\theta > \tan^{-1}\mu$



Net reaction $\sqrt{(mg\cos\theta)^2 + (\mu g\cos\theta)^2} = \sqrt{1+\mu^2}\ \cos\theta$

So curve will be



6.35 10 sec

The block begins to slide if

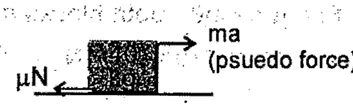
$F\cos 37° = \mu(mg - F\sin 37°)$

$5t[\cos 37° + \mu\sin 37°] = \mu mg$

$5t\left[\frac{4}{5} + \frac{3}{5}\right] = 70$ or $t = 10$ second

6.36 20 m

In the reference frame of the truck FBD of 40 kg block



Net force $\Rightarrow ma - \mu N$ $\Rightarrow 40 \times 2 - \frac{15}{100} \times 40 \times 10$

$ma_{block} \Rightarrow 80 - 60$ $\Rightarrow a_{block} = \frac{20}{40} = \frac{1}{2}$ m/s²

This acceleration of the block in reference frame of truck so time taken by box to fall down from truck

$S_{rel} = u_{rel}t + \frac{1}{2}a_{rel}t^2$ $\Rightarrow 5 = 0 + \frac{1}{2} \times \frac{1}{2} \times t^2 \Rightarrow t^2 = 20$

So distance moved by the truck

$\Rightarrow \frac{1}{2} \times a_{truck} \times t^2$ $\Rightarrow \frac{1}{2} \times 2 \times (20) = 20$ meter

6.37 3

As seen from an inside observer, the forces acting on the block are pseudoforce, frictional force and the applied force.

When the applied force is in the direction of pseudo force (in this case less force will be required to move the block)

$$10 + \text{pseudoforce} = \mu mg \quad ....(1)$$

When the applied force is opposite to the pseudoforce,

$$20 - \text{pseudo force} = \mu mg \quad ....(2)$$

Adding (1) & (2)

$$30 = 2\,\mu mg = 2\mu 50$$

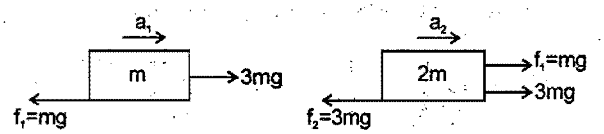
$\therefore \quad \mu = 0.3$

6.38 25

The F.B.D of both blocks is as shown.

$$a_1 = \frac{3mg - mg}{m} = 20 \text{ m/s}^2$$

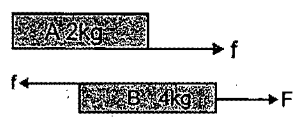
$$a_2 = \frac{4mg - 3mg}{2m} = 5 \text{ m/s}^2$$



$\therefore a_{pulley} = \frac{a_1 + a_2}{2} = \frac{25}{2} = \frac{X}{2}$. Hence X = 25

6.39 15

The F.B.D. of A and B are force of friction



For sliding to start between A and B, the frictional $f = \mu N = \frac{1}{4} \times 2 \times 10 = 5 \text{ N} = f_{max}$

Applying Newton's second law to system of A + B

$$F = (m_A + m_B)\, a = 6a \quad ....(1)$$

Applying Newton's second law to A

$$f = m_A\, a \Rightarrow a_{max} = \frac{f_{max}}{m_A} = \frac{5}{2} = 2.5 \text{ m/s}^2 \quad ....(2)$$

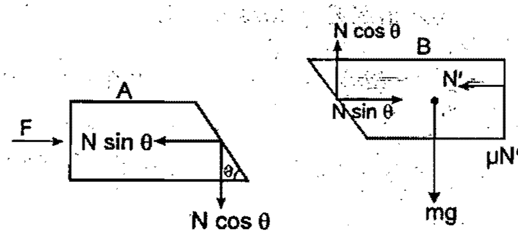
from (1) and (2) $F_{min} = (m_A + m_B)\, 2.5 \text{ m/s}^2 = 6 \times 2.5 = 15 \text{ N}$

6.40 3

The F.B.D. of A and B are

For A to be in equilibrium

$$F = N \sin\theta \quad ....(1)$$



For B to just lift off

$$N \cos\theta = mg + \mu_s N' \quad ....(2)$$

For horizontal equilibrium of B

$$N' = N \sin\theta \quad ....(3)$$

From (2) and (3)

$$N (\cos\theta - \mu_s \sin\theta) = mg$$

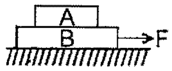
or $$N\left(\frac{4}{5} - \frac{2}{3} \times \frac{3}{5}\right) = mg$$

or $$N = \frac{5}{2}\, mg \quad ....(4)$$

From equation (1) $\quad F = N \times \frac{3}{5} \quad \therefore \quad F = \frac{3}{2}\, mg$

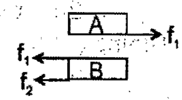
7. WORK, POWER & ENERGY

7.1 (A)



Consider the blocks shown in the figure to be moving together due to friction between them.

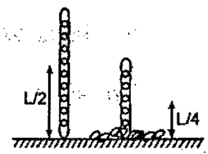
The free body diagrams of both the blocks are shown below.



Work done by static friction on A is positive and on B is negative.

7.2 (C)

The work done by man is negative of magnitude of decrease in potential energy of chain



$$\Delta U = mg\frac{L}{2} - \frac{m}{2}g\frac{L}{4} = 3\,mg\frac{L}{8}$$

$$\therefore W = -\frac{3mg\ell}{8}$$

7.3 (A)

From conservation of energy

$$\text{K.E.} + \text{P.E.} = E \quad \text{or} \quad \text{K.E.} = E - \frac{1}{2}kx^2$$

$$\therefore \quad \text{K.E. at } x = -\sqrt{\frac{2E}{k}} \text{ is } \Rightarrow E - \frac{1}{2}k\left(\frac{2E}{k}\right) = 0$$

$\therefore$ The speed of particle at $x = -\sqrt{\frac{2E}{k}}$ is zero.

7.4 (C)

If A moves down the incline by 1 metre, B shall move up by $\frac{1}{2}$ metre. If the speed of B is v then the speed of A will be 2v.

From conservation of energy:

Gain in K.E. = loss in P.E.

$$\frac{1}{2}m_A(2v)^2 + \frac{1}{2}m_B v^2 = m_A g \times \frac{3}{5} - m_B g \times \frac{1}{2}$$

Solving we get $v = \frac{1}{2}\sqrt{\frac{g}{3}}$ Ans.

7.5 (B)

Internal forces can not change acceleration of centre of mass. Thus internal forces have no effect on velocity of centre of mass.

The kinetic energy of system of two particles of mass m_1 and m_2 having velocities $\vec{v}_1$ and $\vec{v}_2$, in centre of mass frame is:

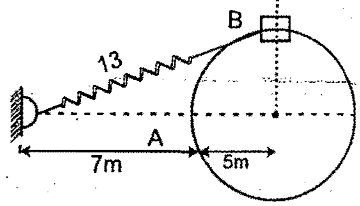
$$k = \frac{1}{2}m_1(\vec{v}_1 - \vec{v}_{cm})\cdot(\vec{v}_1 - \vec{v}_{cm}) + \frac{1}{2}m_2(\vec{v}_2 - \vec{v}_{cm})\cdot(\vec{v}_2 - \vec{v}_{cm}) \text{ है।}$$

Internal forces change velocities $\vec{v}_1$ and $\vec{v}_2$ and hence kinetic energies of constituent particles of the system. Thus internal forces change kinetic energy of the system in centre of mass frame.

$\therefore$ only (i) is correct.

7.6 (C)

Initial extension will be equal to 6 m.

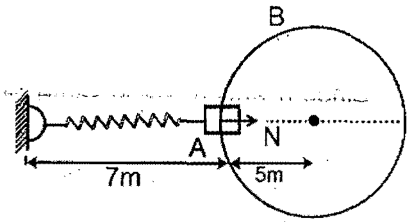


$\therefore$ Initial energy = $\frac{1}{2}(200)(6)^2 = 3600$ J.

Reaching A : $\frac{1}{2}mv^2 = 3600$ J

$\Rightarrow$ $mv^2 = 7200$ J

From F.B.D. at A :



$N = \frac{mv^2}{R} = \frac{7200}{5} = 1440$ N

7.7 (B)

From given graphs :

$a_x = \frac{3}{4}t$ and $a_y = -\left(\frac{3}{4}t+1\right)$ $\Rightarrow$ $v_x = \frac{3}{8}t^2 + C$

At t = 0 : $v_x = -3$ $\Rightarrow C = -3$

$\therefore$ $v_x = \frac{3}{8}t^2 - 3$

$\Rightarrow$ $dx = \left(\frac{3}{8}t^2 - 3\right)dt$ .... (1)

Similarly ; $dy = \left(-\frac{3}{8}t^2 - t + 4\right)dt$ .... (2)

As $dw = \vec{F}.\vec{ds} = \vec{F}.(dx\,\hat{i} + dy\,\hat{j})$

$\therefore$ $\int_0^W dw = \int_0^4 \left[\frac{3}{4}t\,\hat{i} - \left(\frac{3}{4}t+1\right)\hat{j}\right].\left[\left(\frac{3}{8}t^2-3\right)\hat{i} + \left(-\frac{3}{8}t^2 - t + 4\right)\hat{j}\right]dt$

$\therefore$ W = 10 J

Alternate Solution :

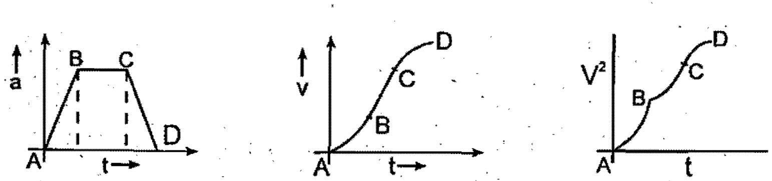
Area of the graph ;

$\int a_x\,dt = 6 = V_{(x)f} - (-3) \Rightarrow V_{(x)f} = 3.$

and $\int a_y\,dt = -10 = V_{(y)f} - (4) \Rightarrow V_{(y)f} = -6.$

Now work done = ΔKE = 10 J

7.8 (D)



The above graphs show v – t graph from a – t graph & Then v² – t graph, which are self explanatory.

7.9 (C)

$$\vec{f} = -\frac{\partial U}{\partial x}\hat{i} - \frac{\partial U}{\partial y}\hat{j} = -[6\hat{i}] + [8]\hat{j} = -6\hat{i} + 8\hat{j}$$

$\therefore \vec{a} = -3\hat{i} + 4\hat{j}$ has same direction as that of $\vec{u} = \frac{-3\hat{i}+4\hat{j}}{2} = \left(\frac{\vec{a}}{2}\right)$

$|\vec{a}| = 5$

$|\vec{u}| = 5/2$

Since $\vec{u}$ and $\vec{a}$ are in same direction, particle will move along a straight line

$\therefore S = \frac{5}{2} \times 2 + \frac{1}{2} \times 5 \times 2^2 = 5 + 10 = 15$ m. **15 m. Ans**

7.10 (D)

Statement I : Work done by gravity is same for motion from A to J and B to M for equal mass. So K.E. will be equal.

Statement II : Acceleration = g sin θ

$\sin\theta_A > \sin\theta_B$

$\frac{h}{\ell} > \frac{h}{2\ell}$

Statement III : $W_g + W_{ext} = 0$ (Because moved slowly)

$W_{ext} = -W_g$

from B to O : W_g is positive so $W_{ext} < 0$

7.11 (A)

Let at any time the speed of the block along the incline upwards be v.
Then from Newton's second law

$$\frac{P}{v} - mg\sin\theta - \mu mg\cos\theta = \frac{mdv}{dt}$$

the speed is maximum when $\frac{dv}{dt} = 0$

$$\therefore \quad v_{max} = \frac{P}{mg\sin\theta + \mu mg\cos\theta}$$

7.12 (D)

$x = x_1$ and $x = x_3$ are not equilibrium positions because $\frac{du}{dx} \neq 0$ at these points.
$x = x_2$ is unstable, as U is maximum at this point.

7.13 (C)

At equilibrium positon $x = \frac{mg}{k}$

$$U_{spring} = \frac{1}{2}kx^2 = \frac{1}{2}k\left(\frac{mg}{k}\right).x = \frac{mgx}{2} = \frac{1}{2}(\text{loss in G.P.E.}) \Rightarrow \quad G = 2S$$

7.14 (B)

$$dU = -\vec{F}.\vec{ds} = -\vec{F}.(dx\,\hat{i} + dy\,\hat{j})$$

Also by reverse method using $F_x = -\frac{\partial U}{\partial X}$ and $F_y = -\frac{\partial U}{\partial Y}$, only (B) option satisfies the criteria.

7.15 (A)

As long as the block of mass m remains stationary, the block of mass M released from rest comes down by $\frac{2Mg}{K}$ (before coming it rest momentanly again).

Thus the maximum extension in spring is

$$x = \frac{2Mg}{K} \quad \text{.................. (1)}$$

for block of mass m to just move up the incline

$$kx = mg\sin\theta + \mu mg\cos\theta \quad \text{.................. (2)}$$

$$2Mg = mg \times \frac{3}{5} + \frac{3}{4} mg \times \frac{4}{5} \quad \text{or} \quad M = \frac{3}{5} m \qquad \text{Ans.}$$

7.16 (B)

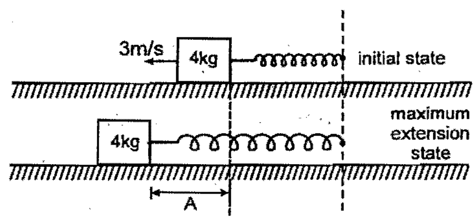
$$F_x = -\frac{\partial U}{\partial x} = \sin(x+y) \qquad F_y = -\frac{\partial U}{\partial y} = \sin(x+y)$$

$$F_x = \sin(x+y)\Big]_{(0,\pi/4)} = \frac{1}{\sqrt{2}} \qquad F_y = \sin(x+y)\Big]_{(0,\pi/4)} = \frac{1}{\sqrt{2}}$$

$$\therefore \quad F = \frac{1}{\sqrt{2}}\left[\hat{i} + \hat{j}\right]$$

7.17 (C)

In the frame (inertial w.r.t earth) of free end of spring, the initial velocity of block is 3 m/s to left and the spring unstretched .



Applying conservation of energy between initial and maximum extension state.

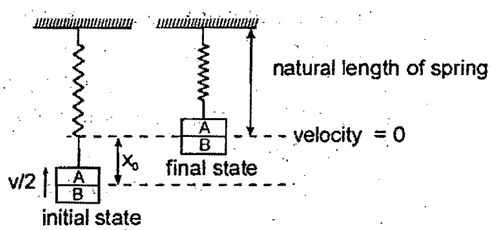
$$\frac{1}{2}mv^2 = \frac{1}{2}kA^2 \quad \text{or} \quad A = \sqrt{\frac{m}{k}}\, v = \sqrt{\frac{4}{10,000}} \times 3 = 6\text{cm}.$$

7.18 (B)

The initial extension in spring is $x_0 = \frac{mg}{k}$

Just after collision of B with A the speed of combined mass is $\frac{v}{2}$.

For the spring to just attain natural length the combined mass must rise up by $x_0 = \frac{mg}{k}$ (sec fig.) and comes to rest.



Applying conservation of energy between initial and final states

$$\frac{1}{2}\, 2m\left(\frac{v}{2}\right)^2 + \frac{1}{2}\, k\left(\frac{mg}{k}\right)^2 = 2mg\left(\frac{mg}{k}\right)$$

Solving we get $v = \sqrt{\frac{6mg^2}{k}}$

Alternative solution by SHM

$$\frac{v}{2} = \sqrt{\frac{k}{2m}}\sqrt{\left(\frac{2mg}{k}\right)^2 - \left(\frac{mg}{k}\right)^2}\; ; \quad v = \sqrt{\frac{2k}{m}}\sqrt{\frac{3m^2g^2}{k^2}} = \sqrt{\frac{6mg^2}{k}}\,]$$

7.19 (A)

Area under P–x graph = $\int p\,dx = \int\left(m\frac{dv}{dt}\right)v\,dx = \int_1^v mv^2\,dV = \left[\frac{mv^3}{3}\right]_1^v = \frac{10}{7\times3}(v^3-1)$

from graph ; area = $\frac{1}{2}(2+4)\times10 = 30$

ALITER :

from graph

$P = 0.2x + 2$

or $mv\frac{dv}{dx}v = 0.2x + 2$

or $mv^2\,dv = (0.2x+2)\,dx$

Now integrate both sides,

$\int_1^v mv^2dv = \int_1^{10}(0.2x+2)dx \Rightarrow v = 4$ m/s.

7.20 (A)

The speed of the water leaving the hose must be $\sqrt{2gh}$ if it is to reach a height h when directed vertically upward. If the diameter is d, the volume of water ejected at this speed is

$(A.v) = \frac{1}{4}\pi d^2\times\sqrt{2gh}\ \frac{m^3}{s} \quad\Rightarrow$ Mass ejected is $\frac{1}{4}\pi d^2\times\sqrt{2gh}\times\rho\ \frac{kg}{s}$.

The kinetic energy of this water leaving the hose = $\frac{1}{2}mv^2 = \frac{1}{8}\pi d^2\times(2gh)^{3/2}\times\rho = 21.5$ kW

7.21 (A)

From work energy theorem

for upward motion

$\frac{1}{2}m(16)^2 = mgh + W$ (work by air resistance)

for downward motion $\frac{1}{2}m(8)^2 = mgh - W$

$\frac{1}{2}[(16)^2 + (8)^2] = 2gh$ or $h = 8$ m

7.22 (A)

When 4 coaches (m each) are attached with engine (2m)

according to question P = K 6mgv ....(1)

(constant power), (K being proportionality constant)

Since resistive force is proportional to weight

Now if 12 coaches are attached

$P = K.14mg.v_1$ ....(2)

Since engine power is constant

So by equation (1) and (2)

$6Kmgv = 14Kmgv_1 \Rightarrow v_1 = \frac{6}{14}\times v = \frac{6}{14}\times20 = \frac{6\times10}{7} = \frac{60}{7} = v_1 = 8.5$ m/sec

Similarly for 6 coaches $\Rightarrow$ $K6mgv = K8mgv_2$

$\Rightarrow v_2 = \frac{6}{8}\times20 = \frac{3}{4}\times20 = 15$ m/sec

7.23 (D)

Increase in KE = work done

$$\frac{1}{2} m v_2^2 - \frac{1}{2} m \times \left(\frac{2F_0x_0}{m}\right) = \frac{1}{2}(2F_0 + F_0)\,3x_0 \Rightarrow v_2 = \sqrt{\frac{11F_0x_0}{m}}$$

7.24 (C)

Let m be minimum mass of ball.

Let mass A moves downwards by x.

From conservation of energy,

$$mgx = \frac{1}{2}kx^2 \Rightarrow x = \left(\frac{2mg}{k}\right)$$

For mass M to leave contact with ground,

$$kx = Mg \Rightarrow K\left(\frac{2mg}{k}\right) = Mg \Rightarrow m = \frac{M}{2}.$$

7.25 (B)

$$W_{spring} + W_{100\,N} = \Delta k \text{ (on A)}$$

$$W_{spring} + (100)\left(\frac{10}{100}\right) = \frac{1}{2}(2)(2)^2$$

$$W_{spring} = 4 - 10 = -6\text{ J}$$

7.26 (ABC)

Since ;$W = \int \vec{F}.\vec{dr}$

Clearly for forces (A) and (B) the integration do not require any information of the path taken.

For (C) : $$W_c = \int \frac{3(x\hat{i} + y\hat{j})}{(x^2+y^2)^{3/2}}.(dx\hat{i} + dy\hat{j}) = 3\int \frac{xdx + y\,dy}{(x^2+y^2)^{3/2}}$$

Taking : $x^2 + y^2 = t$

$2xdx + 2y\,dy = dt$

$$\Rightarrow xdx + ydy = \frac{dt}{2} \Rightarrow W_c = 3\int \frac{dt/2}{t^{3/2}} = \frac{3}{2}\int \frac{dt}{t^{3/2}}$$

which is solvable.

Hence (A), (B) and (C) are conservative forces.

But (D) requires some more information on path. Hence non-conservative.

7.27 (ABCD)

Free body diagram of block is as shown in figure.

From work-energy theorem

$W_{net} = \Delta KE$

or $(40 - 20)s = 40$

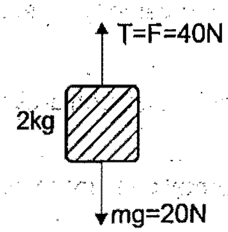
$\therefore$ $s = 2m$

Work done by gravity is

$-20 \times 2 = -40$ J

and work done by tension is

$40 \times 2 = 80$ J



7.28 (AB)

If the springs are compressed to same amount :

$W_A = \frac{1}{2} K_A x^2$; $W_B = \frac{1}{2} K_B x^2$ $\because$ $K_A > K_B \Rightarrow W_A > W_B$

If the springs are compressed by same force.

$F = K_A x_A = K_B x_B$; $x_A = \frac{F}{K_A}$; $x_B = \frac{F}{K_B}$; $\frac{W_A}{W_B} = \frac{\frac{1}{2}K_A.\frac{F^2}{K_A^2}}{\frac{1}{2}K_B\frac{F^2}{K_B^2}} = \frac{K_B}{K_A}$

Hence , $W_A < W_B$

7.29 (BC)

(A) If velocity and acceleration are not in same direction, work done by force perpendicular to acceleration will not be zero.

(B) If the object is at rest no force can do work.

(C) If force is perpendicular to velocity work done will be zero.

(D) If the point on the body has velocity component in direction of application of force work done will be non-zero.

7.30 (ABC)

$U = 3x + 4y$

$a_y = \frac{F_y}{m} = \frac{-(\partial U/\partial x)}{m} = -3$

$a_x = \frac{F_y}{m} = \frac{-(\partial U/\partial y)}{m} = -4$ $\Rightarrow$ $|\vec{a}| = 5\ m/s^2$

Let at time 't' particle crosses y-axis

then $-6 = \frac{1}{2}(-3)t^2$ $\Rightarrow$ $t = 2$ sec.

Along y-direction :

$\Delta y = \frac{1}{2}(-4)(2)^2 = -8$

$\Rightarrow$ particle crosses y-axis at $y = -4$

At (6, 4) : U = 34 & KE = 0

At (0, – 4) : U = – 16 $\Rightarrow$ KE = 50

or $\frac{1}{2}mv^2 = 50$ $\Rightarrow$ **v = 10 m/s** while crossing y-axis

7.31 (D)

The maximum extension is non-zero, while the spring never undergoes compression.
Hence statement-1 is false.

7.32 (D)

When frictional force is opposite to velocity, kinetic energy will decrease.

7.33 (B)

Linear momentum is conserved only in horizontal direction.

7.34 (A)

Net F_{ext} on system is zero in horizontal direction therefore linear momentum is conserved only in horizontal direction

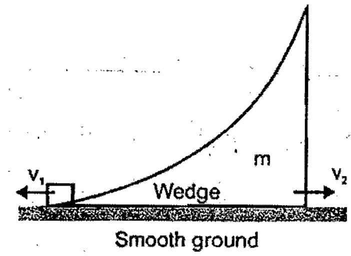
7.35 (A)

$mv_1 = mv_2$ ....(i)

$\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = mgh$ ....(ii)

From (i) & (ii),

$v_2 = 10ms^{-1}$.



7.36 (C)

Applying W-E theorem on the block for any compression x :

$W_{ext} + W_g + W_{spring} = \Delta KE \Rightarrow Fx + 0 - \frac{1}{2}Kx^2 = \frac{1}{2}mv^2$.

$\Rightarrow$ KE vs x is inverted parabola.

7.37 (A)

$W_{ext} = F . x \Rightarrow$ linear variation

7.38 (C)

From beginning to end of motion : $\Delta KE = 0 \Rightarrow x = 2F/K$. (from W-E theorem)

$\therefore$ first half corresponds to $0 \le x \le (F/K)$. $\Rightarrow x = 2F/K$.

$\therefore 0 \le x \le (F/K)$

7.39 (A) p ; (B) p ; (C) s ; (D) q

7.40 (A) s (B) q, t, (C) r, t (D) p, t

Point J $\longrightarrow$ No equilibrium

K $\longrightarrow$ Unstable equilibrium

L $\longrightarrow$ Stable equilibrium

M $\longrightarrow$ Neutral equilibrium

7.41 90

Change in velocity = $\frac{\text{area under F} - \text{T graph}}{\text{mass}} = \frac{40 + (-10)}{5} = 6 \text{ m/s} \Rightarrow W_F = \text{DK.E.} = \frac{1}{2}(5)6^2 = 90 \text{ J}$

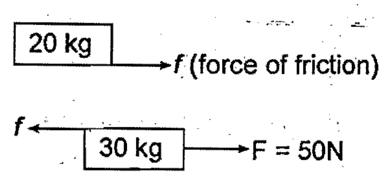
7.42 40

7.43 40

Solutions. (7.42 & 7.43)

Assume 20 kg and 30 kg block move together

$\therefore \quad a = \frac{50}{50} = 1 \text{ m/s}^2$



$\therefore$ frictional force on 20 kg block is

$f = 20 \times 1 = 20$ N

The maximum value of frictional force is $f_{max} = \frac{1}{2} \times 200 = 100$ N

Hence no slipping is occurring.

$\therefore$ The value of frictional force is f = 20 N.

Distance travelled in t = 2 seconds.

$S = \frac{1}{2} \times 1 \times 4 = 2m$.

Work done by frictional force on upper block is

$W_{fri} = 20 \times 2 = 40$ J

Work done by frictional force on lower block is $= -20 \times 2 = -40$ J.

7.44 10

For the block of mass m_2, not to move, the maximum compression in the spring x_o should be such that

$kx_o = \mu m_2 g$ .... (1)

Applying work energy theorem to block of mass m_1 we get

$$\frac{1}{2}m_1 u^2 = \frac{1}{2}k x_o^2 + \mu m_1 g x_o \quad .... (2)$$

From equation (1) and (2) we get

$$\frac{1}{2}m_1 u^2 = \frac{1}{2}\frac{\mu^2 m_2^2 g^2}{K} + \frac{\mu^2 m_1 m_2 g^2}{K}$$

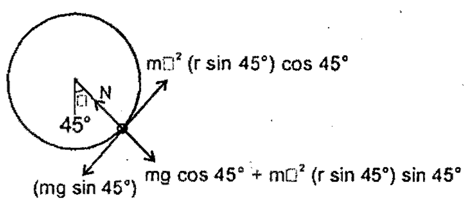
putting the appropriate value we get u=10m/s.

8. CIRCULAR MOTION

8.1 (D)

The maximum angular speed of the hoop corresponds to the situation when the bead is just about to slide upwards.

The free body diagram of the bead is



For the bead not to slide upwards.

$m\omega^2 (r \sin 45°) \cos 45° - mg \sin 45° < \mu N$ .... (1)

where $N = mg \cos 45° + m\omega^2 (r \sin 45°) \sin 45°$ .... (2)

From 1 and 2 we get.

$\omega = \sqrt{30\sqrt{2}}$ rad / s.

8.2 (C)

Let v be the speed of particle at B, just when it is about to loose contact.

From application of Newton's second law to the particle normal to the spherical surface.

$\frac{mv^2}{r} = mg \sin \beta$ .... (1)

Applying conservation of energy as the block moves from A to B..

$\frac{1}{2} mv^2 = mg (r \cos \alpha - r \sin \beta)$ .... (2)

Solving 1 and 2 we get

$3 \sin \beta = 2 \cos \alpha$

8.3 (A)

As the mass is at the verge of slipping

$\therefore$ $mg \sin 37 - \mu\, mg \cos 37 = m\omega^2 r$

$6 - 8\mu = 4.5$

$\therefore$ $\mu = \frac{3}{16}$

8.4 (B)

As when they collide $vt+\frac{1}{2}\left(\frac{72v^2}{25\pi R}\right)t^2-\pi R=vt$

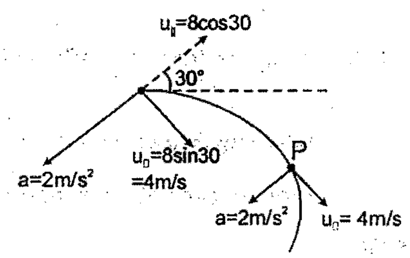
$\therefore t=\frac{5\pi R}{6v}$

Now angle covered by A = $\pi+\frac{vt}{R}$

Put t $\therefore$ angle covered by A = $\frac{11\pi}{6}$

8.5 (C)

The acceleration vector shall change the component of velocity $u_{\parallel}$ along the acceleration vector.



$r=\frac{v^2}{a_n}$

Radius of curvature r_{min} means v is minimum and a_n is maximum. This is at point P when component of velocity parallel to acceleration vector becomes zero, that is $u_{\parallel}=0$.

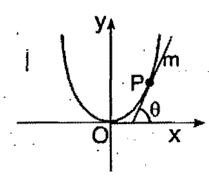
$\therefore R=\frac{u_{\perp}^2}{a}=\frac{4^2}{2}=8$ meters.

8.6 (C)

$x^2 = 4ay$

Differentiating w.r.t. y, we get

$\frac{dy}{dx}=\frac{x}{2a}$



$\therefore$ At (2a, a), $\frac{dy}{dx}=1$ $\Rightarrow$ hence $\theta = 45°$

the component of weight along tangential direction is mg sin θ.

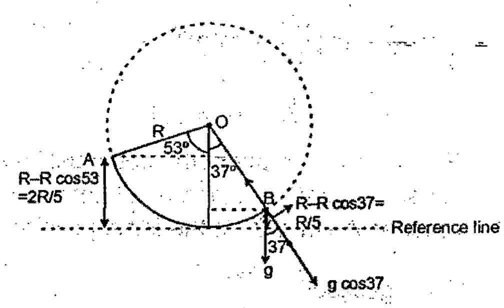
hence tangential acceleration is $g\sin\theta=\frac{g}{\sqrt{2}}$

8.7 (D)

The nature of the motion can be determined only if we know velocity and acceleration as function of time. Here acceleration at an instant is given and not known at other times so D is the correct option

8.8 (C)

By energy conservation between A & B



$\Rightarrow \quad Mg\frac{2R}{5}+0=\frac{MgR}{5}+\frac{1}{2}Mv^2 \Rightarrow v=\sqrt{\frac{2gR}{5}}$

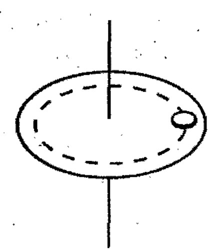
Now, radius of curvature $r=\frac{v_{\perp}^2}{a_r}=\frac{2gR/5}{g\cos 37}=\frac{R}{2}$

8.9 (D)

The friction force on coin just before coin is to slip will be : $f=\mu_s mg$

Normal reaction on the coin ; N = mg

The resultant reaction by disk to the coin is

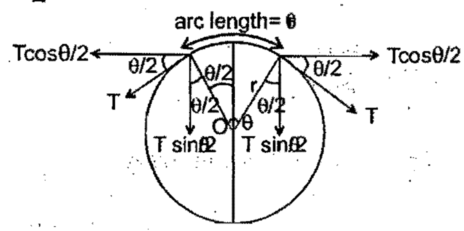


$=\sqrt{N^2+f^2}=\sqrt{(mg)^2+(\mu_s mg)^2}=mg\sqrt{1+\mu_s^2}$

$=40\times10^{-3}\times10\times\sqrt{1+\frac{9}{16}}=0.5$ N

8.10 (D)

As $2T \sin\frac{\theta}{2} = dm\,\omega^2 r$ (for small angle $\sin\frac{\theta}{2} \rightarrow \frac{\theta}{2}$)



but $dm = \frac{m}{\ell}\theta r$

As $\ell = 2\pi r$ $\therefore T = m\omega^2 r/2\pi$

Put $m = 2\pi$ kg $\omega = 10\pi$ radian/s

and $r = 0.25$ m

$\therefore$ $T = 250$ N

8.11 (A)

when he applies brakes $s_1 = \frac{v^2}{2a}$

if μ is the friction coefficient then $a = \mu g$ $\therefore s_1 = \frac{v^2}{2\mu g}$

when he takes turn $\frac{mv^2}{r} = \mu mg$ $r = \frac{v^2}{\mu g}$

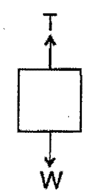
then we can see $r > s_1$ hence driver can hit the wall when he takes turn due to insufficient radius of curvature.

8.12 (A)

As tengential acceleration $a = dv/dt = \omega dr/dt$

but $\omega = 4\pi$ and $dr/dt = 1.5$ (reel is turned uniformly at the rate of 2 r.p.s.)

$\therefore$ $a = 6\pi$, Now by the F.B.D. of the mass.



$$T - W = \frac{W}{g}a$$

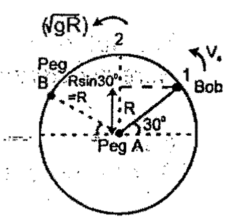
$\therefore$ $T = W(1 + a/g)$ put $a = 6\pi$

$\therefore$ $T = 1.019$ W

8.13 (C)

For anti-clockwise motion, speed at the highest point should be $\sqrt{gR}$.

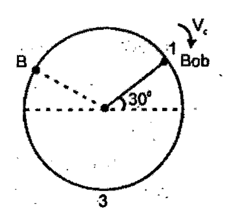
Conserving energy at (1) & (2) :



$$\frac{1}{2}mv_a^2 = mg\frac{R}{2} + \frac{1}{2}m(gR)$$

$\Rightarrow$ $v_a^2 = gR + gR = 2gR$ $\Rightarrow$ $v_a = \sqrt{2gR}$

For clock-wise motion, the bob must have atleast that much speed initially, so that the string must not become loose any where until it reaches the peg B. At the initial position :



$T + mg\cos 60^\circ = \frac{mv_c^2}{R}$;

v_c being the initial speed in clockwise direction.

For $v_{C\,min}$: Put $T = 0$;

$\Rightarrow$ $v_C = \sqrt{\frac{gR}{2}}$ $\Rightarrow$ $v_C/v_a = \frac{\sqrt{\frac{gR}{2}}}{\sqrt{2gR}} = \frac{1}{2}$

$\Rightarrow$ $v_C : v_a = 1 : 2$ Ans.

8.14 (D)

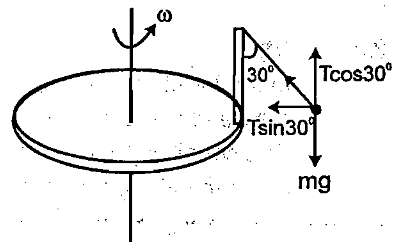
The bob of the pendulum moves in a circle of radius $(R + R\sin 30^0) = \frac{3R}{2}$

Force equations : $T\sin 30^0 = m\left(\frac{3R}{2}\right)\omega^2$

$T\cos 30^0 = mg$

$\Rightarrow \quad \tan 30^0 = \frac{3}{2}\frac{\omega^2 R}{g} = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad \omega = \sqrt{\frac{2g}{3\sqrt{3}R}}$ **Ans.**



8.15 (C)

$v_{min} = \sqrt{5gR} = \sqrt{5\times 10\times 2} = 10$ m/s

8.16 (A)

$T\cos\theta + N = mg$ ...(1)

and $T\sin\theta = m\,\omega^2 r$ ...(2)

but $T = Kx$

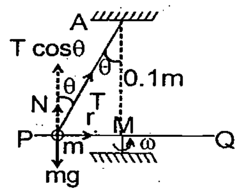
$T = 1.47 \times 10^2\,(0.1\sec\theta - 0.1)$

$(K = 1.47 \times 10^2$ N/m$)$

Also $r = 0.1\tan\theta$

put T, r, m & ω in equation (2)

we have $\cos\theta = 3/5$ and $T = 9.8$ N

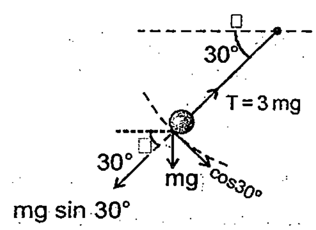


8.17 (C)

$T - mg\sin\theta = \frac{mv^2}{R}$

$\Rightarrow \quad 3\,mg - mg\sin 30° = \frac{m.(u_0^2 + 2g\ell\sin 30°)}{\ell}$

$\therefore \quad u_0 = \sqrt{3g/2}$



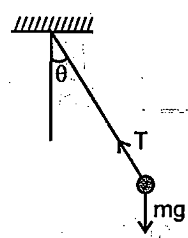
8.18 (B)

When the acceleration of bob is horizontal, net vertical force on the bob will be zero.

$T\cos\theta - mg = 0$

The tangential force at that instant is

$= mg\sin\theta = mg\sqrt{1-\cos^2\theta} = \frac{mg}{T}\sqrt{T^2-(mg)^2}$



8.19 (B)

From length constraint on AB

$a\cos 45° = b\cos 45°$

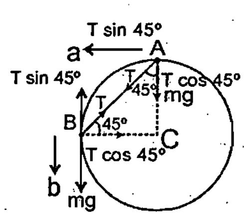
$a = b$

$T\sin 45° = m(a)$ $\qquad mg - T\sin 45° = mb$

$mg - ma = ma$

$2ma = mg$ $\qquad a = \frac{g}{2}$

$\frac{T}{\sqrt{2}} = \frac{mg}{2}$ $\qquad T = \frac{mg}{\sqrt{2}}$



8.20 (C)

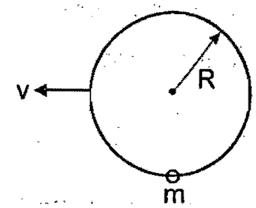
$V = \sqrt{gR\tan\theta} \Rightarrow (20)^2 = 10 \times 100 \times \tan\theta \Rightarrow \tan\theta = \frac{4}{10} = \frac{2}{5} \Rightarrow \theta = \tan^{-1}(2/5)$

8.21 (B)

In the frame of ring (inertial w.r.t. earth), the initial velocity of the bead is v at the lowest position.
The condition for bead to complete the vertical circle is, its speed at top position

$v_{top} \geq 0$

From conservation of energy



$\frac{1}{2} m\ v_{top}^2 + mg\,(2R) = \frac{1}{2} mv^2$ or $v = \sqrt{4gR}$

8.22 (A)

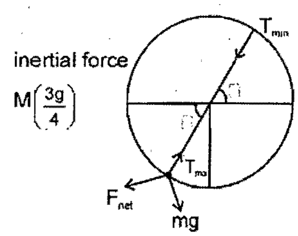
$|\Delta V| = \sqrt{v^2 + v^2 - 2v^2\cos 60^\circ}$
$= v$

$a_{av} = \frac{|\Delta \vec{v}|}{\Delta t} = \frac{v}{t} = \frac{3v^2}{\pi R} \Rightarrow a_i = \frac{v^2}{R}\ ;\ \frac{a_i}{a_{av}} = \frac{v^2\pi R}{R \times 3v^2} = \frac{\pi}{3}$

8.23 (A)

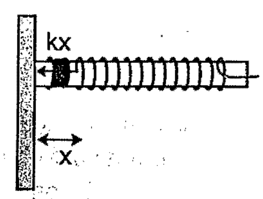
$\tan\theta = \frac{g}{c} = \theta = 53^\circ$

F_{net} is shown in the figure. So, tension will be max. at point A and will be min. at point B



8.24 (B)

For the ring to move in a circle at constant speed the net force on it should be zero. Here spring force will provide the necessary centripetal force.

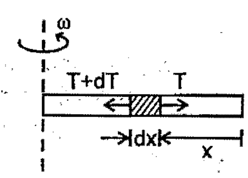


$\therefore \quad kx = mx\omega^2$

$\Rightarrow \quad \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{300}{3}} = 10$ rad/sec. Ans.

8.25 (B)

$dT = dm(\ell - x)\omega^2 \qquad dT = \frac{m}{\ell}.dx(\ell - x)\omega^2$



$\Rightarrow \quad \int_0^T dT = -\int_0^{\ell/2} \frac{m\omega^2}{\ell}(\ell - x)\,dx$

$= \frac{m\omega^2}{\ell}\left[\ell x - \frac{x^2}{2}\right]_0^{\ell/2} = \frac{m\omega^2}{\ell}\left[\frac{\ell^2}{2} - \frac{\ell^2}{8}\right]$

$\therefore$ Tension at mid point is :

$T = \frac{3}{8} m\ell\omega^2 \Rightarrow$ stress $= \frac{3m\ell\omega^2}{8A} \Rightarrow$ strain $= \frac{3m\ell\omega^2}{8AY}$

8.26 (A)

At A $N_A - mg = \frac{mV^2}{R_A}$ $\Rightarrow$ $N_A = mg + \frac{mV^2}{R_A}$

and At B $N_B = mg - \frac{mV^2}{R_B}$ and At C $N_C = mg + \frac{mV^2}{R_C}$

As by energy conservation

$R_A < R_C$

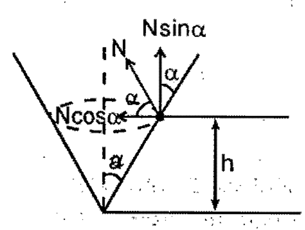
$\therefore$ N_A is greatest among all.

8.27 (A)

As $N \sin \alpha = mg$

$N \cos \alpha = m\omega^2 r$

$\tan \alpha = \frac{g}{\omega^2 r}$ $\therefore T^2 \propto \tan \alpha$



$\therefore$ when α increases T also increases

Also $T^2 \propto r \tan \alpha$

but $r = h \tan \alpha$

$\therefore$ $T^2 \propto h \tan^2 \alpha$

for constant α

$T^2 \propto h$

Thus when h increases T also increases

8.28 (ABCD)

Let N be the normal reaction (Reading of the weighing machine)

at A $\Rightarrow$ $N_A - mg = \frac{mv^2}{r}$

Put v $\therefore N_A - mg = mg$ $\Rightarrow$ $N_A = 2mg = 2W$

Also, at E, $N_E + mg = \frac{mv^2}{r} = mg$

$\therefore N_E = 0$ hence $N_A > N_E$ by 2W

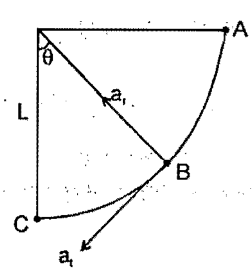
Now at G, $N_G = mg = W = N_C$

Also $\frac{N_E}{N_A} = 0$ and $\frac{N_A}{N_C} = 2$

8.29 (ABC)

Between A and B $mgL \cos\theta = \frac{1}{2}mv_B^2$

$\therefore$ $v_B^2 = 2gL \cos\theta$



Now $a_r = \frac{v_B^2}{L} = 2g \cos\theta$

and $a_t = g \sin\theta$

$\therefore$ $a = \sqrt{a_t^2 + a_r^2} = g\sqrt{1 + 3\cos^2\theta}$

Now, at B $T_B - mg \cos\theta = \frac{mv_B^2}{L}$

Put V_B $\Rightarrow$ $T_B = 3\, mg \cos\theta$

When total acceleration vector directed horizontally

$\tan(90 - \theta) = \frac{a_t}{a_r} = \frac{g\sin\theta}{2g\cos\theta} = \frac{1}{2}\tan\theta$

On solving $\theta = \cos^{-1} 1/\sqrt{3}$

8.30 (AD)

For case : $\omega_1 = \frac{5\pi}{6}$ rad/sec. $\Rightarrow$ $\omega_{A/T} = \frac{5\pi}{6}$ rad/sec.

$\omega_{B/G} = \frac{v}{R} = \frac{3.14}{3} = \frac{\pi}{3}$ rad/sec. $\Rightarrow$ $\omega_{T/G} = -\frac{\pi}{6}$ rad/sec (in opposite direction)

$\omega_{A/G} = \omega_{A/T} + \omega_{T/G} = \frac{5\pi}{6} + \left(-\frac{\pi}{6}\right) = \frac{4\pi}{6} = \frac{2\pi}{3}$ rad/s.

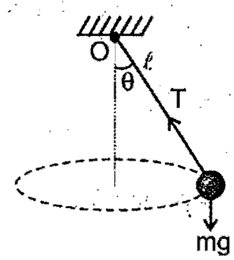
$\omega_{A/B} = \omega_A - \omega_B = \frac{2\pi}{3} - \frac{\pi}{3} = \frac{\pi}{3}$ rad/sec.

and $\theta_{A/B} = 30° = \frac{\pi}{6}$ rad/sec.

Using ; $\theta_{rel} = \omega_{i\,(rel)}\, t + \frac{1}{2}\alpha_{rel}\, t^2$ $\Rightarrow$ $\frac{\pi}{6} = \frac{\pi}{3}t + 0$ $\Rightarrow$ t = 0.5 sec. **Ans.**

8.31 (A)

For conical pendulum of length ℓ, mass m moving along horizontal circle as shown



$T\cos\theta = mg$ .... (1)
$T\sin\theta = m\omega^2\ell\sin\theta$ .... (2)

From equation 1 and equation 2, $\ell\cos\theta = \frac{g}{\omega^2}$

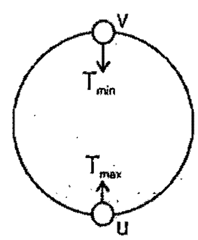
$\ell\cos\theta$ is the vertical distance of sphere below O point of suspension. Hence if ω of both pendulums are same, they shall move in same horizontal plane.
Hence statement-2 is correct explanation of statement-1.

8.32 (A)

(Moderate) Let the minimum and maximum tensions be T_{max} and T_{min} and the minimum and maximum speed be u and v.

$\therefore T_{max} = \frac{mu^2}{R} + mg$

$T_{min} = \frac{mv^2}{R} - mg$



$\therefore \Delta T = m\left(\frac{u^2}{R} - \frac{v^2}{R}\right) + 2\,mg.$

From conservation of energy

$\frac{u^2}{R} - \frac{v^2}{R} = 4g$ $\Rightarrow$ is indepenent of u.

and $\Delta T = 6\,mg.$

$\therefore$ Statement-2 is correct explanation of statement-1.

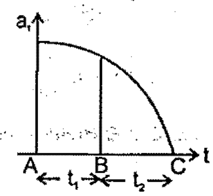
8.33 (B)

$v_B = \sqrt{2gL\sin\theta}$ and $v_C = \sqrt{2gL}$
If $v_C = 2v_B$
Then $2gL = 4\,(2gL\sin\theta)$
or $\sin\theta = \frac{1}{4}$ or $\theta = \sin^{-1}\frac{1}{4}$

8.34 (B)

Tangential acceleration is $a_t = g\cos\theta$,
which decreases with time.
Hence the plot of a_t versus time may be as shown in graph.
Area under graph in time interval $t_1 = v_B - 0 = v_B$
Area under graph in time interval $t_2 = v_C - v_B = v_B$
Hence area under graph in time t_1 and t_2 is same.

$\therefore \quad t_1 < t_2$



8.35 (B)

$$|\vec{v}_B - \vec{v}_C| = \sqrt{v_B^2 + v_C^2 - 2v_Bv_C\sin\theta} = v_B$$

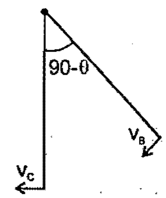
$$\Rightarrow \quad v_B^2 + v_C^2 - 2v_Bv_C\sin\theta = v_B^2$$

$$v_C = 2v_B\sin\theta$$

$$\Rightarrow \quad \sqrt{2g\ell} = 2\sqrt{2g\ell\sin\theta}\,\sin\theta$$

$$\therefore \quad \sin^3\theta = \frac{1}{4} \quad \Rightarrow \sin\theta = \left(\frac{1}{4}\right)^{1/3}$$

$$\theta = \sin^{-1}\left(\frac{1}{4}\right)^{1/3}$$



8.36 (B)

Putting h = 0 and the values we have T = 164 N

8.37 (B)

Putting h = 2R we get T = 144 – 5gR = 44 N.

8.38 (B)

At $\theta = 60°$, $h = R - R\cos 60° = \frac{R}{2}$ $\Rightarrow$ Putting $h = \frac{R}{2}$ in $v^2 = u^2 - 2gh$ $\Rightarrow$ We get the result.

8.39 (A) r, t ; (B) q,s ; (C) p ; (D) q,r

(A) $\vec{F}$ = constant and $\vec{u}\times\vec{F} = 0$

Therefore initial velocity is either in direction of constant force or opposite to it. Hence the particle will move in straight line and speed may increase or decrease. When F and u are antiparallel then particle will come to rest for an instant and will return back

(B) $\vec{u}\cdot\vec{F} = 0$ and $\vec{F}$ = constant

initial velocity is perpendicular to constant force, hence the path will be parabolic with speed of particle increasing.

(C) $\vec{v}\cdot\vec{F} = 0$ means instantaneous velocity is alway perpendicular to force. Hence the speed will remain constant. And also $|\vec{F}|$ = constant. Since the particle moves in one plane, the resulting motion has to be circular.

(D) $\vec{u} = 2\hat{i} - 3\hat{j}$ and $\vec{a} = 6\hat{i} - 9\hat{j}$. Hence initial velocity is in same direction of constant acceleration, therefore particle moves in straight line with increasing speed.

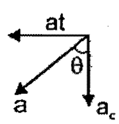
8.40 (A) q, (B) q, t (C) q, t (D) p, s

$v = 2t^2$

Tangential acceleration $a_t = 4t$

Centripetal acceleration $a_c = \frac{v^2}{R} = \frac{2t^4}{R}$

Angular speed $\omega = \frac{v}{R} = \frac{4t}{R}$,



$\tan\theta = \frac{a_t}{a_c} = \frac{4tR}{4t^4} = \frac{R}{t^3}$

8.41 (A) q,s ; (B) p,t ; (C) p,t ; (D) q,r

(Tough) From graph (a) $\Rightarrow$ $\omega = k\theta$ where k is positive constant

angular acceleration = $\omega \frac{d\omega}{d\theta} = k\theta \times k = k^2\theta$

$\therefore$ angular acceleration is non uniform and directly proportional to θ.

$\therefore$ (A) q, s

From graph (b) $\Rightarrow$ $\omega^2 = k\theta$. Differentiating both sides with respect to θ.

$2\omega \frac{d\omega}{d\theta} = k$ or $\omega \frac{d\omega}{d\theta} = \frac{k}{2}$ k is slope of curve hence angular acceleration is uniform.

$\therefore$ (B) p, t

From graph (c)

$\Rightarrow$ $\omega = kt$

angular acceleration = $\frac{d\omega}{dt} = k$

k is slope of curve hence angular acceleration is uniform

$\Rightarrow$ (C) p, t

From graph (d)

$\Rightarrow$ $\omega = kt^2$

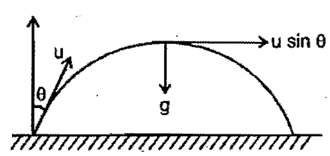
angular acceleration = $\frac{d\omega}{dt} = 2kt$

k is slope of curve hence angular acceleration is non uniform and directly proportional to t. Slope of the curve is constant (can be seen in given graph) but $\alpha = \frac{d\omega}{dt} = 2kt$ increasing with time.

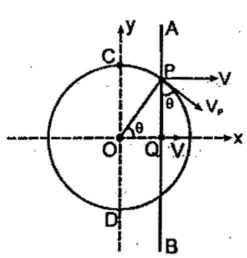
$\therefore$ (D) q,r

8.42 $R = \frac{(v_\perp)^2}{a_\perp} = \frac{u^2 \sin^2\theta}{g}$

Ans. 20 m.



8.43 5



As a rod AB moves, the point 'P' will always lie on the circle.

$\therefore$ its velocity will be along the circle as shown by 'v_P' in the figure. If the point P has to lie on the rod 'AB' also then it should have component in 'x' direction as 'v'.

$\therefore$ $v_P \sin\theta = v$ $\Rightarrow$ $v_P = v \operatorname{cosec}\theta$

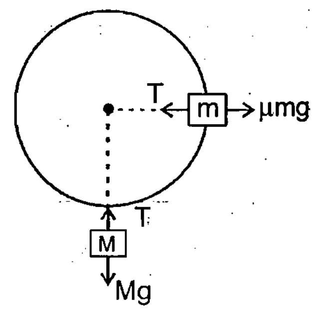
here $\cos\theta = \frac{x}{R} = \frac{1}{R} \cdot \frac{3R}{5} = \frac{3}{5}$

$\therefore$ $\sin\theta = \frac{4}{5}$ $\therefore$ $\operatorname{cosec}\theta = \frac{5}{4}$

$\therefore$ $v_P = \frac{5}{4}v$ **Ans. x = 5**

8.44 As the car travels at a fixed speed 1 m/s, hence tangential acceleration will be zero. Therefore, there will be no component of friction along tangent.

Case I : If $Mg > \frac{mv^2}{r}$; hence friction force on car of mass m will be outwards from the centre.



$$T - \mu mg = \frac{mv^2}{r_{max}}$$

$$Mg - \mu mg = \frac{m}{r_{max}} \quad ..... (1)$$

Case II : If $Mg < \frac{mv^2}{r}$; hence friction force on car of mass m will be towards centre.

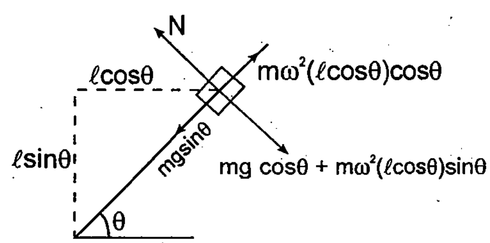
8.45 Distance travelled in climbing 4 feet is $\sqrt{4^2+3^2} = 5$ feet

So, distance travelled in climbing 16 feet will be 20 feet

x = 20 feet

$\frac{x}{10} = 2.$

8.46 The free body diagram of the block is



For block not to slide along wedge,
applying Newton's second law along incline we get
$mg \sin\theta = m\,\omega^2 (\ell \cos\theta) \cos\theta$

$\therefore \quad \omega = \sqrt{\frac{g\sin\theta}{\ell\cos^2\theta}} = 10$ rad/sec.

9. CENTRE OF MASS

9.1 (B)

Since $\Sigma \vec{F}_{ext} = \vec{0}$ ∴ Momentum of system will remain conserved, equal to zero.

9.2 (B)

The line of impact for duration of collision is parallel to x-axis.
The situation of striker and coin just before the collision is given as

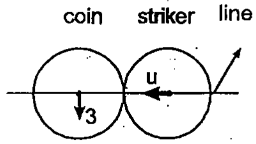


Figure (A)
before collision

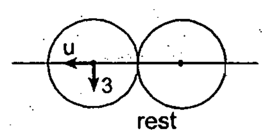
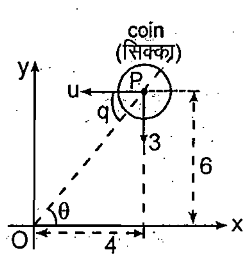


Figure (B)
after collision



Because masses of coin and striker are same, their components of velocities along line of impact shall exchange. Hence the striker comes to rest and the x-y component of velocities of coin are u and 3 m/s as shown in figure.

For coin to enter hole,

its velocity must be along PO $\quad \therefore \tan\theta = \frac{6}{4} = \frac{3}{u} \quad$ or $\quad u = 2$ m/s

9.3 (B)

If we treat the train as a ring of mass 'M' then its COM will be at a distance $\frac{2R}{\pi}$ from the centre of the circle. Velocity of centre of mass is :

$V_{CM} = R_{CM}.\omega$

$= \frac{2R}{\pi}.\omega \quad = \frac{2R}{\pi}\left(\frac{V}{R}\right) \qquad (\because \omega = \frac{V}{R})$

$\Rightarrow \quad V_{CM} = \frac{2V}{\pi} \quad \Rightarrow \quad MV_{CM} = \frac{2MV}{\pi}$

As the linear momentum of any system = MV_{CM}

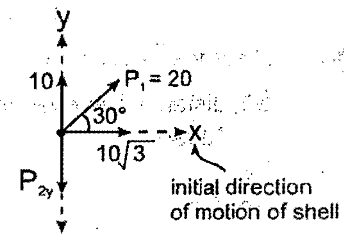
$\therefore$ The linear momentum of the train = $\frac{2MV}{\pi}$ Ans.

9.4 (C)

As shown in figure the component of momentum of one shell along initial direction and perpendicular to initial direction are P_{1X} =Ns and P_{1y}= 10Ns.

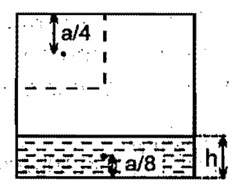
For momentum of the system to be zero in y-direction P_{2y} must be 10 Ns. 2nd part of shell may or may not have momentum in x-direction

$\therefore \quad P_{2min} = 10$ Ns.



9.5 (C)

Let h be the height of water surface, finally



$a^2h = a.\frac{a}{2}.\frac{a}{2}$; $h = \frac{a}{4}$

$\therefore$ C.M. gets lowered by $a - \left(\frac{a}{4} + \frac{a}{8}\right) = a - \frac{3a}{8} = \frac{5a}{8}$

$\therefore$ Work done by gravity = $mg\frac{5a}{8}$

9.6 (C)

Neglecting gravity,

$v = u\ell n\left(\frac{m_0}{m_t}\right)$;

u = ejection velocity w.r.t. balloon.

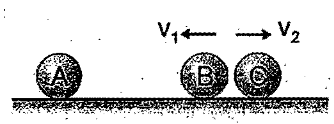
m_0 = initial mass

m_t = mass at any time t.= $2\ell n\left(\frac{m_0}{m_0/2}\right) = 2\ell n2$.

9.7 (B)

For 1st collision

Since



$e = 1$;

$v = v_1 + v_2$

$\Rightarrow \quad v_2 = v - v_1 \qquad$ .....(1)

By momentum conservation :

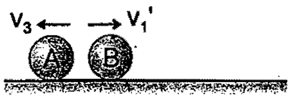
$m_B v = -m_B v_1 + m_C v_2$

or $\quad m_B v = -m_B v_1 + 4 m_B v_2 \qquad (\because m_C = 4 m_B)$

$\Rightarrow \quad v_2 = \frac{v_1 + v}{4} \qquad$ .... (2)

From (1) to (2) : $v_1 = \frac{3}{5}v$ and $v_2 = \frac{2}{5}v$

For second collision :



$$e = 1 \Rightarrow v_1 = v_1' + v_3$$

$$\Rightarrow v_3 = v_1 - v_1' \qquad \text{.... (3)}$$

By momentum conservation :

$$-m_B v_1 = m_B v_1' - m_A v_3$$

or $\quad -m_B v_1 = m_B v_1' - 4\, m_B v_3 \qquad (\because m_A = 4\, m_B)$

$$\Rightarrow \quad v_3 = \frac{v_1' + v_1}{4} \qquad \text{.... (4)}$$

From (3) and (4) :

$$v_1' = \frac{3}{5}v_1 = \frac{3}{5}\left(\frac{3}{5}v\right) = \frac{9}{25}v$$

Clearly $\frac{9}{25}v < \frac{2}{5}v$

Therefore 'B' can not collide with 'C' for the second time.
Hence ; total number of collisions is 2.

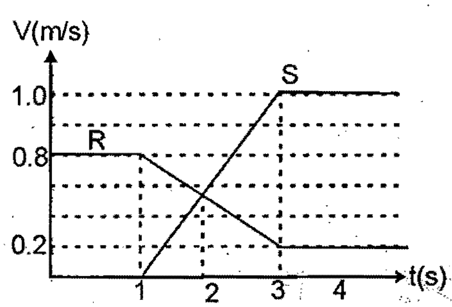
9.8 (B)

Force on table due to collision of balls :

$$F_{dynamic} = \frac{dp}{dt} = 2 \times 20 \times 20 \times 10^{-3} \times 5 \times 0.5 = 2\text{ N}$$

Net force on one leg $= \frac{1}{4}(2 + 0.2 \times 10) = 1\text{ N}$

9.9 (D)



(i) Since, both have positive final velocoties, hence, both moved in the same direction after collision.
(ii) at t = 2 sec, both had equal velocities.
(iii) by conservation of linear momentum, we can say that mass of R was greater than mass of S.

9.10 (D)

By conservation of linear momentum along the string,

$$mu = (m + m + 3m)\,v \quad \text{or} \quad v = \frac{u}{5}$$

and impulse on the block A = 3m (v – 0) $= \frac{3mu}{5}$

9.11 (A)

Let the three mutually perpendicular directions be along x, y and z-axis respectively,

$$\vec{p}_1 = mv_0\hat{i}$$

$$\vec{p}_2 = mv_0\hat{j} \qquad \text{where, } \frac{1}{2}mv_0^2 = E_0$$

$$\vec{p}_3 = mv_0\hat{k} \qquad \text{and} \qquad \vec{p}_4 = m\vec{v}$$

By linear momentum conservation,

$$0 = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \vec{p}_4$$

or $\vec{v} = -v_0(\hat{i} + \hat{j} + \hat{k})$ or $v = v_0\sqrt{1^2 + 1^2 + 1^2} = v_0\sqrt{3}$

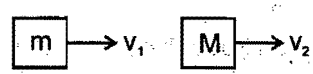
total energy $= 3\left(\frac{1}{2}mv_0^2\right) + \frac{1}{2}mv^2 = 3E_0 + 3E_0 = 6E_0$

9.12 (C)

Let v_1, v_2 and v_3 be velocities of blocks 1, 2 and 3 after suffering collision each.

$mv = mv_1 + Mv_2$ and $v_1 - v_2 = -v$

solving we get $v_1 = \frac{m-M}{m+M} < 0 \because m < M$



$\therefore |v_1| = \frac{M-m}{m+M}v$ .... (1)

and $v_2 = \frac{2mv}{m+M}$

Similarly $v_3 = \frac{2m}{m+M} \times v_2 = \frac{4Mmv}{(m+M)^2}$ ... (2)

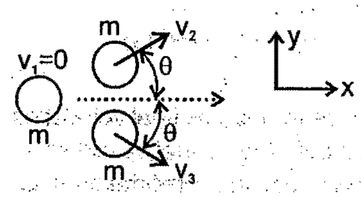
$\therefore \frac{M-m}{M+m}v = \frac{4Mmv}{(M+m)^2}$

or $M^2 - m^2 = 4Mm$.

$\frac{M}{m} = 2 + \sqrt{5}$ Ans.

9.13 (D)

After collision by momentum conservation :



Along y-axis

$0 = 0 + mv_2\sin\theta - mv_3\sin\theta$

$\Rightarrow v_2 = v_3$

Along x-axis

$mv = 0 + mv_2\cos\theta + mv_3\cos\theta$

$mv = 2\,m\,v_2\cos\theta$

$v_2 = \frac{v}{2}\frac{1}{\cos\theta}$

so $v_2 = v_3 > \frac{v}{2}$ $\because$ $\cos\theta < 1$

Q.14 (C)

$\int dp = p_f - p_i = \int F dt$ = Area under the curve.

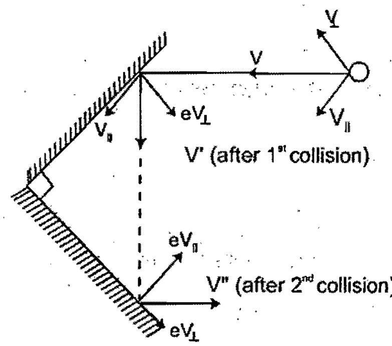
$p_i = 0$

Net Area = 16 – 2 – 1 = 13 N-s

$$\frac{p_f}{M} = V_f = \frac{13}{2} = 6.5\ \hat{i}\ \text{m/s}$$

[As momentum is positive, car is moving along positive x axis.]

Q.15 (C)



During 1st collision perpendicular component of v, $v_\perp$ becomes e times, while IInd component $v_\parallel$ remains unchanged and similarly for second collision. The end result is that both $v_\parallel$ and $v_\perp$ becomes e times their initial value and hence $v'' = -ev$ (the (–) sign indicates the reversal of direction).

Q.16 (D)

It can be shown that

$K_0 = K_{cm} + \frac{1}{2}MV_{cm}^2$ where M is the total mass of the system and V_{cm} is velocity of centre of mass with respect to ground.

Due to internal changes K_{cm} can change but V_{cm} will remain same. Hence only K_{CM} portion of kinetic energy can be transformed to some other form of energy. Thus D is the wrong statement.

Q.17 (C)

For first collision $v = 10$ m/s. $t_1 = \frac{\pi(5)}{10} = \pi/2$ sec.

velocity of sep = e. velocity of opp.

$v_2 - v_1 = \frac{1}{2}(10) \Rightarrow v_2 - v_1 = 5$ m/s

for second collision

$\therefore\ t_2 = \frac{2\pi(5)}{5} = 2\pi$

$\therefore$ total time $t = t + t_2 = \pi/2 + 2\pi \qquad t = \frac{5}{2}\pi$

Q.18 (B)

Just before the particle transfers to inclined surface, we resolve its velocity along and normal to the plane.

For the trajectory of the particle to sharply change from the horizontal line to the inclined line, the impact of the particle with inclined plane should reduce the usinθ component of velocity to zero. Hence the particle starts to move up the incline with speed u cosθ.

Hence as θ increases, the height to which the particle rises shall decrease.

9.19 (B)
Friction force between wedge and block is internal i.e. will not change motion of COM. Friction force on the wedge by ground is external and causes COM to move towards right. Gravitational force (mg) on block brings it downward hence COM comes down.

9.20 (ABD)
$U_Q > U_S > U_P > U_R$

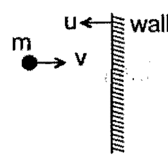
$\because M_Q > M_S > M_P > M_R$ and CM of cone is on smallest height

9.21 (CD)
Since, $F_{ext} = 0$
Hence, momentum will remain conserved equal to mv.

$mv = (m + M)\, v'$ or $v' = \dfrac{mv}{m+M}$

and final kinetic energy is $\frac{1}{2}(m + M)\, v'^2 = \frac{1}{2}(m + M)\left(\dfrac{mv}{m+M}\right)^2 = \dfrac{m^2v^2}{2(m+M)}$

9.22 (BC)
in an elastic collision

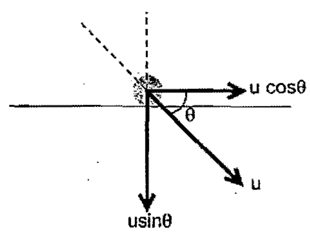


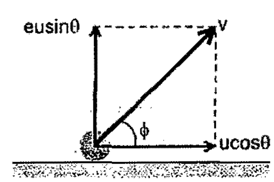
$v_{sep} = v_{app}$
or $v' - u = v + u$
or $v' = v + 2u$
change in momentum of ball is $|p_f - p_i|$
$= |m(-v') - mv| = m(v' + v) = 2m(u + v)$

average force $= \dfrac{\Delta p}{\Delta t} = \dfrac{2m(u+v)}{\Delta t}$

change in KE $= K_f - K_i = \frac{1}{2}mv'^2 - \frac{1}{2}mv^2$

9.23 (ABCD)





Impulse (J) $= \Delta P = mv \sin\phi - m(-u\sin\theta)$
$= m(v\sin\phi + u\sin\theta) = m(V_{sep} + V_{app})$
$= m(eV_{app} + V_{app})$ $[e = \dfrac{V_{sep}}{V_{app}}] = m V_{app}(e + 1)$

$J = m(u\sin\theta)(1 + e)$
In horizontal direction, momentum is conserved :

$u\cos\theta = v\cos\phi$ or $v = \dfrac{u\cos\theta}{\cos\phi}$

or $e = \dfrac{V_{sep}}{V_{app}} = \dfrac{v\sin\phi}{u\sin\theta} = \dfrac{\tan\phi}{\tan\theta}$ or $\tan\phi = e\tan\theta$

in vertical direction, $e = \dfrac{v\sin\phi}{u\sin\theta}$ or $v\sin\phi = eu\sin\theta$,

$v = \sqrt{(eu\sin\theta)^2 + (u\cos\theta)^2} = u\sqrt{e^2\sin^2\theta + \cos^2\theta} \Rightarrow v = u\sqrt{1-(1-e^2)\sin^2\theta}$

final kinetic energy $= \frac{1}{2}mv^2$

initial kinetic energy $= \frac{1}{2}mu^2$

ratio $= \dfrac{v^2}{u^2} = e^2\sin^2\theta + \cos^2\theta$

9.24 (CD)
Sphere A moving with velocity v has a component v/2 along the line joining the centres of the spheres at the time of collision and another component $v\sqrt{3}/2$ perpendicular to the previous direction. After collision the component along the line will interchange i.e. B will move with v/2 velocity i.e. 4 m/s along the line joining the centres and A moves with $v\sqrt{3}/2$ velocity at perpendicular direction to B.

9.25 (C)
For a system of two isolated sphere having non zero initial kinetic energy, the complete kinetic energy can be converted to other forms of energy if the momentum of system is zero. This is due to the fact that for an isolated system, the net momentum remains conserved. If an isolated system has nonzero momentum, for the momentum to remain constant complete kinetic energy of the system cannot become zero. Hence statement 1 is true while statement 2 is false.

9.26 (C)
Statement-2 contradicts Newton's third law and hence is false.

9.27 (A)
For sum of three non null vectors to be zero, they must be coplanar. Hence Statement-2 is a correct explanation for Statement-1.

9.28 (D)
During colision KE of system is not constant, hence statement-1 is false.

9.29 (C)
(a) The acceleration of the centre of mass is

$$a_{COM} = \frac{F}{2m}$$

The displacement of the centre of mass at time t will be

$$x = \frac{1}{2} a_{COM} t^2 = \frac{Ft^2}{4m} \qquad \text{Ans.}$$

9.30 (A)
For explaination see 8.39 solution.

9.31 (D)
Solutions (9.29, 9.30, 9.31)
Suppose the displacement of the right block is x_1 and that of the left is x_2. Then,

$$x = \frac{mx_1 + mx_2}{2m}$$

or $\frac{Ft^2}{4m} = \frac{x_1 + x_2}{2}$ or, $x_1 + x_2 = \frac{Ft^2}{2m}$ ....(i)

Further, the extension of the spring is $x_1 - x_2$. Therefore,

$x_1 - x_2 = x_0$ ....(ii)

From Eqs. (i) and (ii), $x_1 = \frac{1}{2}\left(\frac{Ft^2}{2m} + x_0\right)$

and $x_2 = \frac{1}{2}\left(\frac{Ft^2}{2m} - x_0\right)$ Ans.

9.32 (C)
From conservation of momentum
mv = mv' cos30° + mv' cos30°

$$\therefore \quad v' = \frac{v}{2\cos 30^\circ} = \frac{v}{\sqrt{3}}$$

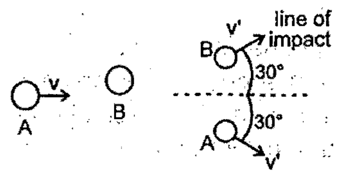
9.33 (D)

Loss in kinetic energy $= \frac{1}{2}mv^2 - 2 \times \frac{1}{2}m\left(\frac{v}{\sqrt{3}}\right)^2 = \frac{1}{6}mv^2$

9.34 (B)

Initially B was at rest, therefore line of impact is along final velocity of B.

$$\therefore\ e = \frac{v'-v'\cos 60^\circ}{v\cos 30} = \frac{\frac{1}{2}\frac{v}{\sqrt{3}}}{v\times\frac{\sqrt{3}}{2}} = \frac{1}{3}$$



9.35 (A)

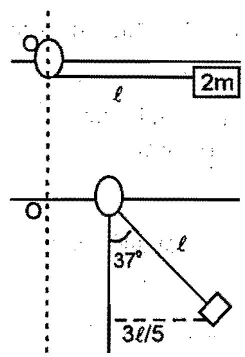
Taking 'O' as the origin ;

$$X_{CM(i)} = \frac{m(0)+2m(\ell)}{3m} = \frac{2\ell}{3}$$

and $X_{CM\,(f)} = \dfrac{m(x)+2m(x+\frac{3\ell}{5})}{m+2m}$

As $\Sigma F_x = 0 \Rightarrow X_{CM}(i) = X_{CM}(f)$

$$\Rightarrow X = \frac{4\ell}{15}\ \textbf{Ans.}$$



9.36 (C)

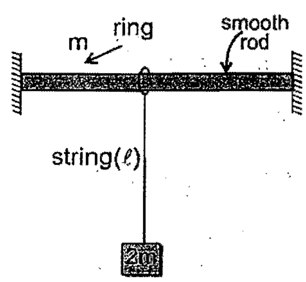
By momentum conservation : (In horizontal direction)

$mv_1 = 2\,mv_2$

& by energy conservation:

$$2\,mg\ell = \frac{1}{2}mv_1^2 + \frac{1}{2}2mv_2^2$$

$$\Rightarrow\ 2\,g\ell = \frac{1}{2}v_1^2 + \left(\frac{v_1}{2}\right)^2 \Rightarrow 2g1 = \frac{3}{4}v_1^2 \Rightarrow v_1 = \sqrt{\frac{8g\ell}{3}}$$



9.37 (B)

From momentum conservation

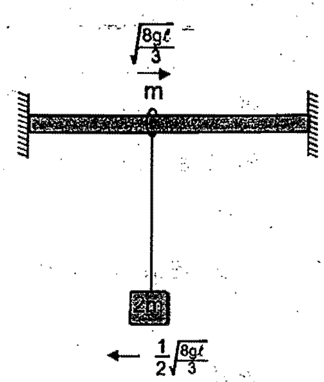
$0 = m\,V_r + 2m\,V_B$

Energy conservation

$$2mg\ell = \frac{1}{2}mV_r^2 + \frac{1}{2}mV_B^2$$

Solving above equations, we get

$$V_r = \sqrt{\frac{8g\ell}{3}},\quad V_B = \frac{1}{2}\sqrt{\frac{8g\ell}{3}}$$



Relative velocity of block with respect to ring is $\frac{3}{2}\sqrt{\frac{8g\ell}{3}}$.

Applying newton's law equations on the block

$$T - 2mg = \frac{(2m)\left(\frac{3}{2}\sqrt{\frac{8g\ell}{3}}\right)^2}{\ell}\qquad T = 14\text{ mg.}\quad \textbf{Ans.}$$

9.38 (A) p (B) q (C) p,r (D) q,s

(A) If velocity of block A is zero, from conservation of momentum, speed of block B is 2u. Then K.E. of block B = $\frac{1}{2}m(2u)^2 = 2mu^2$ is greater than net mechanical energy of system. Since this is not possible, velocity of A can never be zero.

(B) Since initial velocity of B is zero, it shall be zero for many other instants of time.

(C) Since momentum of system is non-zero, K.E. of system cannot be zero. Also KE of system is minimum at maximum extension of spring.

(D) The potential energy of spring shall be zero whenever it comes to natural length. Also P.E. of spring is maximum at maximum extension of spring.

9.39 (A) p,r ; (B) q, s ; (C) p,s ; (D) p,s

A – p,r,t

$\Sigma F = 0$

So, linear momentum conservation and centre of mass will not move.

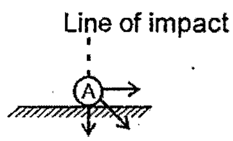
B – q,s

So, linear momentum will not be conserved and centre of mass will accelerate $W_{ext} = \Delta E$.

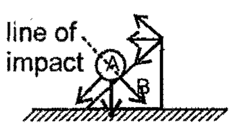
C – p,s,t

D – p,s,t

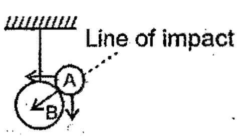
9.40 (A) s ; (B) p,q,r,s,t ; (C) p,q,r,s,t ; (D) qr,s,t

(p) 

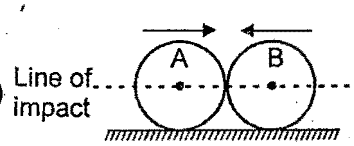
(A) Normal force from ground lies along line of impact. Hence (A) is not answer.
(B) Since no external force act perpendicular to the line of impact. (B) is an answer.
(C) Horizontal direction is same as direction perpendicular to the line of impact. (C) is an answer.
(D) Normal implse from ground lies in vertical direction. (D) is not an answer.

(q) 

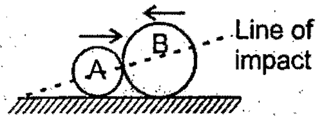
(A) The component of normal force from ground lies along the line of impact. Hence not an answer.
(B) No external force perpendicular to the line of impact for A.
(C) For the system A + B there is no external froce along horizontal direction. Hence an answer.
(D) For B the normal force from ground is balanced by the impulsive force by A. Initial and final momentum is zero. Hence an answer.

(r) 

(A) The component of tension force of thread lies along the line of impact. Hence not an answer.
(B) No external force perpendicular to the line of impact for A.
(C) For system A + B there is no external froce along horizontal direction. Hence an answer.
(D) For B the tension force from thread is balanced by the impulsive force by A. Initial and final momentum is zero. Hence an answer.

(s) 

(A) & (C) are the same direction and there is no external force for the system A + B. Hence answer.
(B) & (D) are the same direction and there is no net force for the system A + B. Hence answer.

(t) 

(A) The component of normal force from ground lies along the line of impact. Hence not an answer.
(B) Is answer because the normal force from the ground is balanced for A. Hence an answer.
(C) For the system A + B there is no external froce along horizontal direction. Hence an answer.
(D) For A the normal force from ground is balanced by the impulsive force by B. Initial and final momentum is zero. Hence an answer.

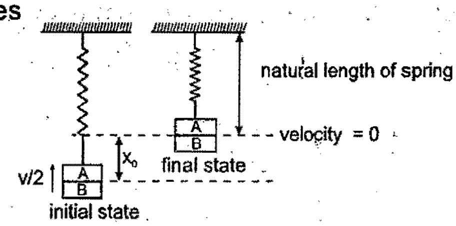
9.41 6

The initial extension in spring is $x_0 = \frac{mg}{k}$

Just after collision of B with A the speed of combined mass is $\frac{v}{2}$.

For the spring to just attain natural length the combined mass must rise up by $x_0 = \frac{mg}{k}$(sec fig.) and comes to rest.

Applying conservation of energy between initial and final states



$$\frac{1}{2}\,2m\left(\frac{v}{2}\right)^2 + \frac{1}{2}\,k\left(\frac{mg}{k}\right)^2 = 2mg\left(\frac{mg}{k}\right)$$

Solving we get $v = \sqrt{\frac{6mg^2}{k}}$

Alternative solution by SHM

$$\frac{v}{2} = \sqrt{\frac{k}{2m}}\sqrt{\left(\frac{2mg}{k}\right)^2 - \left(\frac{mg}{k}\right)^2} \;; \qquad v = \sqrt{\frac{2k}{m}}\sqrt{\frac{3m^2g^2}{k^2}} = \sqrt{\frac{6mg^2}{k}} = 6 \text{ m/sec}$$

9.42 2

Force F on plate = force exerted by dust particles
= force on dust particles by the plate
= rate of change of momentum of dust particles
= mass of dust particles striking the plate per unit time × change in velocity of dust particles.
$= A(v+u)\rho \times (v+u) \quad = A\rho(v+u)^2 \qquad$ Ans. $\rho A(u+v)^2$

9.43 20

Maximum compression will take place when the blocks move with equal velocity. As no net external horizontal force acts on the system of the two blocks, the total linear momentum will remain constant. If V is the common speed at maximum compression, we have,
(1 kg) (2 m/s) = (1 kg)V + (1 kg)V or V = 1 m/s.

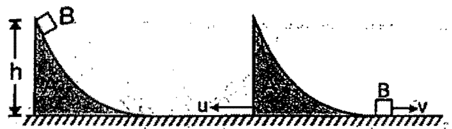
Initial kinetic energy = $\frac{1}{2}$ (1 kg) $(2 \text{ m/s})^2$ = 2 J.

Final kinetic energy = $\frac{1}{2}$ (1 kg) $(1\text{m/s})^2$ + $\frac{1}{2}$ (1 kg) $(1 \text{ m/s})^2$ = 1 J

The kinetic energy lost is stored as the elastic energy in the spring.

Hence $\frac{1}{2}(50 \text{ N/m})\,x^2 = 2\text{J} - 1\text{J} = 1 \text{ J}$ or $x = 0.2$ m.

9.44 25



(figure - 1)

Let u and v be the speed of wedge A and block B at just after the block B gets off the wedge A. Applying conservation of momentum in horizontal direction, we get

$mu = mv$ ....(1)

Applying conservation of energy between initial and final state as shown in figure (1), we get

$mgh = \frac{1}{2}mu^2 + \frac{1}{2}mv^2$ ....(2)

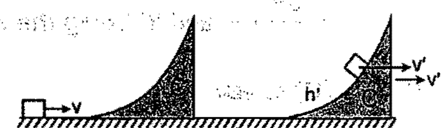
solving (1) and (2) we get

$v = \sqrt{gh}$ ....(3)

At the instant block B reaches maximum height h′ on the wedge C (figure 2), the speed of block B and wedge C are v′.

Applying conservation of momentum in horizontal direction, we get

$mv = (m + m)v'$ ....(4)



Applying conservation of energy between initial and final state

$\frac{1}{2}mv^2 = \frac{1}{2}(m + m)v'^2 + mgh'$ ....(5)

Solving equations (3), (4) and (5) we get

$h' = \frac{h}{4} = 25$ cm **Ans.**

9.45 5

Since $e = \frac{1}{5}$

$\therefore$ Final normal component of velocity $= \frac{v\cos 37^0}{5}$.

As the angle of rebound is equal to the angle before impact.

Therefore, both normal & tangential components of velocities must change by the same factor.

$\therefore$ Tangential velocity after impact becomes $\frac{v\sin 37^0}{5}$.

Let the time of impact be Δt.

$$N = \frac{m\left(v\cos 37^0 + \frac{v\cos 37^0}{5}\right)}{\Delta t} = \frac{6mv\cos 37^0}{5\Delta t}$$

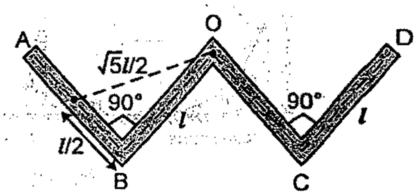
where N is the normal force imparted on the ball by the wall.

Frictional force $= \mu N = \frac{6}{5}\frac{\mu mv\cos 37^0}{\Delta t}$

Also frictional force $= \frac{m\left[v\sin 37^0 - \frac{v\sin 37^0}{5}\right]}{\Delta t}$

$$\Rightarrow \frac{m\left[v\sin 37^0 - \frac{v\sin 37^0}{5}\right]}{\Delta t} = \frac{6}{5}\frac{\mu mv\cos 37^0}{\Delta t} \Rightarrow \mu = \frac{2}{3}\tan 37^0 \Rightarrow \mu = \frac{2}{3}\cdot\frac{3}{4} = \frac{1}{2}$$ **Ans.**

10. RIGID BODY DYNAMICS



10.1 (B)

The given structure can be broken into 4 parts

For AB : $I = I_{CM} + m \times d^2 = \frac{m\ell^2}{12} + \frac{5m}{4}\ell^2$; $I_{AB} = \frac{4}{3}ml^2$

For BO : $I = \frac{m\ell^2}{3}$

∴ For composite frame : (by symmetry)

$$I = 2[I_{AB} + I_{OB}] = 2\left[\frac{4m\ell^2}{3} + \frac{m\ell^2}{3}\right] = \frac{10}{3}m\ell^2.]$$

10.2 (D)

$\omega_{rod} = \omega_{point} = \left(\frac{v_{rel.}}{r}\right)$; $v_{rel.}$ bring the velocity of one point w.r.t. other.

$= \frac{3v - v}{r}$ and 'r' being the distance between them. $= \frac{2v}{r}$

10.3 (D) Given

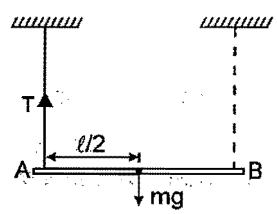
$a_A = 2\alpha = 5\ m/s^2$ ⇒ $\alpha = 5/2\ rad/s^2$ ⇒ $a_B = 1.(\alpha) = 5/2\ m/s^2$

10.4 (C)

Immediately after string connected to end B is cut, the rod has tendency to rotate about point A.

Torque on rod AB about axis passing through A and normal to plane of paper is

$\frac{m\ell^2}{3}\alpha = mg\frac{\ell}{2}$ ⇒ $\alpha = \frac{3g}{2\ell}$



Alternative

Applying Newton's law on center of mass

mg – T =ma .....(i)

Writing $\tau = I\alpha$ about center of mass

$T\frac{\ell}{2} = \frac{m\ell^2}{12}\alpha$ ....(ii)

Also $a = \frac{\ell}{2}\alpha$ ....(iii)

From (i) , (ii) and (iii) $\alpha = \frac{3g}{2\ell}$

10.5 (D)

As the inclined plane is smooth, the sphere can never roll rather it will just slip down.

Hence, the angular momentum remains conserved about any point on a line parallel to the inclined plane and passing through the centre of the ball.

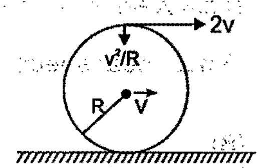
10.6 (D)

As $\Sigma\tau = 0$; Angular momentum, linear momentum remains conserved. F = 0 Linem momntom will re main conword

As the two balls will move radially out, I changes.In order to keep the angular momentum ($L = I\omega$) conserved, angular speed (ω) should change.

10.7 (C)
If the track is smooth (case A), only translational kinetic energy changes to the gravitational potential energy.
But, if the track is rough (case B), both translational and rotational kinetic energy changes to potential energy.
Therefore, potential energy (=mgh) will be more in case B than in case A.
Hence $h_1 > h_2$

10.8 (C)



Radius of Curvature $= \dfrac{(\text{velocity})^2}{\text{Normal Acceleration}} = \dfrac{(2v)^2}{v^2/R} = 4R$

10.9 (B)

Here $u = V_0,\ \omega_0 = -\dfrac{V_0}{2R}$

At pure rolling

$V = V_0 - \left(\dfrac{F_f}{m}\right)t$ & $\dfrac{V}{R} = -\dfrac{V_0}{2R} + \left(\dfrac{F_f}{m.R}\right)t$ (In pure rolling $V = R\omega$) ($\alpha = \dfrac{\tau}{I} = \dfrac{F_f.R}{mR^2}$)

$\Rightarrow \quad V_0 - V = V + \dfrac{V_0}{2} \Rightarrow \quad 2V = \dfrac{V_0}{2} \quad \Rightarrow \quad V = \dfrac{V_0}{4}$ Ans.

10.10 (B)
The two forces along y-direction balance each other.
Hence, the resultant force is 2F along x-direction
Let the point of application of force be at (0, y).
(By symmetry x-coordinate will be zero).
For rotational equilibrium : $F(a) + F(a) + F(a + y) - F(a - y) = 0$
$\Rightarrow \quad y = -a$ Hence (B).
Alternate :
Torque will only be produced by the two forces along y–direction in anti-clockwise direction. To balance this torque we should apply a force 2F in order to produce a torque in the clockwise direction, which is only possible if we apply a force at a point below the x-axis.
Then $\tau = F(a) + F(a) + 2F \times y = 0 \quad \Rightarrow \quad y = a$

10.11 (B)
Let m_1 = mass of the square plate of side 'a'
and m_2 = mass of the square of side 'a/2'

Then $m_1 = \sigma\left(\dfrac{a}{2}\right)^2$; $m_2 = \sigma(a)^2$; (σ being the areal density)

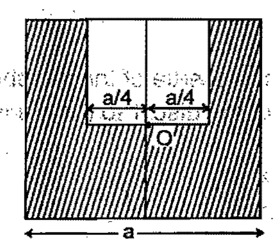
and $m_2 - m_1 = M$.

$\Rightarrow \quad I = \dfrac{m_2a^2}{6} - \left\{\dfrac{m_1(a/2)^2}{6} + m_1\left(\dfrac{a}{4}\right)^2\right\} = \dfrac{\sigma a^4}{6} - \left\{\dfrac{\sigma(a/2)^4}{6} + \sigma\left(\dfrac{a}{2}\right)^2\left(\dfrac{a}{4}\right)^2\right\}$

$= \sigma a^4\left\{\dfrac{1}{6} - \dfrac{1}{16\times6} - \dfrac{1}{4\times16}\right\}$

$= \sigma a^4\left\{\dfrac{(2\times16)-2-3}{16\times12}\right\}$

$I = \sigma a^4\left\{\dfrac{27}{12\times16}\right\}$



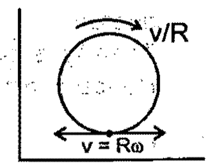
Also; $M = \sigma\left(1-\dfrac{1}{4}\right)a^2 \quad \Rightarrow \sigma = \dfrac{4}{3}\dfrac{M}{a^2} \quad \Rightarrow I = \left(\dfrac{4}{3}\dfrac{M}{a^2}\right).a^4\left\{\dfrac{27}{12\times16}\right\} \quad \Rightarrow I = \dfrac{3Ma^2}{16}$

10.12 (D)

As the disc is in combined rotation and translation, each point has a tangential velocity and a linear velocity in the forward direction.

From figure

v_{net} (for lowest point) = $v - R\omega = v - v = 0$.



and Acceleration = $\frac{v^2}{R} + 0 = \frac{v^2}{R}$

(Since linear speed is constant)

10.13 (B)

$f = 4\ ma$ .... (1)

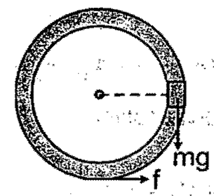
$(mg - f)r = (3\,mr^2 + mr^2)\,\alpha$

$mg - f = 4\ ma$ .... (2)

from (1) and (2)

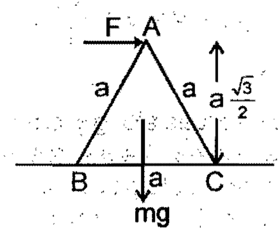
$\Rightarrow$ 8 ma = mg

$\Rightarrow a = \frac{g}{8} \Rightarrow \alpha = \frac{g}{8r}$



10.14 (A)

The tendency of rotating will be about the pont C.
For minimum force, the torque of F about C has to be equal to the torque of mg about C.



$\therefore F\left(a\frac{\sqrt{3}}{2}\right) = mg\left(\frac{a}{2}\right) \Rightarrow F = \frac{mg}{\sqrt{3}}$ Ans.

10.15 (D)

As torque = change in angular momentum

$\therefore$ $F.\Delta t = mv$ (Linear) ..... (1)

and $\left(F\cdot\frac{\ell}{2}\right)\Delta t = \frac{m\ell^2}{12}\cdot\omega$ (Angular) ..... (2)

Dividing : (1) and (2)

$$2 = \frac{12v}{\omega\ell} \Rightarrow \omega = \frac{6v}{\ell}$$

Using : S = ut :

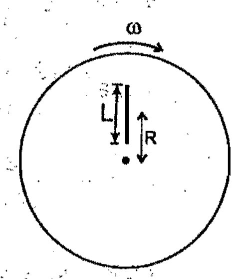
Displacement of COM is : $\frac{\pi}{2} = \omega t = \left(\frac{6v}{\ell}\right)t$

and $x = vt$

Dividing $\frac{2x}{\pi} = \frac{\ell}{6} \Rightarrow x = \frac{\pi\ell}{12}$ $\Rightarrow$ Coordinate of A will be $\left[\frac{\pi\ell}{12} + \frac{\ell}{2}, 0\right]$ Ans.

10.16 (A)

Moment of inertia of the rod about w.r.t. the axis through centre of the disc ⊥ to the plans line is : (by parallel axis theorem).

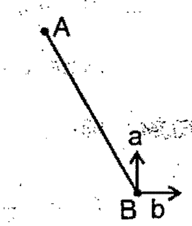


$I = \frac{mL^2}{12} + mR^2$

& K.E. of rod disc = $\frac{1}{2}I\omega^2$ $= \frac{1}{2}m\omega^2\left[R^2 + \frac{L^2}{12}\right]$ Ans.

10.17 (C)

At the initial moment, angular velocity of rod is zero.
Acceleration of end B of rod with respect to end A is shown in figure.
Centripetal acceleration of point B with respect to A is zero ($\because \omega^2\ell = 0$)
So at the initial moment, acceleration of end B with respect to end A is perpendicular to the rod which is equal to $\sqrt{a^2+b^2}$



$a_{rel} = \ell\alpha$

$$\frac{\sqrt{a^2+b^2}}{\ell} = \alpha \text{ where } \alpha \text{ is angular acceleration}$$

10.18 (A)

By conservation of angular momentum about pivot

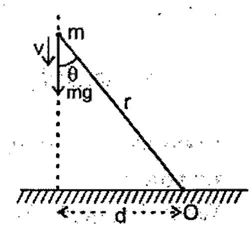
$L = I\omega$

$$mv\frac{d}{2} = \left[\frac{Md^2}{12} + m\left(\frac{d}{2}\right)^2\right]\omega \quad \Rightarrow \quad \frac{mvd}{2} = \left(\frac{md^2}{2} + \frac{md^2}{4}\right)\omega$$

$$\frac{mvd}{2} = \frac{3}{4}md^2\omega \quad \Rightarrow \quad \frac{2}{3}\frac{v}{d} = \omega$$

10.19 (B)

The magnitude of angular momentum of particle about O = mvd
Since speed v of particle increases, its angular momentum about O increases.
Magnitude of torque of gravitational force about O = mgd $\Rightarrow$ constant.
Moment of inertia of particle about O = mr^2
Hence MI of particle about O decreases.



angular velocity of particle about O = $\frac{v\sin\theta}{r}$

$\because$ v and $\sin\theta$ increase and r decreases
$\therefore$ angular velocity of particle about O increases.

10.20 (B)

As the normal force exerted by horizontal surface passes through point B, external torque on the ball is zero about point B. So angular momentum of ball is conserved about point B ($\because \tau = \frac{dL}{dt}$)

10.21 (A)

From conservation of energy, the kinetic energy of ball at lowest portion is (v_c = speed of centre of ball)

$$\frac{1}{2}mv_c^2 + \frac{1}{2}\times\frac{2}{5}mv_c^2 = mgR \quad \text{or} \quad \frac{7}{10}mv_c^2 = mgR$$

Since net tangential force on sphere at lowest point is zero, net force on sphere at lowest position is

$$= \frac{mv_c^2}{R} = \frac{10}{7}mg \text{ upwards}$$

10.22 (B)

Moment of inertia of semicircular portions about x and y axes are same. But moment of inertia of straight portions about x-axis is zero.

$\therefore \quad I_x < I_y$

or $\quad \frac{I_x}{I_y} < 1$

10.23 (A)

$m_2g.\,1 = m_1g.3 \rightarrow m_2 = 3m_1$

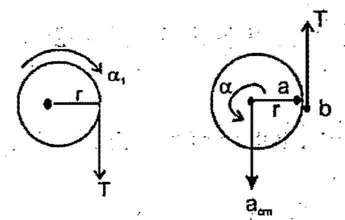
$\rightarrow 4m_1g\,.\,3 = m_3\,g \rightarrow m_3 = 12\,m_1$

$\rightarrow 16\,m_1g\,.\,3 = m_4\,g \rightarrow m_1 = \frac{48}{48} = 1$ kg.

10.24 (B)

$T\,r = \frac{mr^2}{2}\,\alpha_1$ ....(1)

$Tr = \frac{mr^2}{2}\,\alpha$ .... (2)



$\alpha_1 = \alpha$ .... (3)

From (1) & (2)

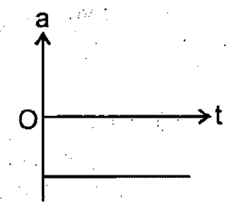
acc^n. of point b = acc^n of point a

$r\alpha_1 = a_{cm} - r\,\alpha$ .... (4)

Hence $2\,r\,\alpha = a_{cm}$ **Ans. (B)**

10.25 (D)

As the sphere rolls up its speed is decreasing and while rolling down its speed is increasing. Hence the acceleration of its centre of mass is down the incline and is thus always negative. Therefore the correct graph is.



10.26 (C)

The direction of $\vec{L}$ is perpendicular to the line joining the bob to point C. Since this line keeps changing its orientation in space, direction of $\vec{L}$ keeps changing however as ω is constant, magnitude of $\vec{L}$ remain constant.

Aliter : The torque about point is perpendicular to the angular momentum vector about point C. Hence it can only change the direction of L, and not its magnitude.

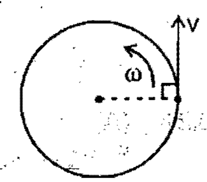
10.27 (D)

(Easy) Let the angular velocity of disc after child jumps off, be ω'

$\therefore$ From conservation of angular momentum

$(I + mR^2)\omega = mvR + I\omega'$

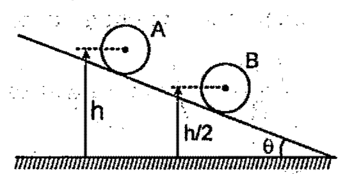
$\therefore\ \omega' = \frac{(I + mR^2)\omega - mvR}{I}$



10.28 (A)

Just before collision Between two Balls

potential energy lost by Ball A = kinetic energy gained by Ball A.



$mg\frac{h}{2} = \frac{1}{2}I_{cm}\omega^2 + \frac{1}{2}mv_{cm}^2$

$= \frac{1}{2} \times \frac{2}{5}mR^2 \times \left(\frac{v_{cm}}{R}\right)^2 + \frac{1}{2}mv_{cm}^2 = \frac{1}{5}\ mv_{cm}^2 + \frac{1}{2}\ mv_{cm}^2$

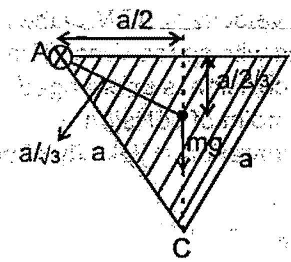
$\Rightarrow \quad \frac{5}{7}mgh = mv_{cm}^2 \quad \Rightarrow \quad \frac{mgh}{7} = \frac{1}{5}\ mv_{cm}^2$

After collision only translational kinetic energy is transfered to ball B

So just after collision rotational kinetic energy of Ball A = $\frac{1}{5}\ mv_{cm}^2 = \frac{mgh}{7}$

10.29 (C)

Torque about A :

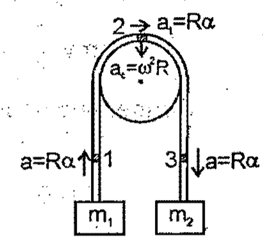


$$mg\frac{a}{2} = I\alpha$$

$$\Rightarrow \quad \alpha = \frac{mga}{2I}$$

$$\Rightarrow \quad \text{acceleration} = \frac{a}{\sqrt{3}}\alpha = \frac{mga^2}{2\sqrt{3}I}$$

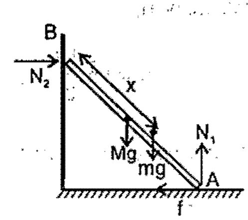
10.30 (BD)



The acceleration of three sections of rope 1, 2, 3 are as shown. Hence for the section of rope in contact with pulley, acceleration increases till the section looses contact with pulley.

Due to friction between string and pulley, the tension in right portion of string is larger in comparision to tension in the left portion of string.

10.31 (ACD)



Let m and M be mass of man and ladder. From FBD normal reaction at A is $N_1 = (m + M)g$ which remains constant. Net torque on man + ladder is zero about B. If x decreases then torque of mg about B will decrease. Hence f must increase.

10.32 (CD)

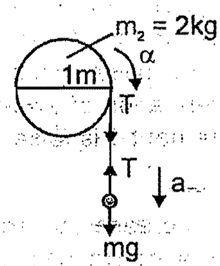
All points in the body, in plane perpendicular to the axis of rotation revolve in concentric circles. All points lying on circle of same radius have same speed (and also same magnitude of acceleration) but different directions of velocity (also different directions of acceleration)

Hence there cannot be two points in the given plane with same velocity or with same acceleration.

As mentioned above, points lying on circle of same radius have same speed.

Angular speed of body at any instant w.r.t. any point on body is same by definition.

10.33 (ABCD)



By FBD of particle

$mg - T = ma$

$10 - T = a$ ....(i)

By FBD of disc

$$TR = I\alpha = I.\frac{a}{R} \quad \Rightarrow \quad T = \frac{m_2R^2}{2}.\frac{a}{R^2}$$

$$T = \frac{ma}{2} = a \quad ....(ii)$$

By eq. (i) and (ii)

(A) $a = 5\ m/s^2$ and $T = 5N$ and $\alpha = \frac{a}{R} = 5\ rad/s^2$ **Ans.**

(B) For angular displacement of disc : $\theta = \omega t + \frac{1}{2}\alpha t^2$

$\theta = \frac{1}{2} \times 5 \times 4^2 = 40$ rad. **Ans.**

(C) Work done by torque = $\int \tau d\theta = \tau \int d\theta = 5 \times 40 = 200$ J **Ans.**

(D) $\Delta K.E. = \Delta w = 200$ J

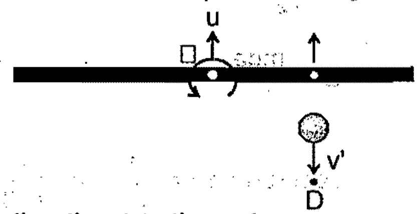
$k_2 - k_1 = 200$ J. **Ans.**

10.34 (ABC)

The ball has V', component of its velocity perpendicular to the length of rod immediately after the collision. u is velocity of COM of the rod and ω is angular velocity of the rod, just after collision. The ball strikes the rod with speed vcos53° in perpendicular direction and its component along the length of the rod after the collision is unchanged.

Using for the point of collision.

Velocity of separation = Velocity of approach



$$\Rightarrow \frac{3v}{5} = \left(\frac{\omega\ell}{4}+u\right) + V' \quad .... (1)$$

Conserving linear momentum (of rod + particle), in the direction ⊥ to the rod.

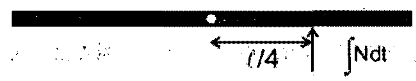
$$mv.\frac{3}{5} = mu - mV' \quad ....(2)$$

Conserving angular moment about point 'D' as shown in the figure

$$0 = 0 + \left[mu\frac{\ell}{4} - \frac{m\ell^2}{12}\omega\right] \Rightarrow u = \frac{\omega\ell}{3} \quad .... (3)$$

By solving

$$u = \frac{24v}{55}, \; w = \frac{72v}{55\ell}$$



Time taken to rotate by π angle $t = \frac{\pi}{\omega}$

In the same time, distance travelled $= u_2.t = \frac{\pi\ell}{3}$

Using angular impulse-angular momentum equation.

$$\int N.dt.\frac{\ell}{4} = \frac{1}{3}\frac{m\ell^2}{4}.\frac{72v}{55\ell} \Rightarrow \int N.dt = \frac{24mv}{55}$$ or $\left\{\text{using impulse - momentum equation on Rod} \; \int Ndt = mu = \frac{24mv}{55}\right.$

10.35 (D)

For a disc rolling without slipping on a horizontal rough surface with uniform angular velocity, the acceleration of lowest point of disc is directed vertically upwards and is not zero(Due to translation part of rolling, acceleration of lowest point is zero. Due to rotational part of rolling, the tangential acceleration of lowest point is zero and centripetal acceleration is non-zero and upwards). Hence statement 1 is false.

10.36 (D)

As x increases, the required component of reaction decreases to zero and then increases (with direction reversed). Hence statement-1 is false.

10.37 (C)

The applied horizontal force F has tendency to rotate the cube in anticlockwise sense about centre of cube. Hence statement-2 is false.

10.38 (A)

The acceleration of centrers of both spheres is μg up the incline. Since initial velocity of centres of both spheres is zero, they shall travel same distance in same time interval. Hence Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

10.39 (C)

$\vec{L} = \vec{r}_1 \times \vec{p}_1 + \vec{r}_2 \times \vec{p}_2 \qquad \because \vec{p}_1 + \vec{p}_2 = 0 \qquad = \vec{r}_1 \times (-\vec{p}_2) + r_2 \times \vec{p}_2 \qquad = (\vec{r}_2 - \vec{r}_1) \times \vec{p}_2$

$\vec{L} = \vec{r}_{rel} \times \vec{p}_2$. Hence Statement-1 is True, Statement-2 is False

10.40 (A)
Since acceleration is same for all the three spheres, they cover equal distances in equal intervals of time in all the cases (A), (B) and (C).
Hence (A).

10.41 (C)
From passage, for case (C)

$$\mu_{min.} = \frac{\tan\theta\left(\frac{k^2}{R^2}\right)}{\left(1+\frac{k^2}{R^2}\right)} \rightarrow \text{(pure rolling)}$$

Putting the values of 'k' for different objects given in the table (in passage) we get ;

$$\mu_{min\ (Ring)} = \frac{\tan\theta}{2}$$

$$\mu_{min\ (Disc)} = \frac{\tan\theta}{3},\ \mu_{min\ (Solid\ sphere)} = \frac{2}{5}\tan\theta,\ \mu_{min\ (hollow\ sphere)} = \frac{2}{7}\tan\theta$$

$\Rightarrow$ $\mu_{min(Ring)}$ is greater than either of

$\mu_{min\ (Disc)}, \mu_{min\ (Solid\ sphere)}, \mu_{min\ (hollow\ sphere)}$.

Therefore, the pure rolling of ring will confirm pure rolling of all other bodies.

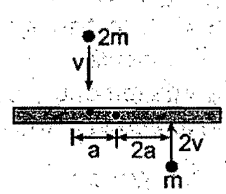
10.42 (C)
As given in the equation of case (B) ;
$\mu NR = Mk^2\alpha$
and $N = Mg\cos\theta$
As ; θ, M, R, μ are same for all, 'α' will be least for that object for which 'k' and hence I is maximum.
Therefore 'α' for ring (k = R) and hence ω for ring at the bottom is minimum.
Also, $Mg\sin\theta - \mu N = Ma$
Since M, μ, θ, N are same for all objects, they have same linear acceleration and hence same linear velocity and hence same $\frac{1}{2}Mv_{cm}^2$.

$\therefore$ K.E. $\left(\frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2\right)$ is least for the ring.

10.43 (B)
For ring $a = g\sin\theta/2$ (for pure rolling) is less than that of disc.
Hence (B).

10.44 (D)
10.45 (A)
10.46 (A)

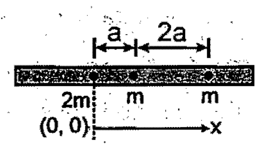
(i) Cons. linear momentum
$-2m.v + 2v.m = 0 = MV_{cm}$
$V_{cm} = 0$



(ii) As ball sticks to Rod
Conserving angular momentum about C

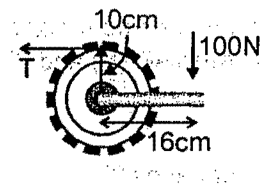
$$2v.m.\,2a + 2mva = I\omega = \left(\frac{8m.\,36a^2}{12} + 2m.\,a^2 + m.4a^2\right)$$

$$6mv.a = 30\,ma^2.\omega \Rightarrow \omega = \frac{v}{5a}$$



(iii) $$KE = \frac{1}{2}I\omega^2 = \frac{1}{2}.\,30\,ma^2 \times \frac{v^2}{25a^2} = \frac{3mv^2}{5}.$$

10.47 (C)



As angular velocity of the disc is constant i.e.

$\Sigma\tau = 0$

100N × 16 cm = T × 10 cm

T = 160 N

10.48 (A)

As angular acceleration of the rear wheel is zero therefore net torque on the wheel is zero.

10.49 (C)

Power delivered = $\vec{F}.\vec{v}$

where $\vec{v}$ is velocity of the point of application of the force.

v = 16 cm × 2π.2 (= Rω) = 0.64 π m/s

P = 100 × 0.64 π = 64 πW.

ALT : P = τω

10.50 (B)

RN = rn ⇒ $n = \frac{10\text{cm} \times 2}{4\text{cm}} = 5$ cy/s

So rear wheel rotates 5 cycles/second.

Hence $V = \frac{35}{100} \times 2\pi \times 5 = 3.5\,\pi$ m/s

10.51 (D)

As $\Sigma\tau = 0$

160 N × 4 cm = f × 35 cm ⇒ $f = \frac{160 \times 4\text{cm}}{35\text{cm}} = 18.3$ N

10.52 (A)p,t (B) q,s, t (C) p, t (D) q,s

(A) Speed of point P changes with time

(B) Acceleration of point P is equal to $\omega^2 x$ (ω = angular speed of disc and x = OP). The acceleration is directed from P towards O.

(C) The angle between acceleration of P (constant in magnitude) and velocity of P changes with time. Therefore, tangential acceleration of P changes with time.

(D) The acceleration of lowest point is directed towards centre of disc and remains constant with time

10.53 (A) p,q,r, s, t (B) p,q,r,s, t (C) p,q, s,t (D) p,q,r, s, t

Since all forces on disc pass through point of contact with horizontal surface, the angular momentum of disc about point on ground in contact with disc is conserved. Also the angular momentum of disc in all cases is conserved about any point on the line passing through point of contact and parallel to velocity of centre of mass.

The K.E. of disc is decreased in all cases due to work done by friction.

From calculation of velocity of lowest point on disc, the direction of friction in case A, B and D is towards left and in case C is towards right.

The direction of frictional force cannot change in any given case.

10.54 (A) p,t (B) q,t (C) p,q,t (D) s,t

(A) If resultant force is zero, $\vec{P}_{system}$ will be constant.

(B) If resultant torque is zero, $\vec{L}_{system}$ will be constant.

(C) If external forces are absent, both $\vec{P}_{system}$ and $\vec{L}_{system}$ will be constant.

(D) If no non conservative force acts, total mechanical energy of system will be constant.

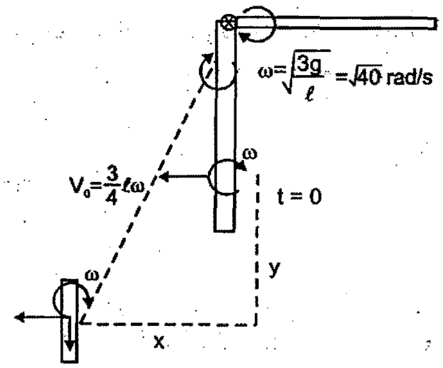
10.55 5

$$t = \frac{\pi}{\omega} = \frac{\pi}{\sqrt{40}} \text{ sec.}$$

$x = v_0 t$

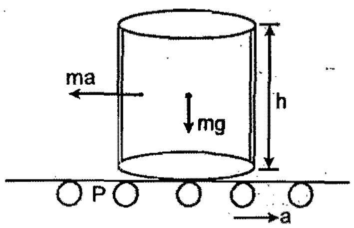
$$y = \frac{1}{2}gt^2$$

$$r = \sqrt{x^2 + \left(y + \frac{3\ell}{4}\right)^2} \simeq 2.5\text{m}.$$



10.56 10

WRT of belt, pseudo force ma acts on cylinder at COM as shown about to cylinder will be just about to topple when torque to weight w.r.t. P.



$$\frac{dv}{dt} = 2t$$

$$m.2t \cdot \frac{h}{2} = mg.r$$

$t = 10$

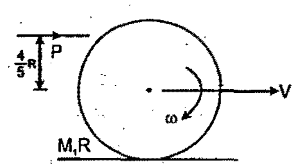
[Ans.: 10]

10.57 13

Using imp - momentum equation.

$P = M.v$

$$\Rightarrow \quad v = \frac{P}{M} \qquad \text{...........(1)}$$



using angular impulse–momentum equation. wrt. centre.

$$P\frac{4}{5}R = \frac{2}{5}MR^2\omega.$$

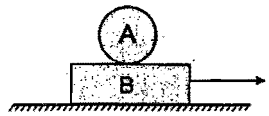
$$\omega = \frac{2P}{MR}$$

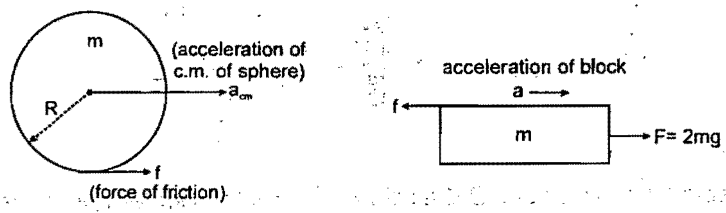
Total K.E. = Trans KE + Rotational KE

$$= \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}M. \times \frac{P^2}{M} + \quad \frac{1}{2} \cdot \frac{2}{5}MR^2 \cdot \frac{4P^2}{M^2R^2} \quad = \frac{13P^2}{10M}.$$

10.58 10

The free body diagrams of sphere A and Block B are as shown





Applying Newton's law to block and sphere

$F - f = m a$ ....(1)

$f = m a_{cm}$ ....(2)

$fr = \frac{2}{5} mr^2 \alpha$ ....(3)

Since the sphere does not slip over the block, therefore from constraint

$a = a_{cm} + r\alpha$ ....(4)

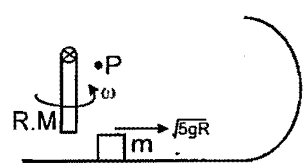
solving equation (1), (2), (3) and (4)

we get the angular acceleration of sphere

10.59 15

Minimum velocity required by block 'm' to complete the motion in $\sqrt{5gR}$

conserving mech. energy



$$\frac{1}{2} I\omega^2 = Mg \times \frac{R}{2} \quad \Rightarrow \quad \omega = \sqrt{\frac{MgR}{I}}$$

Cons. angular momentum wrt P before & after collision.

$$I\omega = m.R\sqrt{5gR}$$

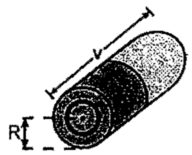
$$I.\sqrt{\frac{MgR}{I}} = mR\sqrt{5gR}$$

$$MgRI = m^2R^2\, 5gR$$

putting $I = \frac{ML^2}{3} = \frac{MR^2}{3}$ (since L = R)

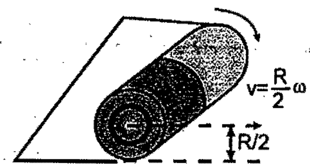
$\frac{M}{m} = \sqrt{15}$ Ans.: 15

10.60 14



$M_{initial} = \pi R^2. L.\rho.$

ρ = density of carpet material



$M_{final} = \pi. \left(\frac{R}{2}\right)^2 . L . \rho = \frac{M_i}{4}.$

Initial PE of carpet $= Mg.R$

Final PE of carpet $= \frac{M}{4}.g. \frac{R}{2} = \frac{MgR}{8}$

ΔPE (decrease) $= \frac{7}{8}MgR$

It's equal to gain in KE $= K_{trans} + K_{Rot} = \frac{1}{2} Mv^2 + \frac{1}{2} I\omega^2$

using mass $= \frac{M}{4}$, $v = \frac{R}{2}.\omega$ $I = \frac{1}{2}\left(\frac{M}{4}\right)\left(\frac{R}{2}\right)^2$

$K = \frac{1}{2}\left(\frac{M}{4}\right) v^2 + \frac{1}{2}\left(\frac{MR^2}{32}\right)\left(\frac{2v}{R}\right)^2 = \frac{3}{16}Mv^2$

Equating $\frac{7}{8}Mg R = \frac{3}{16}Mv^2$

$v = \sqrt{\frac{14gR}{3}}$ [Ans. : 14]]

10.61 5

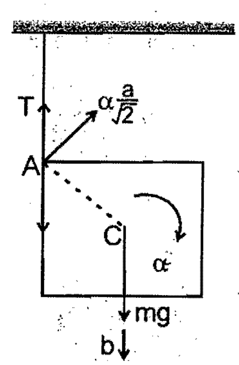
Let b and α are linear acc. of centre of mass and angular acc. of the plane, just after BF is cut.

$mg - T = mb$ .... (1)

Taking torques about CoM

$mg - T = mb$ .... (1)

$\frac{Ta}{2} = \frac{ma^2}{6}.\alpha$ .... (2)



$mg = mb + \frac{ma\alpha}{3} \Rightarrow g = b + \frac{a\alpha}{3}$

and $b = \alpha\frac{a}{2}$

$\therefore \quad g = b + \frac{2b}{3} = \frac{5b}{3} \Rightarrow b = \frac{3g}{5}$

$\therefore \quad T = mg - \frac{m.3g}{5} = \frac{2mg}{5}$ Ans.

11. SIMPLE HARMONIC MOTION

11.1 (D)

At equilibrium $P_1 = \frac{mg}{A}$

Now, lets displace the piston by x distance downward

$P_iV_i^\gamma = P_f V_f^\gamma$

$$\left(\frac{mg}{A}\right)(\ell_0)^\gamma = (P_f)(\ell_0 - x)^\gamma \quad \Rightarrow \quad P_f = \left(\frac{mg}{A}\right)\left(1 - \frac{x}{\ell_0}\right)^{-\gamma} = \frac{mg}{A}\left(1 + \frac{\gamma x}{\ell_0}\right)$$

$$P_f = \frac{mg}{A} + \frac{mg}{A}\frac{\gamma}{\ell_0}x \quad \Rightarrow \quad F_{net} = P_f A - mg = \frac{\gamma mg}{\ell_0}x$$

$$F_{net} = -\left(\frac{\gamma mg}{\ell_0}\right)x$$

$$T = 2\pi\sqrt{\frac{m}{\gamma(mg/\ell_0)}} \quad \Rightarrow \quad T = 2\pi\sqrt{\frac{\ell_0}{\gamma g}} \quad \text{where } \gamma = 1 + \frac{2}{5}.$$

If the piston were spherical, then also the answer would be the same.

11.2 (B)

$K_{max} = 16\pi^2 \qquad T = 0.2$ s

$f = \frac{1}{T} = 5$ Hz

$$\frac{1}{2}mv^2_{max} = K_{max} \quad \Rightarrow \quad v_{max} = 4\pi \quad \Rightarrow \quad A\frac{2\pi}{0.2} = 4\pi \quad \Rightarrow \quad A = 0.4 \text{ m}$$

At t = 0.1 sec the particle is at extreme position so acceleration is not zero.

11.3 (D)

11.4 (B)

$x_1 = a\sin(\omega t + \phi_1)$

$x_2 = a\sin(\omega t + \phi_2)$

$$\Rightarrow \quad |x_1 - x_2| = 2a\sin(\omega t + \frac{\phi_1 + \phi_2}{2})\cos\left(\frac{\phi_1 - \phi_2}{2}\right)$$

To maximize $|x_1 - x_2|$:

$$\sin(\omega t + \frac{\phi_1 + \phi_2}{2}) = 1$$

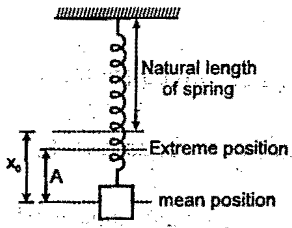
$$\Rightarrow \quad a\sqrt{2} = 2a \times 1 \times \cos\left(\frac{\phi_1 - \phi_2}{2}\right) \quad \Rightarrow \quad \frac{1}{\sqrt{2}} = \cos\left(\frac{\phi_1 - \phi_2}{2}\right)$$

$$\Rightarrow \quad \frac{\pi}{4} = \frac{\phi_1 - \phi_2}{2} \quad \Rightarrow \quad \phi_1 - \phi_2 = \frac{\pi}{2}.$$

Hence (B).

11.5 (A)

The spring is never compressed. Hence spring shall exert least force on the block when the block is at topmost position.



$$F_{least} = kx_0 - kA = mg - m\omega^2 A = mg - 4\frac{\pi^2}{T^2}mA$$

11.6 (C)

Due to impulse, the total energy of the particle becomes :

$$\frac{1}{2}m\omega^2A^2 + \frac{1}{2}m\omega^2A^2 = m\omega^2A^2$$

Let ; A' be the new amplitude. (Apply energy conservation law)

$$\therefore \quad \frac{1}{2}m\omega^2(A')^2 = m\omega^2A^2 \qquad A' = \sqrt{2}\,A. \qquad \textbf{Ans.}$$

11.7 (A)

Conserving momentum : 2V = 3V'

$$\Rightarrow V' = \frac{2}{3}V. \quad \Rightarrow \quad \omega'A' = \frac{2}{3}\omega A \quad \Rightarrow \quad \sqrt{\frac{K}{3}} \times A' = \frac{2}{3}\sqrt{\frac{K}{2}} \times A \quad \Rightarrow \quad A' = \sqrt{\frac{2}{3}}\,A$$

11.8 (D)

The extension developed in the string due to small values of 'θ' is :

$x = h \sin\theta \cong h\theta$

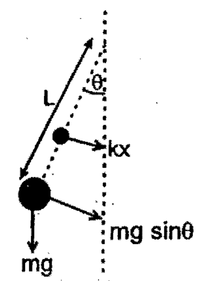
Torque about 'O' :

$\tau_0 = (Mg \sin\theta)\, L + (kx)h$

or, $\tau_0 \cong mg\,\theta L + kh^2\theta = (mgL + kh^2)\theta$ ...(1)

Also;

$\tau_0 = I_0\alpha = mL^2\alpha$ ...(2)



From (1) and (2) :

$mL^2\,\alpha = (mg\,L + kh^2)\theta$

or $\quad \alpha = \frac{1}{L^2}\left(gL + \frac{kh^2}{m}\right)\theta$

$$\text{Now : } T = 2\pi\sqrt{\frac{\theta}{\alpha}} = 2\pi\sqrt{\frac{\theta}{\frac{1}{L^2}\left(gL + \frac{kh^2}{m}\right)\theta}} \quad \Rightarrow \quad \upsilon = \frac{1}{T} = \frac{1}{2\pi L}\sqrt{\left(gL + \frac{kh^2}{m}\right)} \quad \text{Hence (D).}$$

11.9 (B)

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{mg\ell}{I}}$$

where, ℓ is distance between point of suspension and centre of mass of the body.

Thus, for the stick of length L and mass m :

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{m.g.\frac{L}{2}}{(mL^2/3)}} = \frac{1}{2\pi}\sqrt{\frac{3g}{2L}}$$

when bottom half of the stick is cut off

$$f_0' = \frac{1}{2\pi}\sqrt{\frac{\frac{m}{2}.g.\frac{L}{4}}{\frac{m}{2}\frac{(L/2)^2}{3}}} = \frac{1}{2\pi}\times\sqrt{\frac{3g}{L}} = \sqrt{2}\,f_0\,\textbf{Ans.}$$

11.10 (A)

Average kinetic energy with respect to space = $\frac{1}{3}kA^2$

Average potential energy with respect to space = $\frac{1}{6}kA^2$

Average kinetic energy with respect to time = $\frac{1}{4}kA^2$

Average potential energy with respect to time = $\frac{1}{4}kA^2$

Hence (A) is true.

11.11 (BC)

$x = 3 \sin 100\pi t \quad \Rightarrow \quad y = 4 \sin 100\pi t$

Equation of path is

$\frac{y}{x} = \frac{4}{3} \Rightarrow$ i.e. $y = \frac{4}{3}x$

which is equation of a straight line having slope $\frac{4}{3}$

Equation of resulting motion is $\vec{r} = x\hat{i} + y\hat{j} = (3\hat{i} + 4\hat{j}) \sin 100\pi t$

Amplitude is $\sqrt{3^2 + 4^2} = 5$

11.12 (ABCD)

Given $A = 0.4m$, and $a = g$

so $\omega^2 A = g \Rightarrow \omega^2 = \frac{10}{0.4} = 25 \quad \Rightarrow \omega = 5 \quad T = \frac{2\pi}{\omega} = 2\pi/5$ sec.

At lowest position acceleration.= $\omega^2 A + g = g + g = 2g$

So weight = m (2g) = 2mg

at half distance a = g/2

So weight at upper half distance = m(g-g/2) = mg/2

and weight at lower half distance = $m(g + g/2) = \frac{3mg}{2}$

actual weight at equilibrium position (maximum v)

11.13 (D)

PE is related to reference. Only when PE at mean position is taken zero, the assertion is true.

11.14 (A)

Statement-2 itself explains statement-1.

11.15 (D)

The mean position of the particle in statement-1 is $x = -\frac{b}{a}$ and the force is always proportional to displacement from this mean position. The particle executes SHM about this mean position. Hence statement-1 is false

11.16 (B)

When speed of block is maximum, net force on block is zero. Hence at that instant spring exerts a force of magnitude 'mg' on block.

11.17 (C)

At the instant block is in equilibrium position, its speed is maximum and compression in spring is x given by kx = mg .... (1)

From conservation of energy

$$mg\,(L + x) = \frac{1}{2}kx^2 + \frac{1}{2}mv_{max}^2 \quad .... (2)$$

from (1) and (2) we get $v_{max} = \frac{3}{2}\sqrt{gL}$.

11.18 (B)

$$V_{max} = \frac{3}{2}\sqrt{gL} \text{ and } \omega = \sqrt{\frac{k}{m}} = 2\sqrt{\frac{g}{L}}$$

$$\therefore \quad A = \frac{V_{max}}{\omega} = \frac{3}{4}L$$

Hence time taken t, from start of compression till block reaches mean position is given by

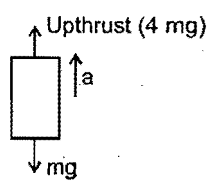
$x = A \sin \omega t_0$ where $x = \frac{L}{4}$

$$\therefore t_0 = \sqrt{\frac{L}{4g}} \sin^{-1}\frac{1}{3}$$

Time taken by block to reach from mean position to bottom most position is $\frac{T}{4} = \frac{2\pi}{4\omega} = \frac{\pi}{4}\sqrt{\frac{L}{g}}$

Hence the required time $= \frac{\pi}{4}\sqrt{\frac{L}{g}} + \sqrt{\frac{L}{4g}} \sin^{-1}\frac{1}{3}$

11.19 (C)



$$4\,mg - mg = ma \quad \Rightarrow \quad a = 3g$$

11.20 (B)

The density of liquid is four times that of cylinder, hence in equlibrium postion one fourth of the cylinder is submerged.

So as the cylinder is released from initial postion, it moves by $\frac{3\ell}{4}$ to reach its equlibrium position. The upward motion in this time is SHM.

$$a = 3g = \omega^2 A = \omega^2 \times \frac{3\ell}{4}, \text{ so } \omega = \sqrt{\frac{4g}{\ell}}.$$

Therefore required velocity is $v_{max} = \omega A$. $\omega = \sqrt{\frac{4g}{\ell}}$ and $A = \frac{3\ell}{4}$. Therefore $v_{max} = \frac{3}{2}\sqrt{g\ell}$

11.21 (C)

The require time is one fourth of time period of SHM. Therefore $t = \frac{T}{4} = \frac{\pi}{2\omega} = \frac{\pi}{4}\sqrt{\frac{\ell}{g}}$

11.22 (A) r , (B) t , (C) q , (D) r

$F = 8 - 2x$

$= -2(x - 4)$

At equilibrium position, $F = 0$

$\Rightarrow \quad x = 4$ m

As particle is released at rest from x = 6 m, i.e. it is one of the extreme position.

Hence, Amplitude A = 2 m.

Here, force constant k = 2 N/m $\Rightarrow \quad m\omega^2 = 2 \quad$ or $\quad \omega = 1$

$\therefore$ Time period, $T = \frac{2\pi}{\omega} = 2\pi$

Time to go from x = 2 m to x = 4 m (i.e. from extreme position to mean position) = $\frac{T}{4} = \frac{\pi}{2}$

Energy of S.H.M. = $\frac{1}{2}kA^2 = \frac{1}{2} \times 2 \times 4$ N-m = 4 J

As particle has started it's motion from positive extreme

$\therefore \quad$ Phase constant = $\frac{\pi}{2}$

11.23 (A) r , (B) p , (C) t , (D) s

$V_{max} = A\omega \quad \Rightarrow \quad A = \frac{V_{max}}{\omega} = \frac{2\pi}{2\pi} \times (0.2) = 0.20$m

$T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow m = \frac{T^2k}{4\pi^2} = 0.2$ kg

At t = 0.1, acc. is maximum $\Rightarrow a_{max} = -\omega^2 A = -\left(\frac{2\pi}{0.2}\right)^2 \times 0.2 = -200$ m/s²

Maximum energy = $\frac{1}{2} mV_{max}^2 = 4$ J

11.24 (A) p,q (B) q,r (C) q,r (D) s

(A) $x = \sqrt{2}\left(\frac{1}{\sqrt{2}}\sin\omega t - \frac{1}{\sqrt{2}}\cos\omega t\right) \quad \Rightarrow x = \sqrt{2}\sin(\omega t - \frac{\pi}{4})$ is periodic with SHM.

(B) $x = \sin^3\omega t$ can not be written
as $x = A\sin(\omega' t + \phi)$ so it is not SHM
but periodic motion.

(C) Linear combination of different periodic function is also periodic function.

$\frac{d^2x}{dt^2}$ is not directly proportional to x i.e. this motion is not SHM

(D) x continuously decreases with time. So x is not periodic function.

11.25 (A) p (B) q (C) p,r (D) q,s,t

(A) If velocity of block A is zero, from conservation of momentum, speed of block B is 2u. Then K.E. of block B = $\frac{1}{2}m(2u)^2 = 2mu^2$ is greater than net mechanical energy of system. Since this is not possible, velocity of A can never be zero.

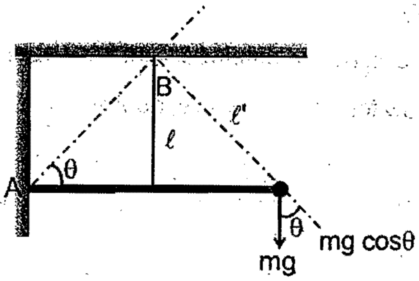
(B) Since initial velocity of B is zero, it shall be zero for many other instants of time.

(C) Since momentum of system is non-zero, K.E. of system cannot be zero. Also KE of system is minimum at maximum extension of spring.

(D) The potential energy of spring shall be zero whenever it comes to natural length. Also P.E. of spring is maximum at maximum extension and maximum compression of spring.

11.26 (4)

The bob will execute SHM about a stationary axis passing through AB. If its effective length is ℓ' then



$T = 2\pi\sqrt{\frac{\ell'}{g'}}$

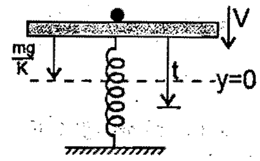
$\ell' = \ell/\sin\theta = \sqrt{2}\,\ell$ (because $\theta = 45^\circ$)

$g' = g\cos\theta = g/\sqrt{2}$

$T = 2\pi\sqrt{\frac{2\ell}{g}} = 2\pi\sqrt{\frac{2\times 0.2}{10}} = \frac{2\pi}{5}$ s. Ans. X = 4

11.27 (20)

Velocity of the particle just before collision



$u = \sqrt{2g\times\frac{4.5mg}{K}} \Rightarrow u = 3g\sqrt{\frac{m}{K}}$

Now it collides with the plate.

Now just after collision velocity (V) of system of "plate + particle"

$mu = 3mV \Rightarrow V = \frac{u}{3} = g\sqrt{\frac{m}{K}}$

Now system perform's SHM with time period $T = 2\pi\sqrt{\frac{3m}{K}}$ and mean position as $\frac{mg}{K}$ distance below the point of collision.

Let the equation of motion be $y = A\sin(\omega t + \phi)$

for t = 0 $y = mg/K$

$\frac{mg}{K} = A\sin\phi$ ....(1)

Now for amplitude $V = \omega\sqrt{A^2 - x^2}$

$g\sqrt{\frac{m}{K}} = \sqrt{\frac{K}{3m}}\sqrt{A^2 - \frac{m^2g^2}{K^2}} \Rightarrow \left(\sqrt{3}\,\frac{mg}{K}\right)^2 = A^2 - \frac{m^2g^2}{K^2} \Rightarrow A = \frac{2mg}{K}$ ...(2)

By (1) & (2)

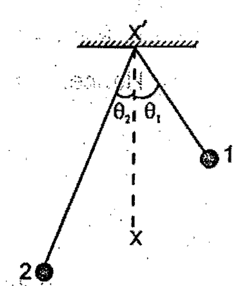
$x = \frac{A}{2}$ to $x = 0 \Rightarrow t = \frac{T}{12}$

$x = 0$ to $x = A \Rightarrow t = T/4$

total time $= \frac{T}{12} + \frac{T}{4} = \frac{2\pi}{3}\sqrt{\frac{3m}{K}} \Rightarrow$ Using values, $t = 20\pi$ms.

11.28 (3)

The angular position of pendulum 1 and 2 are (taking angles to the right of reference line xx' to be positive)



$\theta_1 = \theta\cos\left(\frac{4\pi}{T}t\right)$ [where $T = 2\pi\sqrt{\frac{\ell}{g}}$]

$\theta_2 = -\theta\cos\left(\frac{2\pi}{T}t\right) = \cos\left(\frac{2\pi}{T}t + \pi\right)$

For the strings to be parallel for the first time

$\theta_1 = \theta_2$ or $\cos\left(\frac{4\pi}{T}t\right) = \cos\left(\frac{2\pi}{T}t + \pi\right)$

$\therefore \frac{4\pi}{T}t = 2n\pi \pm \left(\frac{2\pi}{T}t + \pi\right)$ for n = 0, $t = \frac{T}{2}$

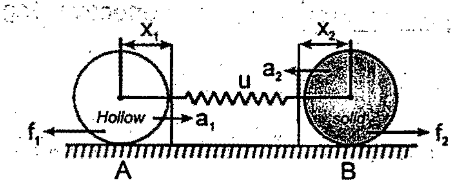
for n = 1, $t = \frac{T}{6}, \frac{3T}{2}$

$\therefore$ Both the pendulum are parallel to each other for the first time after $t = \frac{T}{6} = \frac{\pi}{3}\sqrt{\frac{\ell}{g}}$ Ans.

11.29 (21)

x_1 & x_2 be the displacement from equilibrium position

Now for hollow sphere, applying $\tau_A = I_A \alpha$



$$k(x_1 + x_2)r = \frac{5}{3} m r^2 \frac{a_1}{r} \quad \text{— (1)}$$

By angular momentum conservation (about A) of the system, $\frac{5}{3} m v_1 r = \frac{7}{5} m v_2 r$

$\Rightarrow 25 v_1 = 21 v_2$ — (2) $\Rightarrow$ $25 x_1 = 21 x_2$ — (3)

Using (1) and (3) we get, $k\left(x_1 + \frac{25}{21} x_1\right) = \frac{5}{3} m a_1$ $\Rightarrow a_1 = \frac{46}{35} \frac{k}{m} x_1$

$\Rightarrow f = \frac{1}{2\pi} \sqrt{\frac{46 k}{35 m}}$ Ans. Now for amplitude, $A_1 + A_2 = x_0$ — (4)

From equation (3) we get $A_2 = \frac{25}{21} A_1$ — (5).

By (4) and (5) we get, $A_1 = \frac{21}{46} x_0$

Ans. $A_1 = \frac{21}{46} x_0$; $A_2 = \frac{25}{46} x_0$; $f = \frac{1}{2\pi} \sqrt{\frac{46 k}{35 m}}$

11.30 (8)

$$v\frac{dv}{dx} = 8 - 2x \Rightarrow \int_0^v v\,dv = \int_0^x (8 - 2x)dx \Rightarrow \frac{v^2}{2} = 8x - x^2 \Rightarrow v^2 = 16x - 2x^2$$

At B , v = 0 so, x = 8

Hence, AB = 8

12. STRING WAVE

12.1 (D)

As wave has been reflected from a rarer medium, therefore there is no change in phase. Hence equation for the reflected waves can be written as

$y = 0.5A \sin(-kx - \omega t + \theta) = -0.5A \sin(kx + \omega t - \theta)$

12.2 (B)

Substituting x = 0 we have given wave y = A sin ωt at x = 0 other should have y = – A sin ωt equation so displacement may be zero at all the time Hence (B) is correct option.

12.3 (C)

$f = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}} \quad \Rightarrow \quad \mu = \rho \pi r^2$

If radius is doubled and length is doubled, mass per unit length will become four times. Hence

$f' = \frac{1}{2 \times 2\ell}\sqrt{\frac{2T}{4\mu}} = \frac{f}{2\sqrt{2}}$

12.4 (A)

$\lambda = 2\ell = 3m$

Equation of standing wave (As x = 0 is taken as a node)

$y = 2A \sin kx \cos \omega t$

y = A as amplitude is 2A.

$A = 2A \sin kx$

$kx = \frac{\pi}{6}$ or $\frac{5\pi}{6} \Rightarrow \frac{2\pi}{\lambda}x = \frac{\pi}{6} \Rightarrow x_1 = \frac{1}{4}m$

and $\frac{2\pi}{\lambda}.x = \frac{\pi}{2} + \frac{\pi}{3} \Rightarrow x_2 = 1.25\ m \Rightarrow x_2 - x_1 = 1m$

12.5 (B)

Given $\omega = 3\pi$

$\therefore \quad f = \frac{\omega}{2\pi} = 1.5,$

Also $\Delta x = 1.0$ cm

Given, $\phi = \frac{2\pi}{\lambda}\Delta x \Rightarrow \frac{\pi}{8} = \frac{2\pi}{\lambda} \times 1 \Rightarrow \lambda = 16\ cm \Rightarrow v = f\lambda = 16 \times 1.5 = 24$ cm/sec

12.6 (B)

$\frac{I_1}{I_2} = \frac{a_1^2 f_1^2}{a_2^2 f_2^2} = \frac{(3)^2(8)^2}{(2)^2(12)^2} = 1$

12.7 (B)

$y(x, t = 0) = \frac{6}{x^2}$ then $y(x,t) = \frac{6}{(x-2t)^2}$

$\Rightarrow \frac{\partial y}{\partial t} = \frac{24}{(x-2t)^3}$ at x = 2, t = 2

$V_y = \frac{24}{(-2)^3} = -3$ m/s.

12.8 (C)

(C) $P = \frac{1}{2}\mu\omega^2A^2V$ using $V = \sqrt{\frac{T}{\mu}}$

$P = \frac{1}{2}\omega^2 A^2 \sqrt{T\mu}$ $\Rightarrow$ $\omega = \sqrt{\frac{2P}{A^2\sqrt{T\mu}}}$ $f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{2P}{A^2\sqrt{T\mu}}}$

using the given data, we get f = 30 Hz.

12.9 (B)

In figure, 'C' reaches the position where 'A' already reaches after $\omega t = \frac{\pi}{2}$

and 'A' reaches the position where 'B' already reaches after $\omega t = \frac{\pi}{2}$

Hence (B).

12.10 (A)

$384 = \frac{nv}{2\ell}$ ...(i) $\Rightarrow$ $288 = \frac{mv}{2\ell}$ ...(ii)

from equation (i) & (ii)

$\left(\frac{n}{m}\right) = \left(\frac{4}{3}\right)$

so n = 4

from equation (i)

$384 = \frac{4v}{2\times 3/4} = \frac{10v}{3}$

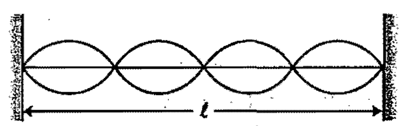
v = 144 m/s

$384 = \frac{nv}{2\ell}$ ...(i) $\Rightarrow$ $288 = \frac{mv}{2\ell}$ ...(ii)

12.11 (C)

For a string vibrating in its n^{th} overtone ($(n + 1)^{th}$ harmonic)

$y = 2A \sin\left(\frac{(n+1)\pi x}{L}\right) \cos\omega t$



For $x = \frac{\ell}{3}$, 2A = a and n = 3; $y = \left[a \sin\left(\frac{4\pi}{\ell}\cdot\frac{\ell}{3}\right)\right] \cos\omega t$

$= a.\ \sin\frac{4\pi}{3} \cos\omega t$ $=$ $-a.\left(\frac{\sqrt{3}}{2}\right)\cos\omega t$

i.e. at $x = \frac{\ell}{3}$; the amplitude is $\frac{\sqrt{3}a}{2}$.

12.12 (D)

In a sonometer,

$v \propto \sqrt{T}$ $\Rightarrow$ $\therefore \frac{v_1}{v_2} = 2 = \sqrt{\frac{T_1}{T_2}}$ $\Rightarrow$ $T_2 = \frac{T_1}{4}$

$\therefore$ % change will be :

$$\frac{T_1 - T_2}{T_1} \times 100 = \frac{T_1 - \frac{T_1}{4}}{T_1} \times 100 = 75\% \text{ Ans.}$$

12.13 (D)

For waves along a string :

$$\upsilon \propto \sqrt{T}$$

$$\Rightarrow \quad \lambda \propto \sqrt{T}$$

Now, for 6 loops : $3\lambda_1 = L \Rightarrow \lambda_1 = L/3$

& for 4 loops : $2\lambda_2 = L \Rightarrow \lambda_2 = L/2$

$$\Rightarrow \quad \frac{\lambda_1}{\lambda_2} = \frac{2}{3} \quad \Rightarrow \quad T_2 = \frac{9}{4} x T_1 = \frac{9}{4} \times 36$$

$= \quad$ 81 N. Ans.

12.14 (C)

Velocity of sound is inversely proportional to the square root of density of the medium.

i.e., $V\sqrt{\rho}$ = constant $\Rightarrow \dfrac{V_1}{V_2} = \sqrt{\dfrac{\rho_2}{\rho_1}} = \sqrt{\dfrac{2\rho}{\rho}} = \sqrt{2}$ Ans.

12.15 (C)

$$y = \log\frac{x^2 - t^2}{x - t} = \log(x + t) \qquad (\because \log a - \log b = \log \frac{a}{b})$$

$$\frac{\partial y}{\partial x} = \frac{1}{(x+t)}$$

$$\frac{\partial^2 y}{\partial x^2} = -\frac{1}{(x+t)^2} \quad \text{and} \quad \frac{\partial y}{\partial t} = \frac{(\partial x/\partial t)}{(x+t)} = \frac{v}{(x+t)}$$

$$\frac{\partial^2 y}{\partial t^2} = -\frac{v^2}{(x+t)^2} \quad \Rightarrow \quad \frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

Which is the general form of wave equation.

12.16 (D)

At t = 2 second, the position of both pulses are separately given by fig.(a) and fig. (b); the superposition of both pulses is given by fig. (c)

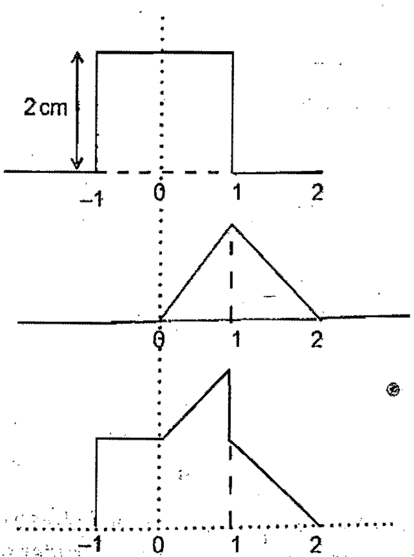
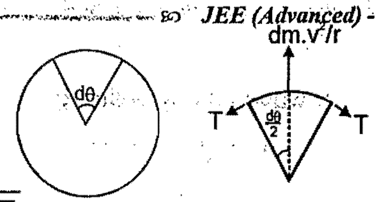


Fig. (a)

Fig. (b)

Fig. (c)

12.17 (B)



$$dm.\omega^2R = 2T\sin\frac{d\theta}{2}$$

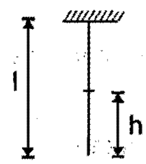
$$\mu R d\theta\, \omega^2 R = 2T\frac{d\theta}{2} \quad \Rightarrow \quad \mu\omega^2R^2 = T \quad \Rightarrow \quad v_w = \sqrt{\frac{T}{\mu}} = \sqrt{\omega^2R^2} = \omega R$$

Also speed of string is ωR

$\therefore$ The velocity of disturbance w.r.t. ground = $\omega R + \omega R = 2\omega R$.

12.18 (C)

Let ℓ be the length of rope. Then tension in the string at height h will be :



$$T = \frac{m}{\ell}hg \qquad u = \sqrt{\frac{T}{\mu}}$$

Here, μ = mass per unit length = $\frac{m}{\ell}$

$\therefore$ $u = \sqrt{gh}$ or $u^2 = gh$

i.e., u versus h graph is a parabola.

12.19 (B)

Let a_i and a_r be the amplitudes of incident and reflected wave. Then

$$\frac{a_i + a_r}{a_i - a_r} = n \text{ (Given)}$$

$\therefore$ $\frac{a_r}{a_i} = \left(\frac{n-1}{n+1}\right)$ $\therefore$ Fraction of energy reflected is

$$\frac{E_r}{E_i} = \left(\frac{a_r}{a_i}\right)^2 = \left(\frac{n-1}{n+1}\right)^2$$

12.20 (A)

$$f_0 = \frac{v}{2\ell}$$

Now beat frequency = $f_1 - f_2$

$$= \frac{v}{2\left(\frac{\ell}{2} - \Delta\ell\right)} - \frac{v}{2\left(\frac{\ell}{2} + \Delta\ell\right)} = \frac{v}{2}\left[\frac{1}{\frac{\ell}{2} - \Delta\ell} - \frac{1}{\frac{\ell}{2} + \Delta\ell}\right]$$

$$= (f_0\ell)\left[\frac{2}{\ell - 2\Delta\ell} - \frac{2}{\ell + 2\Delta\ell}\right] = 2f_0\ell\left[\frac{1 + 2\Delta\ell - \ell + 2\Delta\ell}{\ell^2 - 4(\Delta\ell)^2}\right] \approx 2f_0\ell\left(\frac{4\Delta\ell}{\ell^2}\right) \approx \frac{8f_0\Delta\ell}{\ell}$$

12.21 (C)

$v = \sqrt{T/\mu}$ or $v \propto \frac{1}{\mu}$ $1 \to RP$, $2 \to PQ$, $3 \to QS$

Here μ is mass per unit length.

$$\mu_1 = \frac{0.1}{2} = 0.05 \text{ kg/m} \quad \Rightarrow \quad \mu_2 = \frac{0.1}{3} = 0.067 \text{ kg/m}$$

and $\mu_3 = \frac{0.15}{4} = 0.0375$ kg/m

$\mu_3 < \mu_1 < \mu_2$ $\therefore$ $\mu_3 < \mu_1 < \mu_2$

Between string RP and PQ, medium of string PQ is denser. Therefore, wave-2 will suffer a phase change of π. Between string PQ and QS, medium of string PQ is denser. Therefore wave 4 will not suffer any phase change.

12.22 (AB)

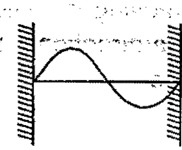
Speed of wave in wire $V = \sqrt{\frac{T}{\rho A}} = \sqrt{\frac{Y\Delta\ell}{\ell}A \times \frac{1}{\rho A}} = \sqrt{\frac{Y\Delta\ell}{\ell\rho}}$

Maximum time period means minimum frequency ; that means fundamental mode.

$f = \frac{V}{\lambda} = \frac{V}{2\ell}$

$\therefore T = \frac{2\ell}{V} = 2\ell\sqrt{\frac{\ell\rho}{Y\Delta\ell}} = \frac{1}{35}$ second **Ans.**

$\therefore$ (f = 35 Hz)



and; frequency of first overtone = $\frac{V}{\ell}$ = 70 Hz.

12.23 (AC)

$y = 2A \sin kx.\ \sin\omega t$

$V_y = \frac{dy}{dt} = 2A \sin kx.\ \cos\omega t$

$V_y = 0 \Rightarrow t = T/4,\ 3T/4$ $\left(T = \frac{2\pi}{\omega}\right)$

(2 times in one time period)

12.24 (BD)

In standing waves, particles may have phase differences only 0 or π.

12.25 (ACD)

$\frac{\lambda}{4} = 0.1 \Rightarrow \lambda = 0.4$ m

from graph $\Rightarrow$ T = 0.2 sec. and amplitude of standing wave is 2A = 4 cm.

Equation of the standing wave

$y(x, t) = -2A \cos\left(\frac{2\pi}{0.4}x\right) \sin\left(\frac{2\pi}{0.2}t\right)$ cm

$y\ (x = 0.05,\ t = 0.05) = -2\sqrt{2}$ cm

$y(x = 0.04,\ t = 0.025) = -2\sqrt{2} \cos 36°$

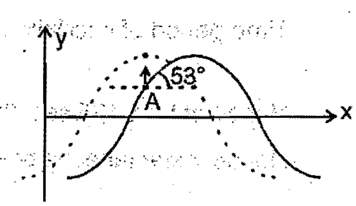
speed = $\frac{\lambda}{T}$ = 2 m/sec.

$V_y = \frac{dy}{dt} = -2A \times \frac{2\pi}{0.2} \cos\left(\frac{2\pi x}{0.4}\right).\cos\left(\frac{2\pi t}{0.2}\right)$

$V_y = (x = \frac{1}{15}\text{m},\ t = 0.1) = 20\pi$ cm/sec.

12.26 (BC)

As shown in the curve, if wave is moving along –x axis, V_p is positive.



$\frac{V_p}{V_w} = -\tan\theta$

$|-\tan\theta| > 1$

$\therefore V_p > V_w$.

12.27 (B) (C)

As $f_1 : f_2 : f_3 = 3 : 5 : 7$, string is fixed at one end, Its fundamental frequency is $f_0 = \frac{f_1}{3} = \frac{105}{3} = 35$ Hz

12.28 (D)
Every small segment is acted upon by forces from both sides of it hence energy is not conserved, rather it is transmitted by the element.

12.29 (D)
(D) Two waves moving in uniform string with uniform tension shall have same speed and may be moving in opposite directions. Hence both waves may have velocities in opposite direction. Hence statement-1 is false

12.30 (D)
(False) at node v = 0, at antinode Tension ⊥ to velocity ∴ at the points power = 0 $(P = \vec{F}.\vec{V})$
At other points $P \neq 0$.

12.31 (D)

12.32 (C)

12.33 (A)

$\mu = \frac{1.2}{2} = 0.6$ kg/m

f = 5 Hz ⇒ $\lambda = 2\ell = 4$m ⇒ V = fλ
= 5 x 4 = 20 m/s Ans. 11.34

using $v = \sqrt{\frac{T}{\mu}} \Rightarrow T = 20^2 \times 0.6 = 240$ N Ans. 11.33

$\left(\frac{\partial y}{\partial t}\right)_{max} = 3.14$ m/s ⇒ $(2A)\omega = 3.14$

Amplitude $2A = \frac{3.14}{2 \times (3.14) \times 5} = 0.1$ m

Equation of standing wave is
$y = (0.1) \sin (\pi/2)x \sin (10\,\pi)\, t$ Ans. 11.35

12.34 (A)
The equation of wave moving in negative x-direction, assuming origin of position at x = 2 and origin of time (i.e. initial time) at t = 1 sec.
$y = 0.1 \sin (4\pi (t-1) + 8(x-2))$
Shifting the origin of position to left by 2m, that is, to x = 0. Also shifting the origin of time backwards by 1 sec, that is to t = 0 sec.
$y = 0.1 \sin (4\pi (t-1) + 8(x-2))$

12.35 (C)
As given the particle at x = 2 is at mean position at t = 1 sec.
∴ its velocity $v = \omega A = 4\pi \times 0.1 = 0.4\,\pi$ m/s.

12.36 (D)

Time period of oscillation $T = \frac{2\pi}{\omega} = \frac{2\pi}{4\pi} = \frac{1}{2}$ sec.

Hence at t = 1.125 sec, that is, at $\frac{T}{4}$ seconds after t = 1 second, the particle is at rest at extreme position.
Hence instantaneous power at x = 2 at t = 1.125 sec is zero.

12.37 (A) p,q,r,t ; (B) p,q,s ; (C) p,r,s,t ; (D) p,s

12.38 (A) p,q,s (B) s (C) q,r,s (D) s,t

(A) Number of loops (of length $\lambda/2$) will be even or odd and node or antinode will respectively be formed at the middle.

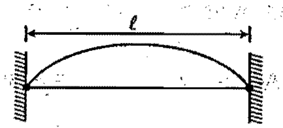
Phase difference between two particle in same loop will be zero and that between two particles in adjacent loops will be π.

(B) and (D) Number of loops will not be integral. Hence neither a node nor an antinode will be formed in in the middle.

Phase difference between two particle in same loop will be zero and that between two particles in adjacent loops will be π.

12.39 9

$$\mu = \frac{3.2\,\text{gm}}{40\,\text{cm}} = \frac{3.2\times10^{-3}}{40\times10^{-2}} = \frac{3.2}{40} = \frac{32}{4000}\,\text{kg/m}$$



$$\ell = \frac{\lambda}{2}$$

$$\Rightarrow \quad \lambda = 2\ell \quad \text{...........(1)}$$

$$f = \frac{v}{\lambda} = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$$

$$\Rightarrow \quad \frac{1000}{64} = \frac{1}{2\times40\times10^{-2}}\sqrt{\frac{T}{32/4000}}$$

$$\Rightarrow \quad \left[\frac{1000}{64}\times2\times40\times10^{-2}\right]^2 \frac{32}{4000} = T \quad \Rightarrow \quad \frac{10000}{64}\times\frac{32}{4000} = T \quad \Rightarrow T = \frac{10}{8}\,\text{N}$$

$$y = \frac{\dfrac{10/8}{10^{-6}}}{\dfrac{.05\times10^{-2}}{40\times10^{-2}}} = \frac{10^7}{8}\,\frac{40}{(.05)} = 10^9\,\text{N/m}^2.$$

[Ans. 1×10^9 Nm²]

12.40 2

$$\mu = Kx = \frac{dM}{dx} \quad \Rightarrow \quad \int_0^M dM = \int_0^\ell Kx\,dx \text{ and } K = \frac{2M}{\ell^2}$$

$$V = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{F}{Kx}} = \frac{dx}{dt} \quad \int_0^\ell \sqrt{x}\,dx = \sqrt{\frac{F}{K}}\int_0^t dt$$

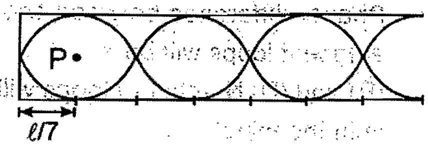
$$\therefore \quad t = \sqrt{\frac{4\ell^3}{9}\cdot\frac{K}{f}} = \sqrt{\frac{4\ell^3}{9F}\cdot\frac{2m}{\ell^2}} = \sqrt{\frac{8M\ell}{9F}} = \sqrt{\frac{8\times45\times1.5}{9\times15}} = 2.$$

12.41 The magnitude of phase difference between the points separated by distance 10 metres $= k \times 10 = [10\pi \times 0.] \times 10 = \pi$

13. SOUND WAVES

13.1 (A)

The figure shows variation of displacement of particles in a closed organ pipe for 3rd overtone.



For third overtone $\ell = \frac{7\lambda}{4}$ or $\lambda = \frac{4\ell}{7}$ or $\frac{\lambda}{4} = \frac{\ell}{7}$

Hence the amplitude at P at a distance $\frac{\ell}{7}$ from closed end is 'a' because there is an antinode at that point

Alternate

Because there is node at x = 0 the displacement amplitude as function of x can be written as $A = a \sin kx = a \sin \frac{2\pi}{\lambda} x$

For third overtone $\ell = \frac{7\lambda}{4}$ or $\lambda = \frac{4\ell}{7}$

$\therefore \quad A = a \sin \frac{7\pi}{2\ell} \frac{\ell}{7} = a \sin \frac{\pi}{2} = a \qquad$ at $x = \frac{\ell}{7} \quad \Rightarrow \quad A = a$

13.2 (A)

When a sound wave gets reflected from a rigid boundary, the particles at the boundary are unable to vibrate. Thus, a reflected wave is generated which interferes with the oncoming wave to produce zero displacement at the rigid boundary. At these points (zero displacement), the pressure variation is maximum. Thus, a reflected pressure wave has the same phase as the incident wave.

13.3 (A)

After a time t, velocity of observer $V_0 = at$

$\therefore \quad f_0 = \left(\frac{V + V_0}{V}\right) f_s \quad = \left(\frac{V + at}{V}\right) f_s$, which is a straight line graph of positive slope.

13.4 (C)

$\left[\left(\frac{v}{v - v_s}\right) - \left(\frac{v}{v + v_s}\right)\right] f_0 = 2 \text{ Hz} \qquad v_s = 0.5 \text{ m/s}$

13.5 (D)

For a stationary observer between wall and source,

freq. from direct source $= \left(\frac{v}{v - v_s}\right) f_0$

frq. from reflected sound $= \left(\frac{v}{v - v_s}\right) f_0$.

So no beats will be heard.

13.6 (B)

(B) $dB = 10 \log \left(\frac{I}{I_0}\right) = 10 \log \left(\frac{K/r^2}{I_0}\right) = 10 [\log (K') - 2 \log r] \qquad \left(K' = \frac{K}{I_0}\right)$

$dB_1 = 10 (\log K' - 2 \log r_1)$

$dB_2 = 10 (\log K' - 2 \log r_2)$

$3 = dB_1 - dB_2 = 20 \log \left(\frac{r_2}{r_1}\right) \Rightarrow (0.3) = \log \left(\frac{r_2}{r_1}\right)^2 \quad \Rightarrow \quad \left(\frac{r_1}{r_2}\right) = \frac{1}{\sqrt{2}}$

13.7 (C)

$$\frac{I_{max}}{I_{min}} = \frac{(a_1+a_2)^2}{(a_1-a_2)^2} = 25 \quad \Rightarrow a_1 + a_2 = 5\,(a_1 - a_2) \quad \Rightarrow \frac{a_1}{a_2} = \frac{3}{2} \Rightarrow \frac{I_1}{I_2} = \left(\frac{a_1}{a_2}\right)^2 = \frac{9}{4}$$

13.8 (A)

$f_1 \lambda_1 = f_2 \lambda_2$ (in same medium)

(300) (1) = (f_2) (1.5)

200 Hz = f_2

13.9 (C)

$v_{max} = \omega A = (2\pi f) A = (2\pi)(440)(10^{-6}) = 2.76 \times 10^{-3}$ m/sec.

13.10 (C)

Apparent frequency

$$n' = n\,\frac{(u+v_w)}{(u+v_w-v_s\cos 60°)} = \frac{510\,(330+20)}{330+20-20\cos 60°} = 510 \times \frac{350}{340} = 525 \text{ Hz Ans.}$$

13.11 (A)

$$\lambda_i = \text{wavelength of the incident sound} = \frac{10u-\frac{u}{2}}{f} = \frac{19u}{2f}$$

$$f_i = \text{frequency of the incident sound} = \frac{10u-u}{10u-\frac{u}{2}} f = \frac{18}{19} f = f_r = \text{frequency of the reflected sound}$$

$$\lambda_r = \text{wavelength of the reflected sound} = \frac{10u+u}{f_r} = \frac{11u}{18f} \times 19 = \frac{11\times 19}{18} \cdot \frac{u}{f}$$

$$\frac{\lambda_i}{\lambda_r} = \frac{19u}{2f} \times \frac{18f}{11\times 19u} = \frac{9}{11} \text{ Ans.}$$

13.12 (A)

For minimum,

$$\Delta x = (2n-1)\frac{\lambda}{2}$$

The maximum possible path difference = distance between the sources = 3m.

For no minimum

$$\frac{\lambda}{2} > 3$$

$\lambda > 6 \qquad \therefore \qquad f = \frac{V}{\lambda} < \frac{330}{6} = 55.$

$\therefore$ If $f < 55$ Hz, no minimum will occur.

13.13 (A)

The speed of sound in air is $v = \sqrt{\frac{\gamma RT}{M}}$

$\frac{\gamma}{M}$ of H_2 is greatest in the given gases, hence speed of sound in H_2 shall be maximum.

13.14 (B)

As $y = A_b \sin(2\pi n_{av} t)$ $\Rightarrow$ where $A_b = 2A\cos(2\pi n_A t)$ $\Rightarrow$ where $n_A = \frac{n_1 - n_2}{2}$

13.15 (B)

For interference at A : S_2 is behind of S_1 by a distance of $100\lambda + \frac{\lambda}{4}$.(equal to phase difference $\frac{\pi}{2}$).

Further S_2 lags S_1 by $\frac{\pi}{2}$. Hence the waves from S_1 and S_2 interfere at A with a phase difference of $200.5\,\pi + 0.5\pi = 201\pi \Rightarrow \pi$

Hence the net amplitude at A is 2a – a = a

For interference at B : S_2 is ahead of S_1 by a distance of $100\lambda + \frac{\lambda}{4}$.(equal to phase difference $\frac{\pi}{2}$).

Further S_2 lags S_1 by $\frac{\pi}{2}$.

Hence waves from S_1 and S_2 interfere at B with a phase difference of $200.5\,\pi - 0.5\pi = 200\pi \Rightarrow 0\pi$.

Hence the net amplitude at A is 2a + a = 3a

Hence $\left(\frac{I_A}{I_B}\right) = \left(\frac{a}{3a}\right)^2 = \frac{1}{9}$

13.16 (D)

To get beat frequency 1, 2, 3, 5, 7, 8, it is possible when other three tuning fork have frequencies 551, 553, 558, etc.

13.17 (B)

$V_s = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{10^{11}}{10.0 \times 10^4}} = 10^3$ m/sec. $\Rightarrow$ $t = \frac{2\ell}{V} = \frac{2 \times 100}{1000} = 0.2$ sec Ans. is (B)

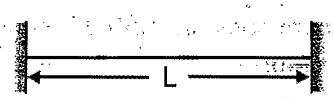
13.18 (D)

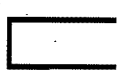
$\xi = A\sin(kx - \omega t)$ $\Rightarrow$ $P_{ex} = -B\frac{d\xi}{dx} = -BAk\cos(kx - \omega t)$

amplitude of P_{ex} = BAk $= (5 \times 10^5)(10^{-4})\left(\frac{2\pi}{0.2}\right) = 5\pi \times 10^2$ Pa So correct ans is (D)

13.19 (B)

Fundamental frequency of wire $(f_{wire}) = \frac{v}{2\ell}$

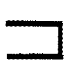


(A) 

$f = \frac{v}{4\ell}, \frac{3v}{4\ell}, \frac{5v}{4\ell}$ cannot match with f_{wire}

(B) $f = \frac{v}{2(2\ell)}, \frac{2v}{2(2\ell)}, \frac{3v}{2(2\ell)}$ its second harmonic $\frac{2v}{2(2\ell)}$ matches with f_{wire}.

(C) , $f = \frac{v}{2(\ell/2)}, \frac{2v}{2(\ell/2)}$ cannot match with f_{wire}

(D)  $f = \frac{v}{4(\ell/2)}, \frac{3v}{4(\ell/2)}$ ..... cannot match with f_{wire}

13.20 (B)

$v_s = 4$ km/sec $\quad v_p = \sqrt{\frac{y}{\rho}} = \sqrt{\frac{12.8 \times 10^{10}}{2000}} = 8000$ m/sec. $= 8$ km/sec

$\frac{\ell}{v_s} - \frac{\ell}{v_p} = 3$ min $\quad = 3 \times 60$ sec.

$\frac{\ell}{4} - \frac{\ell}{8} = 3 \times 60 \quad \ell = 1440$ km

13.21 (B)

Towards right wavelength gets compressed, towards left, wavelength gets expended

13.22 (D)

x_1 and x_2 are in successive loops of std. waves.

so, $\phi_1 = \pi$

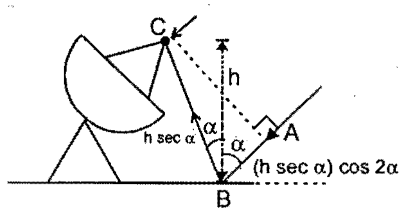
and $\phi_2 = K(\Delta x) = K\left(\frac{3\pi}{2K} - \frac{\pi}{3K}\right) = \frac{7\pi}{6} \quad \frac{\phi_1}{\phi_2} = \frac{6}{7}$

13.23 (A)

$\ell_1 + \varepsilon = \frac{v}{4f_0} \qquad \ell_2 + \varepsilon = \frac{3v}{4f_0} \qquad \ell_3 + \varepsilon = \frac{5v}{4f_0}$

Solving get $\ell_3 = 2\ell_2 - \ell_1$

13.24 (D)



radio wave are electromagnetic wave.

So it get extra phase after reflection

total path difference = AB + BC + $\lambda/2$ = λ for maxima

h sec α cos 2α + h sec α = $\lambda/2$

h sec α $(2\cos^2 \alpha)$ = $\lambda/2$

$h = \frac{\lambda}{2\cos\alpha}$

13.25 (A)

If detector moves x distance ,

distance from direct sound increases by x and distance from reflected sound decreases by x so path difference created = 2x

$2(0.14) = 14\lambda = 14$ c/f $\quad \Rightarrow \quad f = \frac{14 \times 3 \times 10^8}{0.14 \times 2} = 1.5 \times 10^{10}$ Hz.

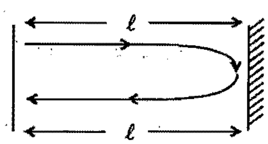
13.26 (D)

Drumming frequency = 40 cycle/min = 40 cycle/ 60 sec

Drumming time period = $\frac{1}{f} = \frac{60\,\text{sec}}{40\,\text{cycle}} = \frac{3}{2}$ sec/cycle

(time duration between consecutive drumming)

During this time interval, if sound goes to mountain and comes back then echo will not be heard distinctly.



$\frac{3}{2} = \frac{2\ell}{v}$ ...... (1)

Now if he moves 90 m. This situation arises at t = 60 cycle/min, $T = \frac{1}{f} = 1$ sec/cycle

so for this case $1 = \frac{2(\ell - 90)}{v}$ ......(2)

Solving equation (1) and (2)

set $\ell = 270$ m

$v = 360$ m/sec.

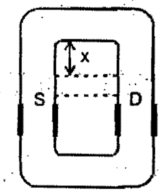
13.27 (B)

$P_0 = B.K.S_0 = B\left(\frac{2\pi}{\lambda}\right)S_0 \qquad \Rightarrow P_0 \propto \frac{1}{\lambda}$

Thus, pressure amplitude is highest for minimum wavelength, other parameters B and S_0 being same for all. From given graphs.

$\lambda_3 < \lambda_2 < \lambda_1$. Hence (B).

13.28 (B)



Path difference introduced due to displacement of tube

= 2x = 10 cm due to one wavelength change maxima / minima will be attained once hence for 10 maxima's

path difference $\Delta P = 10\lambda = 10$ cm so $\lambda = 1$ cm. **Ans.**

13.29 (A)

$x = x_0 \sin(\omega t + \phi) = x_0 \sin \omega t \cos\phi + x_0 \cos\omega t \sin\phi$. Comparing with given equation.

Thus $x_0 \cos\phi = 3$ and $x_0 \sin\phi = 4$

Dividing we get $\tan\phi = \frac{4}{3}$ or $\phi = 53°$

$x_1 = 4\cos\omega t = 4\sin(\omega t + 90°)$

$\Delta\theta = 90° - 53° = 37°$

13.30 (C)

The wavelength of sound source = $\frac{330}{110}$ = 3 metre.

The phase difference betwen interfering waves at P is = $\Delta\phi = \frac{2\pi}{\lambda}(S_2P - S_1P) = \frac{2\pi}{3}(5-4) = \frac{2\pi}{3}$

$\therefore$ Resultant intensity at P = $I_0 + 4I_0 + 2\sqrt{I_0}\sqrt{4I_0}\cos\frac{2\pi}{3} = 3I_0$

13.31 (C)

This problem is a Doppler effect analogy

where f = 20/min, v = 300 m/min

and v_s = 30 m/min $\therefore \quad f' = f\left(\frac{v}{v - v_s}\right) = (20)\left(\frac{300}{300-30}\right) = 22.22\ \text{min}^{-1}$

13.32 (C)

$f = \left(\frac{v + v_0}{v}\right) f_0 = f_0 + \frac{f_0 v_0}{v} \qquad v_0 = gt$

$\therefore \quad f = f_0 + \left(\frac{f_0 g}{v}\right) t$

i.e., f-t graph is a straight line of slope $\frac{f_0 g}{v}$

or $\frac{f_0 g}{v}$ = slope

or $v = \frac{f_0 g}{\text{slope}} = \frac{(10^3)(10)}{\left(\frac{10^3}{30}\right)}$ = 300 m/s

13.33 (B)

let Δl be the end correction.

Given that, fundamental tone for a length 0.1m = first overtone for the length 0.35 m.

$$\frac{v}{4(0.1+\Delta l)} = \frac{3v}{4(0.35+\Delta l)}$$

Solving this equation, we get Δl = 0.025m = 2.5 cm

13.34 (B)

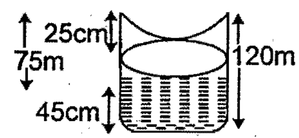
When the skater is approaching the observer. $f_1 = f\left(\frac{v}{v-v_s}\right) > f$ and constant

When it receds from the observer. $f_2 = f\left(\frac{v}{v+v_s}\right) < f$ and constant.

13.35 (ABC)

As $V = \nu\lambda$

$$\lambda = \frac{V}{\nu} = \frac{340}{340} = 1\text{m}$$



first Resonance depth (from upper end)

$$R_1 = \frac{\lambda}{4} = \frac{1}{4}\text{ m} = 25\text{ cm}$$

$$\therefore \quad R_2 = \frac{3\lambda}{4} = \frac{3}{4}\text{m} = 75\text{ cm}$$

$$\therefore \quad R_3 = \frac{5\lambda}{4} = \frac{5}{4}\text{m} = 125\text{ cm}$$

i.e. third resonance does not establish

Now Water is poured,

∴ Minimum length of water column to have the resonance = 45 cm

∴ Distance between two successive nodes = $\frac{\lambda}{2} = \frac{1}{2}$ m = 50 cm

& maximum length of water column to create resonance

i.e. 120 – 25 = 95 cm.

13.36 (ABCD)

$$n = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$$

on increasing or decreasing (T_1 & T_2) significantly we can get result of higher beats.

13.37 (D)

Doppler formula for sound a wave is not symmetric w.r.t speed of source and speed of observer.

13.38 (A)

(A) Propagation of sound in air is an adiabatic process.

13.39 (D)

13.40 (A)

(13.39 to 13.40)

(26) $f_{1i} = f_{1r} = \dfrac{v}{v - v_c} f$ $\Rightarrow$ $f_{2i} = f_{2r} = \dfrac{v}{v + v_c} f$

Now, for driver

$$f_{dr1} = \frac{v + v_c}{v} f_{1r} \qquad \text{and } f_{dr2} = \frac{v - v_c}{v} f_{2r}$$

So, beat frequency = $| f_{dr1} - f_{dr2} |$

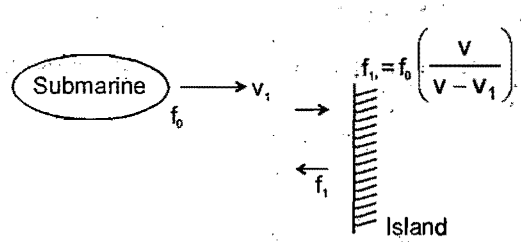
$$= \left| \frac{v + v_c}{v} f_{1r} - \frac{v - v_c}{v} f_{2r} \right| = \left\{ \frac{(v + v_c)^2 - (v - v_c)^2}{(v + v_c)(v - v_c)} \right\} f = \left(\frac{4vv_c}{v^2} \right) f = \left(\frac{4v_c}{v} \right) f .$$

(27) $\lambda_1 = \dfrac{v + v_c}{f}$ $\Rightarrow$ $\lambda_2 = \dfrac{v - v_c}{f}$

$$\lambda_1 - \lambda_2 = \frac{2v_c}{f} \Rightarrow \lambda_1 + \lambda_2 = \frac{2v}{f} \Rightarrow \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2} = \frac{v_c}{v}.$$

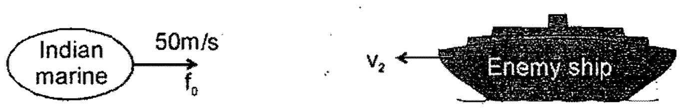
13.41 (B)

$f'' = f_0 \left(\dfrac{v + v_1}{v - v_1} \right)$, v = 1050 $\left[\dfrac{f'' - f_0}{f_0} = 0.1 \right]$



$\dfrac{f'' - f_0}{f_0} = \dfrac{2V_1}{V - V_1} = 0.1$ $v_1 = 50$ m/sec.

13.42 (A)



$f'' = f' \left(\dfrac{v + 50}{v - v_2} \right)$ $f' = f_0 \left(\dfrac{v + v_2}{v - 50} \right)$

$f'' = f_0 \left(\dfrac{(v + v_2)(v + 50)}{(v - v_2)(v - 50)} \right) = 1.21 f_0$ [21% greater then sent waves]

get $v_2 = 50$ m/sec toward Indian submarine

13.43 (B)

$$\lambda' = \frac{\text{v wrt. to observer}}{f'} = \frac{v + v_2}{f_0 \dfrac{(v + v_2)}{(v - 50)}} = \frac{v - 50}{f_0}$$

$$\lambda'' = \frac{v + 50}{f_0 \dfrac{(v + v_2)(v + 50)}{(v - v_2)(v - 50)}} = \frac{(v - v_2)(v - 50)}{f_0 (v + v_2)}$$

$$\frac{\lambda'}{\lambda''} = \frac{v + v_2}{v - v_2} = \frac{1050 + 50}{1050 - 50} = 1.1$$

13.44 (B)

$v = \sqrt{\frac{B}{\rho}} \Rightarrow 1050 = \sqrt{\frac{B}{1000}} \qquad B \approx 10^9\ N/m^2$

13.45 (A)

At $t = 0$, $y = 10^{-2} \sin 2\pi \left(\frac{50}{17}x\right)$

Change in pressure will be maximum where y = 0 at t = 0,

$\frac{2\pi \times 50}{17} x = 0, \pi, 2\pi....$

or $x = 0, 0.17\ m, 0.34m...$

13.46 (D)

$v = \frac{\omega}{k} = \sqrt{\frac{B}{\rho}} \qquad \therefore \qquad B = \rho \left(\frac{\omega}{k}\right)^2$

$\therefore \qquad (\Delta P)_0 = BAK = \frac{\rho\omega^2}{K^2} AK = \frac{\rho A\omega^2}{K}$

Substituting the values, we get

$(\Delta P)_0 = \frac{10^{-2} \times 10^{-2} \times (2\pi \times 1000)^2}{(2\pi \times 50/17)} = 21.36\ N/m^2$

13.47 (a) r, (b) q, (c) p,s, (d) t

13.48 (A) p, q (B) q, s, t (C) r (D) s,q

(A) $y = 4 \sin (5x - 4t) + 3 \cos (4t - 5x + \pi/6)$
is super position of two coherent waves moving in positive direction, so their equivalent will be an another travelling wave.

(B) $y = 10 \cos \left(t - \frac{x}{330}\right) \sin (100) \left(t - \frac{x}{330}\right)$ lets check at any point, say at x = 0,

$y = (10 \cos t) \sin (100\ t)$

at any point amplitude is changing sinusoidally. so this is equation of beats.

(C) $y = 10 \sin (2\pi x - 120t) + 10 \cos (120t + 2\pi x)$ = superposition of two coherent waves travelling in opposite direction.

⇒ equation of standing waves.

(d) $y = 10 \sin (2\pi x - 120\ t) + 8 \cos (118t - 59/30\pi x)$ = superposition of two waves whose frequencies are slightly different ($\omega_1 = 120$, $\omega_2 = 118$)

⇒ equation of Beats.

13.49 1

$3 = 3.\frac{\lambda}{2} \Rightarrow \lambda = 2\ m$

$P_m = 100\ N/m^2$, $V = 330\ m/s$, $\rho_0 = 1\ kg/m^3$

$P_m = B s_0 k = \rho_0 v^2 s_0 \frac{2\pi}{\lambda} \Rightarrow s_0 = \frac{\lambda P_m}{\rho_0 v^2 2\pi} = \frac{2 \times 100}{1 \times 330 \times 330 \times 2\pi} \qquad s_0 = \frac{1}{1089\pi}\ m$

13.50 1

$$3 = 3.\frac{\lambda}{2} \Rightarrow \lambda = 2 \text{ m}$$

$P_m = 100 \text{ N/m}^2$, $V = 330$ m/s, $\rho_0 = 1 \text{ kg/m}^3$

$$B = -\frac{dp}{dv/v} = \frac{dP}{d\rho/\rho} \qquad [\because m = \rho v \Rightarrow O = \frac{d\rho}{\rho} + \frac{dv}{v}]$$

$$d\rho = \frac{\rho . dp}{B} \Rightarrow (d\rho)_{max} = \frac{\rho}{B} (dp)_{max} = \frac{\rho P_m}{B} \qquad (d\rho)_{max} = \frac{\rho . P_m}{\rho v^2} = \frac{1}{1089} \text{ kg/m}^3$$

13.51 4

Let the velocities of car 1 and car 2 be V_1 m/s and V_2 m/s.

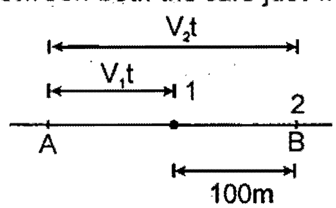
$\therefore$ Apparent frequencies of sound emitted by car 1 and car 2 as detected at end point are

$$f_1 = f_0 \frac{V}{V - V_1}, \quad f_2 = f_0 \frac{V}{V - V_2}$$

$$\Rightarrow \quad 330 = 300 \frac{330}{330 - V_1}, \quad 360 = 300 \frac{330}{330 - V_2}$$

$\Rightarrow$ $V_1 = 30$ m/s and $V_2 = 55$ m/s.

The distance between both the cars just when the 2nd car reach and point B (as shown in figure is)



$100\text{m} = V_2t - V_1t \quad \Rightarrow \quad t = 4$ sec.

13.52 117

$$\lambda_{air} = \frac{V_{air}}{f} = \frac{330}{1000} = 0.33 \text{ m}$$

$$V_{water} = \sqrt{\frac{\beta}{\rho}} = \sqrt{\frac{2.25 \times 10^9}{1000}} = 1.5 \times 10^3 = 1500$$

$$\lambda_{water} = \frac{1500}{1000} = 1.5 \text{ m}$$

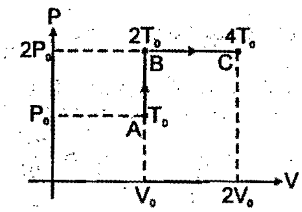
$$\lambda_{water} - \lambda_{air} = 1.5 - 0.33 = 1.17\text{m}.$$

14. HEAT & THERMODYNAMICS

14.1 (B)

Let initial pressure, volume, temperature be P_0, V_0, T_0 indicated by state A in P-V diagram. The gas is then isochorically taken to state B ($2P_0$, V_0, $2T_0$) and then taken from state B to state C ($2P_0$, $2V_0$, $4T_0$) isobarically.

Total heat absorbed by 1 mole of gas



$$\Delta Q = C_v (2T_0 - T_0) + C_P (4T_0 - 2T_0)$$

$$= \frac{5}{2} R T_0 + \frac{7}{2} R \times 2T_0 = \frac{19}{2} RT_0$$

Total change in temperature from state A to C is

$$\Delta T = 3T_0$$

$$\therefore \quad \text{Molar heat capacity} = \frac{\Delta Q}{\Delta T} = \frac{\frac{19}{2} RT_0}{3T_0} = \frac{19}{6} R.$$

14.2 (D)

Heat absorbed by gas in three processes is given by

$$Q_{ACB} = \Delta U + W_{ACB}$$

$$Q_{ADB} = \Delta U$$

$$Q_{AEB} = \Delta U + W_{AEB}$$

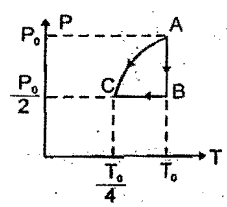
The change in internal energy in all the three cases is same. And W_{ACB} is +ve, W_{AEB} is –ve.

Hence $Q_{ACB} > Q_{ADB} > Q_{AEB}$

14.3 (C)

Process AB is isothermal expansion, BC is isobaric compression

In process CA



$$P \propto \frac{nRT}{P} \quad \Rightarrow P^2 \propto T.$$

14.4 (B)

Higher is the temperature greater is the most probable speed.

14.5 (B)

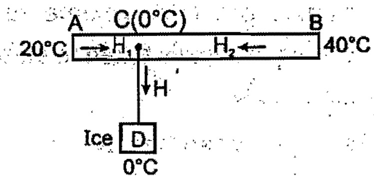
The rate of heat loss by a thin hollow sphere of thickness 'Δx', mean radius 'r' and made of density 'ρ' is given by

$$mS \frac{dT}{dt} = -\varepsilon\sigma A(T^4 - T_0^4)$$

$$(\rho\, 4\pi r^2 \Delta x)\ S \frac{dT}{dt} = -\varepsilon\sigma\, 4\pi r^2 (T^4 - T_0^4) \quad \Rightarrow \quad \frac{dT}{dt} = -\frac{\varepsilon\sigma(T^4 - T_0^4)}{S\Delta x} \text{ is independent of radius}$$

Hence rate of cooling is same for both spheres.

14.6 (D)



Thermal resistance of AC $\left(=\frac{L}{KA}\right)$

$$= \frac{0.1}{336\times1\times10^{-4}} = \frac{10^3}{336} = R \text{ (suppose)}$$

thermal resistance of BC = $\frac{0.2}{336\times10^{-4}} = 2R$

temperature of point C = 0°C

$$\therefore \quad H_1 = \frac{20}{R} \quad ; \quad H_2 = \frac{40}{2R} = \frac{20}{R}$$

$$\therefore \quad H = H_1 + H_2 = \frac{40}{R} = \frac{40\times336}{10^3} = \frac{13440}{10^3} = 13.44 \text{ watt}$$

Rate of melting of ice = $\frac{H}{L_f} = \frac{13.44/4.2}{80}$ g/s = 40 mg/s

14.7 (D)

Since tension is the two rods will be same, hence :

$A_1Y_1\alpha_1\,\Delta\theta = A_2Y_2\alpha_2\,\Delta\theta \Rightarrow \quad A_1Y_1\alpha_1 = A_2Y_2\alpha_2$.

14.8 (B)

$$V_{r.m.s.} = \sqrt{\frac{1^2+0^2+2^2+3^2}{4}} = \sqrt{3.5}.$$

14.9 (A)

We have $\qquad \rho = \frac{PM}{RT}$

$$\frac{P_1M}{RT_1} = \frac{P_2M}{RT_2} \qquad \frac{P_1}{T_1} = \frac{P_2}{2T_1}$$

$$P_2 = 2P_1$$

i.e., change in pressure is 100%.

14.10 (C)

Strain developed :

$\varepsilon = \alpha\,\Delta T = (12\times10^{-6})(50) = 6\times10^{-4}$

Strain will be **negative**, as the rod is in a compressed state.

14.11 (D)

Heat current : $i = -kA\frac{dT}{dx}$

$$i\,dx = -kA\,dT$$

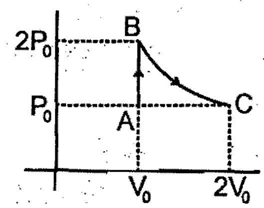
$$i\int_0^{\ell} dx = -A\alpha\int_{T_1}^{T_2} T\,dT$$

$$\Rightarrow \quad i\ell = -A\alpha\frac{(T_2^2-T_1^2)}{2} \quad \Rightarrow \quad i = \frac{A\alpha(T_1^2-T_2^2)}{2\ell}$$

4.12 (D)

BC is isochoric. $V_B > V_A$, $V_B = V_C$, $V_D > V_C$

14.13 (C)



$Q_{AB} = \Delta U_{AB} + W_{AB}$

$W_{AB} = 0$

$\Delta U_{AB} = \frac{f}{2} nR\Delta T \quad \Rightarrow \frac{f}{2}(\Delta PV) \quad \Delta U_{AB} = \frac{5}{2}(\Delta PV)$

$Q_{AB} = 2.5\, P_0V_0$

Process BC $\quad Q_{BC} = \Delta U_{BC} + W_{BC}$

$Q_{BC} = 0 + 2P_0V_0 \log 2$

$= 1.4\, P_0V_0$

$Q_{net} = Q_{AB} + Q_{BC} = 3.9\, P_0V_0$

14.14 (C)

$$\frac{X-(-125)}{500} = \frac{Y-(-70)}{40}$$

For $Y = 50$ $\quad X = 1375.0°X$

14.15 (B)

Rate of radiation per unit area is proportional to (T^4)

$\therefore \quad P \propto AT^4 \quad \Rightarrow \quad P \propto r^2$.

Also $\quad ms\frac{dT}{dt} \propto AT^4 \quad \therefore \quad \frac{dT}{dt} = R \propto \frac{1}{r}$

(because $m = (V\rho) \propto r^3$ and $A \propto r^2$)

$\therefore \quad PR \propto r$

14.16 (A)

For a block body, wavelength for maximum intensity :

$\lambda \propto \frac{1}{T}$ & $P \propto T^4 \Rightarrow P \propto \frac{1}{\lambda^4} \Rightarrow P' = 16\, P. \quad \therefore \quad P'T' = 32PT$

14.17 (A)

Since , e = a = 0.2

(Since , a = (1 – r – t) = 0.2 for the body B)

E = (100) (0.2) = 20 W/m²

14.18 (C)

Thermal equilibrium means same temperature.

14.19 (C)

$\frac{C}{5} = \frac{F-32}{9} \Rightarrow \frac{\Delta C}{5} = \frac{\Delta F}{9} \Rightarrow \quad \Delta C = \frac{5}{9}\Delta F \quad \Rightarrow \quad \Delta C = \Delta K$

14.20 (A)

(A) PV = RT for 1 mole

$W = \int PdV = \int \frac{RT}{V} dV$

$V = CT^{2/3}$

$\therefore \quad dV = \frac{2}{3} CT^{-1/3} dT \quad$ or $\quad \frac{dV}{V} = \frac{2}{3}\frac{dT}{T}$

$\therefore \quad W = \int_{T_1}^{T_2} RT\left(\frac{2}{3}\right)\frac{dT}{T} = \frac{2}{3}R(T_2 - T_1) = 166.2\ J$

14.21 (A)

$$\Delta s\Delta U = \frac{nfR}{2}\Delta T$$

$$\Delta U \propto \Delta T_i$$

Since volume is same in all three process therefore temperature will be least having least pressure.

14.22 (C)

log P = m log V where m is slope

$$m = \frac{2.38-2.10}{1.1-1.3} = -1.4$$

$$\log P = -1.4 \log V$$

$$\log PV^{1.4} = 0$$

$$PV^{1.4} = k$$

14.23 (B)

Rate of heat loss = $\sigma eA(T^4 - T_s^4)$

$$-ms\frac{dT}{dt} = \sigma eA(T^4 - T_s^4)$$

$$-\frac{dT}{dt} = \frac{5.8\times10^{-4}\times i\times T(0.08)^2((500)^4(300)^4)}{10\times4.2\times90} \Rightarrow \frac{-dT}{dt} = 0.066\ ^\circ C/sec.$$

14.24 (ABCD)

In the equilibrium position the net force on the partition will be zero.
Hence pressure on both sides are same.
Hence, (A) is correct.
Initially, PV = nRT

$$n_1 = \frac{P_1V_1}{RT_1} = \frac{PV}{RT}$$

& $$n_2 = \frac{(2P)(2V)}{RT} = 4\frac{PV}{RT} \Rightarrow n_2 = 4n_1$$

Moles remains conserved.
Finally, pressure becomes equal in both parts.
Using, $P_1V_1 = n_1RT_1$
$P_2V_2 = n_2RT_2$
$\because$ $P_1 = P_2$ & $T_1 = T_2$

$$\therefore \frac{V_1}{V_2} = \frac{n_1}{n_2} = \frac{1}{4} \Rightarrow V_2 = 4V_1$$

Also $V_1 + V_2 = 3V \Rightarrow V_1 + 4V_1 = 3V \Rightarrow V_1 = \frac{3}{5}V$ And $V_2 = \frac{12}{5}V$

Hence (B) and (C) are correct.
In compartment (I) :
$P_1'V_1 = n_1RT_1$

$$P_1'\left(\frac{3V}{5}\right) = \left(\frac{PV}{RT}\right)R(T)$$

$$P_1' = \frac{5PV}{3V} = \frac{5}{3}P$$

Hence (D) is also correct.

14.25 (BD)

$$\frac{P^2}{\rho} = k \quad \Rightarrow \quad \frac{P^2RT}{PM} = k$$

$$\Rightarrow \quad PT = \left(\frac{kM}{R}\right) \quad \text{.........(i)}$$

$$\frac{P^2}{\rho} = \frac{P'^2}{\rho/2} \qquad P' = \frac{P}{\sqrt{2}}$$

Hence from (i) $T' = T\sqrt{2}$. PT = constant hence P–T curve is a hyperabola.

14.26 (AD)

$$V_{r.m.s.} = \sqrt{\frac{3kT}{m}}$$

Since PV = nRT therefore P and V both can change simultaneously keeping temperature constant.

14.27 (AC)

Slope of graph is smaller in the solid state i.e., temperature is rising slower, hence higher heat capacity.

The transition from solid to liquid state takes lesser time, hence latent heat is smaller.

14.28 (ACD)

$$\frac{\Delta A}{A} \times 100 = 2\left(\frac{\Delta \ell}{\ell}\right) \times 100$$

$\Rightarrow$ % increase in Area = 2 × 0.2 = 0.4

$$\frac{\Delta V}{V} \times 100 = 3 \times 0.2 = 0.6\ \%$$

Since $\Delta l = l\,\alpha\,\Delta T$

$$\frac{\Delta \ell}{\ell} \times 100 = \alpha\,\Delta T \times 100 = 0.2$$

$$\Rightarrow \quad \alpha = 0.25 \times 10^{-4}\ /°C$$

14.29 (AD)

$$\Delta V_L = \Delta V_V \Rightarrow Y_LV_L = Y_VV_V \quad \text{or} \quad \frac{Y_L}{Y_V} = \frac{V_V}{V_L} \Rightarrow \text{but} \quad V_V > V_L \Rightarrow \quad Y_L > Y_V$$

14.30 (BD)

Every object emit and absorb the radiations simultaneously, if energy emitted is more than energy absorbed temperature falls and vice versa.

14.31. (D)

Equivalent thermal conductivity of two identical rods in series is given by

$$\frac{2}{K} = \frac{1}{K_1} + \frac{1}{K_2}$$

If $K_1 < K_2$, then $K_1 < K < K_2$

Hence statement 1 is false.

14.32 (C)

From Wein's law $\lambda_m T$ = constant i.e., peak emission wavelength $\lambda_m \propto \frac{1}{T}$. Hence as T increase λ_m decreases.

14.33 (A)

If the rate at which molecules of same mass having same rms velocity striking a wall decreases, then the rate at which momentum is imparted to the wall decreases. This results in lowering of pressure. Hence statement-2 is correct.
In statement-1 the rms velocity of gas remains same on increasing the volume of container by piston, since the given process is isothermal. Now the piston is at a greater distance from opposite wall and hence time taken by gas molecules from near the opposite wall to reach the piston will be more. Thus rate of molecules striking the piston decreases. Hence statement-1 is correct and statement-2 is correct explanation.

14.34 (C)

Heat given : $\Delta Q = n_1 C_{V_1} \Delta T \rightarrow$ For gas A

& for Gas B $\rightarrow$ $\Delta Q = n_2 C_{V_2} \Delta T$

($\because$ For same heat given, temperature rises by same value for both the gases.)

$\Rightarrow \quad n_1 C_{V_1} = n_2 C_{V_2}$ ....(1)

Also, $(\Delta P_B)V = n_2 R \Delta T$ and $(\Delta P_A)V = n_1 R \Delta T$

$$\Rightarrow \quad \frac{n_1}{n_2} = \frac{\Delta P_A}{\Delta P_B} = \frac{2.5}{1.5} = \frac{5}{3} \quad \Rightarrow \quad n_1 = \frac{5}{3} n_2$$

Substituting in (1)

$$\frac{5}{3} n_2 C_{V_1} = n_2 C_{V_2} \quad \Rightarrow \quad \frac{C_{V_2}}{C_{V_1}} = \frac{5}{3} = \frac{(\frac{5}{2}R)}{(\frac{3}{2}R)}$$

Hence, Gas B is diatomic and Gas A is monoatomic.

14.35 (D)

$$\text{Since} \quad n_1 = \frac{5}{3} n_2 \qquad \text{Therefore} \quad \frac{125}{M_A} = \frac{5}{3}\left(\frac{60}{M_B}\right)$$

(From experiment 1 : W_A = 25 gm & W_B = 60 gm)

$\Rightarrow \quad 5M_B = 4M_A$

The above relation holds for the pair–Gas A : Ar and Gas B : O_2.

14.36 (B)

$$n_A C_{V_A} \times 300 + n_B C_{V_B} \times 300 = n_A C_{V_A} T + n_B C_{V_B} T$$

(number of moles remains same)

$\Rightarrow \quad T = 300$ K

(It could also be seen directly that temperature finally will be 300 K, since no heat exchange takes place between those gases as their initial temperatures are same)
Since, volume remains same but number of moles increases.
Therefore, pressure increases.

14.37 (A) p,r,s (B) q,t ; (C) p,r,s (D) q,r,t

(A) If $P = 2V^2$, from ideal gas equation we get

$2V^3 = nRT$

$\therefore$ with increase in volume

(A) Temperature increases implies dU = +ve

$\therefore$ dW = +ve

Hence dQ = dU + W = +ve

(B) If PV^2 = constant, from ideal gas equation we get VT = K (constant)

On decreasing temperature, as volume of an ideal gas increases

$\therefore$ dW = +ve

Now $dQ = dU + PdV = nC_vdT - \frac{PK}{T^2}dT$ $[\because dV = -\frac{K}{T^2}dT]$

$= nC_vdT - \frac{PV}{T}dT = n(C_v - R)\,dT$

$\therefore$ with increase in temperature dT = +ve

and since $C_v > R$ for monoatomic gas. Hence dQ = +ve as temperature is increased

(C) $dQ = nC\,dT = nC_v\,dT + PdV$

$\Rightarrow$ $n(C_v + 2R)\,dT = nC_vdT + PdV$

$\therefore$ $2nRdT = PdV$ $\therefore \frac{dV}{dT} = +ve$

Hence with increase in temperature volume increases and vice versa.

$\therefore$ dQ = dU + W = +ve

(D) $dQ = nC\,dT = nC_v\,dT + PdV$

or $n(C_v - 2R)dT = nC_rdT + PdV$

or $-2nRdT = PdV$ $\therefore \frac{dV}{dT} = -ve$

On decreasing temperature, as volume of an ideal gas increases

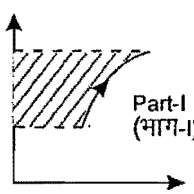
$\therefore$ dW = +ve

Also $dQ = n(C_v - 2R)dT$

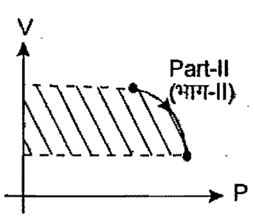
with increase in temperature dT = +ve but $C_v < 2R$ for monoatomic gas. Therefore dQ = –ve with increase in temperature.

14.38 (A) s ; (B) q,t ; (C) r,t ; (D) q,t

in (A), V is on vertical axis.



As V is icreasing, W is positive.



V is decreasing, W is negative.

As negative work in part-II is greater than positive work in part-I, net work during the process is negative. Using PV = nRT and as $V_{remains}$ same for initial and final points of the process, it is obvious that final temp. is greater than initial temperature as pressure has increased. Therefore dU is positive. Hence option (S) is connected with (A).

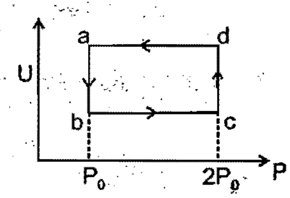
As $T = \frac{PV}{nR}$

$\therefore$ From graph we can say

$T_{final} < T_{initial}$ in part B,C & D

Similar arguments can be applied to other graphs.

14.39 40



Change in internal energy for cyclic process $(\Delta U) = 0$

for process $a \to b$, (P – constant)

$$W_{a\to b} = P\Delta V = nR\Delta T = -400\ R$$

for process $b \to c$ (T – constant)

$$W_{b\to c} = -2R((300)\ell n2$$

for process $c \to d$ (P – constant)

$$W_{c\to d} = +400\ R$$

for process $d \to a$ (T – constant)

$$W_{d\to a} = +2R\ (500)\ell n2$$

Net work $(\Delta W) = W_{a\to b} + W_{b\to c} + W_{c\to d} + W_{d\to a}$

$$\Delta W = 400\ R\ell n2$$

∴ $dQ = dU + W$, first law of thermady names.

∴ $dQ = 400\ R\ell n2$.

14.40 4

Energy supply by resistance

$$= i^2.R$$

By first law of thermodynamics

$$\Delta Q = \Delta U + \Delta W$$

$$\Delta U = 0$$

∴ $$i^2 R = mg\left(\frac{dx}{dt}\right)$$

$$v = \frac{F^2R}{mg}$$

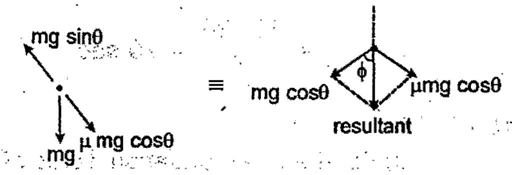
$$= \frac{2\times2\times10}{1\times10}$$

$$v = 4\ m/s.$$

15. FLUIDS & PROPERTIES OF MATTER

15.1 (A)

figure shows forces acting on a 'particle' on the surface, with respect to vessel.



(mg sin θ & μ mg cos θ are pseudo forces).

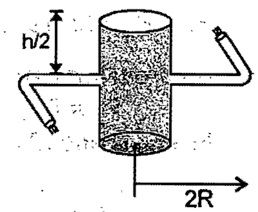
$\tan\phi = \mu$

$\therefore \quad \phi = \tan^{-1}\mu$.

ϕ is angle between normal to the inclined surface and the resultant force. The same angle will be formed between the surface of water & the inclined surface.

{∵ free surface is ⊥ to the resultant force acting on it.}

15.2 (A)



Velocity of efflux of water (v) = $\sqrt{2g\left(\frac{h}{2}\right)} = \sqrt{gh}$

force on ejected water = Rate of change of momentum of ejected water.

$= \rho\,(av)\,(v) \qquad = \rho\, av^2$

Torque of these forces about central line

$= (\rho av^2)\, 2R \,.\, 2 \qquad = 4\rho av^2 R = 4\rho\, agh\, R$

15.3 (A)

(A) Let ρ_S, ρ_L be the density of silver and liquid. Also m and V be the mass and volume of silver block.

$\therefore$ Tension in string = mg – bouyant force

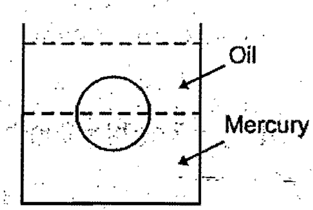
$T = \rho_S Vg - \rho_L Vg = (\rho_S - \rho_L)\, Vg$

Also $\quad V = \dfrac{m}{\rho_S}$

$$\therefore \quad T = \left(\frac{\rho_S - \rho_L}{\rho_S}\right) mg = \frac{(10-0.72)\times 10^3}{10\times 10^3} \times 4 \times 10 = 37.12 \text{ N.}$$

15.4. (C)

Weight = Buoyant force



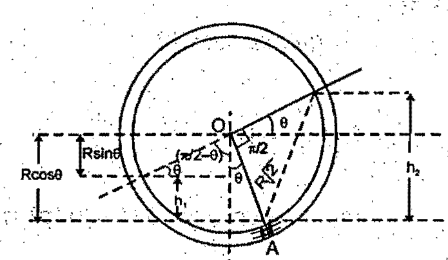
$$V\rho_m g = \frac{V}{2}\rho_{Hg} g + \frac{V}{2}\rho_{oil} g$$

$$\rho_m = \frac{\rho_{Hg} + \rho_{oil}}{2} = \frac{13.6+0.8}{2} = \frac{14.4}{2} = 7.2$$

15.5 (A)

Pressure at 'A' from both side must balance.

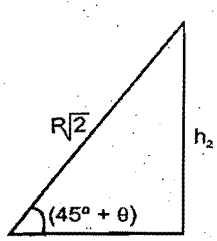
Figure is self–explanatory.



$\sigma h_2 g = \rho h_1 g$

$\sigma \sqrt{2}R \sin(45° + \theta) = \rho R [\cos\theta - \sin\theta]$

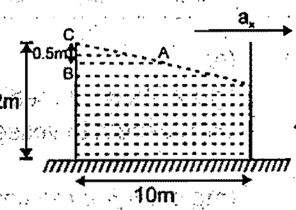
$\sigma [\cos\theta + \sin\theta] = \rho[\cos\theta - \sin\theta]$



$$\tan\theta = \frac{\rho - \sigma}{\rho + \sigma}$$

15.6 (A)

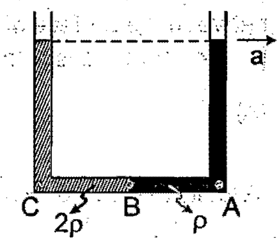
$v = u + a_x t$, $a_x = \dfrac{v}{t}$ $\tan\theta = \dfrac{a_x}{g} = \dfrac{v}{tg} = \dfrac{0.5}{5}$ (in triangle ABC)



$\Rightarrow \quad t = \dfrac{10 \times 20}{10} = 20$ sec.

15.7 (B)

(B) For the given situation, liquid of density 2 ρ should be behind that of ρ.
From right limb :



$P_A = P_{atm} + \rho g h$

$P_B = P_A + \rho a \dfrac{\ell}{2} = P_{atm} + \rho g h + \rho a \dfrac{\ell}{2}$

$P_C = P_B + (2\rho) a \dfrac{\ell}{2} = P_{atm} + \rho g h + \dfrac{3}{2} \rho a \ell$ .... (1)

But from left limb :

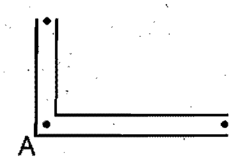
$P_C = P_{atm} + (2\rho) g h$ .... (2)

From (1) and (2) :

$P_{atm} + \rho g h + \dfrac{3}{2} \rho a \ell = P_{atm} + 2 \rho g h \quad \Rightarrow \quad h = \dfrac{3a}{2g} \ell$ Ans.

15.8 (A)

(A) No sliding ⇒ pure rolling



Therefore, acceleration of the tube = 2a (since COM of cylinders are moving at 'a')

$P_A = P_{atm} + \rho (2a) L$ (From horizontal limb)

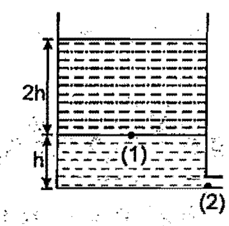
Also ; $P_A = P_{atm} + \rho g H$ (From vertical limb)

$\Rightarrow \quad a = \dfrac{gH}{2L}$ Ans.

15.9 (B)

(B) Pressure at (1) :

(B) (1)



$P_1 = P_{atm} + \rho g (2h)$

Applying Bernoulli's theorum between points (1) and (2)

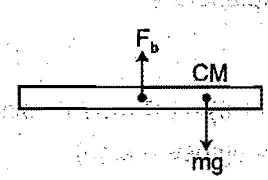
$[P_{atm} + 2 \rho g h] + 2\rho g(h) + \dfrac{1}{2} (2 \rho) (0)^2$

$= P_{atm} + (2 \rho) g (0) + \dfrac{1}{2} (2 \rho) v^2 \quad \Rightarrow \quad v = 2\sqrt{gh}$ Ans.

15.10 (B)

Torque about CM :

$F_b \cdot \dfrac{\ell}{4} = I\alpha$



$\Rightarrow \quad \alpha = \dfrac{1}{I} (\pi r^2) (\ell) (\rho) (g) . \dfrac{\ell}{4}$

$\alpha = \dfrac{\pi r^2 \ell^2 g \rho}{4 I}$

'α' will be same for all points.
Hence (B).

15.11 (C)
by dimensional analysis, (c) is the only correct answer.

15.12 (D)

$$V_0 = \sqrt{2gh} \quad \Rightarrow \quad V_2 = \sqrt{2g\frac{h}{\sqrt{2}}} = \frac{V_0}{\sqrt[4]{2}}$$

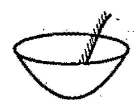
15.13 (C)
Pressure at all points in stream will be atmospheric.

15.14 (D)
Volume of water filled in tank in
t = 15 sec.

$$V = \int_0^{15} A \times 10[1 - \sin\frac{\pi}{30}t]dt \qquad V = 10A[t + \left[\frac{\cos\pi/30t}{\pi/30}\right]_0^{15}$$

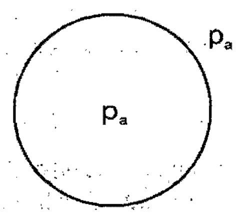
$$V = 10[15 - \frac{30}{\pi}]A \qquad h = \frac{V}{10A} = \left[15 - \frac{30}{\pi}\right]m$$

15.15 (B)
Figure shows one of the legs of the mosquito landing upon the water surface.
Therefore, T. $2\pi a \times 8 = W$ = weight of the mosquito.



15.16 (B)

Inside pressure must be $\frac{4T}{r}$ greater than outside pressure in bubble. This excess pressure is provided by charge on bubble.



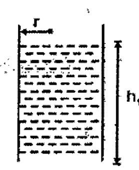
$$\frac{4T}{r} = \frac{\sigma^2}{2\varepsilon_0}$$

$$\frac{4T}{r} = \frac{Q^2}{16\pi^2 r^4 \times 2\varepsilon_0} \quad \ldots\ldots \left[\sigma = \frac{Q}{4\pi r^2}\right]$$

$$Q = 8\pi r\sqrt{2rT\varepsilon_0}$$

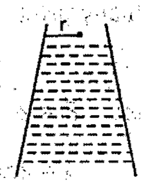
15.17 (C)
The force exerted by film on wire or thread depends only on the nature of material of the film and not on its surface area. Hence the radius of circle formed by elastic thread does not change.

15.18 (C)
As weight of liquid in capillary is balanced by surface tension,
then $T \times 2\pi r = \pi r^2 h_1 \rho g$ (for uniform r radius tube)



$$h_1 = \frac{2T}{r\rho g}$$

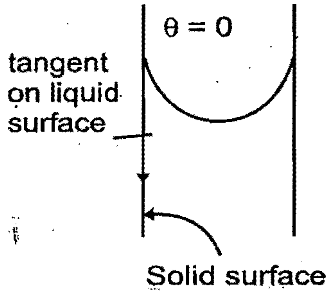
but weight of liquid in tapered tube is more than uniform tube of radius r, then in order to balance $h < h_1$



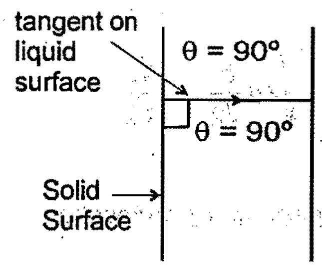
$$h < \frac{2T}{r\rho g}$$

15.19 (B)

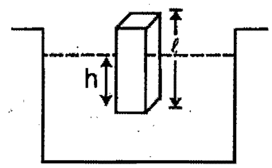
For hemispherical shape –



For flat surface -



15.20 (D)



Balancing the force :

$T.4a \cos 120° + \ell \rho a^2 g = a^2 h \rho g$

$T.2a = a^2 \rho g (\ell - h)$

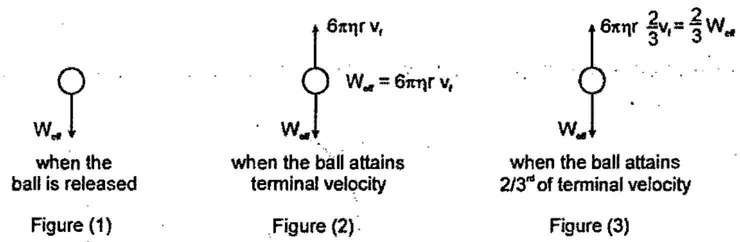
$(\ell - h) = \dfrac{2T}{a\rho g}$

15.21 (A)

Viscous force = mg sin θ

$\therefore \quad \eta A \dfrac{v}{t} = mg \sin\theta \quad$ or $\quad \eta a^2 \dfrac{v}{t} = a^3 \rho g \sin\theta \quad \Rightarrow \quad \eta = \dfrac{t\rho g \sin\theta a}{v}$

15.22 (A)



When the ball is just released, the net force on ball is W_{eff} (= mg – buoyant force)

The terminal velocity 'v_f' of the ball is attained when net force on the ball is zero.

$\therefore$ Viscous force $6\pi\eta r v_f = W_{eff}$

When the ball acquires $\dfrac{2}{3}$rd of its maximum velocity v_f

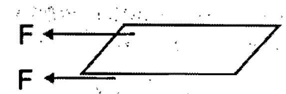
the viscous force is $= \dfrac{2}{3} W_{eff}$

Hence net force is $W_{eff} - \dfrac{2}{3}W_{eff} = \dfrac{1}{3}W_{eff}$

$\therefore$ required acceleration is = $\dfrac{a}{3}$

15.23 (B)

Velocity gradient = $\dfrac{0.5 \times 2}{2.5 \times 10^{-2}}$



Also, $F = 2\eta A \dfrac{dv}{dz} = 2 \times \eta \times (0.5) \dfrac{0.5}{1.25 \times 10^{-2}}$

$\Rightarrow \quad \eta = 2.5 \times 10^{-2}$ kg – sec/m²

15.24 (A)

(A) From continuity equation, velocity at cross-section (1) is more than that at cross-section (2).

Hence ; $P_1 < P_2$

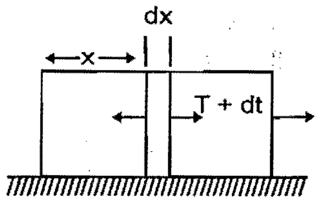
Hence (A)

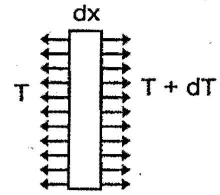
15.25 (A)

(A) $\Delta l = \frac{F\ell}{AY}$ $\quad \frac{\Delta \ell}{(F/A)} = \frac{\ell}{Y}$ = slope of curve $\Rightarrow \quad \frac{\ell}{Y} = \frac{(4-2)\times10^{-3}}{4000\times10^{3}}$

Given $l = 1\text{m} \rightarrow Y = \frac{4000\times10^{3}}{2\times10^{-3}} = 2\times10^{9}\ \text{N/m}^2$

15.26 (B)





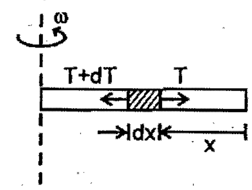
Acceleration $\quad a = F/m$

then $\quad T = \frac{mx}{\ell}\times\frac{F}{m} = \frac{Fx}{\ell}$

Extension in 'dx' element – $d\delta = \frac{Tdx}{AY} = \frac{Fxdx}{\ell AY}$

Total extension $\delta = \int_0^{\ell}\frac{Fxdx}{\ell AY} = \frac{F\ell}{2AY}$

15.27 (B)



$dT = dm(\ell - x)\omega^2 \qquad dT = \frac{m}{\ell}.dx(\ell - x)\omega^2$

$$\Rightarrow \int_0^{T} dT = \int_0^{\ell/2}\frac{m\omega^2}{\ell}(\ell - x)\,dx$$

$$= \frac{m\omega^2}{\ell}\left[\ell x - \frac{x^2}{2}\right]_0^{\ell/2} = \frac{m\omega^2}{\ell}\left[\frac{\ell^2}{2} - \frac{\ell^2}{8}\right]$$

∴ Tension at mid point is :

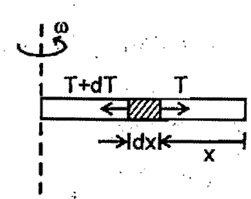
$T = \frac{3}{8}m\ell\omega^2 \quad \Rightarrow$ stress $= \frac{3m\ell\omega^2}{8A} \quad \Rightarrow$ strain $= \frac{3m\ell\omega^2}{8AY}$

<u>ALTERNATIVELY</u>

Tension at mid point can be found by using $F_{ext} = ma_{cm}$

$T = \frac{m}{2}.\left(\omega^2\frac{3\ell}{4}\right) = \frac{3}{8}m\omega^2\ell$

(B)



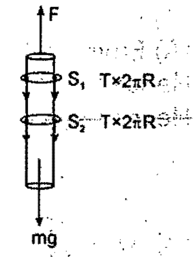
$dT = dm(\ell - x)\omega^2 \qquad dT = \frac{m}{\ell}.dx(\ell - x)\omega^2$

$$\Rightarrow \int_0^{T} dT = \int_0^{\ell/2}\frac{m\omega^2}{\ell}(\ell - x)\,dx$$

$$= \frac{m\omega^2}{\ell}\left[\ell x - \frac{x^2}{2}\right]_0^{\ell/2} = \frac{m\omega^2}{\ell}\left[\frac{\ell^2}{2} - \frac{\ell^2}{8}\right]$$

15.28 (B)

The free body diagram of the capillary tube is as shown in the figure.
Net force F required to hold tube is :

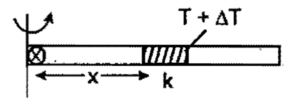


Free body daigram of capillary tube

F = force due to surface tension at cross-section
$(S_1 + S_2)$ + weight of tube = $(2\pi RT + 2\pi RT) + mg = 4\pi RT + mg$

15.29 (A)

$$-\int_T^0 \Delta T = \int_0^\ell \frac{m}{\ell} dx\omega^2 x \quad \Rightarrow \quad T = \frac{m}{\ell}\omega^2\frac{x^2}{2}$$



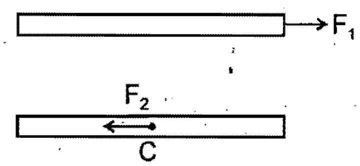
$$\Rightarrow \quad Y = \frac{F\ell}{A\Delta\ell} \qquad \Delta\ell = \frac{F\ell}{Ay}$$

$$\Delta\ell = \frac{\frac{m}{\ell}\frac{\omega^2 x^2}{2}dx}{AY} \qquad \Delta\ell = \frac{m}{\ell}\frac{\omega^2\ell^3}{6AY} \qquad \Delta\ell = \frac{\rho\omega^2\ell^3}{6y}$$

$\Delta\ell \propto \omega^2$
$\omega_2 = 2\omega_1$

15.30 (C)

The force F_1 causes extension in rod.
F_2 causes compression in left half of rod and an equal extension in right half of rod. Hence F_2 does not effectively change length of the rod.



15.31 (AB)

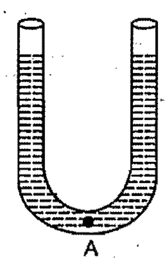
The maximum horizontal distance from the vessel comes from hole number 3 and 4

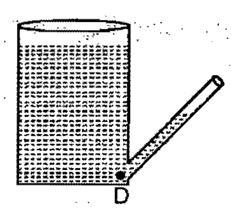
$v = \sqrt{2gh} \rightarrow$ h is height of hole from top.

horizontal distance $x = vt = \sqrt{2gh}\sqrt{\frac{2(H-h)}{g}}$ $\quad x = 2\sqrt{h(H-h)}$

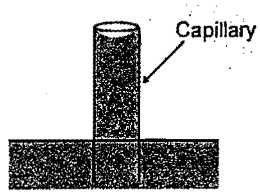
15.32 (AC)

The pressure at any point can never have different values. Hence (A) & (D) are not possible. (Calculate the pressures at points A & D from both their left and right)





In case of insufficient length of capillary tube the shape of meniscus is as below :



(Stationary fluid)

15.33 (A)

Since the net bouyant force on the brick completely submerged in water is independent of its depth below the water surface, the man will have to exert same force on both the bricks. Hence Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

15.34 (C)

Tension at a point on rod of (length L) at a distance x from point of application of force is

$$T = F\left(1 - \frac{x}{L}\right) \text{ in both cases.}$$

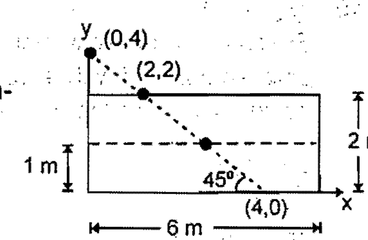
Hence weight has no effect on tension in situation of figure (ii).
Extension in rod occurs due to force acting at any point on the rod. In certain cases when net force acts at the centre of rod like weight, extension due to this force may not occur like the given case.

15.35 (D)

$$\frac{dy}{dx} = \frac{a_x}{a_y + g} = \frac{g/2}{-g/2 + g} = 1 \quad \ldots\ldots \quad \text{(effective g will be } g - a = g/2\text{]}$$

15.36 (C)

As the slope of free surface is 45°. Thus free surface passes through centre of box and having co–ordinates (2,2) at top of box. Thus length of exposed top part- = 6 – 2 = 4 m.



15.37 (B)

$P = P_A + \rho g_{eff} h = 10^5 + 1000 \times (10/2) \times 1 = 0.105$ MPa

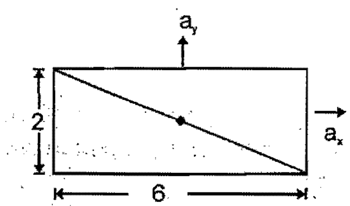
15.38 (B)

$p = (10^5 + 10^3 \times 10/2 \times 4)\ N/m^2 = [0.1 + 0.02]$ MPa $= 0.12$ MPa

15.39 (A)

As maximum slope of free surface is $\frac{1}{3}$ for the condition of non–exposure of bottom of box, then

$$\frac{a_x}{a_y + g} = \frac{1}{3}$$



as $a_x = g/2$
$3a_x = a_y + g$
$a_y = g/2$, thus g/2 upward.

15.40 (B)

$F = \rho A(V_0 - 0)^2 [1 - \cos 180°] = 2\rho A v^2 = 2 \times 1000 \times 2 \times 10^{-4} \times 10 \times 10 = 40$ N

15.41 (C)

$F = 2\rho A (V_0 - u)^2$ $\quad$ u = speed of cart

$$m\frac{du}{dt} = 2\rho A (v_0 - u)^2$$

$$\int_0^u \frac{du}{(V_0 - u)^2} = \frac{2\rho A}{m}\int_0^t dt \Rightarrow \left[\frac{2\rho A}{m} = \frac{2 \times 10^3 \times 2 \times 10^{-4}}{10} = \frac{4}{100}\right] \Rightarrow \left[\frac{1}{V_0 - u}\right]_0^u = \frac{2\rho At}{m}$$

$$\frac{1}{V_0 - u} - \frac{1}{V_0} = \frac{2\rho At}{m} = \frac{4t}{100} \quad \ldots\ldots\ldots\ldots(1)$$

at $\quad t = 10$ sec. $\rightarrow \quad \frac{1}{V_0 - u} = \frac{4}{10} + \frac{1}{10} = \frac{1}{2}$

$V_0 - u = 2 \quad \Rightarrow \quad u = 8$ m/sec.

15.42 (D)

$F = 2\rho A (V_0 - u)^2 = 2 \times 10^3 \times 2 \times 10^{-4} (10 - 8)^2 = 2 \times 10^3 \times 2 \times 10^{-4} \times 4 = 1.6$ N

$a = \frac{F}{M} = 0.16$ m/sec^2

15.43 (A)

From equation (1)

$\frac{1}{V_0 - u} - \frac{1}{V_0} = \frac{4t}{100} \Rightarrow \frac{1}{8} - \frac{1}{10} = \frac{4t}{100} \Rightarrow \frac{2}{80} = \frac{4t}{100}, \quad t = \frac{10}{16}$ sec.

15.44 (C)

$F = 2\rho A(V_0 - u)^2$

$= 2 \times 10^3 \times 2 \times 10^{-4} \times 25 = 10$ N

$P = F.u = 10 \times 5 = 50$ W.

15.45 (A) p ; (B) q ; (C) t ; (D) s

Pressure varies with height $\Rightarrow P = \rho gh$

and is horizontal with acceleration $\Rightarrow P = \rho \ell a$

so on (A) ρgh part is zero while average of ρax is

$$\left[\frac{0 + \rho \ell a}{2}\right][\ell^2] = \frac{\ell \rho a}{2}(\ell^2) = \frac{(\rho \ell^3)}{2} a = \frac{ma}{2}$$

In (B) $\rho \ell a$ part is zero while average of ρgx is

$$\left[\frac{0 + \rho g \ell}{2}\right]\left[\ell^2\right] = \frac{\rho g}{2}(\ell^3) = \frac{\rho(\ell^3)}{2}(g) = \frac{mg}{2}$$

Similarly for other part.

15.46 (A) q ; (B) p ; (C) r ; (D) s

(A) On ABCD avg pressure $= \left[\frac{0 + \rho_1 gh}{2}\right]$

So $F = \left[\frac{\rho_1 gh}{2}\right][\ell h] = \frac{\rho_1 gh^2 \ell}{2}$

(B) No contact of ρ_2 and not any pressure on ABCD due to ρ_2

(C) On CDEF due to ρ_1, at every point pressure is $\rho_1 gh$ so average is also $\rho_1 gh$

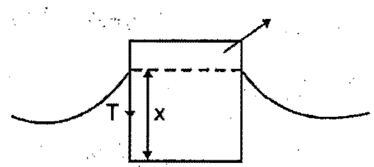
so $F = (\rho_1 gh)(h\ell) = \rho_1 gh^2 \ell$

(D) On CDEF due to ρ_1 constant but ρ_1 is variable so average is ρ_1 will be taken.

$$\left[\rho_1 gh + \left\{\frac{0 + \rho_2 gh}{2}\right\}\right][h\ell]$$

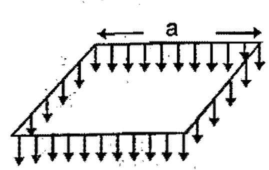
15.47 23

Downward force = Buoyant force



$Mg + 4Ta = a^2 x\rho g \qquad x = \frac{Mg + 4Ta}{a^2 \rho g}$

$= \frac{20 \times 10^{-3} \times 10 + 4 \times 70 \times 10^{-3} \times 30 \times 10^{-2}}{9 \times 10^{-4} \times 10^3 \times 10}$



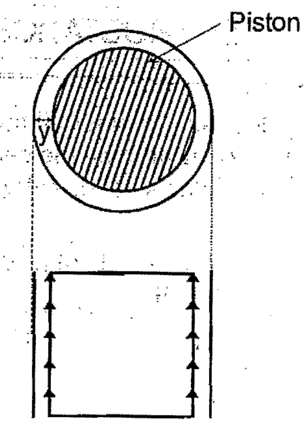
$= \frac{0.2 + 84}{9} = \frac{2084}{9} = 2315$ m $= 2315$ cm

Ans. $\left[x = \frac{mg + 4aT}{a^2 \rho g}\right] = 23$ cm

15.48 25

Thickness of annular space = $\frac{20.0628 - 20}{2}$ = .0314 cm

In steady state, gravitational force = viscous force



$mg = \eta A \frac{v}{y}$

$1 \times 10 = 10 \times 10^{-1} \times 2\pi r l \frac{v}{y}$

$1 \times 10 = 10 \times 10^{-1} \times 2 \times 3.14 \times 10 \times 10^{-2} \times 20 \times 10^{-2} \frac{v}{.0314 \times 10^{-2}}$

$1 = 40v$

$v = \frac{1}{40} = 0.025$ m/sec. = 2.5 cm/sec.

15.49 5

Due to rotation, Let the shift of liquid is x cm.

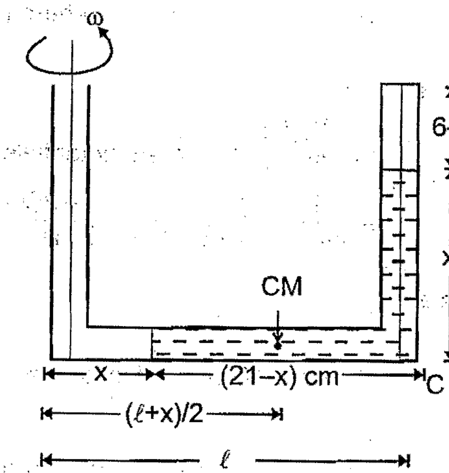
Let cross–section area of tube = A

In the right limb for compressed air -

$p_1V_1 = p_2V_2$

$p_0A \times 6 = p_2A(6 - x)$

$p_2 = \frac{6p_0}{6 - x}$ ....(1)



Force at the corner 'C' of right limb climb due to liquid above,

$F_1 = \left[\frac{6p_0}{6-x} + x\rho g\right]A$

Mass of the liquid in horizontal arm

$m = \rho(l - x)A$

It is rotated about left limb, then centripeted force

$F_2 = m\,\omega_0^2 r$

$= \rho(l - x)A\,\omega_0^2 \frac{\ell + x}{2} = \frac{\rho A \omega_0^2}{2}(l^2 - x^2)$

But $F_1 = F_2$

$\frac{\rho A \omega_0^2(\ell^2 - x^2)}{2} = \left[\frac{6p_0}{6-x} + x\rho g\right]A = \frac{10^3 \times 100 \times (21^2 - x^2) \times 10^{-4}}{2} = \left[\frac{6 \times 10500}{(6-x)} + x \times 10^3 \times 10 \times 10^{-2}\right]$

On solving x = 1 cm

then length of air column = 6 – 1 = 5 cm

15.50 40

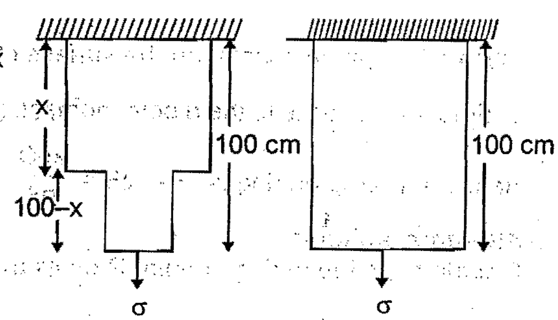
Maximum stress lies in stepped bar in the portion of lesser area (5 cm²)

For the stress σ in lesser area, the stress in larger cross–section = $\frac{\sigma A/2}{A} = \frac{\sigma}{2}$

Strain energy of stepped bar –

$= \frac{\sigma^2}{2y} \times 5 \times (100 - x) + \left(\frac{\sigma}{2}\right)^2 \frac{1}{2y} \times 10 \times x$

$= \frac{\sigma^2}{2y}[500 - 5x + 2.5x] = \frac{\sigma^2}{2y}[500 - 2.5x]$



Strain energy of uniform bar = $\frac{\sigma^2}{2y} \times 10 \times 100$

As per given condition

$\frac{\sigma^2}{2y}[500 - 2.5x] = \frac{40}{100} \times \frac{\sigma^2}{2y} \times 10 \times 100$

500 – 2.5x = 400

2.5 x = 100

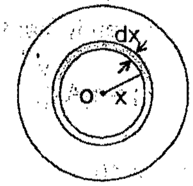
x = 40 cm Ans.

16. ELECTROSTATICS

161 (B)

We can consider all the charge inside the sphere to be concentrated on the center of sphere

Consider an elementry shell of radius x and thickness dx.



$$E = \frac{K\int dq}{r^2} = \frac{K\int_0^r 4\pi x^2 dx\,(\alpha x)}{r^2} = \frac{k4\pi\alpha}{r^2}\int_0^r x^3 dx = \frac{\alpha r^2}{4\varepsilon_0}$$

16.2 (A)

$W_{net} = q\vec{E}.\vec{d}$ $\Rightarrow$ where ($\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{i}$ & $\vec{d} = (x_1 - x_2)\hat{i}$) $\therefore$ $W_{net} = q\frac{\sigma}{2\varepsilon_0}.(x_1 - x_2)$

16.3 (A)

$V = V_1 + V_2 + V_3$

(where V_1, V_2 & V_3 are potentials due to the three parts of ring)

$$= \frac{1}{4\pi\varepsilon_0}.\frac{Q}{R} + \frac{1}{4\pi\varepsilon_0}\left(\frac{-2Q}{R}\right) + \frac{1}{4\pi\varepsilon_0}\left(\frac{3Q}{R}\right) = \frac{1}{4\pi\varepsilon_0}.\left(\frac{2Q}{R}\right) = \frac{Q}{2\pi\varepsilon_0 R}$$

16.4 (A)

Potential difference due to inner 10C charge $= K\,10\left(\frac{1}{.1} - \frac{1}{.2}\right) = 9 \times 10^{10}(5) = 45 \times 10^{10} = 4.5 \times 10^{11}$V

Potential difference due to outer charge $= \left(\frac{K \times 20}{0.2} - \frac{K \times 20}{0.2}\right) = 0$ V

$\therefore$ P.d. $= 4.5 \times 10^{11}$V

16.5 (B)

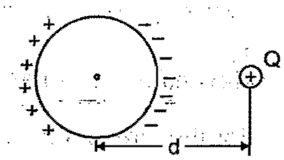
By argument of symmetry, it will be half of the potential produced by the full sphere

$\because$ Charge on hemisphere = Q, so charge on sphere = 2Q

$$\Rightarrow \quad \frac{1}{2}.\frac{K(2Q)}{R} = \frac{KQ}{R} \Rightarrow V = \frac{KQ}{R} = \frac{9\times10^9 \times 5\times10^{-9}}{15\times10^{-2}} = 300\text{ V}$$

16.6 (B)

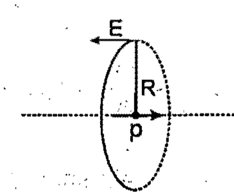
Net potential of the sphere due to the induced charge is zero. Therefore potential is due to the point charge only, equal to potential at the centre of the sphere.



16.7 (B)

Electric field at each point on the surface of ring due to dipole is $E = \frac{kp}{R^3}$

in direction opposite to the dipole moment. (figure below)

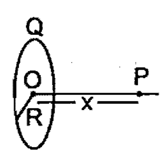


Hence net force on ring is $F = QE = \frac{kpQ}{R^3}$

Alternate solution

Electric field due to ring at point P on its axis at a distance x from centre O of ring is

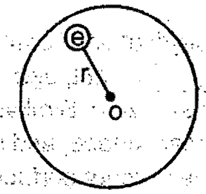
$$E = k.\frac{Qx}{(x^2 + R^2)^{3/2}}\;;\; \left.\frac{dE}{dx}\right]_{at\ x=0} = \frac{kQ}{R^3}$$



$\therefore$ Force on dipole $= \frac{dE}{dx} = \frac{k\,Qp}{R^3}$

16.8 (B)

$eE = m_e\omega^2 r$ (Balancing of forces on e^-)



Also, $V_O - V_R = -\int_R^0 E.dr = \int_0^R E.dr = \frac{m_e\omega^2}{e}\int_0^R r\,dr$

$\Rightarrow$ So, $V = \frac{m_e\omega^2 R^2}{2e}$

16.9 (B)

$u = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\frac{\varepsilon_0 K^2Q^2}{2r^4} \Rightarrow v = \frac{KQ}{r} \Rightarrow \frac{u}{v^2} = \frac{\frac{1}{2}\varepsilon_0 K^2\frac{Q^2}{r^4}}{\frac{K^2Q^2}{r^2}} = \frac{1}{2}\frac{\varepsilon_0}{r^2}$

because $\frac{u}{v^2} \propto \frac{1}{r^2}$

so the correct option is B.

16.10 (C)

$\phi = \vec{E}\cdot\vec{ds}$

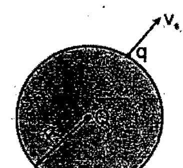
since r<< R so we can consider electric field is constant throughout the surface of smaller ring, hence

$\phi \propto E \propto \frac{x}{(R^2+x^2)^{3/2}}$

So, the best represnted graph is C.

16.11 (C)

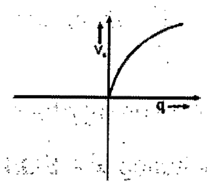
Let q be positive.



If it escapes then from energy conservation principle, $\frac{1}{2}mv_s^2 + \frac{KQq}{R} = 0$

$v = \sqrt{\frac{-2KQq}{Rm}}$ { Note that Q is negative, therefore the quantity within the root is positive.}

$\therefore \quad v \propto \sqrt{q}$



When q is negative, escape velocity will be zero due to electrostatic repulsion from negative Q.

16.12 (B)

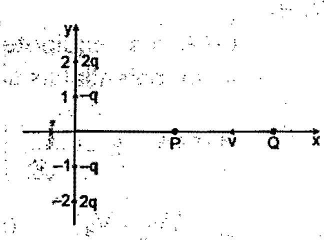
$\vec{E} = -\nabla v = +\frac{1}{x^2}\hat{i} + \frac{1}{y^2}\hat{j} + \frac{1}{z^2}\hat{k} = \hat{i}+\hat{j}+\hat{k}$

16.13 (B)

There exists a point P on the x-axis (other than the origin), where net electric field is zero. Once the charge Q reaches point P, attractive forces of the two –ve charge will dominate and automatically cause the charge Q to cross the origin.

Now if Q is projected with just enough velocity to reach P, its K.E. at P is zero.

But while being attracted towards origin it acquires K.E. & hence its net energy at the origin is positive. (P.E. at origin = zero).



16.14 (B)

$V = \frac{K\vec{p}.\vec{r}}{r^3} = \frac{9\times10^9\times(2\hat{i}-3\hat{j}+4\hat{k}).(2\hat{i}+2\hat{j}-\hat{k})}{[2^2+2^2+1^2]^{3/2}} = \frac{9\times10^9\times(4-6-4)}{27}$ V = – 2 × 10⁹ volts Ans.

16.15 (A)

The distribution of charge on the outer surface, depends only on the charges outside, and it distributes itself such that the net, electric field inside the outer surface due to the charge on outer surface and all the outer charges is zero. Similarly the distribution of charge on the inner surface, depends only on the charges inside the inner surface, and it distributes itself such that the net, electric field outside the inner surface due to the charge on inner surface and all the inner charges is zero.

Also the force on charge inside the cavity is due to the charge on the inner surface. Hence answer is option

16.16 (C)

Using the formula for electric field produced by large sheet $E = \frac{Q}{2A\epsilon_0}$.

We get ;

$E_A = \frac{4Q}{2A\varepsilon_0}(-\hat{i})$; $E_B = \frac{2Q}{2A\varepsilon_0}.(-\hat{i})$; $E_C = \frac{4Q}{2A\varepsilon_0}.(+\hat{i})$

16.17 (D)

$V_B - V_A = -\int E_x\,dx = -$ [Area under $E_x - x$ curve]

$V_B - 10 = -\frac{1}{2}.2.(-20) = 20$

$V_B = 30$ V.

16.18 (C)

$F = \frac{kq^2}{r^2}$; $(k = \frac{1}{4\pi\varepsilon_0})$ $\Rightarrow$ $\frac{kq^2}{r^2} = \frac{mv^2}{R_C}$ $\Rightarrow$ $R_C = \frac{mv^2r^2}{kq^2}$

$R_C = \frac{4\pi\varepsilon_0 v^2 r^2 m}{q^2}$

16.19 (B)

Speed will be maximum when acceleration becomes zero. ie when $Kx = EQ$ $\Rightarrow x = \frac{EQ}{K}$

By work–energy theorem : $w_{all} = \Delta KE$ $\Rightarrow$ $EQX - \frac{1}{2}KX^2 = \frac{1}{2}mv^2$

Substituting $x = EQ/K$,

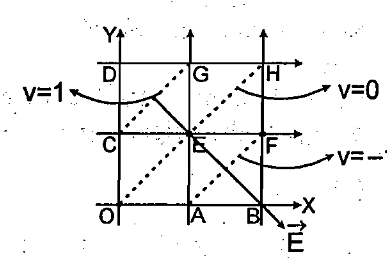
$V_{max} = QE/\sqrt{mK}$

Compression will be maximum when velocity becomes zero.

$W_{all} = \Delta KE$ $\Rightarrow$ $EQX - \frac{1}{2}KX^2 = 0$; $X_{max} = \frac{2EQ}{K}$

16.20 (B)

OEH is an equipotential surface, the uniform E.F. must be perpendicular to it pointing from higher to lower potential as shown



Hence, $\hat{E} = \left(\frac{\hat{i}-\hat{j}}{\sqrt{2}}\right)$.

$E = \frac{(V_E - V_B)}{EB} = \frac{0-(-2)}{\sqrt{2}} = \sqrt{2}$ V/m

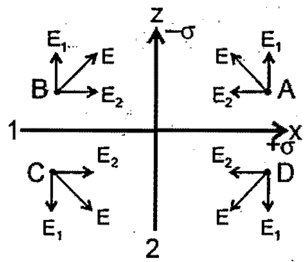
$\therefore$ $\vec{E} = E.\hat{E} = \sqrt{2}\frac{(\hat{i}-\hat{j})}{\sqrt{2}} = (\hat{i}-\hat{j})$ V/m

16.21 (C)

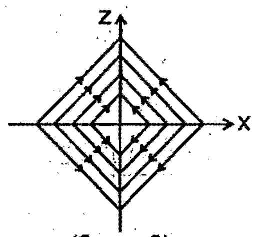
$V_p = \frac{kq}{r'} + V_{in} = V_C = \frac{kq}{r}$ $\therefore$ $V_{in} = \frac{kq}{r} - \frac{kq}{r'}$

16.22 (C)

The electric field intensity due to each uniformly charged infinite plane is uniform. The electric field intensity at points A, B, C and D due to plane 1, plane 2 and both planes are given by E_1, E_2 and E as shown in figure 1. Hence the electric lines of forces are as given in figure 2.



(figure 1)



(figure 2)

Aliter :

Electric lines of forces originate from positively charged plane and terminate at negatively charged plane. Hence the correct representation of ELOF is as shown figure 2.

16.23 (C)

The acceleration of centre of mass of system of particles is

$$a_{cm} = \frac{(q_1 + q_2)}{2m}E$$

$\therefore$ x-coordinate of centre of mass at t = 2 second is

$$x_{cm} = \frac{1}{2}a_{cm}t^2 = \frac{1}{2}\frac{(q_1+q_2)}{2m}E \times 2^2 = \frac{q_1+q_2}{m}E$$

Let the x-coordinates of q_1 and q_2 at t = 2 sec be x_1 and x_2 ; [x_1 = 2a at t = 2 sec.]

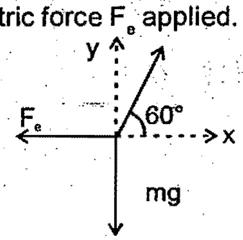
$$\therefore x_{cm} = \frac{mx_1 + mx_2}{2m} = \frac{x_1 + x_2}{2} \quad \text{or} \quad x_2 = 2x_{cm} - x_1 = 2\frac{(q_1+q_2)}{m}E - 2a$$

16.24 (A)

The bowl exerts a normal force N on each bead, directed along the radial line or at 60° above the horizontal. Consider the free-body diagram of the bead on the left with the electric force F_e applied.

$\Sigma F_y = N \sin 60° - mg = 0, \Rightarrow N = mg/\sin 60°$

$\Sigma F_x = -F_e + N \cos 60° = 0, \Rightarrow \frac{Kq^2}{R^2} = N \cos 60° = \frac{mg}{\tan 60°} = \frac{mg}{\sqrt{3}}$



Thus $q = R\left(\frac{mg}{K\sqrt{3}}\right)^{1/2}$

16.25 (A)

The potential at centre of sphere in which q charge is uniformly distributed throughout the volume is -

$$V_c = \frac{1}{4\pi\varepsilon_0}\frac{3q}{2R}$$

By symmetry the potential at centre due to half sphere will be half of the complete sphere.

$$\therefore V_c = \frac{1}{4\pi\varepsilon_0}\frac{3q/2}{2R} = \frac{1}{4\pi\varepsilon_0}\frac{3Q}{2R} \quad [\because \frac{q}{2} = Q]$$

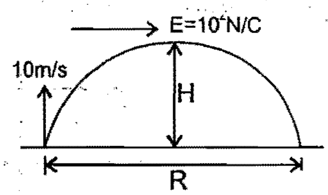
16.26 (ABD)

Increasing the accelerating voltage means increasing speed of the electron, thereby decreasing time spent between the plates. It will reduce X.

Increasing deflecting voltage means increasing electric field between the plates, making acceleration of electron greater. Increasing distance once again will change electric field between the plates.

16.27 (ABC)

Time of flight $(t) = \frac{2u}{g} = \frac{2\times10}{10} = 2$ sec



$H = \frac{u^2}{2g} = \frac{10\times10}{2\times10} = 5$m

$R = 0 + \frac{1}{2}\left(\frac{qE}{m}\right) t^2 = \frac{1}{2} \times \frac{10^{-3}\times10^{4}\times2\times2}{2} = 10$ m

16.28 (ABCD)

Charge on $a_1 = (r_1\,\theta)\,\lambda$

Charge on $a_2 = (r_2\,\theta)\,\lambda$

$\therefore$ Ratio of charges $= \frac{r_1}{r_2}$

E_1, Field produced by $a_1 = \frac{K[(r_1\,\theta)\lambda]}{r_1^2} = \frac{KQ_1}{r_1^2}$

E_2, Field produced by $a_2 = \frac{KQ_2}{r_2^2}$

So, $\frac{E_1}{E_2} = \frac{KQ_1}{r_1^2}\times\frac{r_2^2}{KQ_2} = \frac{r_2}{r_1}$ So, $\frac{E_1}{E_2} = \frac{KQ_1}{r_1^2}\times\frac{r_2^2}{KQ_2} = \frac{r_2}{r_1}$

As $r_2 > r_1$

Therefore $E_1 > E_2$

i.e. Net field at A is towards a_2.

$V_1 = \frac{K.(r_1\theta)\lambda}{r_1} = K\theta\lambda \quad\Rightarrow\quad V_2 = \frac{K.(r_2\,\theta)\lambda}{r_2} = K\theta\lambda \;\Rightarrow V_1 = V_2$.

16.29 (ABD)

$0 \le x \le a : V_x = \left[-\int_0^x E_x dx\right] + V_{(0)} = 0$ (as $E_x = 0$)

$x \ge a$; $V_x = -\int_a^x E_x dx + V_{(a)} = \left[-\int_a^x \frac{\sigma}{\epsilon_0} dx\right] + V_{(a)} = -\frac{\sigma}{\epsilon_0}(x-a)$; (As, $V_a = 0$) $\quad x \le 0$

; $V_x = -\int_0^x E_x dx + V_{(0)} = -\left(-\frac{\sigma}{\epsilon_0}.x\right) + V_{(0)} = \frac{\sigma}{\epsilon_0}.x$; (As, $V_0 = 0$)

16.30 (BCD)

V at origin $\neq 0$

$E(r = 2\text{ m}) = \frac{K(-q)r}{(R_1^2+r^2)^{3/2}} + \frac{K.Q.r}{(R_2^2+r^2)^{3/2}} = K.rq\left[-\frac{1}{10^{3/2}} + \frac{2\sqrt{2}}{2^{3/2}.10^{3/2}}\right] = 0$

From origin to r = 2, field is towards origin.

6.31 (ABC)
Charge is distributed over the surface of conductor in such a way that net field due to this charge and outside charge q is zero inside. Field due to only q is non-zero.

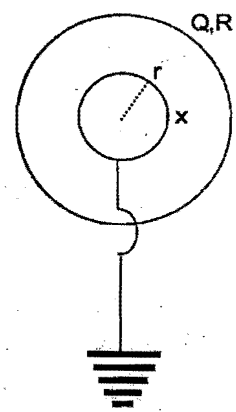
6.32 (D)
Potential at a point is zero does not imply that electric field at same point should be zero. For instance in the equatorial plane of a dipole, potential at any point is zero but electric field is not zero. Hence statement 1 is false.
No electric field in space means, potential at all points in space is same. Hence potential difference between any two points is zero. Hence statement 2 is true.
Statement–1 is False, Statement–2 is True.

16.33 (D)
For a non-uniformly charged thin circular ring with net zero charge, electric potential at each point on its axis is zero. Hence electric field at each point on its axis must be perpendicular to the axis. Therefore statement 1 is false and statement 2 is true.

16.34 (A)
The electric field due to disc is superposition of electric field due to its constituent ring as given in statement-2. Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

16.35 (C)

16.36 (C)



$V_{ball} = 0$

$$\frac{KX}{r} + \frac{KQ}{R} = 0$$

$$x = -\frac{Qr}{R}$$

Potential difference $V_s - V_b = \frac{KQ}{R} + \frac{Kx}{R}$

because potential difference depend only on charge on inner surface after electrostatic condition is reached after grounding.

$$V_s - V_b = \frac{KQ}{R}\left[1 - \frac{r}{R}\right]$$

16.37 (D)
Let the speed of charges A and B be V_A and V_B when the separation between them is ℓ_o. Then from conservation of momentum
$-mV_A + 2\,mV_B = 0$ or $V_A = 2\,V_B$
Applying conservation of energy, as the separation increases from ℓ_o to $2\ell_o$.
Gain in K.E of system of charges = Loss in electrostatic potential energy of the system of charges.

$$\frac{1}{2}mV_A^2 + \frac{1}{2}2mV_B^2 = \frac{1}{4\pi\varepsilon_o}\frac{2q^2}{\ell_o} - \frac{1}{4\pi\varepsilon_o}\frac{2q^2}{2\ell_o}$$

$$\text{or } \frac{1}{2}mV_A^2 + \frac{1}{2}2m\left(\frac{V_A}{2}\right)^2 = \frac{1}{4\pi\varepsilon_o}\frac{q^2}{\ell_o}.$$

Solving we get the speed of charge A is $V_A = \sqrt{\frac{1}{3\pi\varepsilon_o}\frac{q^2}{m\ell_o}}$.

16.38 (B)

The work done by electrostatic force on charge A, from work energy theorem, in the given duration is

= Final kinetic energy of charge A – Initial kinetic energy of charge A = $\frac{1}{2}mV_A^2 - 0$

$= \frac{1}{6\pi\varepsilon_0}\frac{q^2}{\ell_0}$. The sign of work done is positive

16.39 (C)

The net work done by electrostatic force on system of two charged particle is equal to change in electrostatic potential energy of the system

$= \frac{1}{4\pi\varepsilon_0}\frac{2q^2}{\ell_0} - \frac{1}{4\pi\varepsilon_0}\frac{2q^2}{2\ell_0} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{\ell_0}$. The sign of work done is positive

16.40 (B)

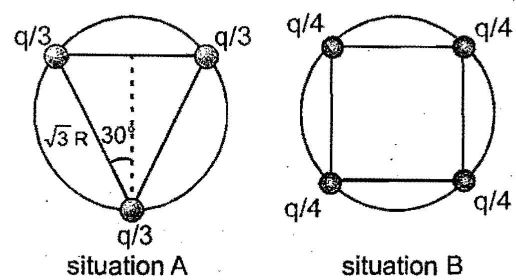
16.41 (C)

16.42 (D)

Potentials at the centre

$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$; $V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

Potential energy in situation I is



$U_1 = 3 \times \frac{1}{4\pi\varepsilon_0}\frac{(q/3)^2}{(\sqrt{3}R)} = \frac{1}{12\sqrt{3}\pi\varepsilon_0}\frac{q^2}{R}$

When one charge is removed, the field intensity at the centre is due to the removed charge only.

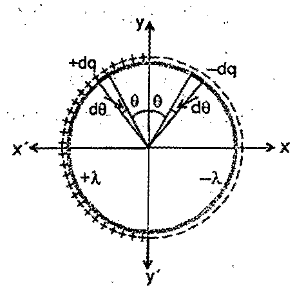
$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{q/3}{r^2} \Rightarrow E_2 = \frac{1}{4\pi\varepsilon_0}\frac{q/4}{r^2}$ ∴ $\frac{E_1}{E_2} = \frac{4}{3}$

16.43 (B)

Consider two small elements of ring having charges +dq and – dq symmetrically located about y-axis.

The potential due to this pair at any point on y-axis is zero. The sum of potential due to all such possible pairs is zero at all points on y-axis.

Hence potential at P(0, $\frac{R}{2}$) is zero.



16.44 (A)

Since all charge lies in x-y plane, hence direction of electric field at point P should be in x-y plane. Also y-axis is an equipotential (zero potential) line. Hence direction of electric field at all point on y-axis should be normal to y-axis.

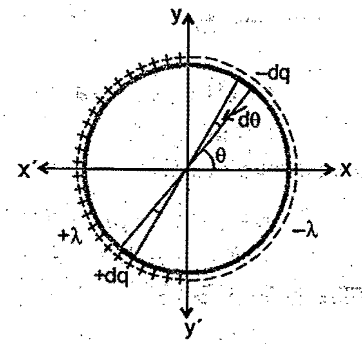
∴ The direction of electric field at P should be in x-y plane and normal to y-axis. Hence direction of electric field is along positive-x direction.

16.45 (C)

Consider two small elements of ring having charge +dq and –dq as shown in figure. The pair constitutes a dipole of dipole moment.

$dp = dq\,2R = (\lambda R d\theta)\,2R$

The net dipole moment of system is vector sum of dipole moments of all such pairs of elementary charges. By symmetry, the resultant dipole moment is along negative x-direction.



$\therefore$ net dipole moment $= -\int_{-\pi/2}^{+\pi/2}(dp\cos\theta)\,\hat{i} = -\int_{-\pi/2}^{+\pi/2}(2\lambda R^2\cos\theta\,d\theta)\,\hat{i}$

$= -4R^2\lambda\,\hat{i}$

16.46 (A) p, s (B) q, s (C) q, s (D) s

(A) Electrostatic potential energy $= \dfrac{1}{4\pi\epsilon_0}\dfrac{(-Q)^2}{2a} = \dfrac{Q^2}{8\pi\epsilon_0 a}$

(B) Electrostatic potential energy $= \dfrac{1}{4\pi\epsilon_0}\left[\dfrac{(-Q)\times(-Q)}{5a/2} + \dfrac{(-Q)^2}{2(5a/2)}\right] = \dfrac{3}{20}\dfrac{Q^2}{\pi\epsilon_0 a}$

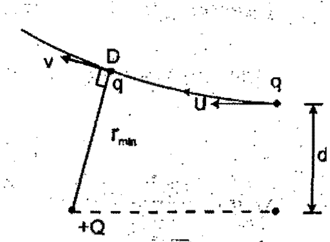
(C) Electrostatic potential energy $= \dfrac{1}{4\pi\epsilon_0}\dfrac{3Q^2}{5a} = \dfrac{3}{20}\dfrac{Q^2}{\pi\epsilon_0 a}$

(D) Electrostatic potential energy $= \dfrac{1}{4\pi\epsilon_0}\left[\dfrac{3Q^2}{5a} + \dfrac{(-Q)^2}{2(2a)} + \dfrac{(-Q)\times(-Q)}{2a}\right] = \dfrac{27Q^2}{80\pi\epsilon_0 a}$

16.47 1

The path of the particle will be as shown in the figure. At the point of minimum distance (D) the velocity of the particle will be ⊥ to its position vector w.r. to +Q.

Now by conservation of energy :-



$\dfrac{1}{2}mu^2 + 0 = \dfrac{1}{2}mv^2 + \dfrac{KQq}{r_{min}}$ ......(1)

$\because$ Torque on q about Q is zero hence angular momentum about Q will be conserved

$\Rightarrow$ $m v r_{min} = m u d$ ......(2)

by (2) in (1) $\Rightarrow$ $\dfrac{1}{2}mu^2 = \dfrac{1}{2}m\left(\dfrac{ud}{r_{min}}\right)^2 + \dfrac{KQq}{r_{min}}$

$\Rightarrow$ $\dfrac{1}{2}mu^2\left(1-\dfrac{d^2}{r_{min}^2}\right) = \dfrac{mu^2 d}{r_{min}}$ { $\because$ $KQq = mu^2 d$ (given) }

$\Rightarrow$ $r_{min}^2 - 2r_{min}d - d^2 = 0$ $\Rightarrow$ $r_{min} = \dfrac{2d \pm \sqrt{4d^2+4d^2}}{2} = d(1 \pm \sqrt{2})$

$\because$ distance cannot be negative

$\therefore$ $r_{min} = d(1+\sqrt{2})$ Ans.

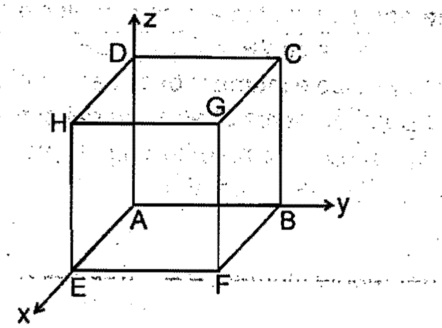
16.48 11

Flux through ABCD.

$$\phi_1 = \vec{E}\vec{A}$$

$$= (x^2\hat{i} + y\hat{j}).(-a^2\hat{i})$$

$$= 0 \text{ as } x = 0$$



Flux through EFGH

$$\phi_2 = (x^2\hat{i} + y\hat{j}).(+a^2\hat{i})$$

$$= x^2.a^2 = a^4 = 1.0 \times 10^{-4}\ Nm^2/C$$

Flux through BCGF

$$\phi_3 = (x^2\hat{i} + y\hat{j}).(a^2\hat{j})$$

$$= a^3 = 1.0 \times 10^{-3}\ Nm^2/C$$

Flux through EADH

$$\phi_4 = (x^2\hat{i} + y\hat{j}).(-a^2\hat{j}) \qquad = 0 \text{ as } y = 0$$

Flux through ABFE

$$\phi_5 = (x^2\hat{i} + y\hat{j}).(-a^2\hat{k}) \qquad = 0$$

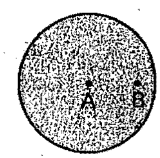
Flux through CDHG

$$\phi_6 = 0$$

Net flux = $(1.0 \times 10^{-4} + 1.0 \times 10^{-3})$ N-m²/C = 11×10^{-4} N-m²/C

16.49 6

Assume a solid sphere without cavity.

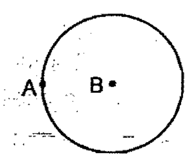


Potential at A due to this solid sphere ⇒ $V_A' = \frac{3}{2} \cdot \frac{1}{4\pi\epsilon_0} \frac{\left(\frac{4}{3}\pi R^2 \rho\right)}{R} = \frac{\rho R^2}{2\epsilon_0}$

Electric field at 'C' due to this solid sphere = $\vec{E}_C' = \frac{\rho}{3\epsilon_0}\overrightarrow{AC}$

Now consider the cavity filled with negative charge

$$V_A'' = \frac{1}{4\pi\epsilon_0} \frac{\frac{4}{3}\pi\left(\frac{R}{2}\right)^3(-\rho)}{\frac{R}{2}} = \frac{-\rho R^2}{12\epsilon_0}$$



$$\vec{E}_C'' = \frac{-\rho}{3\epsilon_0}\overrightarrow{BC}$$

Now net values for the solid sphere with the cavity can be given by superposition of the above two cases

Hence, $V_A = V_A' + V_A'' = \frac{\rho R^2}{\epsilon_0}\left(\frac{1}{2} - \frac{1}{12}\right) = \frac{5\rho R^2}{12\epsilon_0}$

$$\vec{E}_C = \vec{E}_C' + \vec{E}_C'' = \frac{\rho}{3\epsilon_0}(\overrightarrow{AC} - \overrightarrow{BC}) = \frac{\rho}{3\epsilon_0}\overrightarrow{AB}$$

$$\therefore \quad E_C = \frac{\rho}{3\epsilon_0}\left(\frac{R}{2}\right) = \frac{\rho R}{6\epsilon_0}$$

Ans. $V_A = \frac{5\rho R^2}{12\epsilon_0}$, $E_C = \frac{\rho R}{6\epsilon_0}$

17. GRAVITATION

17.1 (A)

Let the minimum speed imparted to the particle of mass m so that it just reaches surface of earth is v.
Applying conservation of energy

$$\frac{1}{2}mv^2 + \left(-\frac{3}{2}\frac{GM}{R}m\right) = -\frac{GM}{R}m + 0$$

Solving we get $v = \sqrt{\frac{GM}{R}}$

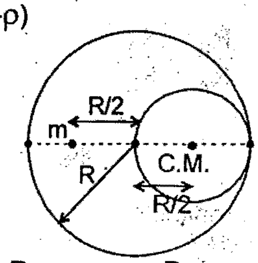
17.2 (B)

Gravitation field at mass 'm' due to full solid sphere

$$\vec{E}_1 = \frac{\rho\vec{r}}{3\varepsilon_0} = \frac{\rho R}{6\varepsilon_0} \ldots \left[\varepsilon_0 = \frac{1}{4\pi G}\right]$$

Gravitational field at mass 'm' due to cavity (−ρ)

$$\vec{E}_2 = \frac{(-\rho)(R/2)^3}{3\varepsilon_0 R^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \left[\frac{\rho a^3}{3\varepsilon_0 r^2}\right]$$



$$= \frac{(-\rho)R^3}{24\varepsilon_0 R^2} = \frac{-\rho R}{24\varepsilon_0}$$

Net gravitational field $\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{\rho R}{6\varepsilon_0} - \frac{\rho R}{24\varepsilon_0} = \frac{\rho R}{8\varepsilon_0}$

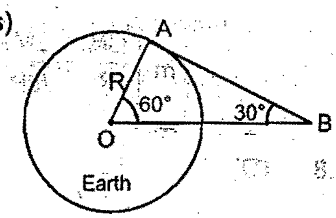
Net force on 'm' → $F = m\vec{E} = \frac{m\rho R}{8\varepsilon_0}$

Here $\rho = \frac{M}{4/3\pi R^3}$ & $\varepsilon_0 = \frac{1}{4\pi G}$

then $F = \frac{3mg}{8}$

17.3 (A)

In Δ AOB : - $\cos 60° = \frac{R}{OB} \Rightarrow OB = 2R$ (where OB is orbital radius)



Here gravitational force will provide the required centripetal force.

Hence $\frac{GMm}{(OB)^2} = m(OB)\,\omega^2$

$$\Rightarrow \omega = \sqrt{\frac{GM}{(OB)^3}} = \sqrt{\frac{GM}{(2R)^3}} \Rightarrow \omega = \sqrt{\frac{GM}{8R^3}}$$

17.4 (C)

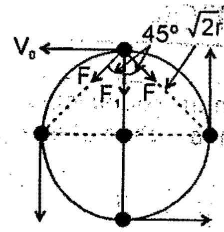
Given $8 = \frac{2\pi}{\omega_1 + \omega_2} = \frac{2\pi}{\frac{2\pi}{T_1} + \frac{2\pi}{T_2}}$, T_1 = 24 hours for earth.

($\omega_1 + \omega_2$ is the relative angular velocity for opposite direction)
⇒ T_2 = 12 hours (T_2 being the time period of satellite, it will remain same as the distance from the centre of the earth remains constant).

⇒ $T = \frac{2\pi}{\omega_2 - \omega_1} = \frac{2\pi}{\frac{2\pi}{T_2} - \frac{2\pi}{T_1}}$ = 24 hours.($\omega_2 - \omega_1$ is the relative angular velocity for same direction)

17.5 (A)

Centripetal force = net gravitational force



$$\frac{mv_0^2}{r} = 2F\cos 45° + F_1$$

$$= \frac{2Gm^2}{(\sqrt{2}r)^2}\frac{1}{\sqrt{2}} + \frac{Gm^2}{4r^2}$$

$$\frac{mv_0^2}{r} = \frac{Gm^2}{4r^2}[2\sqrt{2}+1] \quad \Rightarrow \quad \left(\frac{Gm(2\sqrt{2}+1)}{4r}\right)^{1/2}$$

17.6 (C)

$M_A = \sigma 4\pi R_A^2$, $M_B = \sigma 4\pi R_B^2$ where σ is surface density.

$$V_A = \frac{-GM_A}{R_A}, \quad V_B = \frac{-GM_B}{R_B}$$

$$\frac{V_A}{V_B} = \frac{M_A}{M_B}\frac{R_B}{R_A} = \frac{\sigma 4\pi R_A^2}{\sigma 4\pi R_B^2}\frac{R_B}{R_A} = \frac{R_A}{R_B}$$

Given $\frac{V_A}{V_B} = \frac{R_A}{R_B} = \frac{3}{4}$, then $R_B = \frac{4}{3}R_A$

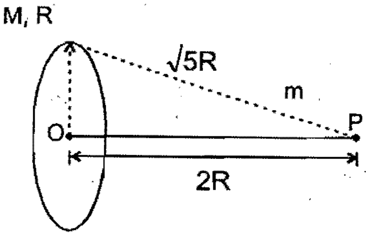
for New shell of mass M and radius R -

$M = M_A + M_B = \sigma 4\pi R_A^2 + \sigma 4\pi R_B^2$ $\sigma 4\pi R^2 = \sigma 4\pi(R_A^2 + R_B^2)$

then $$\frac{V}{V_A} = \frac{M}{R}\frac{R_A}{M_A} = \frac{\sigma 4\pi[R_A^2+R_B^2]}{(R_A^2+R_B^2)^{1/2}}\frac{R_A}{\sigma 4\pi R_A^2} = \frac{\sqrt{R_A^2+R_B^2}}{R_A} = \frac{5}{3}$$

17.7 (D)

Gravitational potential at 'P'



$$V_P = \frac{-GM}{\sqrt{5}R}$$

Gravitational potential at 'O'

$$V_0 = -\frac{GM}{R}$$

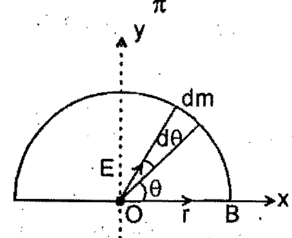
work energy theorem $W = \Delta K \Rightarrow m[V_P - V_O] = 1/2\, mv^2$

$$m\left[\frac{GM}{R} - \frac{GM}{\sqrt{5}R}\right] = \frac{1}{2}mv^2 \qquad \sqrt{\frac{2GM}{R}\left[1-\frac{1}{\sqrt{5}}\right]} = v$$

17.8 (D)

Let mass per unit length of wire, $\lambda = \frac{m}{\ell}$ and $\pi r = \ell$, $r = \frac{\ell}{\pi}$

mass of element, $dm = \lambda r d\theta$



then $$dE = \frac{Gdm}{r^2}$$

$$\int_0^{\pi} dE = \int_0^{\pi}\frac{G\lambda r d\theta}{r^2}(\hat{i}\cos\theta + \hat{j}\sin\theta)$$

$$E = \frac{G\lambda}{r}\left[\int_0^{\pi}\hat{i}\cos\theta\, d\theta + \int_0^{\pi}\hat{j}\sin\theta\, d\theta\right]$$

$$= \frac{2G\lambda}{r}\hat{j} = \frac{2Gm}{\ell r}\hat{j} = \frac{2Gm\pi}{\ell^2}\hat{j} \text{ (along y-axis)}$$

17.9 (C)

During total eclipse-

Total attraction due to sun and moon, $F_1 = \frac{GM_sM_e}{r_1^2} + \frac{GM_mM_e}{r_2^2}$

When moon goes on the opposite side of earth.

Effective force of attraction, $F_2 = \frac{GM_sM_e}{r_1^2} - \frac{GM_mM_e}{r_2^2}$

Changein force, $\Delta F = F_1 - F_2 = \frac{2GM_mM_e}{r_2^2}$

Change in acceleration of earth $\Delta a = \frac{\Delta F}{M_e} = \frac{2GM_m}{r_2^2}$

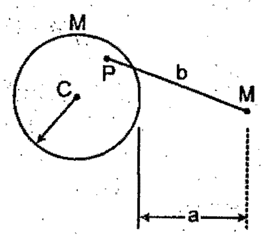
Average force on earth, $F_{av} = \frac{F_1 + F_2}{2} = \frac{GM_sM_e}{r_1^2}$

Average acceleration of earth, $a_{av} = \frac{F_{av}}{M_e} = \frac{GM_s}{r_1^2}$

%age change in acceleration $= \frac{\Delta a}{a_{av}} \times 100 = \frac{2GM_m}{r_2^2} \times \frac{r_1^2}{GM_s} \times 100 = 2\left(\frac{r_1}{r_2}\right)^2 \frac{M_m}{M_s} \times 100$

17.10 (D)

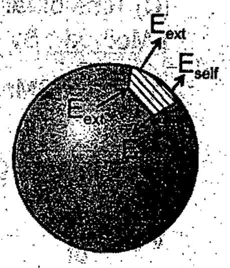
$V_c = -\frac{GM}{a} - \frac{GM}{2a}$; $E_c = \frac{GM}{(2a)^2}$; At any point P inside $V_P = -\frac{GM}{a} - \frac{GM}{b}$



$E_P = \frac{GM}{b^2}$ {only due to outside mass M}

17.11 (B)

Consider a small area (shaded strip)
here E_{self} = Gravitational field due to this strip
and E_{ext} = Gravitational field due to the rest of spherical shell.
E_{in} = Gravitational field just inside the strip
E_{out} = Gravitational field just outside the strip
$E_{in} = E_{ext} - E_{self} = 0$
$\Rightarrow E_{ext} = E_{self}$



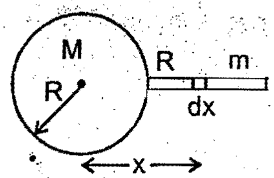
$E_{out} = E_{ext} + E_{self} = \frac{GM}{R^2}$ $\Rightarrow E_{ext} = \frac{GM}{2R^2}$

After the shaded area has been removed there is no E_{self} and only E_{ext}.

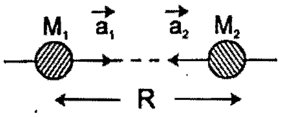
hence, $E_{net} = E_{ext} = \frac{GM}{2R^2}$

17.12 (A)

$$F = \int_R^{2R} \frac{GM\left(\frac{m}{R}\right)dx}{x^2} = \frac{GMm}{2R^2}$$



17.13 (B)



$a_1 = \frac{GM_1M_2}{R^2} / M_1$ $\qquad$ $a_2 = \frac{GM_1M_2}{R^2} / M_2$

acceleration of M_1 w.r.t. M_2

$a_{rel.} = a_1 + a_2 \qquad = \frac{G(M_1+M_2)}{R^2} \qquad = \frac{GM}{R^2}.$

17.14 (C)

$v = \frac{50}{100} V_e = \frac{1}{2}\sqrt{\frac{2GM}{R}}$

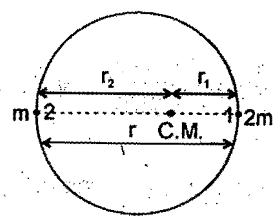
Applying energy conservation $\Rightarrow -\frac{GMm}{R} + \frac{1}{2}mv^2 = -\frac{GMm}{(R+h)}$

$v^2 = \frac{2GM}{R} - \frac{2GM}{R+h} \quad \Rightarrow \quad \frac{1}{4}.\frac{2GM}{R} = 2GM\left(\frac{1}{R} - \frac{1}{R+h}\right) \quad \Rightarrow \quad \frac{1}{4R} = \frac{h}{R(R+h)}$

$\Rightarrow \quad R + h = 4h \quad \Rightarrow \quad h = R/3$

17.15 (AD)

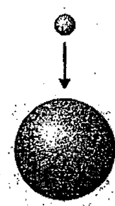
$r_2 = \frac{2mr}{m+2m} = \frac{2r}{3}$ $\qquad$ $T_2^2 = \frac{4\pi^2 r_2^3}{Gm}$



$T_2^2 = \frac{32\pi^2 r^3}{27Gm}$

$T_2 \propto r^{3/2}$ $\qquad$ $T_2 \propto m^{-1/2}$

17.16 (AC)



(A) It will fall because mg is acting on it towards the centre of planet and initial velocity is zero. It'll move in straight line.

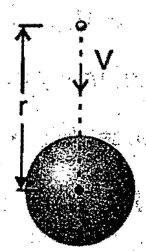
(C) Time of fall can be found by two methods :

<u>I Method :</u> By energy conservation

$\frac{1}{2}mv^2 - \frac{GMm}{r} = 0 - \frac{GMm}{R}$ – (1)

using this we get V = f(r). Now use

$V = -\frac{dr}{dt} \quad \Rightarrow f(r) = -\frac{dr}{dt}$



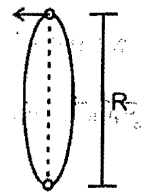
$\Rightarrow \int_R^{R'} \frac{dr}{f(r)} = -\int_0^t dt$; R' = radius of the planet.

In the final expression (or in the beginning itself) $R' \to 0$ $\{\because R >> R'\}$

you will get $\quad t = \frac{T}{4\sqrt{2}}$

Here $\frac{GMm}{R^2} = m\left(\frac{2\pi}{T}\right)^2 R$

Note : This method is longer. If a student gets idea of solving the question only by this method then it is better to leave this question because it will consume more time.



II Method : Kepler's Law : $T^2 \alpha r^3$.

Assume that the satellite moves in elliptical path with maximum and minimum distances from centre as R and R'.

$\because$ R >> R'

$\therefore$ velocity at R is very small ($\simeq$ 0). When it reaches R' then it touches the surface of the planet. This motion (from R to R') is almost, same as given in the question.

Now $\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$, $T_1 = T$, $r_1 = R$

$$r_2 = \frac{R+R'}{2} \simeq \frac{R}{2}$$

$$\therefore \quad T_2 = \frac{T}{4\sqrt{2}}$$

17.17 (A)

Till the particle reaches the centre of planet, force on both bodies are in direction of their respective velocities, hence kinetic energies of both keep on increasing . After the particle crosses the centre of planet, forces on both are retarding in nature. Hence as the particle passes through the centre of the planet, sum of kinetic energies of both the bodies is maximum. Therefore statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

17.18 (B)

The total mechanical energy of the system after firing the rocket will increase by 10%.

Hence (B)

Note : $-0.9\,E_0 > -E_0$

17.19 (A)

Because the mechanical energy is negative, a decrease in magnitude is increase in energy.

$$\frac{E_{ell}}{E_{cir.}} = \frac{-\frac{GMm}{2a}}{-\frac{GMm}{2r}} = \frac{r}{a} \quad \Rightarrow \quad 0.9 = \frac{(6400+300)}{a} \quad \Rightarrow \quad a = \frac{6.7\times10^4}{9}\text{ Km.}$$

17.20 (D)

Maximum distance from the centre of the Earth will occur when the spacecraft is at apogee thus

$$r_{max} = 2a - h - r \quad = \quad 2\times\frac{6.7\times10^4}{9} - 6700 = \frac{7.37\times10^4}{9}\text{ km.}$$

$$h_{max} = r_{max} - R_E \quad = \frac{7.37\times10^4}{9} - 6400 \quad = \frac{1.61\times10^4}{9}\text{ km.}$$

17.21 (A) p ; (B) (t) ; (C) (r)

(B)

Angular momentum of particle $= m(v_0 + v)\, a$

$$= \sqrt{\frac{5}{4}}\, mv_0 a \quad .... \; v_0 = \sqrt{\frac{GM_e}{a}}$$

Total energy of particle $= \frac{1}{2} m(v_0 + v)^2 - \frac{GMem}{a}$

$$= \frac{1}{2} \times \frac{5}{4} mv_0^2 - \frac{GM_e m}{a}$$

$$= \frac{5}{8}\frac{GM_e m}{a} - \frac{GM_e m}{a}$$

$$= -\frac{3GM_e m}{a}$$

At any distance 'r' T.E. $= \frac{1}{2} mu^2 - \frac{GM_e m}{r}$

but angular momentum conservation

$$mur = m\sqrt{\frac{5GM_e}{4a}}\, a$$

$$u = \sqrt{\frac{5}{4}\frac{GM_e a}{r^2}}$$

T.E. at any distance 'r'

$$= \frac{1}{2} m \frac{5}{4}\frac{GM_e a}{r^2} - \frac{GM_e m}{r}$$

but through conservation of total energy

$$= \frac{1}{2} m \frac{5}{4}\frac{GM_e a}{r^2} - \frac{GM_e m}{r} = -\frac{3GM_e m}{a}$$

on solving $3r^2 - 8ar + 5a^2 = 0$

$(r - a)(3r - 5a) = 0$

$r = a, \quad r = 5a/3$

minimum distance $= a$

maximum distance $= 5a/3$

17.22 (A) p,r (B) p,r (C) q,r (D) p,r

(A) At centre of thin spherical shell $V \neq 0$, $E = 0$.

(B) At centre of solid sphere $V \neq 0$, $E = 0$.

(C) At centre of spherical cavity inside solid sphere $V \neq 0$, $E \neq 0$.

(D) At centre of two point masses $V \neq 0$, $E=0$.

17.23 4

Speed of the ball which can cross 10 m wide river is

$$R = \frac{V^2 \sin(2\times 45^\circ)}{g} = 10, \qquad v = \sqrt{10g}$$

Let the radius of planet is 'R', then

Mass of planet $M = \frac{4}{3}\pi R^3 \times 2\rho \quad = \frac{4}{3}\pi R^3 \times \frac{2\times M_e}{4/3\pi R_e^3} \quad = \frac{2M_e R^3}{R_e^3}$

Escape velocity on planet $V = \sqrt{\frac{2GM}{R}} = \sqrt{10g}$

$$\sqrt{\frac{2G\times 2\times M_e R^3}{R_e^3 R}} = \sqrt{\frac{10GM_e}{R_e^2}} \quad \ldots\ldots \left[g = \frac{GM_e}{R_e^2}\right]$$

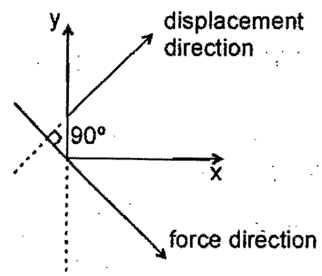
$$2R = \sqrt{10R_e} \qquad 2R = \sqrt{10\times 6.4\times 10^6}$$

$$R = \frac{8\times 10^3}{2} = 4\times 10^3 \qquad = 4 \text{ Km.}$$

17.24 0

Slope of displacement vector

$$m_1 = \frac{3}{4}$$



Slope of force vector

$$m_2 = -\frac{4}{3}$$

$$m_1 \times m_2 = \frac{3}{4} \times -\frac{4}{3} = -1$$

i.e. force and displacement directions are perpendicular. The work done is zero

17.25 8

Conserving angular momentum $m.(V_1 \cos 60°).\ 4R = m.V_2.R$; $\frac{V_2}{V_1} = 2.$

Conserving energy of the system

$$-\frac{GMm}{4R} + \frac{1}{2}mV_1^2 = -\frac{GMm}{R} + \frac{1}{2}mV_2^2$$

$$\frac{1}{2}V_2^2 - \frac{1}{2}V_1^2 = \frac{3}{4}\frac{GM}{R} \qquad \text{or} \qquad V_1^2 = \frac{1}{2}\frac{GM}{R}$$

$$V_1 = \frac{1}{\sqrt{2}}\sqrt{64\times 10^6} \qquad = \frac{8000}{\sqrt{2}} \text{ m/sec}^{-1} \qquad \textbf{Ans. 8}$$

18. CURRENT ELECTRICITY

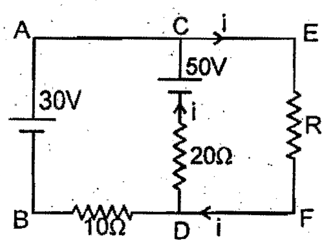
18.1 (D)

$i = \frac{7V}{7\Omega} = 1$ A.

Current flows in anticolockwise direction in the loop.
Therefore $0 - 1 \times 2 - 1 \times 2 - 5 = V_1$
$V_1 = -9$ V.

18.2 (C)

$i = \frac{50}{20+R}$

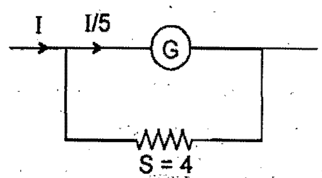


Potential drop across R = Potential drop across AB

$\Rightarrow \frac{50}{20+R}.R = 30 \quad \Rightarrow \quad R = 30\ \Omega$

18.3 (B)
$I_G = 10$ mA
$G = 10\Omega$
$S(I - I_G) = I_G G$ where S is shunt is parallel
$S = 0.1\Omega$

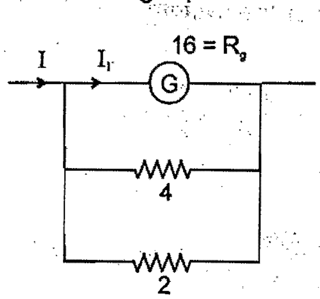
18.4 (A)
Case I



$R_g \times \frac{I}{5} = \left(I - \frac{I}{5}\right) \times 4$

$\Rightarrow \quad R_g = 16\ \Omega$

Case II

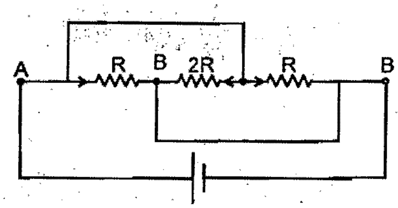


$16\ I_1 = \frac{4\times2}{6}(I - I_1)$

$\Rightarrow \quad I_1 = I/13$
so decrease in current to previous current

$= \frac{I/5 - I/13}{I/5} = \frac{8}{13}$ Ans.

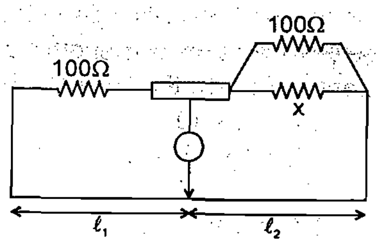
18.5 (B)
In figure all resistance are connected in parallel.



So $R_{eq} = \frac{2R \times R/2}{2R + R/2}$ and current in all resistance flow from positive terminal of battery (means A end) to negative terminal of battery (means B end).

18.6 **(B)**

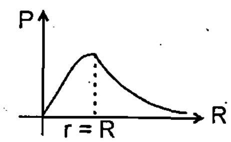
$\because$ wheat stone bridge is in balanced condition



So $\dfrac{100}{\ell_1} = \dfrac{\frac{100x}{100+x}}{\ell_2}$

$\because$ $\dfrac{\ell_1}{\ell_2} = 2$ $\Rightarrow$ $x = 100\ \Omega$

18.7 **(A)**



Power maximum when r = R.

So, power consumed by it will decrease. for R > r.

18.8 **(B)**

$$V = E - ir = -\frac{Er}{R+r} = E\left[\frac{R+r-r}{R+r}\right]$$

$V = \dfrac{ER}{(R+r)}$ $\Rightarrow$ V = 0 at R = 0 R = 0 V = 0

V = E at R = ∞ ⇒ so (B) is correct option.

18.9 **(D)**

Voltage across each bulb will be

$$V_1 = iR = \frac{V}{nR}.\,R = (V/n)$$

so power developed by each bulb $= iV_1 = \dfrac{V}{nR}.\dfrac{V}{n} = \dfrac{V^2}{n^2R}$ & $P = \dfrac{V^2}{R}$

so power consumed by one bulb $= \dfrac{P}{n^2}$

18.10 **(D)**

For maximum current, net resistance of cells must be equal to 2.5Ω

i.e. $\dfrac{n\,(0.5)}{m} = 2.5$ ....(1)

& $m \times n = 45$ ....(2)

solving, we get n = 15, m = 3

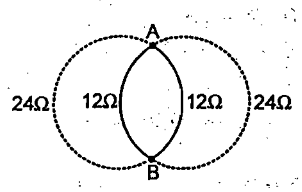
18.11 **(B)**

From the figure.

$AC_1 = AC_2 = C_1C_2$ = radius

$\therefore$ $\angle AC_1B = 120°$

Hence the resistance of four sections are

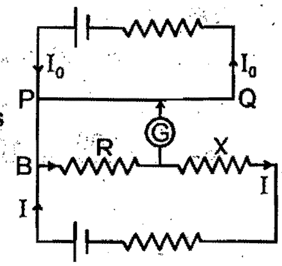


Hence equivalent resistance R across AB is

$\dfrac{1}{R} = \dfrac{1}{24} + \dfrac{1}{12} + \dfrac{1}{12} + \dfrac{1}{24}$ or $R = 4\Omega$

$\therefore$ Power $= \dfrac{V^2}{R} = \dfrac{(20)^2}{4} = 100$ watt.

18.12 (A)



In potentiometer wire potential difference is directly proportional to length
Let potential drop per unit length a potentiometer wire be K.
For zero deflection the current will flow independently in two closed circuits

$\therefore$ $IR = K \times 10$ .... (1)

$IR + IX = K \times 30$ .... (2)

(2) – (1)

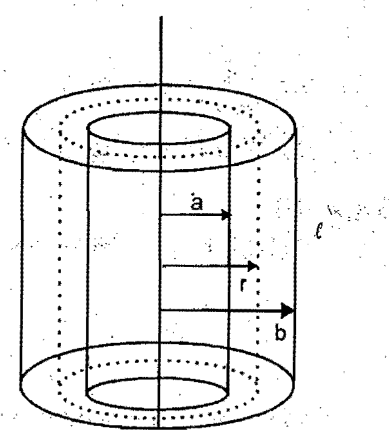
$\Rightarrow$ $IX = k \times 20$ .... (3)

Divide (1) & (3)

$\frac{R}{X} = \frac{1}{2}$ $\Rightarrow$ $x = 2R$

18.13 (C)

$E = \frac{\lambda}{2\pi\varepsilon_0 r}$, where λ is the linear charge density on the inner cylinder.



and $V = \int_a^b E.d\ell = \frac{\lambda}{2\pi\varepsilon_0}\ell n\left(\frac{b}{a}\right)$ .... (1)

Now ; $I = \int \vec{J}.\vec{dA} = \sigma \int \vec{E}.\vec{dA}$

$= \sigma \frac{\lambda}{2\pi\varepsilon_0 r}.2\pi r \ell$

Current per unti length will be :

$I = \frac{\sigma\lambda}{\varepsilon_0}$ .... (2)

From (1) (1)

$I = \frac{2\sigma\pi\varepsilon_0}{\varepsilon_0 \ell n(b/a)}V = \frac{2\pi\sigma}{\ell n(b/a)}V$

$I_b = \frac{V}{R}$

$R = \int_{x=a}^{b} \frac{1}{\sigma}\frac{dx}{2\pi x.1} = \frac{1}{2\pi\sigma}\ell n\left(\frac{b}{a}\right)$

$\therefore$ $I = \frac{2\pi\sigma V}{\ell n(b/a)}$

18.14 (B)

$50 = 10\,[R + r]$

$R + r = 5\,\Omega$

$\eta = \frac{R}{R+r}$ $\Rightarrow$ $0.25 = \frac{R}{R+r}$

$R + r = 4R$

$r = 3R$

then $R = \frac{5}{4} = 1.25\,\Omega$, and $r = 3.75\,\Omega$.

18.15 (D)

$P = VI,\quad 50 = 5 \times I$

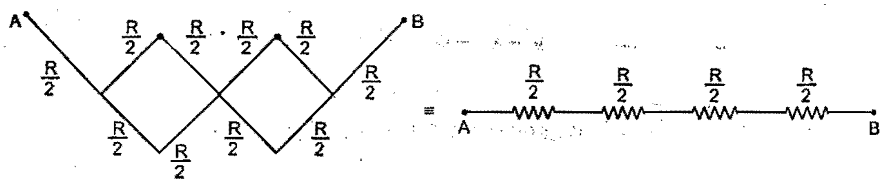
$I = 10\ A$

Power lost in cable = $I^2R = 10 \times 10 \times 0.02 = 2\ W$

Power supplied to T.R. = 50 W – 2 W = 48 W

18.16 (B)

The circuit can be folded about B and redrawn as



Hence equivalent resistance between A and B is 2R.

18.17 (D)

$$R = \frac{1}{\sigma} \times \frac{t}{4\pi r^2}$$

Using values $R = 5 \times 10^{-11}\ \Omega$

18.18 (B)

Since current $I = neAv_d$ through both rods is same

$$2(n)\,e\,A\,v_L = n\,e\,(2A)\,v_R \quad \text{or} \quad \frac{v_L}{v_R} = 1$$

18.19 (C)

$$i = \frac{dq}{dt} = \text{slope of } q - t \text{ graph} = -5 \text{ (which is constant)}$$

Amount of heat generated in time t

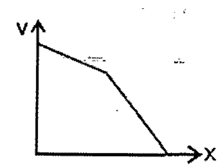
$H = i^2RT$

$H \propto t.$

18.20 (B)

From relation $E = \rho J$, the magnitude of electric field is greater in right rod as compared to left rod. There fore magnitude of potential gradient in the right rod is greater. (remember potential is continous).

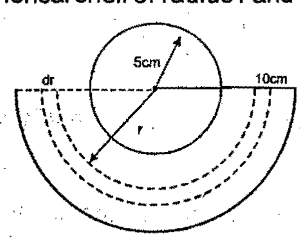
Therefore the variation is shown by figure.



18.21 (A)

The arrangement is shown in figure. Consider the hemispherical shell of radius r and thickness dr as shown.

Resistance of this shell is ;



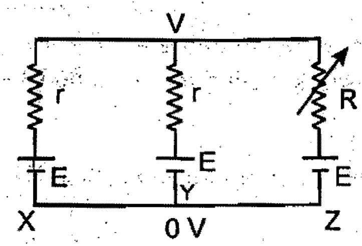
$$dR = \frac{dr}{\sigma \times 2\pi r^2}$$

$$R = \frac{1}{2\pi\sigma} \times \int_{r=5cm}^{r=10cm} \frac{dr}{r^2} = 1591.6\ \Omega.$$

18.22 (C)

Redrawing the given circuit diagram as shown below :

Using point potential theory,



$$\frac{V-E}{r}+\frac{V-E}{r}+\frac{V-E}{R}=0$$

$$\Rightarrow \quad (V-E)\left(\frac{2}{r}+\frac{1}{R}\right)=0$$

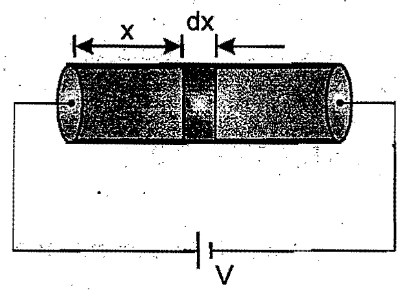
As $\frac{2}{r}+\frac{1}{R}\neq 0$ so $V-E=0$

So, current through R, $i=\frac{V-E}{R}=0$ whatever be the value of R.

18.23 (A)

Consider an elemental part of solid at a distance x from left end of width dx.

Resistance of this elemental part is,



$$dR=\frac{\rho dx}{\pi a^2}=\frac{\rho_0 x dx}{\pi a^2}$$

$$R=\int dR=\int_0^L\frac{\rho_0 x dx}{\pi a^2}=\frac{\rho_0 L^2}{2\pi a^2}$$

Current through cylinder is, $I=\frac{V}{R}=\frac{V\times 2\pi a^2}{\rho_0 L^2}$

Potential drop across element is, $dV=IdR=\frac{2V}{L^2}x\,dx$

$$E(x)=\frac{dV}{dx}=\frac{2V}{L^2}x.$$

18.24 (ABCD)

(A) p.d. across each cell = $V_P - V_Q$

(B) If i is clockwise then E_2 is source and for anti–clockwise current E_1 is source.

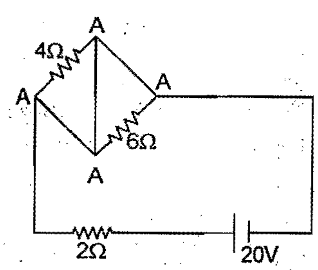
(C) P.D. = E – ir (when battery supplies energy) = E + ir (when battery consumes energy).

By KVL $\quad i=\frac{E_1-E_2}{r_1+r_2}$ (Anticlockwise)

$$\therefore \quad V_P-V_Q=E_1-ir_1=\frac{E_1r_2+E_2r_1}{r_1+r_2}$$

18.25 (AC)

there is zero potential difference across 4 Ω and 6 Ω resistance.



$$i=\frac{20}{2}=10\text{ A}$$

power by battery

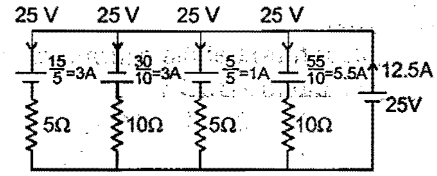
$p_b=\varepsilon i=20\times 10 = 200$ W **Ans.**

18.26 (AC)

The area of cross-section of conductor at point A is less than that at point B. So current density at A is higher. Hence, the electric field at A is more than at B and the thermal power generated at A is more than at B in an element of small same width. since resistance at A is greater

18.27 (BD)

Power supplied by 20 V cell = (–1) (20) = – 20 W
as the cell is not supplying the power,
it is eating the power (getting charged).



18.28 (BC)

As the length is doubled, the cross section area of the wire becomes half. thus the resistance of the wire $R = \rho\frac{L}{A}$ becomes four times the previous value. Hence after the wire is elongated the current becomes one fourth. Electric field is potential difference per unit length and hence becomes half the initial value. The power delivered to resistance is $P = \frac{V^2}{R}$ and hence becomes one fourth.

18.29 (ABD)

Total charge $= \int Idt$ = Area under the curve = 10 C

Average current $= \frac{\int Idt}{\int dt} = 5A$

Total heat produced $= \int I^2Rdt = \int_0^2 (-5t+10)^2 \, . \, 1 \, . \, dt = \frac{200}{3}$ J

Maximum Power = I^2R when I is maximum current. = 100 x 1 = 100 W

18.30 (ABC)

Let a be the radius of left end side cross–section, then radius of cross–section at distance x from left end is a + bx where b is a constant.

From $J = \sigma E \quad \Rightarrow \quad \frac{i}{A} = \sigma E$

$\Rightarrow \quad E = \frac{i}{A} \times \frac{1}{\sigma}$ as i and σ are same for all cross–section

$E \propto \frac{1}{A} = \frac{1}{\pi(a+bx)^2}$

Rate of heat generation per unit length, $H = \frac{i^2\rho}{A}$, So $H \propto \frac{1}{A}$

$\Rightarrow \quad \frac{H}{E} = \frac{i^2\rho}{A} \times \frac{A}{i\rho} = i$ = constant $\Rightarrow \; dV = -\vec{E} \bullet d\vec{x} \quad \Rightarrow \quad \int_\varepsilon^V dV = \int_0^x \frac{\rho \, i \, dx}{\pi(a+bx)^2}$

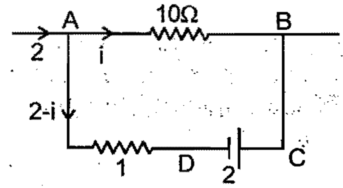
$\Rightarrow \quad V = \varepsilon + \frac{\rho i}{\pi b}\left[\frac{1}{a+bx} - \frac{1}{a}\right] = \varepsilon - \frac{\rho i}{\pi ab}\left(\frac{bx}{a+bx}\right).$

18.31 (BD)

Resistance absorbs energy at the rate of 2W.
Potential difference across AB $\Rightarrow V_{AB}.I = 50$ W
$V_{AB} = 50$ V
Drop across resistor is 2V, therefore EMF of E is 48 V.
As AB is absorbing energy at the rate of 50 W, 48 W is being absorbed by E. Thus E is on charging mode i.e. current is entering from +ve terminal of E.

18.32 (BD)

(Moderate) Let the currents be as shown in the figure



KVL along ABCDA

$\Rightarrow -10\,i - 2 + (2 - i)1 = 0$

$\therefore i = 0$

Potential difference across S = (2 – i)1 = 2 × 1 = 2 V.

18.33 (B)

Both statements 1 and 2 are true. In statement-1 R is varied while in statement-2 R is kept constant. Hence both statements are independent.

18.34 (C)

From relation $\vec{J} = \sigma\vec{E}$, the current density $\vec{J}$ at any point in ohmic resistor is in direction of electric field $\vec{E}$ at that point. In space having non-uniform electric field, charges released from rest may not move along ELOF. Hence statement 1 is true while statement 2 is false.

18.35 (D)

As the length of wire is doubled, the cross-section area of wire becomes half. Therefore resistance of wire becomes four times and current becomes $\frac{1}{4}$th of the initial value.

also $v_d = \frac{I}{neA}$

Since current becomes one fourth and cross-section area of wire becomes half, therefore from above equation the drift velocity of electron becomes half. Hence statement I is false.

18.36 (A)

The potential difference across the resistance is always $|E_1 - E_2|$ in magnitude. Hence statement 1 and 2 are true and statement 2 is correct explanation of statement 1 .

18.37 (A)

Just after switching ON the bulb, the filament of bulb is cold and its resistance is low. But after some time as filament gets hot, its resistance increases and hence withdraws less power from the source as compared to initial duration.

18.38 (C)

The points A and B are at same potential, then under given conditions points A and B on the circuit can be connected by a conducting wire. Hence the circuit can be redrawn as shown in figure 2.

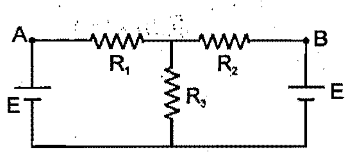
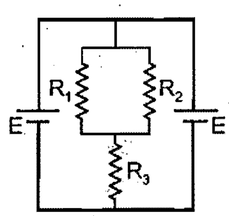


Fig-1



(fig.2)

Therefore statement 1 is true. Statement 2 is obviously false.

18.39 (A)

$R_A = \frac{R.R_V}{R+R_V} < R$

18.40 (A)

$R_B = R + R_G > R$

18.41 (D)

% error in case A.

$$\frac{R_A - R}{R} \times 100 = \left(\frac{R_V}{R + R_V} - 1\right) \times 100 = \frac{-R}{R + R_V} \times 100 \approx -1\%$$

% error in case B $\quad \frac{R_B - R}{R} \times 100 = \frac{R_G}{R} \times 100 \approx 10\%$

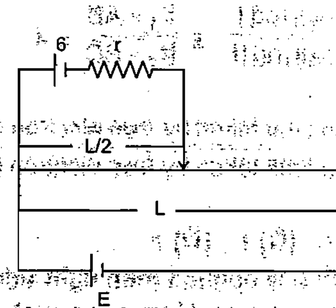
Hence percentage error in circuit B is more than that in A.

18.42 (C)

18.43 (B)

(42 to 43)

case-I S_2 is open

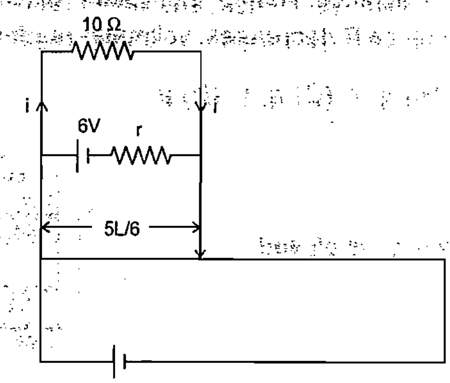


Potential gradient = $\frac{E}{L}$

so $\quad 6 = \frac{E}{L} \frac{L}{2}$

E = 12 V

case-II S_2 is closed



$$i = \frac{6}{10 + r}$$

$$6 - ir = \frac{E}{L} \frac{5L}{12}$$

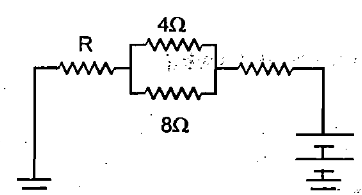
$$6 - \frac{6}{10 + r} r = 5$$

$$\frac{6r}{r + 10} = 1$$

6r = r + 10

r = 2Ω

18.44 (A)



4Ω Current in 4Ω resistance = 4A

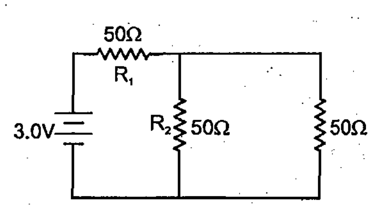
Total current = 4 + 2 = 6A

18.45 (A)

18.46 (A)

18.47 (C)

18 [Moderate], 19 [Easy], 20[Easy] The equivalent circuit can be redrawn as shown in figure1. From figure 1 it is obivious that power dissipated by R_1 is maximum.



Potential difference across R_2 is $= \frac{25}{25 + 50} \times 3 \text{ volt} = 1 \text{ volt}$

Therefore potential difference across R_3 or $R_4 = \frac{20}{20 + 30} \times 1 \text{volt} = 0.4 \text{ volt}$

The equivalent resistance of circuit across the cell is 50+25 = 75 ohms

Therefore current drawn through cell is $\frac{3}{75} \times 1000 \text{mA} = 40 \text{ mA}$.

18.48 (A) q (B) s (C) s (D) q

(A) Since current in both rods is same.

$\therefore n_1 e v_1 A_1 = n_2 e v_2 A_2$

$$\therefore \frac{v_1}{v_2} = \frac{n_2 A_2}{n_1 A_1} = \frac{1}{2} \times \frac{2}{1} = 1$$

(B) $\because E = \rho J = \rho \frac{I}{A}$ $\quad \therefore \frac{E_1}{E_2} = \frac{\rho_1}{\rho_2} \times \frac{A_2}{A_1} = \frac{2}{1} \times \frac{2}{1} = 4$

(C) $\frac{\text{p.d. across rod I}}{\text{p.d. across rod II}} = \frac{E_1 \times AB}{E_2 \times BC} = 4$

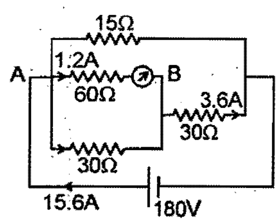
(D) $\frac{\text{Average time taken by free electron to move from A to B}}{\text{Average time taken by free electron to move from B to C}} = \frac{AB}{v_1} \times \frac{v_2}{BC} = 1$

18.49 (A) q, (B) r, (C) r, (D) r

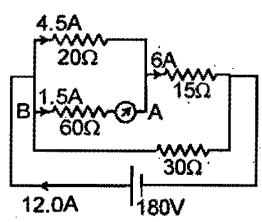
When switch S is opened then right side resistance R which was short circuited earlier contributes to equivalent resistance. Hence, equivalent resistance across the battery increases, power dissipated by left resistance R decreases, voltmeter reading decreases and ammeter reading decreases.

18.50 (A) p, s, t (B) q, r (C) q, t (D) p

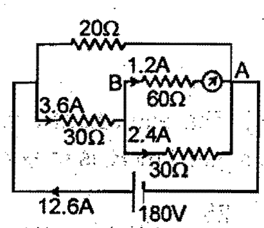
When switch S_1 is closed



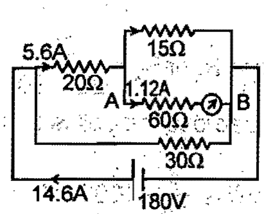
When switch S_2 is closed



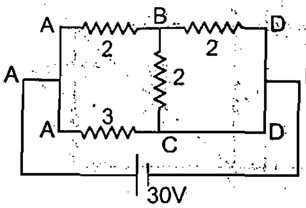
When switch S_3 is closed

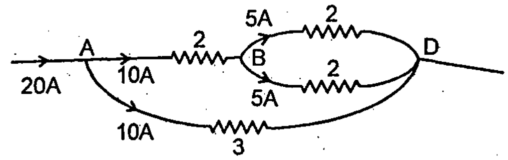


When switch S_4 is closed



18.51 15

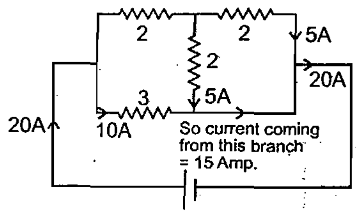




$\Rightarrow R_{eq} = 3/2$

$$i = \frac{30}{3/2} = 20 \text{ Amp.}$$

From figure current through B $\rightarrow$ D branch = 5 Amp.



18.52 20

I_2=20mA towards **left**

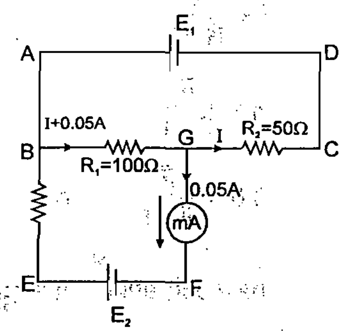
Applying KVL in loop ABCD

$E_1 = (I + 0.05)\, R_1 + IR_2$

$\Rightarrow$ $I = -20$ mA

$\therefore$ Current through R_1 = 30 mA towards right

Current through R_2 = 20 mA towards left



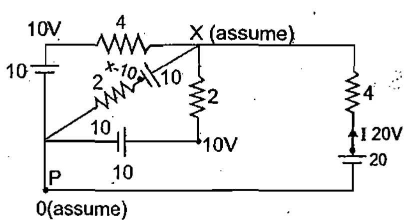
Applying KVL in loop BGFE

$E_2 = (I + 0.05)\, 100 + (0.05)20 = 4$ volts

18.53 70

The simplified circuit is.

We have to find I.



Let potential of point 'P' be '0'. Potential at other points are shown in the figure apply kirchoff's current law at point x.

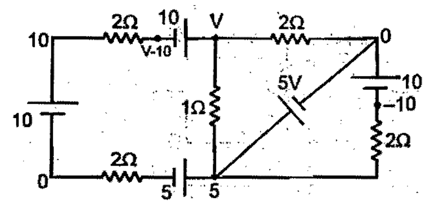
$$\frac{x-10}{4} + \frac{x-10}{2} + \frac{x-20}{4} + \frac{(x-10)-0}{2} = 0$$

$$\Rightarrow x - 10 + 2x - 20 + x - 20 + 2x - 20 = 0 \Rightarrow 6x = 70 \Rightarrow x = \frac{35}{3} \text{ volt.}$$

$$\therefore \quad I = \frac{20 - \frac{35}{3}}{4} = \frac{25}{12} \text{ A.}$$

18.54 5

$\frac{5}{2}$ A

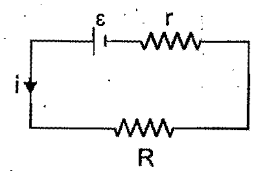


$$\frac{v-20}{2} + \frac{v}{2} + v - 5 = 0 \quad ; \quad v - 20 + v + 2(v - 5) = 0 \Rightarrow 4v - 20 - 10 = 0$$

$$v = \frac{30}{4} = \frac{15}{2}; \quad v - 5 = \frac{15}{2} - 5 = \frac{15-10}{2} = \frac{5}{2} \Rightarrow i = \frac{5/2}{1} = \frac{5}{2} \text{ amp. Ans.}$$

18.55 90%

$$\text{Efficiency} = \eta = \frac{\text{output power}}{\text{input power}}$$



$$\Rightarrow \quad \eta = \frac{i^2R}{\varepsilon i}$$

$$\because \quad i = \frac{\varepsilon}{R+r}$$

$$\eta = \frac{R}{R+r}$$

$$0.6 = \frac{R}{R+r} \quad \Rightarrow \quad 3R + 3r = 5R \; 2R = 3r$$

$$\therefore \quad \text{new efficiency} \quad \eta = \frac{6R}{6R+r} = 0.9 = 90\% \text{ Ans.}$$

18.56 1

V = Potential difference across the cell = Electric field × width of the cell
= 8 × 0.1 = 0.8 volt ....Ans.
ε = emf of the cell = 10 × 0.1 = 1.0 volt ....Ans.
e = 10 ×0.1 = 1volt

18.57 32

When Jockey is not connected. tc tkWdh tqM+h gqbZ ugha gSA

$$I = \frac{E}{13r} \quad ....(i)$$

Resistance per unit length

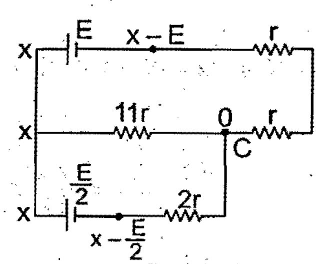
$$\lambda = \frac{12r}{300} \; \Omega/\text{cm}$$

$\therefore$ $\ell = 157.5$ cm

Let potential at C is zero

Then apply Kirchoff's Ist law

$$\frac{x-0}{11r} + \frac{x - \frac{E}{2} - 0}{2r} + \frac{(x-E-0)}{2r} = 0 \quad \Rightarrow x = \frac{11E}{16}$$



$$I_g = \frac{x - \frac{E}{2}}{2r} = \frac{\left(\frac{11\varepsilon}{16}\right) - \frac{E}{2}}{2r} = \frac{3E}{32r}$$

<u>Alternate method</u>

$\ell = 300$ cm

$\therefore \quad r' = (275) \times \frac{12r}{300}$

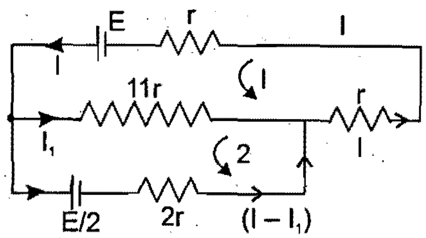
$r' = 11r$

Using KVL in loop (i)

$E - I_1 .11r - Ir - Ir = 0$ ....(i)

and in loop (2)

$-I_1\, 11r + (I - I_1)2r + \frac{E}{2} = 0$ ....(ii)



Solving equation (i) and (ii) we have

$I_1 = \frac{E}{16r}$ and $I = \frac{5E}{32r}$

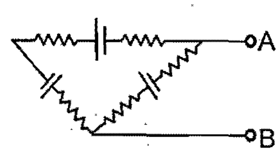
So current in galvanometer

Branch $= (I - I_1 = \frac{5E}{32r} - \frac{E}{16r} = \frac{3E}{32r}$ $\quad I_g = \frac{3E}{32r}$ **Ans. 32**

18.58 42

$R = 100\,\Omega$, $\varepsilon = 3V$

In open circuit



$i = \frac{3+3-3}{5\times200}$ So $V_{AB} = \varepsilon + ir$

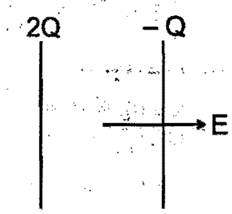
$= 3 + \frac{3}{5\times200} \times 400$ $\quad V_{AB} = 4.2$ V Ans.

19. CAPACITANCE

19.1 (D)

$$E = \frac{2Q}{2A\epsilon_0} + \frac{Q}{2A\epsilon_0} \Rightarrow E = \frac{3Q}{2A\epsilon_0}$$

$$E = \frac{3}{2}\frac{Q}{Cd} \Rightarrow Ed = \frac{3Q}{2C} = V$$



(ii) F = EQ/2

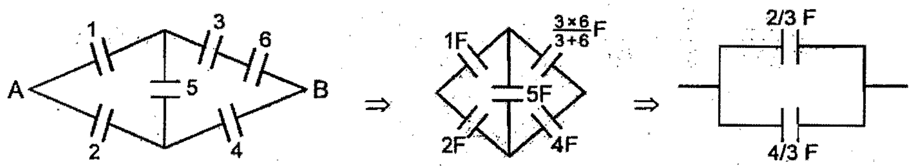
$$F = \left(\frac{2Q}{2A\epsilon_0}\right) \times \frac{(-Q)}{1} = -\frac{Q^2}{A\epsilon_0}$$

$$F = \frac{Q^2}{A\epsilon_0}$$

(iii) $\text{Energy} = \frac{1}{2}\epsilon_0 E^2 Ad \qquad = \frac{1}{2}\epsilon_0\left(\frac{3Q}{2Cd}\right)^2 Ad = \frac{9}{8}\frac{Q^2}{C}$

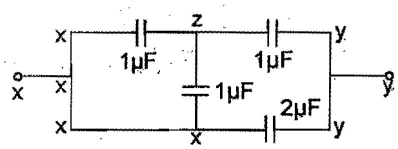
19.2 (B)

Equivalent circuit is

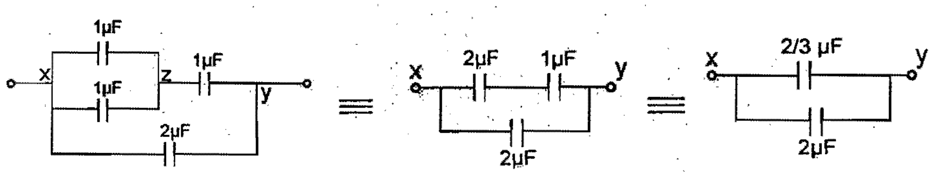


= 2 F

19.3 (C)



Rearrange the circuit



$$C_{eq} = \frac{8}{3}\,\mu F$$

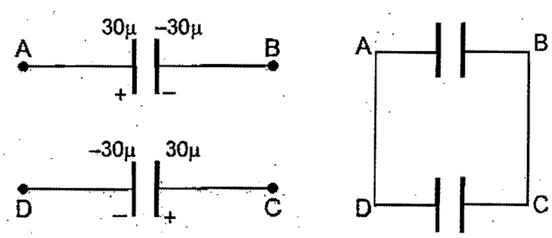
19.4 (D)

$$V = \frac{Q_1 + Q_2}{C_1 + C_2} = 0$$

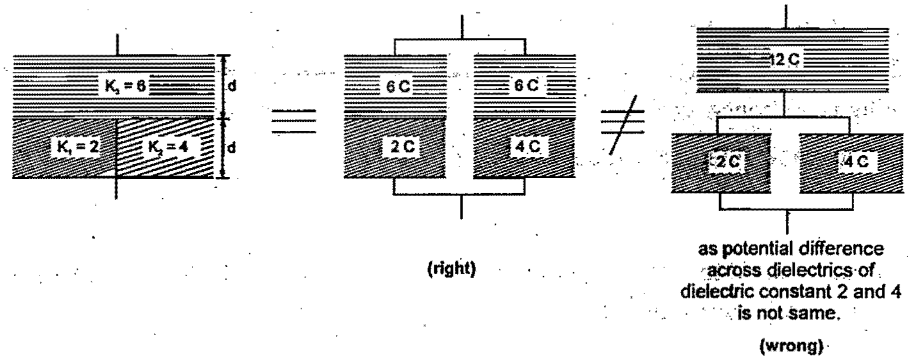
Final potential difference = zero

Final charge = Zero

Charge flow 30 μc from A to D



19.5 (A)



The equivalent capacitance $C_{eq} = \frac{2C \times 6C}{2C + 6C} + \frac{6C \times 4C}{6C + 4C} = 3.9$ C

19.6 (D)

Let the capacitance before insertion of dielectric be C and the resistance be R.

$\therefore \quad q = q_0 e^{-\frac{t}{RC}}$ and $i = \frac{q/C}{R} = \frac{q}{RC}$

$\because$ Just after insertion of dielectric the capacitance increases.

$\therefore$ The charge just after insertion of dielectric remains same, but the current decreases. $\Rightarrow$ (A) and (B) are false

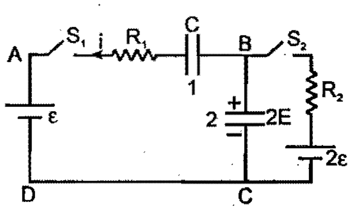
The energy stored in capacitor is $\frac{q^2}{2C}$, hence energy decreases

$\Rightarrow$ (C) is false

The time constant is RC and hence increases.

$\Rightarrow$ (D) is true

19.7 (B)



just before S_1 is closed the potential difference across capacitor 2 is 2E.

just after S_1 is closed the potential difference across capacitors 1 and 2 are 0 and 2E respectively.

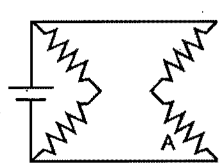
Applying KVL to loop ABCD immediately after S_1 is closed.

$E = -iR_1 + 0 + 2E$

or $\quad i = \frac{E}{R_1}$ towards left

19.8 (D)

This is a DC circuit because the battery is the only source of voltage. Hence the capacitors behave like open circuits. An equivalent circuit is then two parallel sets of two identical series resistors, see figure. The voltage drop across each parallel branch must be the battery voltage of 3V. Since the resistors are identical there is an equal voltage drop of 1.5V across each resistor. In particular there is a drop of 1.5 V across resistor A.



19.9 (A)

Rate of change of energy = V.I.
Initially V = 0 hence VI = 0
finally I = 0 hence VI = 0
∴ first increases then decreases

19.10 (A)

As the key is connected to 1 and 2 frequently and at equal intervals, the two emf's E_1 & E_2 behave as d.c. sources in continuous contact.

The potential due to the two cells is : $V = \left(\frac{E_1R_2 + E_2R_1}{R_1 + R_2}\right)$

Hence the charge on the capacitor is $q = CV = \frac{(E_1R_2 + E_2R_1)C}{(R_1 + R_2)}$ **Ans.**

19.11 (C)

$U_i = \frac{1}{2}C\varepsilon_1^2$

$U_f = \frac{1}{2}C\varepsilon_2^2 \qquad \Delta U = \frac{1}{2}C(\varepsilon_2^2 - \varepsilon_1^2)$

$Q_{in} = +C\varepsilon_1$; $Q_{final} = -C\varepsilon_2$
$\Delta Q = c(\varepsilon_1 + \varepsilon_2)$
work done by battery $W_b = E_2 \cdot \Delta Q = C(\varepsilon_2 + \varepsilon_1)\varepsilon_2$
Heat generated = $W_b - \Delta U$

$= c\varepsilon_2^2 + c\varepsilon_1\varepsilon_2 - \frac{1}{2}C(\varepsilon_2^2 - \varepsilon_1^2) = \frac{1}{2}C(\varepsilon_2^2 + \varepsilon_1^2 + 2\varepsilon_1\varepsilon_2) = \frac{1}{2}C(\varepsilon_1 + \varepsilon_2)^2$

19.12 (A)

The equivalent circuit is as shown in figure (b).

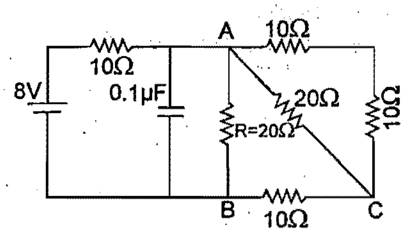


Fig. (a)

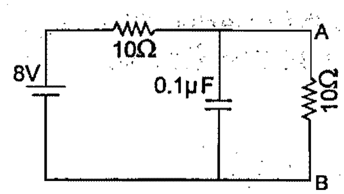


Fig. (b)

In the steady state the potential difference across AB is 4 volts.
∴ Charge on capacitor in steady state is
$q = CV = 0.4\ \mu C$

Current through resistor R is $I = \frac{V}{R} = \frac{4}{20} = 0.2$ A

19.13 (D)

$U = \frac{1}{2}C_{eq}V^2$ $\quad C_1 = \frac{k\epsilon_0 A}{d/2} = \frac{2\epsilon_0 A}{(d/2)}$

$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$ $\quad C_2 = \frac{\epsilon_0 A}{d/2}$; $\quad C_{eq} = \frac{\left(2\frac{\epsilon_0 A}{d/2}\right)\times\frac{\epsilon_0 A}{d/2}}{3\frac{\epsilon_0 A}{d/2}} = \frac{4}{3}\frac{\epsilon_0 A}{d}$

$U = \frac{1}{2}\left(\frac{4}{3}\frac{\epsilon_0 A}{d}\right)V^2 = \frac{2}{3}\left(\frac{\epsilon_0 A}{d}\right)V^2$

19.14 (B)

Method I

Force between plates

$$F = \frac{Q^2}{2A\epsilon_0} = \frac{\left(\frac{\epsilon_0 A}{x}V\right)^2}{2A\epsilon_0} = \frac{\epsilon_0 AV^2}{2x^2}$$ where x is separation between plates

$dW = F\,dx$

$$W = \int_d^{2d}\frac{\epsilon_0 AV^2}{2x^2}dx = \left[\frac{\epsilon_0 AV^2}{4x}\right]_d^{2d} = \frac{CV^2}{4} = 200\,\mu J$$

Method II

$U_i + W_B + W_{ext} = U_f + \text{loss}$

Process is slow so energy loss is zero work done by battery = $W_B = QE$

$Q = Q_f - Q_i = 20 - 40 = -20$

$W_B = -20 \times 20$

$\frac{1}{2}2 \times 20^2 - 20 \times 20 + W_{ext} = \frac{1}{2}1 \times 20^2 + 0$

$W_{ext} = 200\,\mu J$

19.15 (A)

Force on metal plate S due to electrostatic attraction by plate T is $F = \frac{Q^2}{2A\epsilon_0}$

Force exerted on plate S by spring is = mg

In equilibrium $\Rightarrow$ $\frac{Q^2}{2A\epsilon_0} = mg$ or $Q = \sqrt{2mgA\epsilon_0}$

19.16 (C)

$C_{eff} = \frac{\varepsilon_0 A}{d}$ since effective capacitance between plates A and E is zero.

$\therefore \quad U = \frac{1}{2}CV^2 = \frac{\varepsilon_0 A}{2d}V^2$

19.17 (D)

When switch S_2 is closed, due to symmetry no charge will flow through S_2.

Alternate solution :

After closing and before closing the switch there is no change in potential of any point.

19.18 (D)

$I_{max} = \frac{2\varepsilon}{R}$ at t = 0

$I = \frac{\varepsilon}{R}$ at t = ∞

so charge on the capacitor is Cε, when current is 50% of maximum current.

19.19 (D)

In the given cross-section which lies inside the capacitor plates, no charge flows. hence the required charge is 0.

19.20 (C)

Energy density = $\frac{1}{2} k\epsilon_0 E^2$

Since the cell remains connected, V remains unchanged (and therefore E remains unchanged)

⇒ Energy density will increase k times.

19.21 (C)

Charge on outer surface of C = – charge on inner surface of C

Hence potential at B due to charge on conductor C = 0

charge on outer surface of dielectric = – charge on inner surface of dielectric

∴ Potential at B due to charge on dielectric = 0

Potential at B due to charge on A $= \frac{Q}{4\pi \epsilon_0 b}$

∴ net potential at B $= \frac{Q}{4\pi \epsilon_0 b}$.

19.22 (C)

Let σ be the charge density of conducting plate and V be the volume of either dielectric

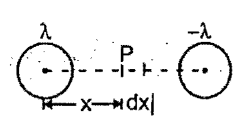
$$\therefore \frac{U_1}{U_2} = \frac{\left(\frac{1}{2} K_1 \epsilon_0 \; E_1^2\right) V}{\left(\frac{1}{2} K_2 \epsilon_0 \; E_2^2\right) V} = \frac{K_1}{K_2} \frac{\left(\frac{\sigma}{K_1 \epsilon_0}\right)^2}{\left(\frac{\sigma}{K_2 \epsilon_0}\right)^2} = \frac{K_2}{K_1}$$

19.23 (C)

Let us give equal and opposite charges to two wires so that they would have linear charge density as + λ and – λ.

Electric field at point P,

$$E = \frac{\lambda}{2\pi \epsilon_0 x} + \frac{\lambda}{2\pi \epsilon_0 (\eta a - x)} \qquad \int dV = -\int E dx = -\int_a^{\eta a - a} E dx$$



where a is radius of wire ⇒ $C = \frac{\lambda}{|V|} = \frac{\pi \epsilon_0}{\ln \eta}$

19.24 (D)

The resultant force acting per unit area of each plate can be written as $F = F_0 - F'$.
where F_0 is the force acting on unit area of plate due to other plate and F' is the force acting on unit area of plate from the dielectric.

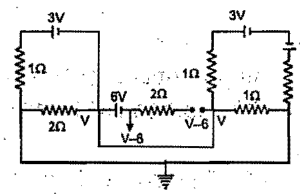
Now, $F = \dfrac{\dfrac{q^2}{2\varepsilon_0\varepsilon A}}{A} = \dfrac{\left(\dfrac{\varepsilon_0\varepsilon A}{d}V\right)^2}{2\varepsilon_0\varepsilon A} \times \dfrac{1}{A} \quad \Rightarrow \quad F = \dfrac{\varepsilon_0\varepsilon V^2}{2d^2}$

Also $F_0 = F \times \varepsilon$

So $F' = F_0 - \dfrac{F_0}{\varepsilon} = F_0\left(1-\dfrac{1}{\varepsilon}\right) = \varepsilon F\left(1-\dfrac{1}{\varepsilon}\right) = \dfrac{\varepsilon(\varepsilon-1)\varepsilon_0 V^2}{2d^2}$

19.25 (B)

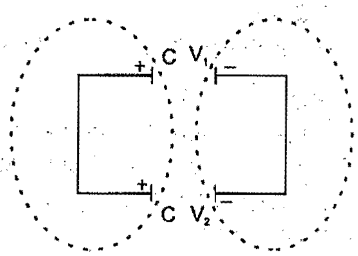
In steady state the capacitor is fully charged and is treated as open circuit, so no current flows through branch containing capacitor in steady state. So the circuit can be redrawn as :



Potential difference across the capacitor in steady state
$= V - 6 - V = -6V$
(–ve sign signifies that left hand plate is of negative polarity)
Charge $= CV = 1 \times 6 = 6\ \mu C$

19.26 (A)

As the charge of isolated system remains conserved, so the sum of charges of plates having-ve polarity remains constant. As potential of two capacitors are different so some charge flows into the circuit till both acquire the same potential.



As charge flows, $\Delta H \neq 0$, and hence $\sum u_i \neq \sum U_f$

Let final common potential be V, then

$V = \dfrac{C_1V_1 + C_2V_2}{C_1 + C_2} = \dfrac{V_1 + V_2}{2}$ [as $C_1 = C_2 = C$]

19.27 (ABCD)

$V_0 = I_0R = 10 \times 10 = 100$ volts (since, $I_0 = 10$amp from figure) Hence (A) is correct
Also : $I = I_0 e^{-t/RC}$

Taking log ; $\ell n\left(\dfrac{I_0}{I}\right) = \dfrac{t}{RC} \quad \Rightarrow \quad C = \dfrac{t}{R\ell n(I_0/I)}$

At ; t = 2 sec, I = 2.5 A

$C = \dfrac{2}{10\ell n\left(\dfrac{10}{2.5}\right)}$

$C = \dfrac{2}{10\ell n4} = \dfrac{2}{10 \times 2\ell n2} = \dfrac{1}{10\ell n2}$ F Hence (B) is correct.

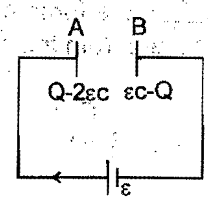
Heat produced $= \dfrac{1}{2}CV^2 = \dfrac{1}{2}\left(\dfrac{1}{10\ell n2}\right)(100^2) = \dfrac{500}{\ell n2}$ joules.

Hence (C) is correct

Thermal power in the resistor will decrease with a time constant $\dfrac{1}{2\ell n2}$ second. Hence (D) is correct

19.28 (ACD)

Suppose charge flown through the battery is Q, then charge distribution will be as :



The electric field in the region between A and B is

$$= \frac{Q-2\varepsilon C}{2A \epsilon_0} - \frac{\varepsilon C - Q}{2A \epsilon_0} = \frac{2Q-3\varepsilon C}{2A \epsilon_0}$$

∴ Potential differnce between the plates,

$$\frac{2Q-3\varepsilon C}{2A \epsilon_0}.d = \varepsilon \Rightarrow \frac{2Q-3\varepsilon C}{2}\frac{1}{C} = \varepsilon$$

$\Rightarrow$ $2Q = 5\,\varepsilon C$ $\quad \Rightarrow Q = \frac{5\varepsilon C}{2}$

∴ work done by battery = $\varepsilon Q = \frac{5\varepsilon^2 C}{2}$

19.29 (ACD)

(A) $E = \frac{1}{2}CV^2$

As potential difference source between the plates is connected, p.d. remains constant. But capacitance C becomes KC hence energy stored is increased by factor K.

(B) Electric field $\frac{V}{d}$ is not changed.

(C) Charge on each plate is increased by factor K hence force between them increases by factor K^2. For effect of the medium, they must completely lie in the medium.

(D) Q = CV

Hence charge becomes KQ as C becomes KC and V remain unchanged.

19.30 (ABCD)

Charge on capacitor before insertion of dielectric slab = 100 μC

Charge on capacitor after insertion of dielectric slab = 300 μC

Increase in charge on the capacitor = 300 – 100 = 200 μC

Heat produced = 0

Energy supplied by the cell = increase in stored potential energy + work done on the person who filling the dielectric slab + heat produced.

19.31 (B D)

The instantaneous charge on the capacitor is $q(t) = q_0\,[1 - e^{-t/RC}] = CV\,[1 - e^{-t/RC}]$

The instantaneous current

$$i = \frac{dq}{dt} = CV\left(\frac{1}{RC}\right)e^{-t/RC}$$

∴ $\quad i = \frac{V}{R}e^{-t/RC} \quad$ or $\quad i_0 = \frac{V}{R} \quad (\because t = 0)$

Given that V and R are same for both capacitors, so the initial current in both condensers is same moreover this is not zero.

During discharge, the instantaneous charge q is

$$q(t) = q_0 e^{-t/RC}$$

Let $q' = q_0/2$ at t = t, then $\quad \frac{q_0}{2} = q_0 e^{-1t/RC}$

or $\quad t = RC \log_e 2$

If t_1 and t_2 be the times in which the two capacitors lose 50% of their charge, then

$$\frac{t_1}{t_2} = \frac{RC_1 \log_e 2}{RC_2 \log_e 2} = \frac{C_1}{C_2} = \frac{1}{2}$$

$$t_1 = t_2/2$$

This shows that C_1 loses 50% charge sooner than C_2 because it takes time t_1 which is half of t_2]

19.32 (D)

If potential difference across an isolated charged capacitor is doubled by doubling separation between plates, the energy stored is capacitor from $U = \frac{Q^2}{2C}$ becomes double of previous value. Hence statement 1 is false.

19.33 (D)

Let the electric field in region I and II be E_1 and E_2. The potential difference across left half capacitor and right half capacitor is same. Therefore $E_1 d = E_2 d$ where d = inter planar gap.

$\therefore \quad E_1 = E_2$

Hence statement 1 is false, statement 2 is correct by definition.

19.34 (D)

The electrostatic force on metal of capacitor is = pressure × area of plate = $\frac{\sigma^2}{2\epsilon_0}A$

σ = charge per unit area on plate.

Since charge on metal plate of an "isolated" capacitor does not change, force on metal plate remains same.

Electric field decreases due to induced charges in dielectric, but this does not effect the charge distribution on isolated metal plate.

19.35 (D)

The battery energizes the circuit and maintains the flow of electron from positive plate of capacitor to positive terminal of battery through wires and from wires to negative plate on other side.

No transfer of charge takes place within the plates in spite of having the electric field in between the plates.

19.36 (B)

$i = 2 \times 10^{-2}$ A

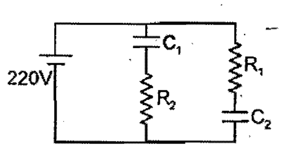
$P_{R_1} = i^2 R_1 = (2 \times 10^{-2})^2 \times 4 \times 10^3 = 1.6$ W

19.37 (C)

$Q_{C_1} = V_{R_1} \times C_1 = 80 \times 3 \times 10^{-6} = 240\ \mu C$

$Q_{C_2} = V_{R_2} \times C_2 = 140 \times 6 \times 10^{-6} = 840\ \mu C$

19.38 (D)



$Q_{C_1} = EC_1 = 220 \times 3 \times 10^{-6} = 660\ \mu C$

19.39 (C)

19.40 (D)

19.41 (C)

39 to 41

39. & 40 (Moderate)

41. (Tough)

For $t = 0$ to $t_o = RC$ seconds, the circuit is of charging type. The charging equation for this time is

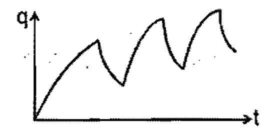
$$q = CE(1 - e^{-\frac{t}{RC}})$$

Therefore the charge on capacitor at time $t_0 = RC$ is $q_o = CE(1 - \frac{1}{e})$

For $t = RC$ to $t = 2RC$ seconds, the circuit is of discharging type. The charge and current equation for this time are

$$q = q_o e^{-\frac{t-t_o}{RC}} \quad \text{and } i = \frac{q_o}{RC} e^{-\frac{t-t_o}{RC}}$$

Hence charge at t = 2 RC and current at t = 1.5 RC are



$$q = q_o e^{-\frac{2RC-RC}{RC}} = \frac{q_o}{e} = \frac{1}{e} CE(1 - \frac{1}{e})$$

and $$i = \frac{q_o}{RC} e^{-\frac{1.5RC-RC}{RC}} = \frac{q_o}{\sqrt{e}RC} = \frac{E}{\sqrt{e}R}(1 - \frac{1}{e}) \text{ respectively}$$

Since the capacitor gets more charged up from t = 2RC to t= 3RC than in the interval t=0 to t=RC, the graph representing the charge variation is as shown in figure

19.42 (A) p, r (B) q; s (C) p, r (D) t

(A) By inserting dielectric slab, capacitance of 1 increases there by increasing charge on capacitor 2 as more charge is flown through the battery. Energy stored in capacitor also increases.

(B) By increasing separation between the plates, capacitor C_1 decreases. Charge on C_2 also decreases.

(C) By shorting capacitor-1, only capacitor 2 remains in the circuit. Potential difference across C_2 increases thereby increasing charge on 2 as well as energy stored.

(D) By earthing plate of capacitor 1 potentials will change but there will be no potential difference change, making no overall change in the circuit.

19.43 (A) p,r, t (B) q, s, t (C) q, s (D) p, r

(A) At constant potential difference, when interplanar separation is increased, the capacitance decreases.

From $U = \frac{1}{2}CV^2$, the potential energy decreases.

Also from $E = \frac{V}{d}$ electric field decreases

(B) At constant charge when interplanar separation is increased the capacitance decreases.

From $U = \frac{Q^2}{2C}$, the potential energy increases

Since charge density on plates is constant, electric field remains same.

(C) At constant potential difference, when area of plate increases the capacitance increases.

Hence from $U = \frac{1}{2}CV^2$, the potential energy increases

Also from $E = \frac{V}{d}$, the electric field remains same.

(D) At constant charge on increase in area of plates

From $U = \frac{1}{2}\frac{Q^2}{C}$, the potential energy decreases and since charge density on plate decreases electric field decreases.

19.44 (A) p (B) r (C) q (D) p

(Moderate) The initial charge on capacitor = $CV_i = 1 \times 2\ \mu C = 2\ \mu C$

The final charge on capacitor = $CV_f = 1 \times 4\ \mu C = 4\ \mu C$

∴ Net charge crossing the cell of emf 4V is

$q_f - q_i = 4 - 2 = 2\ \mu C$

The magnitude of work done by cell of emf 4V is

$$W = (q_f - q_i)\, 4 = 8\ \mu J$$

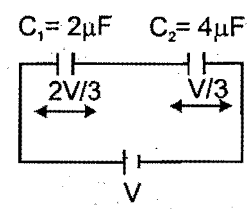
The gain in potential energy of capacitor is

$$\Delta U = \frac{1}{2}C(V_f^2 - V_i^2) = \frac{1}{2}\ 1 \times [4^2 - 2^2]\ \mu J = 6\ \mu J$$

Net heat produced in circuit is

$$\Delta H = W - \Delta U = 8 - 6 = 2\ \mu J$$

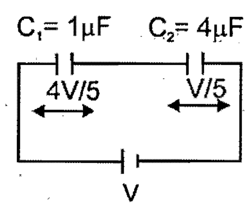
19.45 (A) p, r (B) q (C) q, s (D) q, t



U_i for $C_1 (C_1 U_i) = C_1 \times \frac{4}{9} V^2 \times \frac{1}{2} = \frac{4V^2}{9}$

U_i for $C_2 (C_2\ U_i) = 4 \times \frac{V^2}{9} \times \frac{1}{2} = \frac{2V^2}{9}$

When separation of plates of C_1 gets doubled, its capacity becomes half :



U_f for $C_1 (C_1 U_f) = 1 \times \frac{16V^2}{25} \times \frac{1}{2} = \frac{8V^2}{25}$

U_f for $C_2 (C_2\ U_f) = 4 \times \frac{V^2}{25} \times \frac{1}{2} = \frac{2V^2}{25}$

19.46 75

H (in 2Ω) 225 μJ, H (in 3Ω) 150μJ, H (in 6Ω) 75μJ

Energy taken from cell = 20 × 30 μJ

= 600 μJ

Energy stored in capacitors = $\frac{1}{2}$ 3. 10^2 = 150 μJ

∴ Heat produced in resistors = 600 – 150 = 450 μJ

Divide this heat in 2Ω and (equivalent of 3Ω and 6Ω)

i.e., in 2Ω and 2Ω

which is 225 μ J, 225 μJ

∴ Heat produced in 2Ω = 225 μJ

Further divide 225 μJ in 3Ω and 6Ω in inverse ratio of resistance $\left(\because P = \frac{V^2}{R}\right)$

Heat in 3Ω = $\frac{225}{9} \times 6 = \frac{225 \times 2}{3}$ = 75 × 2 = 150 μJ

Heat in 6Ω = $\frac{225}{9}$ × 3 = 75 μJ

19.47 4

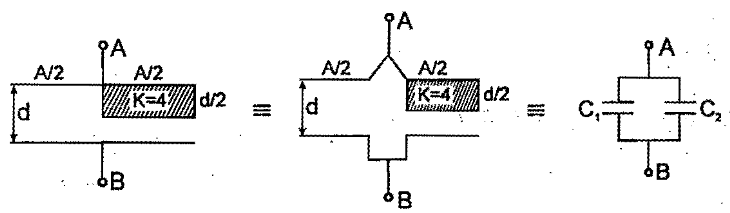
The charge on the capacitor when current reaches I_0

$q_0 = (I_0R).C_1$

When the switch is in position 2, this charge is shared with capacitor C_2 and at steady state potential across C_1 is equal to that across C_2. The energy lost in this process

$$\Delta U = \frac{1}{2}.\frac{C_1C_2}{C_1+C_2}\times\left(\frac{q_0}{C_1}\right)^2 = \frac{1}{2}.\frac{C_1C_2}{C_1+C_2}\times(I_0R)^2 = 4\ J$$

19.48 13



$$C_1 = \frac{\varepsilon_0 A/2}{d},\ C_2 = \frac{\varepsilon_0 A/2}{\frac{d/2}{k}+\frac{d}{2}} = \frac{4\varepsilon_0 A}{5d} \Rightarrow C = C_1 + C_2 = \frac{13}{10}\frac{\varepsilon_0 A}{d}$$

19.49 300

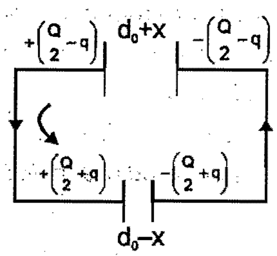
$$E < 10^6 \quad\Rightarrow\quad \frac{10^3}{d} < 10^6$$

$$d > 10^{-3}\ m^2 \quad\Rightarrow\quad C = \frac{k\varepsilon_0 A}{d}$$

$$d = \frac{k\varepsilon_0 A}{C} > 10^{-3} \quad\Rightarrow\quad A > \frac{10^{-3}\times C}{k\varepsilon_0} \quad\Rightarrow\quad A > \frac{10^{-3}\times 50\times 10^{-12}}{(6\pi)\times\left(\frac{1}{36\pi}\times 10^{-9}\right)} = 300\ mm^2$$

19.50 Let each plate moves a distance 'x' from its initial position.

Let q charge flows in the loop. using KVL



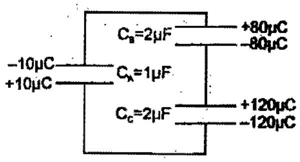
$$\frac{\left(\frac{Q}{2}-q\right)(d+x)}{\epsilon_0 A} - \frac{\left(\frac{Q}{2}+q\right)(d-x)}{\epsilon_0 A} = 0$$

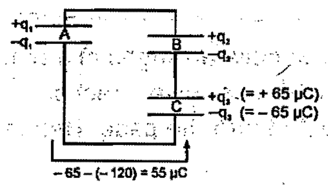
$$\therefore q = \frac{Qx}{2d_0};\quad I = \frac{dq}{dt} = \frac{Q}{2d_0}\left(\frac{dx}{dt}\right);$$

Ans, $I = \dfrac{Qv_0}{2d_0}$

19.51 75

Initial state Final state





From conservation of charge

$-q_1 - q_3 = 10 - 120 = -110\ \mu C$ ....(1)

$-q_2 + q_3 = -80 + 120 = +40\ \mu C$ ....(2)

In the final state

$$\frac{q_1}{C_A} = \frac{q_2}{C_B} + \frac{q_3}{C_C} \Rightarrow \frac{q_1}{1} = \frac{q_2}{2} + \frac{q_3}{2} \quad \text{at } q_2 + q_3 = 2q_1 \Rightarrow \frac{q_1}{1} = \frac{q_2}{2} + \frac{q_3}{2} \;;\; q_2 + q_3 = 2q_1$$

Solving we get $q_3 = 65\ \mu C$.

The charge on lower plate of capacitor C_C changes from $-120\ \mu C$ to $-65\ \mu C$.

Hence the charge flowing through shown connecting wire is

$(120 - 65) = 55\ \mu C$.

final charges

$q_3 = 65\mu C$; $q_2 = 25\mu C$; $q_1 = 45\mu C$

Heat produced = $U_i - U_f$

$$= \left[\frac{(120\mu C)^2}{2\times 2\mu F} + \frac{(80\mu C)^2}{2\times 2\mu F} + \frac{(10\mu C)^2}{2\times 1\mu F}\right] - \left[\frac{(65\mu C)^2}{2\times 2\mu F} + \frac{(25\mu C)^2}{2\times 2\mu F} + \frac{(45\mu C)^2}{2\times 1\mu F}\right] = 3025\ \mu J \qquad \text{Ans. } 3025$$

19.52 5

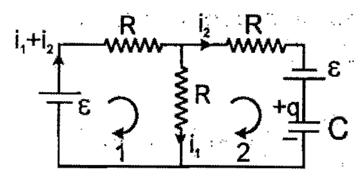
Ans. A+B = 2 + 3 = 5

Applying Kirchoff's law in

Loop1 1

$\varepsilon - (i_1 + i_2) R - i_1 R = 0$ ...(1)

Loop 2 2



$$-i_2 R + \varepsilon - \frac{q}{C} + i_1 R = 0 \quad ....(2)$$

eliminating i_1 from (1) and (2)

$$\varepsilon - \frac{q}{C} - i_2 R + \frac{\varepsilon - i_2 R}{2} = 0 \quad \text{or} \quad \frac{3\varepsilon}{2} - \frac{q}{C} - \frac{3}{2} i_2 R = 0$$

$$i_2 = \frac{dq}{dt}$$

$$\Rightarrow \frac{3C\varepsilon - 2q}{2C} = \frac{3}{2} R \frac{dq}{dt} \quad \text{or} \quad \int_0^q \frac{dq}{3C\varepsilon - 2q} = \int_0^t \frac{dt}{3RC}$$

$$\text{or} \quad -\frac{1}{2} \ln\left(\frac{3C\varepsilon - 2q}{3C\varepsilon}\right) = \frac{t}{3RC} \quad \text{or} \quad 1 - \frac{2q}{3C\varepsilon} = e^{-\frac{2t}{3RC}}$$

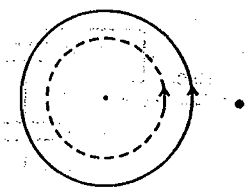
$$\Rightarrow q = \frac{3C\varepsilon}{2}\left(1 - e^{-\frac{2t}{3RC}}\right) \qquad i_2 = \frac{dq}{dt} = \left(\frac{\varepsilon}{R}\right) e^{-\frac{2t}{3RC}}$$

$$\text{from (1)}, \quad i_1 = \frac{\varepsilon - i_2 R}{2R} = \frac{\varepsilon}{2R}\left(1 - e^{-\frac{2t}{3RC}}\right)$$

20. MAGNETIC EFFECT OF CURRENT & MAGNETIC FORCE OF ON CHARGE CURRENT

20.1 (A)

Disc behaves like made up of coils arranged in a plane in which current is flowing in anti-clockwise direction.
Hence, the field at A is directed into the page. Hence (A).



20.2 (A)

$$\vec{B} = \frac{\mu_0}{4\pi} q \frac{\vec{v} \times \vec{r}}{r^3} \quad \text{and} \quad \vec{E} = \frac{1}{4\pi \epsilon_0} \frac{q\vec{r}}{r^3} \quad \therefore \quad \vec{B} = \mu_0 \epsilon_0 (\vec{v} \times \vec{E}) = \frac{\vec{v} \times \vec{E}}{c^2}$$

20.3 (A)

Since

$$\vec{B} = \frac{\mu_0}{4\pi} q \frac{\vec{v} \times \vec{r}}{r^3}, \ \vec{v} \times \vec{r} \text{ must be same}$$

where $\vec{v}$ = velocity of charge with respect to observer

Let A and B are the observers

then $(\vec{v}_C - \vec{v}_A) \times \vec{r} = (\vec{v}_C - \vec{v}_B) \times \vec{r}$

or $(\vec{v}_A - \vec{v}_B) \times \vec{r} = 0$. or $(\vec{v}_A - \vec{v}_B) \parallel \vec{r}$

20.4 (C)

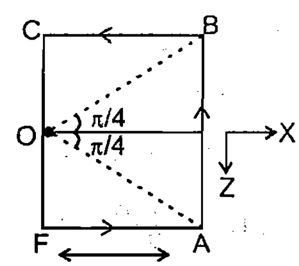
Due to FABC the magnetic field at O is along y – axis and due to CDEF the magnetic field is along x–axis .

Hence the field will be of the form $A\left[\hat{i} + \hat{j}\right]$

Calculating field due to FABC :

due to AB :

$$\vec{B}_{AB} = \frac{\mu_0 i}{4\pi\left(\frac{\ell}{2}\right)} (\sin 45° + \sin 45°)\ \hat{j} \quad = \sqrt{2}\frac{\mu_0 i}{2\pi\ell}\hat{j}$$



Due to BC :

$$\vec{B}_{BC} = \frac{\mu_0 i}{4\pi\left(\frac{\ell}{2}\right)} (\sin 0° + \sin 45°)\hat{j} \quad = \frac{\mu_0 i}{2\sqrt{2}\pi\ell}\hat{j}$$

Similarly due to FA :

$$\vec{B}_{FA} = \frac{\mu_0 i}{2\sqrt{2}\pi\ell}\hat{i}$$

Hence $\vec{B}_{PABC} = \frac{\mu_0 i}{\pi\ell}\left[\frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} + \frac{\sqrt{2}}{2}\right]\hat{j}$

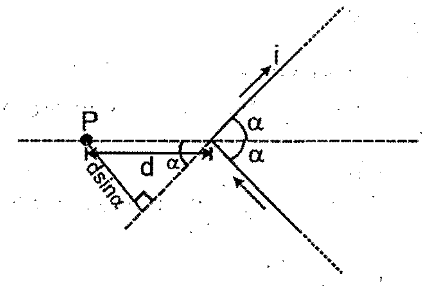
$$\vec{B}_{FABC} = \frac{\sqrt{2}\mu_0 i}{\pi\ell}\left(\hat{j}\right)$$

Similarly due to CDEF:

$$\vec{B}_{CDEF} = \frac{\sqrt{2}\mu_0 i}{\pi\ell}\left(\hat{i}\right) \quad \Rightarrow \vec{B}_{net} = \frac{\sqrt{2}\mu_0 i}{\pi\ell}\left(\hat{i} + \hat{j}\right)$$

20.5 (B)

Let us compute the magnetic field due to any one segment :



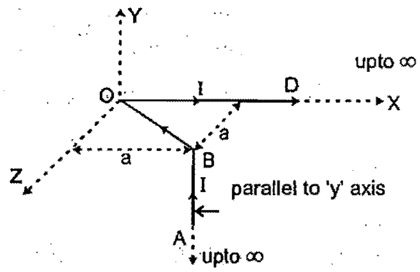
$$B = \frac{\mu_0 i}{4\pi(d\sin\alpha)}(\cos 0^0 + \cos(180-\alpha))$$

$$= \frac{\mu_0 i}{4\pi(d\sin\alpha)}(1-\cos\alpha) = \frac{\mu_0 i}{4\pi d}\tan\frac{\alpha}{2}$$

Resultant field will be :

$$B_{net} = 2B = \frac{\mu_0 i}{2\pi d}\tan\frac{\alpha}{2} \Rightarrow k = \frac{\mu_0 i}{2\pi d}$$

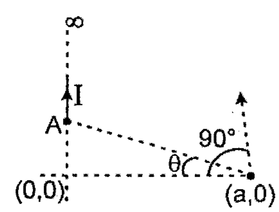
20.6 (C)



$B_{OD} = 0$ $B_{OB} = 0$

$$B_{AB} = \frac{\mu_0 I}{4\pi a\sqrt{2}}[\cos 45°(-\hat{i}) + \cos 45°\hat{k}] = \frac{\mu_0 I}{8\pi a}(-\hat{i}+\hat{k})$$

20.7 (B)



$$B = \frac{\mu_0}{4\pi}\frac{i}{a}(\sin 90 + \sin(-\theta))$$

$$= \frac{\mu_0}{4\pi}\frac{i}{a}(1-\sin\theta) = \frac{\mu_0}{4\pi}\frac{i}{a}\left(1-\frac{b}{\sqrt{a^2+b^2}}\right)$$

20.8 (B)

Magnetic field strength at P due to I_1

$$\vec{B}_1 = \frac{\mu_0 I_1}{2\pi(AP)}\hat{k} = \frac{4\pi\times10^{-7}\times2}{2\pi\times1\times10^{-2}}\hat{k} = (4\times10^{-5}T)\hat{k}$$

Magnetic field strength at P due the I_2 $\vec{B}_2 = \frac{\mu_0 I_2}{2\pi(BP)}\hat{j} = \frac{4\pi\times10^{-7}\times3}{2\pi\times2\times10^{-2}}\hat{j} = (3\times10^{-5}T)\hat{j}$

Hence, $\vec{B} = (3\times10^{-5}T)\hat{j} + (4\times10^{-5}T)\hat{k}$

20.9 (C)

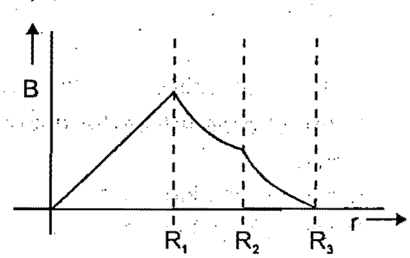
By symmetry, the magnetic field at the centre P is zero.

20.10 (D)

$\oint_{ABCDA} \vec{B}.\vec{d\ell} = \mu_0(i_1 + i_3 + i_2 - i_3) = \mu_0(i_1 + i_2)$ [since for the given direction of circumlation i_3 entering at PSTU is positive while i_3 at PQRS is negative]

20.11 (C)

From ampere's law, the field at the axis is zero.
From x = zero to R_1, the field increases linearly as the current enclosed increases.
From x = R_1 to R_2 and from x = R_2 to R_3, the field decreases hyperbolicaly but with different slopes as the media are different.
Hence the required graph is



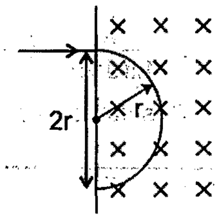
20.12 (D)

$F = q[\, v\,(-\hat{i})] \times B(\hat{i})] = 0$

Because B as well as v is are along axis of circular loop.

20.13 (B)

Electromagnetic force will provide the necessary centripetal force.



$$eBv = \frac{mv^2}{r}$$

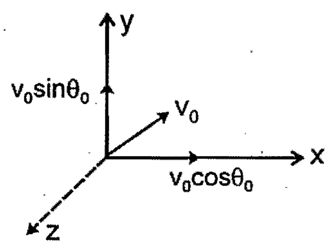
$$\Rightarrow \quad r = \frac{mv}{eB} = \frac{v}{B\alpha} = \frac{(2\alpha d)(B)}{(B\alpha)} = 2d$$

i.e. the electron will move out after travelling on a semicircular path of radius r = 2d.

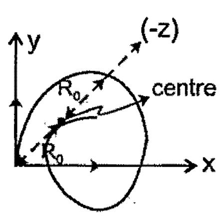
Hence (B)

20.14 (D)

As the magnetic field is along the x-axis, the magnetic force will be along (–) z-axis from $t = 0$ to $t = T_0$ and along (+)ve z-axis from $t = T_0$ to $t = 2T_0$.



For $t = 0$ to $t = T_0$:–



At $t = \frac{T_0}{2}$;

x-coordinate $= \frac{(V_0\cos\theta)T_0}{2} = \frac{P_0}{2}$ (Since pitch $= P_0 = (V_0\cos\theta)T_0$)

y-coordinate = 0 (from figure)

and z-coordinate $= -2R_0$ (from figure)

Hence (A) is correct.

Similarly at $t = \frac{3T_0}{2}$;

coordinates are $\left(\frac{3P_0}{2}, 0, 2R_0\right)$ Hence (B) is correct.

Note : z -coordinate will be $+2R_0$, because from $t = T_0$ to $t = 2T_0$, direction of $\vec{B}$ changes.

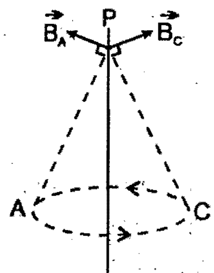
As the charge will perform circular motion about x-axis, the two extremes from x-axis are $2R_0$ from each other.

Hence (C) is also correct.

Hence only (D) is incorrect.

20.15 (A)

The point charge moves in circle as shown in figure. The magnetic field vectors at a point P on axis of circle are $\vec{B}_A$ and $\vec{B}_C$ at the instants the point charge is at A and C respectively as shown in the figure.



Hence as the particles rotates in circle, only magnitude of magnetic field remains constant at the point on axis P but its direction changes.

Alternate solution

The magnetic field at point on the axis due to charged particle moving along a circular path is given by

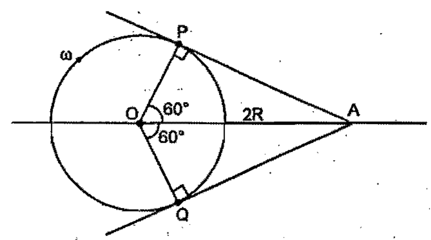
$$\frac{\mu_o}{4\pi}\frac{q\vec{v}\times\vec{r}}{r^3}$$

It can be seen that he magnitude of the magnetic field at an point on the axis remains constant. But the direction of the of the field keeps on changing.

20.16 (B)

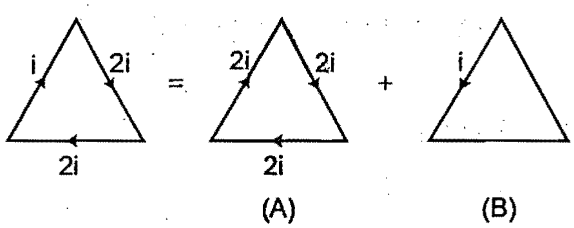
Point A shall record zero magnetic field (due to α-particle) when the α-particle is at position P and Q as shown in figure.

The time taken by α-particle to go from P to Q is



$$t = \frac{1}{3}\frac{2\pi}{\omega} \quad \text{or} \quad \omega = \frac{2\pi}{3t}$$

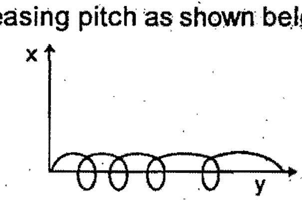
20.17 (A)



force in figure (A) is zero, and force in figure (B) = i ℓ B.

20.18 (D)

The particle will move in a non-uniform helical path with increasing pitch as shown below:



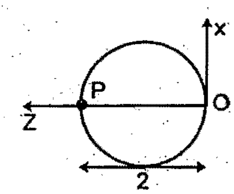
Its time period will be :

$$T = \frac{2\pi m}{qB} = 2\pi \text{ seconds}$$

Changing the view, the particle is seemed to move in a circular path in (x – z) plane as below

After π-seconds the particle will be at point 'P', hence x coordinate will be 0

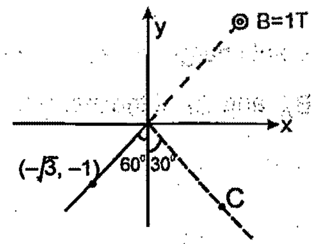
For linear motion along y-direction.



$$y(\pi) = 0(\pi) + \frac{1}{2}\frac{Eq}{m}(\pi)^2$$

$$y(\pi) = \frac{\pi^2}{2} \quad \text{and OP = 2} \quad \text{Hence the coordinate } \left(0, \frac{\pi^2}{2}, 2\right).$$

20.19 (C)



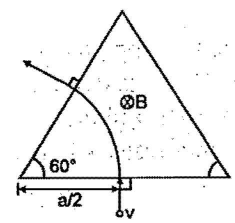
The centre will be at 'C' as shown :

Coordinates of the centre are $(r\cos 60^0, -r\sin 60^0)$

where r = radius of circle = $\frac{mv}{Bq} = \frac{1\times1}{1\times1} = 1$

i.e. $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ Ans.

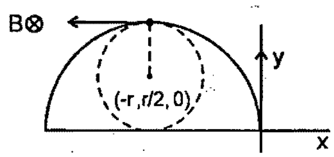
20.20 (B)



The charged particle moves in a circle of radius $\frac{a}{2}$

$\therefore \quad qvB = \frac{mv^2}{a/2} \quad$ or $\quad B = \frac{2mv}{qa}$

20.21 (B)



Hence (B).

20.22 (C)

After two and half time periods, it is at a distance $2R_0$ on the negative z-axis. y-coordinate will be zero And the x-coordinate = 2.5 P_0.

i.e. it is at a distance 7.5 P_0 from the mirror, hence its image will be at $2(7.5\,P_0) + 2.5\,P_0 = 17.5\,P_0$.

Hence (C).

20.23 (A) $\tau = \vec{m}\times\vec{B} = I_0L^2\hat{k}\times\left(\frac{B}{\sqrt{2}}\hat{i}+\frac{B}{\sqrt{2}}\hat{j}\right) = \frac{I_0L^2B}{\sqrt{2}}(\hat{j}-\hat{i})$.

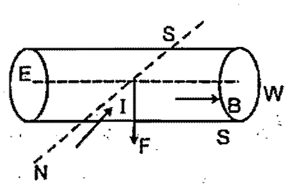
20.24 (C)

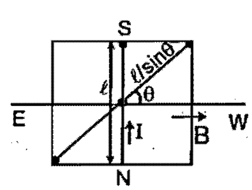
$\int E.d\ell = \frac{-d\phi}{dt} = -\pi r^2\frac{dB}{dt}$ $\qquad$ $N = \int \lambda E d\ell = -R\lambda\pi r^2\frac{dB}{dt}$

$\int N dt = -R\lambda\pi r^2 \int dB$ $\qquad$ $= R\lambda\pi r^2 B = I\omega$

$\Rightarrow \quad \pi r^2 B\frac{q}{2\pi r} = mr^2\omega$ $\qquad$ $\Rightarrow \quad \omega = \frac{qB}{2m}$

20.25 (C)





Initially,

$1.2\text{ N} = I.(\vec{\ell}\times\vec{B})\ \downarrow$

In the given condition –

$F = I\frac{\ell}{\sin\theta}B\sin\theta \quad = \quad I\ell B = 1.2\text{ N}\downarrow$.

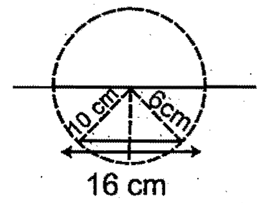
20.26 (C) Force on wire is :

$F = I \times 16 \times B$ ....(1)

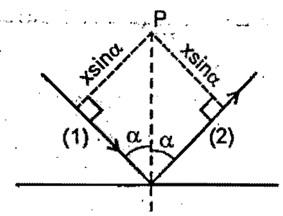
but $1.2 = I \times 20 \times B$ ....(2)

$\frac{F}{1.2} = \frac{16}{20} \Rightarrow F = \frac{4.8}{5}$

0.96 N ↓ downward



20.27 (C) Magnetic field at 'P' due to wires (1) and (2) is :



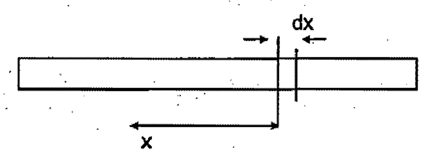
$$B_1 = \frac{\mu_0 I}{2\pi(x\sin\alpha)} + \frac{\mu_0 I}{2\pi(x\sin\alpha)} = \frac{2\mu_0 I}{2\pi(x\sin\alpha)} \text{ (out of the paper)}$$

Now if a current of $\frac{2I}{\sin\alpha}$ is flowing in the third wire then the magnetic field due to the same will be :

$B_2 = \frac{\mu_0}{2\pi x}\left(\frac{2I}{\sin\alpha}\right)$, which will cancel B_1 if it is out of paper which is possible if the current $\frac{2I}{\sin\alpha}$ in the third wire is from right to left.

20.28 (C)

At a distance x consider small element of width dx.



Magnetic moment of the small element is : $dm = \frac{\left(\frac{q}{\ell}dx\right)\omega}{2\pi}.\pi x^2$

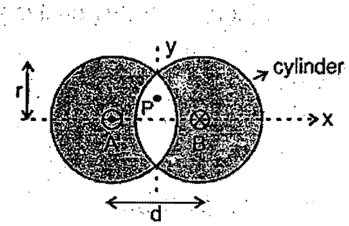
$$M = \int_{-\ell/2}^{\ell/2} \frac{q\omega}{2\ell} x^2 dx, \qquad M = \frac{q\omega\ell^2}{24} = \frac{q\pi f\ell^2}{12}.$$

20.29 (A)

$$B = \frac{\mu_0 I}{4\pi . \frac{R}{\sqrt{2}}} (\sin 90° + \sin 135°) = \frac{\mu_0 I}{4\pi R}\left(\sqrt{2}-1\right)$$

20.30 (A)

Let the current density in complete left cylinder is $\vec{J}$, then current density in complete right cylinder is $-\vec{J}$. Then magnetic field at any point P in the region of overlap is



$$\vec{B} = \frac{\mu_0}{2}\ \vec{J}\times\overrightarrow{AP} + \frac{\mu_0}{2}(-\vec{J}\times\overrightarrow{BP})$$

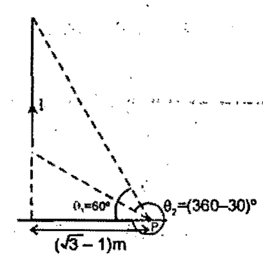
$$= \frac{\mu_0}{2}\ \vec{J}\times(\overrightarrow{AP}+\overrightarrow{PB}) = \frac{\mu_0}{2}\ (\vec{J}\times\overrightarrow{AB})$$

Therefore magnitude of field at any point in region of overlap is = $\frac{\mu_0}{2}Jd$ and its direction is along positive y-direction at any point P in overlap region.

20.31 (C) $\vec{B}_{\text{due to first loop}} = 4\dfrac{\mu_0 i}{4\pi\frac{a}{2}}$ [cos45° + cos45°]

$= \dfrac{2\sqrt{2}\mu_0 i}{\pi a} \Rightarrow \vec{B} = \dfrac{2\sqrt{2}\mu_0 i}{\pi a}\left[1 - \dfrac{1}{2} + \ldots\ldots\ldots\ldots \infty\right] = \dfrac{2\sqrt{2}\mu_0 i}{\pi a} \ln 2$

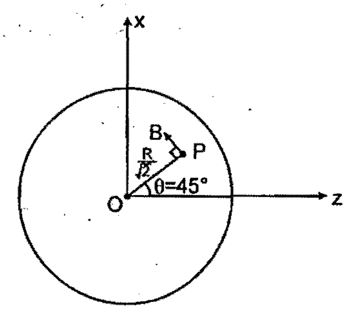
20.32 (A)



$B = \dfrac{\mu_0 . i}{4\pi d}(\sin\theta_1 + \sin\theta_2)$

$= \dfrac{10^{-7} \times 100}{\sqrt{3}-1}\left[\dfrac{\sqrt{3}}{2} - \dfrac{1}{2}\right] = 5 \times 10^{-6}$ T

20.33 (A)



The magnitude of magnetic field at P $\left(\dfrac{R}{2}, y, \dfrac{R}{2}\right)$ is

$B = \dfrac{\mu_0 J r}{2} = \dfrac{\mu_0 i}{2\pi R^2} \times \dfrac{R}{\sqrt{2}} = \dfrac{\mu_0 i}{2\sqrt{2}\pi R}$

unit vector in direction of magnetic field is $\hat{B} = \dfrac{\hat{i} - \hat{k}}{\sqrt{2}}$

$\therefore \quad \vec{B} = B\hat{B} = \dfrac{\mu_0 i}{4\pi R}(\hat{i} - \hat{k})$

Alternate solution

$\vec{B} = \dfrac{\mu_0}{2}\vec{J} \times \vec{r} = \dfrac{\mu_0}{2}\dfrac{i}{\pi R^2}\hat{j} \times \left(\dfrac{R}{2}\hat{i} + \dfrac{R}{2}\hat{k}\right) = \dfrac{\mu_0 i}{4\pi R}(\hat{i} - \hat{k})$

20.34 (A)

$\Rightarrow \quad \vec{F} = M_x \dfrac{\partial B_x}{\partial x}\hat{i} + M_y \dfrac{\partial B_y}{\partial y}\hat{j}$

$\vec{F} = A(2cx)\hat{i} + B(2Dy)\hat{j}$

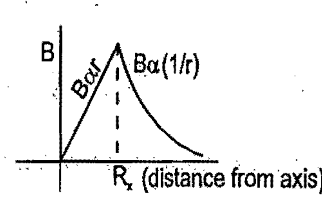
at $\quad \vec{r} = (E\hat{i} + F\hat{j}) \qquad \vec{F} = 2AEC\hat{i} + 2BDF\hat{j}$

20.35 (A)

The resultant magnetic dipole moment of toroid is zero. $d\vec{\mu}$ of small parts of toroid turn along a circle and hence there resultant is zero.

$\therefore$ Torque acting on it is zero.

20.36 (AD)



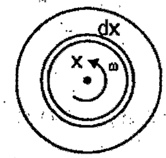
$B_{in} \propto r \quad B_{out} \propto \dfrac{1}{r}$.

Alternate Solution

B (inside the conductor) $\propto r$

B (outside the conductor) $\propto \dfrac{1}{r} \quad \therefore u = \dfrac{B^2}{2\mu_0} \propto \dfrac{1}{r^2}$

20.37 (A D)
Consider a ring of radius x and thickness dx.

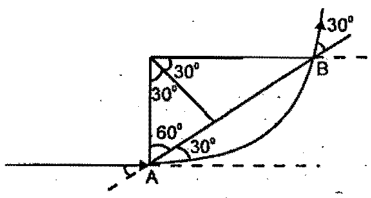


Equivalent current in this ring = $\frac{\omega}{2\pi}$ × charge on ring = $\frac{\omega}{2\pi} \times (2\pi x\, dx)\frac{Q}{\pi R^2}$

dB (due to this ring) = $\frac{\mu_0}{2x}\left(\frac{\omega}{2\pi}\frac{2xQ}{R^2}dx\right)$

$\therefore\ B = \int_0^R \frac{\mu_0\omega}{2\pi}\frac{Q}{R^2}dx. \quad = \frac{\mu_0\omega\theta}{2\pi R^2}\cdot R = \frac{\mu_0\omega\theta}{2\pi R}.$

20.38 (ABC)

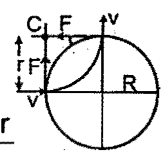


Arc AB = $\frac{\pi}{3}r = \frac{\pi mV}{3qB}$ Time 't' = $\left(\frac{T}{2\pi}\right)\left(\frac{\pi}{3}\right) = \frac{T}{6} \quad = \frac{\pi m}{3qB}.$

20.39 (ABC)
The particle will move along an arc which is part of a circle of radius

$r = \frac{mv}{Bq}$

From the figure we can see
r = R



$\therefore \quad R = \frac{mv}{Bq} \quad ; \quad T = \frac{\pi r/2}{v} = \frac{\pi r}{2v}$

$\because \quad r = R = \frac{mv}{Bq}$

$\therefore \quad T = \frac{\pi m}{2Bq}$

20.40 (CD)
For cylinder ;

$B = \frac{\mu_0 ir}{2\pi R^2} \quad ; \quad r < a = \frac{\mu_0 i}{2\pi r} \quad ; \quad r \geq a$

We can consider the given cylinder as a combination of two cylinders. One of radius 'R' carrying current I in one direction and other of radius $\frac{R}{2}$ carrying current $\frac{I}{3}$ in both directions.

At point A : $B = \frac{\mu_0 (I/3)}{2\pi(R/2)} + 0 = \frac{\mu_0 I}{3\pi R}$

At point B : $B = \frac{\mu_0}{2}\left(\frac{4I/3}{\pi R^2}\right)\left(\frac{R}{2}\right) + 0 = \frac{\mu_0 I}{3\pi R}$

20.41 (D) The magnitude of magnetic force on charged particle undergoing uniform circular motion in uniform magnetic field is

$$F = qvB$$

$\therefore$ If v is doubled keeping q and B constant, the force F just doubles. Hence statement 1 is false.

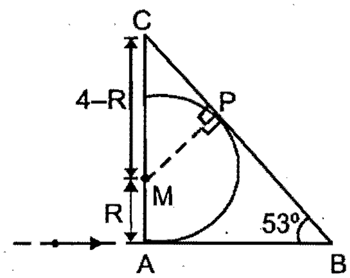
20.42 (B) Both statements are correct, but statement 2 is not a correct explanation of statement 1.

20.43 (A) Solenoid tends to contract because the current in all the circular turns is in same direction. Hence all turns(can be assumed as a ring) attract each other.

20.44 (D) The current through solid metallic cylinder also produces magnetic field inside the cylinder. Hence statement-1 is false

20.45 (C)

In triangle PMC



$\cos 53° = \dfrac{MP}{MC}$

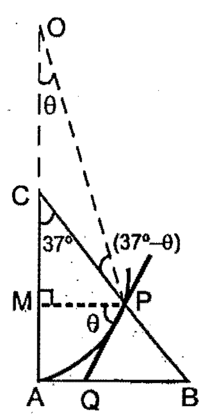
$\dfrac{3}{5} = \dfrac{R}{4-R}$

$12 = 8R$

$R = \dfrac{3}{2}$ m (R is the maximum radius of half–circle)

$R_{max} = \dfrac{mu_{max}}{qB} \quad \Rightarrow \quad U_{max} = 3$ m/s.

20.46 (B)



$R = \dfrac{mu}{qB} = 24$ m

Let, $\angle MPQ = \theta$

By geometry,

$\angle CPO = (37 - \theta)$

In ΔCPO,

$$\frac{OC}{\sin(\angle CPO)} = \frac{OP}{\sin(\angle PCO)}$$

$$\frac{20}{\sin(37°-\theta)} = \frac{24}{\sin(180°-37°)}$$

$$\frac{5}{\sin(37°-\theta)} = \frac{5\times 6}{3}$$

$$\sin(37° - \theta) = \frac{1}{2}$$

$$\theta = \frac{7\pi}{180} \text{ rad.}$$

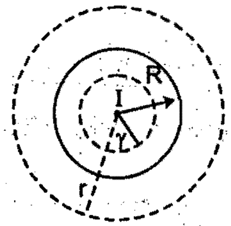
$$\omega = \frac{qB}{m}$$

$\omega = 2$ rad/sec.

$$t = \frac{7\pi}{360} \text{ sec.}$$

20.47 (C)

Inside the cylinder



$$B.2\pi r = \mu_0 \cdot \frac{I}{\pi R^2}\pi r^2$$

$$B = \frac{\mu_0 I}{2\pi R^2}.r \qquad \text{.............(1)}$$

outside the cylinder

$B.2\pi r = \mu_0 I$

$$\therefore \quad B = \frac{\mu_0 I}{2\pi r} \qquad \text{.............(2)}$$

Inside cylinder $B \propto r$ and outside $B \propto \frac{1}{r}$

So from surface nature of mag field changes.

Hence it is clear from the graph that wire 'c' has greatest radius.

20.48 (A)

Magnitude of mag field is maximum at the surface of wire 'a'.

20.49 (A)

Inside the wire

$$B(r) = \frac{\mu_0}{2\pi}.\frac{I}{R^2}.r$$

$$\frac{dB}{dr} = \frac{\mu_0}{2\pi}.\frac{I}{R^2}$$

i.e. slope $\propto \frac{I}{\pi R^2}$

$\propto$ current density

It can be seen that slope of curve for wire a is greater than wire c.

$\propto$ a

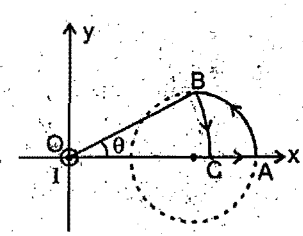
20.50 (D)

Since there is no current passing through circular path, the integral $\oint \vec{B}\cdot d\vec{\ell}$ along the dotted circle is zero.

20.51 (B)

Let segment OB = OC and arc BC is a circular arc with centre at origin. Since the shown closed path ABCA encloses no current, the path integral of magnetic field over this path is zero.

Hence $\int_A^B \vec{B}\cdot d\vec{\ell} + \int_B^C \vec{B}\cdot d\vec{\ell} + \int_C^A \vec{B}\cdot d\vec{\ell} = 0.$



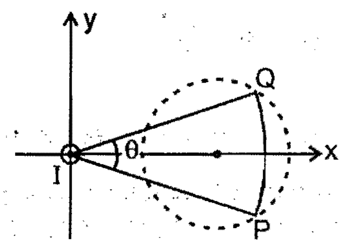
Because $\vec{B}$ is perpendicular to segment AC at all points, therefore $\int_C^A \vec{B}\cdot d\vec{\ell} = 0.$

Hence $\int_A^B \vec{B}\cdot d\vec{\ell} = \int_C^B \vec{B}\cdot d\vec{\ell} = \frac{\mu_0 I}{2\pi}\frac{OB(\theta)}{OB} = \frac{\mu_0 I}{2\pi}\tan^{-1}\frac{1}{2}$

20.52 (C)

Consider two points P and Q lying on dotted circle and equidistant from origin O. We draw a circular arc QP with centre at origin O. The path integral of magnetic field, that is, $\int \vec{B}\cdot d\vec{\ell}$ along the dotted circle between two points P and Q is also is equal to path integral $\int \vec{B}\cdot d\vec{\ell}$ along the arc QP whose centre is at origin.

Therefore the path integral of magnetic field $\int \vec{B}\cdot d\vec{\ell}$ along the dotted circle between two points P and Q



$$= \frac{\mu_0 I}{2\pi}\frac{OP(\theta)}{OP} = \frac{\mu_0 I}{2\pi}\theta.$$

The value of θ will be maximum when chord OQ and chord OP will be tangent to the dotted circle, that is, $\theta = \frac{\pi}{3}$.

Hence the required maximum value = $\frac{\mu_0 I}{6}$.

20.53 (A) s (B) p (C) q (D) r

R = mv/qB

$R_B > R_A$

and $R_A = R_C$ (in opposite sense)

and R_D is smallest

20.54 (A) → (q, s)

(B) → (p, s)

(C) → (q, s)

(D) → (p, s)

Work done by magnetic force on the charge = 0 in any part of its motion.

∴ 'S' is matching for all parts (i), (ii), (iii), (iv)

For loop 1 $\Sigma I_{in} = -i + i - i = -i$ ∴ $\oint \vec{B}.d\vec{\ell} = \mu_0(-i)$

For loop 2 $\Sigma I_{in} = i - i + i = i$ ∴ $\oint \vec{B}.d\vec{\ell} = \mu_0(i)$

For loop 3 $\Sigma I_{in} = -i - i + i = -i$ $\oint \vec{B}.d\vec{\ell} = \mu_0(-i)$

For loop 4 $\Sigma I_{in} = +i + i - i = +i$ $\oint \vec{B}.d\vec{\ell} = \mu_0(i)$

(Note : That current will be taken as positive which produces lines of magnetic field in the same sense in which $\vec{d\ell}$ is taken)

20.55 (A) p,q, r, t (B) p, q, r, s, t (C) r (D) p, q, r, s, t

The magnetic field is along negative y-direction in case A,B and C

Hence z-component of magnetic field is zero in all cases.

The magnetic field at P is $\frac{\mu_0}{4\pi}\frac{i}{d}$ for case (r)

The magnetic field at P is less than $\frac{\mu_0}{2\pi}\frac{i}{d}$ for all cases.

20.56 A – r,s B – r,s C – q,r D – p,r, t

(A) Because the magnetic field is parallel to x-axis, the force on wire parallel to x-axis is zero. The force on each wire parallel to y-axis is $B_o\frac{i}{2}\ell$. Hence net force on loop is $B_o i\ell$. Since force on each wire parallel to y-axis passes through centre of the loop net torque about centre of the loop is zero.

(B) Because the magnetic field is parallel to y-axis, the force on wire parallel to y-axis is zero. The force on each wire parallel to x-axis is $B_o\frac{i}{2}\ell$. Hence net force on loop is $B_o i\ell$. Since force on each wire parallel to x-axis passes through centre of the loop, net torque about centre of the loop is zero.

(C) Since net displacement of current from entry point in the loop to exit point in the loop is along the diagonal of the loop.The direction of external uniform magnetic field is also along the same diagonal. Hence net force on the loop is zero. Since force on each wire on the loop passes through centre of the loop net torque about centre of the loop is zero.

(D)The net displacement of current from entry point in the loop to exit point in the loop is along the diagonal (of length $\sqrt{2}\,\ell$.) of the loop.The direction of external uniform magnetic field is also perpendicular to the same diagonal. Hence magnitude of net force on the loop is $B_o i(\sqrt{2}\,\ell)$. Since force on each wire on the loop passes through centre of the loop net torque about centre of the loop is zero.

20.57 (A) r, (B) q,t (C) t, (D) s

$\vec{F} = q\vec{E} + q(\vec{V}\times\vec{B})$

If $\vec{u}=0$, $\vec{B}=B_x\hat{i}$ and $\vec{E}=E_y\hat{j}$

then charge particle will start to move in y-direction due to electric field and as it aquires velocity it will experience force due to magnetic field and will move in a cycloid path. Similarly, one can find path for other cases.

(A) If $B_y = B_z = E_x = E_z = 0$; $u = 0$ then path is cycloid.

(B) If $E = 0$, $u_xB_x + u_yB_y \neq -u_zB_z$ then path is helix with uniform pitch and constant radius or straight line.

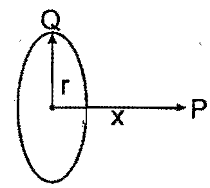
(C) If $\vec{u}\times\vec{B} = 0$, $\vec{u}\times\vec{E} = 0$ then path is straight line.

(D) If $\vec{u}\perp\vec{B}$, $\vec{B}\,||\,\vec{E}$ then path is helix with variable pitch and constant radius.

20.58 91

Electric field at P is

$$E = \frac{Q\,x}{4\pi\epsilon_0 (x^2 + r^2)^{3/2}}$$



Magnetic field at P is $B = \frac{\mu_0}{4\pi}\frac{2\pi i r^2}{(x^2+r^2)^{3/2}} = \frac{\mu_0}{4\pi}\frac{2\pi Q^2 f^2 r^2}{(x^2+r^2)^{3/2}}$

f = frequency of revolution.

Electric energy density = $\frac{1}{2}\varepsilon_0 E^2$; Magnetic energy density $\frac{B^2}{2\mu_0}$

$$\frac{\text{Electric energy density}}{\text{magnetic energy density}} = \frac{\frac{1}{2}\varepsilon_0 E^2}{\frac{B^2}{2\mu_0}} = \frac{E^2}{c^2B^2} = \frac{x^2c^2}{4\pi^2 f^2 r^4} = \frac{9}{\pi^2} \times 10^{10} = 9.1 \times 10^9$$

20.59 1

Since, total charge is zero initially thus the two particle will be of opposite charges. Initially the neutral particle is at rest, so both will have same speed. As both particle move in opposite directions, magnetic force on them will be in the same direction and of same magnitude.

Using $R = \frac{mV}{qB}$, both will be moving in the circle of same radius. So they will meet at point Q. i.e. diametrically opposite to starting point P. So time taken will be [Made 2004]

$t = \frac{\pi R}{V} = \frac{\pi m}{qB}$.................. Ans. 1

20.60 45

Torque of magnetic force about PQ .... (1)

τ_m = (I L B) L cosθ = I L² B cosθ .... (2)

Torque of gravitational force about PQ

τ_g = [(λL) g L sinθ + 2(λL) g (1/2) L sinθ]

= 2λL²g sinθ .... (1)

$\tau_m = \tau_g \Rightarrow \tan\theta = \frac{IB}{2\lambda g} = \frac{10\sqrt{3}\times 2}{2\times\sqrt{3}\times 10} = 1 \qquad \Rightarrow \theta = 45$

21. ELECTROMAGNETIC INDUCTION

21.1 (B)

$$q = \int I dt = \int -\frac{1}{r}\frac{d\phi}{dt}\,dt = -\frac{\Delta\phi}{r} = \frac{\mu_0 I a}{\pi r}\ell n2.$$

21.2 (C)

When the rod rotates, there will be an induced current in the rod. The given situation can be treated as if a rod 'A' of length '3ℓ' rotating in the clockwise direction, while an other say rod 'B' of length '2ℓ' rotating in the anti clockwise direction with same angular speed 'ω'.

As we know that, $e = \frac{1}{2}B\omega\ell^2$

For 'A' : For 'B' :

$e_A = \frac{1}{2}B\omega(3\ell)^2$ & $e_B = \frac{1}{2}B(-\omega)(2\ell)^2$

Resultant induced emf will be :

$e = e_A + e_B = \frac{1}{2}B\omega\ell^2(9-4)$ $e = \frac{5}{2}B\omega\ell^2$

21.3 (A)

Induced emf $\int_a^b Bvdx = \int_a^b \frac{\mu_0 I}{2\pi x}vdx$

$\Rightarrow$ Induced emf $= \frac{\mu_0 I v}{2\pi}\ell n\left(\frac{b}{a}\right)$ $\Rightarrow$ Power dissipated $= \frac{E^2}{R}$

Also, power = F.V $\Rightarrow$ $F = \frac{E^2}{VR}$ $\Rightarrow$ $F = \frac{1}{VR}\left[\frac{\mu_0 I V}{2\pi}\ell n\left(\frac{b}{a}\right)\right]^2$ Ans.

21.4 (A)

Rate of increment of energy in inductor $= \frac{dU}{dt} = \frac{d}{dt}\left(\frac{1}{2}Li^2\right) = Li\frac{di}{dt}$

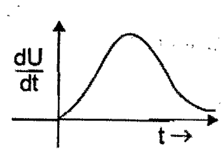
Current in the inductor at time t is:

$i = i_0\left(1 - e^{-\frac{t}{\tau}}\right)$ and $\frac{di}{dt} = \frac{i_0}{\tau}e^{-\frac{t}{\tau}}$

$\therefore$ $\frac{dU}{dt} = \frac{Li_0^2}{\tau}e^{-\frac{t}{\tau}}\left(1 - e^{-\frac{t}{\tau}}\right)$

$\frac{dU}{dt} = 0$ at t = 0 and t = ∞

Hence E is best represented by :



21.5 (A)

$L = \frac{\mu_0 N^2 \pi r^2}{\ell}$

length of wire $= N\,2\pi r$ = Constant (= C, suppose)

$\therefore$ $L = \mu_0\left(\frac{C}{2\pi r}\right)^2\frac{\pi r^2}{\ell}$ $\therefore$ $L \propto \frac{1}{\ell}$

$\therefore$ Self inductance will become 2L.

21.6 (B)

$$\int \vec{E}.d\vec{r} = -\frac{d\phi}{dt}$$

and take the sign of flux according to right hand curl rule.

$$\int \vec{E}.d\vec{r} = -(-(-\alpha A) - (-\alpha A) + (-\alpha A)) = -\alpha A$$

21.7 (A)
Given :
Voltage in primary V_p = 200 volt
Current in primary i_p = 2 amp
Voltage in secondary V_s =2000 volt
The relation for the current in the secondary is

$$\frac{V_s}{V_p} = \frac{i_p}{i_s} \Rightarrow \frac{2000}{200} = \frac{2}{i_s} \text{ or } i_s = \frac{2\times200}{2000} = 0.2 \text{ amp.}$$

21.8 (C)

[Ans.: $\frac{2B\pi R^2}{L}$]
Flux can't change in a superconducting loop. $\Delta\phi = 2\pi R^2.B$
Initially current was zero. So self flux was zero.

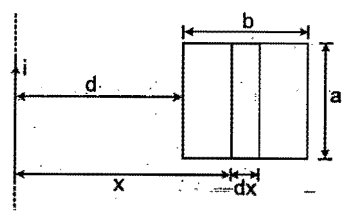
$\therefore$ Finally $Li = 2\pi R^2 \times B$. $i = \frac{2\pi R^2 \times B}{L}$

21.9 (B)

$BlV = iR + \frac{q}{C}$ or $BlV = \left(\frac{dq}{dt}\right)R + \frac{q}{C}$

Hence the charge on capacitor increases with time

21.10 (A)
The flux in rectangular loop due to current i in wire is



$$\phi = \int_d^{d+b} \frac{\mu_0 i}{2\pi x} a\,dx = \frac{\mu_0 ia}{2\pi} \ell n \frac{b+d}{d}$$

Mutual inductance is

$$M = \frac{\phi}{i} = \frac{\mu_0 a}{2\pi} \ell n \frac{b+d}{d}$$

$\therefore$ Mutual inductance is proportional to 'a'.

21.11 (C)
When the magnetic goes away from the ring the flux in the ring decreases hence the induced current will be such that it opposes the decreasing flux in it hence ring will behave like a magnet having face A as north pole and face B as south pole.

21.12 (C)

Just before opening the switch, the current in the inductor is ε/R. Energy stored in it = $\frac{1}{2}L\left(\frac{\varepsilon}{R}\right)^2$.

This energy will dissipate in the resistors R_1 and R_2 in the ratio $\frac{1}{R_1}$ & $\frac{1}{R_2}$.

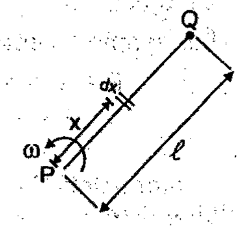
21.13 (B)

Charge on the differential element dx, $dq = \frac{Q}{\ell}dx$

equivalent current $di = f\,dq$

$\therefore$ magnetic moment of this element $d\mu = (di)\,NA$ (N = 1)

$= \left(\pi x^2\right) f \frac{Q}{\ell} dx$



$\Rightarrow \quad \mu = \int_0^{\mu} d\mu = \frac{\pi f Q}{\ell}\int_0^{\ell} x^2\,dx \; ; \; \mu = \frac{1}{3}\pi f Q \ell^2$ ....Ans.

21.14 (A)

$B_H = B_v \cot\theta = B\cot\theta$

Hence the induced e.m.f. in the rod is $B\,\ell\,v\cot\theta$

21.15 (D)

Since all the wires are connected between rim and axle so they will generate induced emf in parallel, hence it is same for any number of spokes.

21.16 (D)

$\frac{dP}{dt} = \frac{dF.v}{dt} = \frac{Fdv}{dt} = Fa$

as 'a' is decreasing continuously hence the rate of power delivered by external force will be decreasing continuously.

21.17 (B)

$e = (\vec{v}\times\vec{B}).\ell \quad \Rightarrow \quad e = [\hat{i}\times(3\hat{i}+4\hat{j}+5\hat{k})].5\hat{j} \quad \Rightarrow \quad e = 25$ volt **Ans.**

21.18 (B)

Time constant $= \frac{1}{20} = 50$ msec

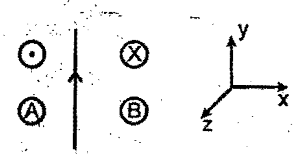
so $\quad i = 0.633\, i_{max} = 0.633\,\frac{E}{R} \quad \Rightarrow E = \frac{3.165\times 20}{0.633} = 100$ V

21.19 (A)

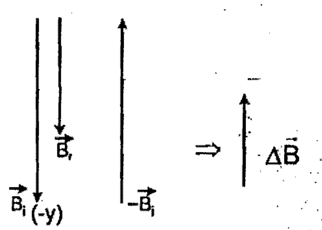
$W = (L)F = L \times ILB = L \times \frac{L^2B^2V}{R} = 1$ J

21.20 (B)

The field at A and B are out of the paper and inside the paper respectively.



As the current in the straight wire decreases the field also decreases.
For B :



The change in the magnetic field which causes induced current $(\Delta\vec{B})$ is along (+)z direction.

Hence, induced emf and hence current should be such as to oppose this change $\Delta\vec{B}$.

Hence, induced emf should be along – z direction which results in a clockwise current in 'B'. Similarly, there will be anticlockwise current in 'A'. Hence (B).

21.21 (B)

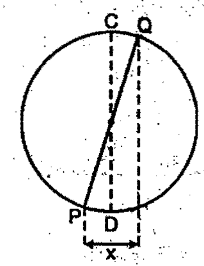
When the ring falls vertically, there will be an induced emf across A & B ($e = Bv(2r)$).

Note that there will be a potential difference across any two points on the ring of line joining them has a projected length in the horizontal plane.

For example, between points 'P' & 'Q' there is a projected length 'x' in the horizontal plane.

∴ P.d. across P & Q is :

$V = Bvx$.



But for points C and D : $x = 0$.

Therefore; P.d. = 0.

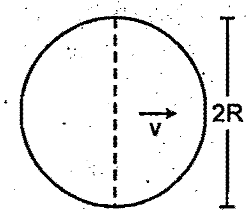
Hence (B).

21.22 (C)

Considering a projected length 2R on the ring in vertical plane.

This length will move at a speed v perpendicular to the field. This results in an induced emf :

$e = Bv(2R)$ in the ring.



In Ring "A" : $eA = B(-v)(2R)$

In Ring "B" : $eB = B(v)(2R)$

Therefore, potential differnce between

A & B = $B(v)(2R) - B(-v)(2R) = 4BvR$.

Note : there wil be no p.d. across a diameter due to rotation.

Alternate – Considering rotation of diameter about lowest point :

$$e = \frac{B\omega(2r)^2}{2} = 2Bvr \text{ in A (since pure rotation).}$$

and $e = -2Bvr$ in B. Hence (C)

21.23 (A)

The graph of current is given by :

$$i = i_0(1 - e^{-t/\tau}) \Rightarrow \frac{di}{dt} = \frac{i_0}{\tau}e^{-t/\tau}$$

Energy stored in the form of magnetic field energy is :

$$U_B = \frac{1}{2}Li^2$$

∴ Rate of increase of magnetic field energy is : $R = \frac{dU_B}{dt} = Li\frac{di}{dt} = \frac{Li_0^2}{\tau}(1 - e^{-t/\tau})\,e^{-t/\tau}$

This will be maximum when $\frac{dR}{dt} = 0 \Rightarrow e^{-t/\tau} = 1/2$

Substituting :

$$R_{max} = \frac{Li_0^2}{\tau}\left(1-\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{Li_0^2}{4\tau} = \left[\frac{L(E/R)^2}{4(L/R)}\right] = \frac{E^2}{4R}$$

21.24 (B)

When the switch is at position 1 :

$$U_B = \frac{1}{2}Li_0^2 = \frac{LE^2}{2R^2}$$

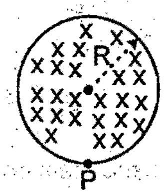
Just after the switch is shifted to position 2, current $I = \frac{E}{R}$ is flowing across the resistance. Hence, at that instant P.d. across reisistance will be :

$$\Delta V = IR = \frac{E}{R}.R = E \Rightarrow \text{Hence (B).}$$

21.25 (A)

If we consider the cylindrical surface to be a ring of radius R, there will be an induced emf due to changing field.

$$\int \vec{E}.\vec{d\ell} = \frac{d\phi}{dt} = -A\frac{dB}{dt}$$



$$\Rightarrow \quad E(2\pi R) = -A\frac{dB}{dt} = -\pi R^2\frac{dB}{dt} \Rightarrow E = \frac{R}{2}\frac{dB}{dt}$$

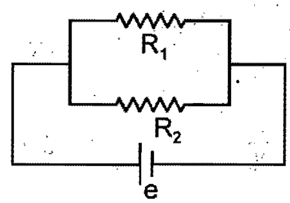
∴ Force on the electron.

$$F = -Ee = -\frac{eR}{2}\frac{dB}{dt} \quad \Rightarrow \quad \text{acceleration} = \frac{1}{2}\frac{eR}{m}\frac{dB}{dt}$$

As the field is increasing being directed inside the paper, hence there will be anticlockwise induced current (in order to oppose the cause) in the ring (assumed). Hence there will be a force towards left on the electron.

21.26 (A)

The equivalent diagram is :



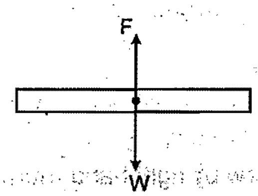
The induced emf across the centre and any ponit on the circumference is :

$$|\vec{e}| = \frac{1}{2}B\omega\ell^2 = \frac{B\omega r^2}{2}$$

∴ Current through $R_1 = \dfrac{B\omega r^2}{2R_1}$ Ans.

21.27 (D)

There is a force $\vec{F}_M = I(d\vec{\ell} \times \vec{B})$ acting on the rod carrying a current I.



By the rule of cross product, this force is vertically upward.

F.B.D. of the rod :

F – W = ma

$$a = \frac{F - W}{m}$$

The magnitude of acceleration will be constant, but the direction will depend on the mass of the rod. Hence (D) is correct option.

21.28 (C)

Considering pure rolling of OA about A: the induced emf across OA will be :

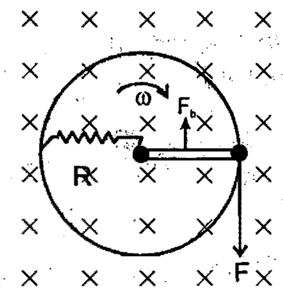
$$|\vec{e}| = \frac{B\omega(r)^2}{2}$$

From Lenz law, O will be the negative end, while A will be the positive end.

Hence $\quad V_0 - V_A = -\dfrac{B\omega r^2}{2}$

Hence (C) is correct option.

21.29 (D)



$F_b = BIL$

Induced current :

$$I = \frac{(B\omega r^2/2)}{R}$$

$$\therefore \quad F_b = B\left(\frac{B\omega r^2}{2R}\right)r = \frac{B^2\omega r^3}{2R}$$

To maintain constant angular velocity :

$$F(r) = F_B(r/2) \quad \Rightarrow \quad F = \frac{F_B}{2} = \frac{B^2\omega r^3}{4R}. \quad \textbf{Ans.}$$

21.30 (C)

Flux through a closed circuit containing an inductor does not change instantaneously.

$$\therefore \quad L\left(\frac{E}{R}\right) = \frac{L}{4}(i) \quad \Rightarrow \quad i = \frac{4E}{R} \ \textbf{Ans.}$$

21.31 (A)

$$E = \frac{d\phi}{dt} = \frac{Bd(b\ell)}{dt} = Bbv = B \times 2 \times 10^{-2} \times 20 = 0.40\ B$$

$$\Delta t = \frac{1\times10^{-2}}{20} = 5 \times 10^{-4}\ \text{sec} = 500\ \mu\ \text{sec}$$

$$t = \frac{6\times10^{-2}}{20} = 3 \times 10^{-3}\ \text{sec} = 3000\ \mu\ \text{sec}$$

21.32 (D)

$$i = \frac{|E|}{R} = \frac{B}{R}\frac{dA}{dt} \Rightarrow \quad Q = \int i dt = \frac{B}{R}\int dA = \frac{B}{R}A$$

using values $Q = 1.2 \times 10^{-6}$ C

21.33 (C)

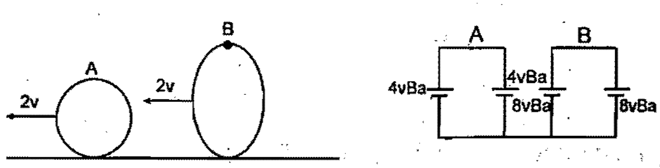
$|\varepsilon| = B\ell v$ where l is the edge perpendicular to both B and $\vec{v}$ i.e.c.

$\therefore |\varepsilon| = B\,v\,c.$

Now by right hand thumb rule magnetic force an a positive charge moving towards right is in down ward direction Hence end P will be positive.

$\therefore \quad V_P - V_Q$ is positive $\quad \Rightarrow \quad \varepsilon = +Bv\ell.$

21.34 (B)



$V_{AB} = 4vBa$

21.35 (B)

The magnetic filux in inner loop due to current in outer loop is

$$\phi = BA = \left(\frac{\mu_0}{4\pi}\frac{2\pi i}{b}\right).\pi a^2 = \left(\mu_0\frac{\pi a^2}{2b}\right)i_0\cos\omega t \quad \therefore \quad e = \frac{d\phi}{dt} = \mu_0\frac{\pi a^2\omega i_0}{2b}\sin\omega t$$

21.36 (D)

$L = \mu_0 n^2 \pi r^2 \ell$

$A = \pi r^2$

$n = \frac{N}{\ell} \quad \Rightarrow \quad L = \mu_0 \frac{N^2}{\ell} A$

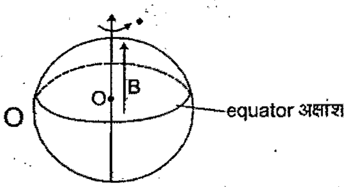
By putting the given values, it can be seen that it is maximum for solenoid no.4.

21.37 (A)

the equator can be seen as a conducting ring of radiur R_e revolving with angular velocity ω in a perpendicular magnetic field B.

$\therefore$ Potential difference. across its center and periphery = $\frac{B\omega R_e^2}{2}$

Potential at pole = potential of the axis of earth i.e. potential at point O



$\therefore \quad V_{eqvator} - V_{pole} = \frac{B\omega R_e^{\,2}}{2}$.

21.38 (A)

When switch K_1 is opended and K_2 is closed it becomes L–C circuit so applying energy conservation

$\frac{Q_0^2}{2C} = \frac{1}{2} Li^2$; $Q_0 = C_{eq}V = \frac{C_1C_2}{C_1+C_2} \cdot V = (20 \times 10^{-6})$

$\frac{(20\times10^{-6})^2}{2\times2\times10^{-6}} = \frac{1}{2}(0.2 \times 10^{-3})\, i^2 \Rightarrow \quad i = 1\text{ A}$

21.39 (D)

$i = \frac{\varepsilon}{R}(1-e^{-Rt/L})$

charge passing through battery

$q = \int i dt = \frac{\varepsilon}{R}\int_0^{L/R}(1-e^{-Rt/L})dt \quad = \frac{\varepsilon}{R}\left\{\frac{L}{R}+\frac{L}{R}\left(e^{-1}-1\right)\right\} \quad = \frac{\varepsilon L}{eR^2}$

$\therefore \quad W_b = q\,\varepsilon = \frac{\varepsilon^2 L}{eR^2}$

21.40 (ABD)

Rate of work done by external agent is :

$\frac{dw}{dt} = \frac{BIL.dx}{dt}$ = BILv & thermal power dissipated in the resistor = eI = (BvL) I

clearly both are equal, hence (A) .

If applied external force is doubled, the rod will experience a net force and hence acceleration. As a result velocity increases, hence (B)

Since ; $I = \frac{e}{R}$; $I = \frac{e}{R}$

On doubling 'R', current and hence required power becomes half.

Since, P = BILv

Hence (D).

21.41 (BD)

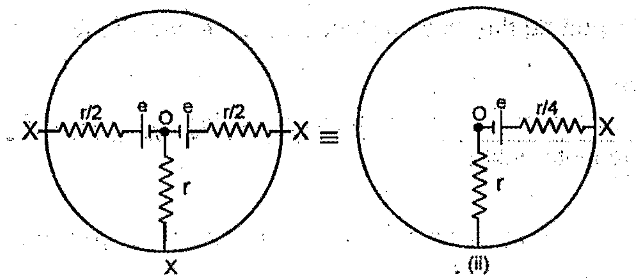
Equivalent circuit :

Induced emf $e = \frac{B\omega r^2}{2} = \left(\frac{B\omega a^2}{2}\right)$

($\because$ Radius= a)

By nodal equation :

nodal



$4\left(\frac{X-e}{r}\right) + \left(\frac{X-0}{r}\right) = 0$

$5X = 4e$

$\Rightarrow \quad X = \frac{4e}{5} = \frac{2B\omega a^2}{5}$

and $I = \frac{X}{r} = \frac{2B\omega a^2}{5r}$

also direction of current in 'r' will be towards negative terminal i.e. from rim to origin.

Alternatively;by equivalent of cells (figure (ii)) :

$I = \frac{e}{r + \frac{r}{4}} = \frac{4e}{5r} = \frac{2B\omega a^2}{5r} \qquad (\because \quad e = \frac{B\omega r^2}{2})$

21.42 (AC)

Since curve 'b' is below curve 'a' hence it is posssible that E & R are kept constant and L is increased or E & R are both halved and L is kept constant

21.43 (A,D)

At steady state no resistance will be offered by inductor hence, time constant is L/R and steady state current in inductor is E/R.

21.44 (ABCD)

$|e| = Ba\ \omega\ \sin\omega t$

$\phi = Ba\ \cos\omega t$;

* $|e|$ is maximum when $\omega t = \pi/2$

so ϕ is zero.

* $|e|$ is zero then $\omega t = 0$ So ϕ = is maximum.

21.45 (AC)

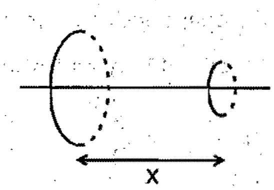
By principal of energy conservation. $P_B = P_R + P_L$

Near the starting of the circuit $P_R = i^2 R$ and $P_L = L\, i\, \frac{di}{dt}$.

As $\frac{di}{dt}$ has greater value at the starting of the circuit, $P_L > P_R$

21.46 (A)

It is obvious that flux linkage in one ring due to current in other coaxial ring is maximum when x = 0 (as shown) or the rings are also coplanar. Hence under this condition their mutual induction is maximum.M



21.47 (D)

Magnetic field cannot do work, hence statement-1 is false.

21.48 (A)
Obiviously statement 2 is correct explanation of statement-1

21.49 (A)

$\oint \vec{B}\cdot d\vec{\ell}$ along any closed path within a uniform magnetic field is always zero. Hence the closed path can be chosen of any size, even very small size enclosing a very small area. Hence we can prove that net current through each area of infinitesimally small size within region of uniform magnetic field is zero. Hence we can say no current (rather than no net current) flows through region of uniform magnetic field. Hence statement-2 is correct explanation of statement-1.

21.50 (C) 21.51 (C) 21.52 (C)

21.53 (B)
51 to 53

The fan is operating at 200 V, consuming 1000 W, then $I = \frac{1000}{200} = 5A$

But as coil resistance is 1Ω then power dissipated by internal resistance heat is $P_1 = I^2R = 25W$
If V is net emf across coil then

$$\frac{V^2}{R} = 25 \text{ W} \qquad V = 5 \text{ volt}$$

Net emf = source emf – back emf
$V = V_s - e \Rightarrow e = 195$ V
The work done $P_2 = 1000 - 25 = 975$ W.

21.54 (A)

$$\frac{dB}{dt} = 2T/s \Rightarrow E = -\frac{AdB}{dt} = -800 \times 10^{-4}\ m^2 \times 2 = -0.16\ V$$

$$i = \frac{0.16}{1\Omega} = 0.16 \text{ A, clockwise}$$

21.55 (B)

At t = 2s $\quad B = 4T; \quad \frac{dB}{dt} = 2T/s$

t = 2 s $\quad B = 4T; \quad \frac{dB}{dt} = 2T/s$

$$A = 20 \times 30\ cm^2 = 600 \times 10^{-4}\ m^2; \quad \frac{dA}{dt} = -(5 \times 20)\ cm^2/s$$

$$= -100 \times 10^{-4}\ m^2/s$$

$$E = -\frac{d\phi}{dt} = -\left[\frac{d(BA)}{dt}\right] = -\left[\frac{BdA}{dt} + \frac{AdB}{dt}\right] = -[4\times(-100\times10^{-4}) + 600 \times 10^{-4} \times 2]$$

$$= -[-0.04 + 0.120] = -0.08\ v$$

Alternative ;
$\phi = BA = 2t \times 0.2\ (0.4 - vt) = 0.16t - 0.4\ vt^2$

$$E = -\frac{d\phi}{dt} = 0.8\ vt - 0.16$$

at t = 2s
t = 2s
E = – 0.08 V

21.56 (C)

At t = 2s, length of the wire = (2 × 30 cm) + 20 cm = 0.8 m

Resistance of the wire = 0.8 Ω

Current through the rod = $\frac{0.08}{0.8} = \frac{1}{10}$ A

Force on the wire = $i\ell B = \frac{1}{10} \times (0.2) \times 4 = 0.08$ N

Same force is applied on the rod in opposite direction to make net force zero.

21.57 (B)

$$\text{Power} = \frac{\mu m g.v}{\text{fractional efficiency}} = \frac{0.45\times10^2\times10^3\times10\times20}{0.9} = 10^6 \text{ watt}$$

21.58 (C)

$\varepsilon = B_\perp V\ell = B_V.V\ell = 2 \times 10^{-5} \times 20 \times 1 = 40 \times 10^{-5}$ Volts

21.59 (D)

Time taken = $\frac{324}{72} = \frac{9}{2}$ hour

extra power engine = Power dissipated in resistor = $\frac{\varepsilon^2}{R} = \frac{(40\times10^{-5})^2}{10^{-3}} = 16 \times 10^{-5}$ watt.

For this the extra power consumed by the train engine will be $\frac{16\times10^{-5}}{0.9}$ watt.

$\therefore$ energy consumed = $\frac{16\times10^{-5}}{0.9}$ watt × $\frac{9}{2}$ hour = 8×10^{-7} KW hour

21.60 (A) r, (B) s , (C) p, (D) q

Magnetic field is along x axis because when the cube is moved along x-axis, there is no motional emf as $\vec{v} \times \vec{B} = 0$. When the block is moved along y axis, force on the electrons is in direction

$-(j \times i) = k$

Therefore electric field will be created along z-axis.

Now, c . v. B = 24mV ⇒ c = 20 cm

similarly b vB = 36 mV ⇒ b = 30 cm

$\therefore$ a = 25 cm

21.61 (A) p, r,t (B) p, r , t (C) q,s (D) p, q, r, s, t

(A) Speed of charged particle cannot be changed by magnetic force because magnetic force does no work on charged particle. Only electric field in case (p), (t) and induced electric field in case (r) can change speed of charged particle.

(B) Magnetic field cannot exert a force on charged particle at rest. Only electric field in case (p), (t) and induced electric field in case (r) can exert force on charge initially at rest.

In case (r) after the charge starts moving even the magnetic field can exert force on charge.

(C) A charged particle can move in a circle within uniform speed due to uniform and constant magnetic field in case . Even within a region of non uniform magnitic field, at all points on a circle field may be uniform for example on any circle coaxial with a current carring ring

(D) A moving charged particle is accelerated by electric field and also accelerated by magnetic field (provided v is not parallel to B).

21.62 (A) q,s (B) p,r (C) p,r (D) q,s

(A) Due to current carrying wire, the magnetic field in loop will be inwards w.r.t. the paper. As current is increased, magnetic flux associated with loop increases. So a current will be induced so as to decrease magnetic flux inside the loop. Hence Induced current in the loop will be anticlockwise. The current in left side of loop shall be downwards and hence repelled by wire. The current in right side of loop is upwards and hence attracted by wire. Since left side of loop is nearer to wire, repulsive force will dominate. Hence wire will repel the loop

(B) Options in (B) will be oppsite of that in (A)

(C) When the loop is moved away from wire, magnetic flux decreases in the loop. Hence the options for this case shall be same as in (B)

(D) When the loop is moved towards the wire, magnetic flux increases in the loop. Hence the options for this case shall be same as in (A)

21.63 8

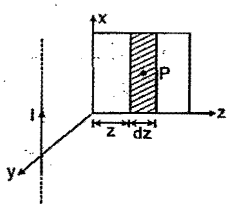


Figure - 1

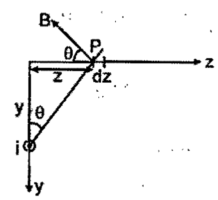


Figure - 2

The magnetic field at point P (figure -2) is $B = \frac{\mu_0}{2\pi}\frac{i}{\sqrt{y^2+z^2}}$

The magnetic flux through the shaded strip in figure -1 is $d\phi = (W\,dz)\frac{\mu_0}{2\pi}\frac{i}{\sqrt{y^2+z^2}}\sin\theta$

where $\sin\theta = \frac{z}{\sqrt{y^2+z^2}}$

∴ total magnetic flux through rectangular loop is

$$\phi = \int_0^L \frac{\mu_0}{2\pi}\frac{i_0 \sin\omega t\, W\, z\,dz}{y^2+z^2} = \frac{\mu_0}{4\pi} W \ln\left(\frac{y^2+L^2}{y^2}\right) i_0 \sin\omega t$$

∴ induced emf in the loop is $e = \frac{d\phi}{dt} = \frac{\mu_0}{4\pi} i_0 W \omega \cos\omega t \ln\left(\frac{L^2+y^2}{y^2}\right)$

Ans : $\frac{\mu_0 i_0 W\omega\cos\omega t}{4\pi}\ell n\left(\frac{L^2}{y^2}+1\right)$

21.64 240

$$\int \vec{E}\cdot d\vec{\ell} = -A\cdot\frac{dB}{dt}$$

As $B = 17 + (0.2)\sin(\omega t + \phi)$;

$E(2\pi r) = -\pi r^2 (0.2)\omega \,.\, \cos(\omega t + \phi)$

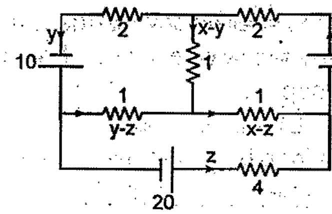
$$E = -\frac{r}{2}(0.2)\omega\,.\,\cos(\omega t + \phi)$$

Magnitude of the amplitude $= \frac{r}{2}(0.2)\,.\,\omega = 240$ mN/C

21.65 40

The induced emf in loop ABHFG $= \frac{d}{dt}(BA) = A\frac{dB}{dt} = 2 \times 10 = 20$ V

The induced emf in loop BCDH & DEFH



$= 1 \times 10 = 10$ volt . KVL in top left loop is ,

$10 - (y - z) + (x - y) - 2y = 0$

$\Rightarrow \quad x - 4y + z = -10$ — (1)

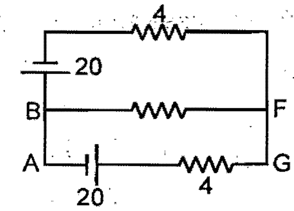
KVL in right loop : $10 - 2x - (x - y) - (x - z) = 0$

$\Rightarrow \quad -4x + y + z = -10$ — (2)

By equation (1) & (2) it is seen that $x - y = 0 \Rightarrow$ no current in DH

{This can also be seen by symmetry}

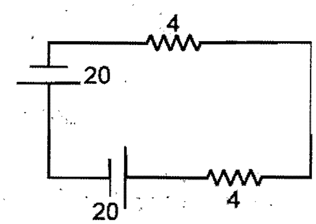
This makes solution very simple now the circuit is ,



Assume $v_B = 0$, & $v_F = v$, then $\frac{v+20}{4} + \frac{v-20}{4} + \frac{v-0}{2} = 0$

$\Rightarrow \quad v = 0 \quad \Rightarrow$ no current in FB

$\therefore$ circuit is :



$\therefore$ rate of heat production $= (40)^2/8 = 200$ watt . Ans.

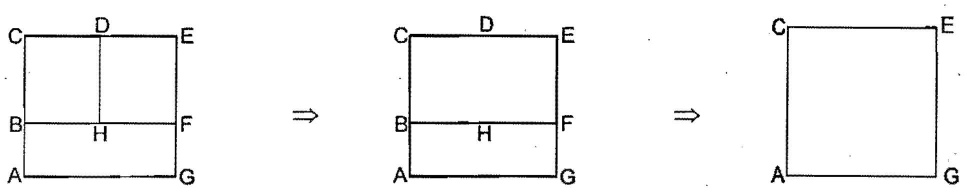
Note : If you are not able to observe the symmetry or decide $x - y = 0$, then write kVL in the lower loop .

It will be $x + y - 6z = -20$ — (3)

Solving (1) , (2) and (3) you will get $x = +5$, $y = 5$, $z = 5$ A . Heat rate will be ,

$10x + 10y + 20z = 40 \times 5 = 200$ W

Method II



$H = I^2R = \frac{\varepsilon^2}{R}$

where $R = 2 \times 4 \times 1 = 8\,\Omega$ and $\varepsilon = 4 \times 10 = 40$

21.66 140

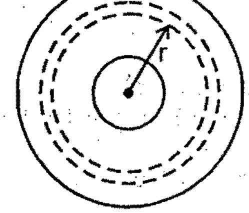
The magnetic field inside is only due to current of inner cylinder.

$B = \frac{\mu_0 i}{2\pi r}$

Magnetic field energy density is not uniform in the space in between cylinders. At distance r from the centre

$u_B = \frac{B^2}{2\mu_0} = \frac{\mu_0 i^2}{8\pi^2 r^2}$

Energy in volume of element (length ℓ)



$dU_B = u_B \cdot dV = \frac{\mu_0 i^2}{8\pi^2 r^2} \cdot (2\pi r\ell)\, dr = \frac{\mu_0 i^2 \ell}{4\pi} \cdot \frac{dr}{r}$

$U_B = \int dU_B = \frac{\mu_0 i^2 \ell}{4\pi} \int_a^b \frac{dr}{r} = \frac{\mu_0 i^2 \ell}{4\pi} \ell n \frac{b}{a}$

Using values

U = 140 nJ

22. ALTERNATING CURRENT

22.1 (B)

$\varepsilon = \varepsilon_0 \sin \omega t$

If $i = i_m \sin(\omega t - \phi)$ then $v_C = \left(\frac{1}{\omega C}\right) i_m \sin(\omega t - \phi - \pi/2)$

and $v_L = (\omega L)\, i_m \sin(\omega t - \phi + \pi/2)$.

So $v_C + v_L + v_R = \varepsilon_0 \sin \omega t$

$\Rightarrow$ $0 + v_R = \varepsilon_0 \sin \omega t \Rightarrow$ $v_R = \varepsilon_0 \sin \omega t$

Also $\tan \phi = \dfrac{\omega L - \frac{1}{\omega C}}{R} = 0$, so $i = i_m \sin \omega t$

Hence answer is (B) $z = \sqrt{\left(\omega L - \frac{1}{\omega C}\right)^2 + R^2} = R$

22.2 (D)

When capcitance is removed

$\tan \theta = \dfrac{\omega L}{R}$ or $\omega L = 100 \tan 60°$ ...(1)

when inductance is removed

$\tan \phi = \dfrac{1}{(\omega C)(R)}$ or $\dfrac{1}{\omega C} = 100 \tan 60°$ ...(2)

From equation (1) & (2) $\omega L = \dfrac{1}{\omega C}$

So it is condition of resonance.

so $z = R = 100\ \Omega$

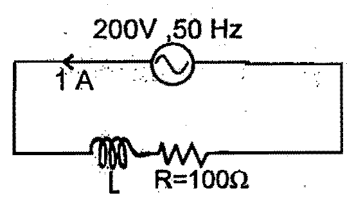
$I = v/R = 200/100 = 2A$

Power $P = I^2 R = 4 \times 100 = 400$ W

22.3 (D)

From the rating of the bulb , the resistance of the bulb can be calculated.

$R = \dfrac{V_{rms}^2}{P} = 100\ \Omega$



For the bulb to be operated at its rated value the rms current through it should be 1A

Also, $I_{rms} = \dfrac{V_{rms}}{Z}$

$\therefore\ 1 = \dfrac{200}{\sqrt{100^2 + (2\pi 50 L)^2}}$ $L = \dfrac{\sqrt{3}}{\pi}$ H

22.4 (A)

According to given problem,

$$I = \frac{V}{Z} = \frac{V}{[R^2 + (1/C\omega)^2]^{1/2}} \quad \text{... (1)}$$

and, $$\frac{I}{2} = \frac{V}{[R^2 + (3/C\omega)^2]^{1/2}} \quad \text{...(2)}$$

Substituting the value of I from Equation (1) in (2),

$$4\left(R^2 + \frac{1}{C^2\omega^2}\right) = R^2 + \frac{9}{C^2\omega^2}. \text{ i.e., } \frac{1}{C^2\omega^2} = \frac{3}{5}R^2$$

So that $$\frac{X}{R} = \frac{(1/C\omega)}{R} = \frac{\left(\frac{3}{5}R^2\right)^{1/2}}{R} = \sqrt{\frac{3}{5}}$$ **Ans.**

22.5 (C)

$$i = 3 \sin \omega t + 4 \cos \omega t$$

$$= 5\left[\frac{3}{5}\sin\omega t + \frac{4}{5}\cos\omega t\right]$$

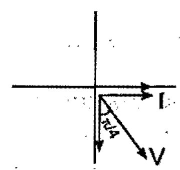
$$= 5\,[\sin(\omega t + \delta)] \quad \text{..........(1)}$$

$\Rightarrow$ rms value $= \dfrac{5}{\sqrt{2}}$ $\Rightarrow$ mean value $= \dfrac{\int_{T_1}^{T_2} i\,dt}{\int_{T_1}^{T_2} dt}$

$\therefore$ Initial value of time is not given hence the mean value will be different for various time intervals.
If voltage applied is $V = V_m \sin\omega t$ then i given by equation (1) indicates that it is ahead of V by δ where $0 < \delta < 90$ which indicates that the circuit contains R & C.
Hence (C).

22.6 (A)

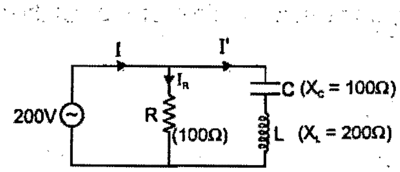
$v = v_0 \sin(\omega t + \pi/4) = v_0 \cos(\omega t - \pi/4)$
Since V lags current, an inductor can bring it in phase with current.

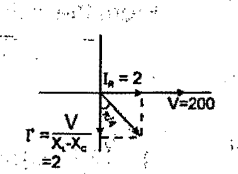


22.7 (B)

$$I_R = \frac{V}{R} = \frac{200}{100} = 2A \quad I' = \frac{V}{X_L - X_C} = \frac{200}{100}$$

$= 2A \quad I = \sqrt{I_R^2 + I'^2} = 2\sqrt{2}$ Amp.





22.8 (C)

The circuit will have inductive nature if $\omega > \dfrac{1}{\sqrt{LC}}$ $\left(\omega L > \dfrac{1}{\omega C}\right)$.
Hence A is false. Also if circuit has inductive nature the current will lag behind voltage. Hence D is also false.

If $\omega = \dfrac{1}{\sqrt{LC}}$ $\left(\omega L = \dfrac{1}{\omega C}\right)$ the circuit will have resistance nature. Hence B is false

Power factor $\cos\phi = \dfrac{R}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}} = 1$ it $\omega L = \dfrac{1}{\omega C}$. Hence C is true.

22.9 (C)

$X_L = X_C$ at resonance

$\therefore \quad \frac{X_L}{X_C} = 1.$ for both circuits

22.10 (D)

Since, $\cos\theta = \frac{R}{Z} = \frac{IR}{IZ} = \frac{8}{10} = \frac{4}{5}$ (Also $\cos\theta$ can never be greater than 1)

Hence (C) is wrong.

Also, $IX_C > IX_L \quad \Rightarrow \quad X_C > X_L$.

$\therefore$ Current will be leading.

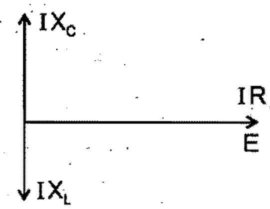
In a LCR circuit

$v = \sqrt{(v_L - v_C)^2 + v_R^2} = \sqrt{(6-12)^2 + 8^2}$

v = 10 ; which is less than voltage drop across capacitor.

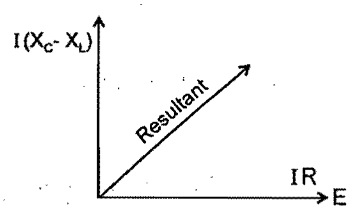
22.11 (C)

If we have all R,L and C then I vs. E will be :



To obtain a leading phase difference of $\pi/4$:

if $X_L < X_C$ and we use all R, L and C in the circuit, then the resultant graph will be :

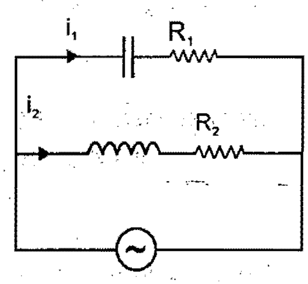


which can give a leading phase difference of $\pi/4$:

Similarly, if we have only resistance and capacitor then we can obtain a phase difference of $\pi/4$ (leading) for suitable values of I, X_C and R. But we cannot obtain a leading phase difference of $\pi/4$ if we use only capacitor (phase difference of $\pi/2$), or only (inductor and resistor) (phase difference of $\pi/2$), or only resistor (phase difference of 0).

2.12 (A) $z = \sqrt{3^2 + 4^2} = 5$ Ans.

2.13 (C)



$i_{1_{rms}} = \frac{E_{rms}}{\sqrt{X_C^2 + R_1^2}} = \frac{130}{13} = 10$ A

$i_{2_{rms}} = \frac{E_{rms}}{\sqrt{X_L^2 + R_2^2}} = 13$ A

Power dissipated $= i_{1rms}^2 R_1 + i_{2rms}^2 R_2 = 10^2 \times 5 + 13^2 \times 6$

= power delivered by battery = 500 + 169 × 6 = 1514 watt

2.14 (B)

P = VI

For secondary :

$V_2 = \frac{P_2}{I_2} = \frac{500}{12.5} = 40$ volts

For an ideal transformer (100% efficient) $P_{input} = P_{output}$

$\Rightarrow \quad V_1 I_1 = V_2 I_2$

$\Rightarrow \quad I_1 = \frac{V_2 I_2}{V_1} = \frac{40(12.5)}{40 \times 5} = 2.5$ A $\quad [\because \frac{n_1}{n_2} = \frac{V_1}{V_2} \Rightarrow \frac{5}{1} = \frac{V_1}{40}]$.

22.15 (BC)

It is apparant from the graph that emf attains its maximum value before the current does, therefore current lags behind emf in the circuit. Nature of the circuit is inductive.

Value of power factor cos ϕ increases by either decreasing L increasing C.

22.16 (A D)

Since the circuit is at resonance so current in the circuit is in the phase with applied voltage.

Voltage across inductor leads the current by $\pi/2$ and across a capacitor lags by $\pi/2$. So the voltage across resistance is lagging by 90° than the voltage across capacitor.

22.17. (ACD)

$V_1 = V_2$

$$\Rightarrow \quad x_L = x_C \quad \Rightarrow \quad f = \frac{1}{2\pi\sqrt{LC}} = 125 \text{ Hz}$$

$$I_0 = \frac{v_0}{R} = \frac{200}{100} \qquad (\because X = 0 \ \therefore \ Z = R) = 2A$$

$V_1 = V_2 = IX_L = I.(\omega L) = 2 \times 2\pi \times 125 \times 2/\pi = 1000$ volt **Ans.**

22.18 (C)

Statement–1 is true but Statement–2 is false

Both A.C. and D.C. produce heat, which is proportional to square of the current. The reversal of direction of current in A.C. is immaterial so far as production of heat is concerned.

22.19 (D)

Statement 1 is false because the given relation is true if all voltages are instantaneous.

22.20 (D)

In resonance condition when energy across capacitor is maximum, energy stored in inductor is zero, vice versa is also true. Hence statement 1 is false.

22.21 (A)

When current through inductor decreases, the magnetic energy stored in inductor decreases and this energy is absorbed by the ac source.

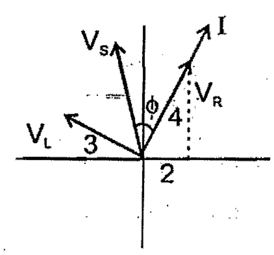
22.22 (A)

Let at an instant $v_R = (V_R)_m \sin(\omega t + \theta)$

$\therefore 2 = 4 \sin(\omega t + \theta)$

$\sin(\omega t + \theta) = ½$

$\therefore \quad \omega t + \theta = 30°.$



Since V_L is 90° ahead of V_R

$v_L = (V_L)_m \sin(\omega t + \theta + 90)$

$\therefore |(V_L)_m| = 3 \cos 30°$

22.23 (B)

From phasor diagram $(V_S)_m = \sqrt{(V_R)_m{}^2 + (V_L)_m{}^2} = 5$ volt.

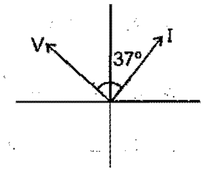
$$\tan\phi = \frac{V_L)_m}{V_R)_m} = \frac{3}{4} \qquad \therefore \phi = 37°$$

$\therefore |v_S| = |(V_S)_m \sin(\omega t + \theta + 37°)|$

$= 5 |\sin(30° + 37°)| = 5 \sin 67°$

22.24 (D)

From phasor diagram it is clear that instantaneous current will decrease or increases, we cannot say.



22.25 (C)

Current drawn is maximum at resonant angular frequency. $L_{eq} = 4$ mH, $C_{eq} = 10\ \mu F$

$L_{eq} = 4$ mH, $C_{eq} = 10\ \mu F$

$$\omega = \frac{1}{\sqrt{LC}} = 5000 \text{ rad/s}$$

22.26 (D)

C_{eq} decreases thereby increasing resonant frequency.

22.27 (B)

At resonance $i_{rms} = \frac{100}{100} = 1A$

Power supplied = $V_{rms} I_{rms} \cos\phi$ ($\phi = 0$ at resonance $\phi = 0$) P = 100 W

22.28 (B)

Average energy stored $= \frac{1}{2} Li_{rms}^2 = \frac{1}{2} (2.4 \times 10^{-3} \text{ H}) . (1 \text{ A})^2 = 1.2$ mJ

22.29 (D)

As 1μs time duration is very less than time period T at resonance, thermal energy produced is not possible to calculate without information about start of the given time duration.

22.30 (A) s ; (B) p, r, s ; (C) q, s, t ; (D) q, t

(A) For sinusodial curve $i_{rms} = \frac{i_0}{\sqrt{2}}$

(B) $$i_{rms}^2 = \frac{\int_0^T i^2 dt}{T} = \frac{4\int_0^{T/4} i^2 dt}{T} = \frac{\int_0^{T/4} i^2 dt}{\frac{T}{4}} = \frac{\int_0^{T/4}\left(\frac{i_0 t}{T/4}\right)^2 dt}{\frac{T}{4}} = \frac{i_0^2}{\left(\frac{T}{4}\right)^3}\int_0^{T/4} t^2 dt = \frac{i_0^2}{3} \Rightarrow i_{rms} = \frac{i_0}{\sqrt{3}}$$

For positive half cycle average current $= \frac{\int i dt}{\int dt} = \frac{½(i_0)(T/2)}{(T/2)} = \frac{i_0}{2}$

Full cycle average current is zero.

(C) For positive half cycle average current $= \frac{\int i dt}{\int dt} = \frac{i_0(T/2)}{T/2} = i_0$

$$i_{rms} = \left[\frac{\int_0^{T/2} i^2 dt}{T/2}\right]^{1/2} = i_0$$

(D) For full cycle average current $= \frac{\int i dt}{\int dt} = \frac{i_0(T/2)+0}{T} = \frac{i_0}{2}$

$$i_{rms} = \left[\frac{\int_0^{T/2} i^2 dt}{T/2}\right]^{1/2} = i_0$$

22.31 (A) q,r (B) p,t (C) p,r (D) q, t

(a) $\tan\phi = \dfrac{1/\omega C}{R} \Rightarrow \phi = \dfrac{\pi}{4}$, current leads source voltage because reactance is capacitive

(b) Pure inductive circuit $\phi = \pi/2$, current lags behind source voltage because reactance is inductive

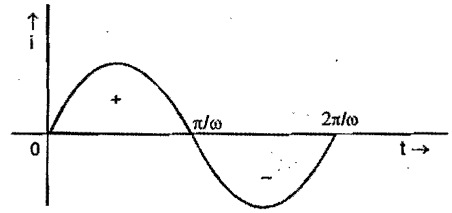
(c) as R = 0, $\tan\phi = \infty$

$\phi = \pi/2$, current leads source voltage because reactance is capacitive

(d) $\tan\phi = \dfrac{\omega L}{R} = 1 \Rightarrow \phi = \dfrac{\pi}{4}$, current lags behind source voltage because reactance is inductive

22.32 0

$$<i> = \frac{\int_0^{\frac{2\pi}{\omega}} I_m \sin\omega t dt}{\frac{2\pi}{\omega}} = \frac{\frac{I_m}{\omega}\left(1-\cos\omega\frac{2\pi}{\omega}\right)}{\frac{2\pi}{\omega}} = 0$$



It can be seen graphically that the area of i – t graph of one cycle is zero.

∴ < i > in one cycle = 0.

22.33 10

impedance of circuit $= \sqrt{R^2 + (X_C - X_L)^2}$

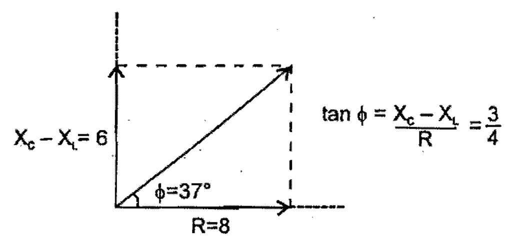
$$Z = \sqrt{8^2 + (8-2)^2} = 10\Omega$$

22.34 24

The current leads in phase by $(\because X_C > X_L)$

$\phi = 37°$

$$\therefore\ i = \frac{10\cos(100\pi t + 37°)}{Z} = \cos(100\pi t + 37°)$$



The instantaneous potential difference across A B is

$= I_m (X_C - X_L)\cos(100\pi t + 37° - 90°)$

$= 6\cos(100\pi t - 53°)$

The instantaneous potential difference across A B is half of source voltage.

$\Rightarrow$ $6\cos(100\pi t - 53°) = 5\cos 100\pi t$

solving we get $\cos 100\pi t = \dfrac{1}{\sqrt{1+(7/24)^2}} = \dfrac{24}{25}$

∴ instantaneous potential difference $= 5 \times \dfrac{24}{25} = \dfrac{24}{5}$ volts

22.35 128

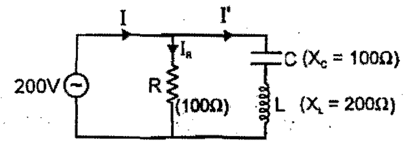
$$V_{rms} = \sqrt{16^2 + 20^2} = 25.6\text{ V} = \frac{128}{5}\text{V}$$

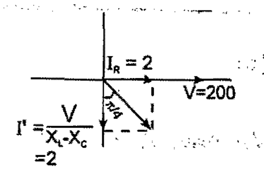
22.36 3

$$V = \frac{V_0}{T/4}t \Rightarrow V = \frac{4V_0}{T}t \Rightarrow V_{rms} = \sqrt{<V^2>} = \frac{4V_0}{T}\sqrt{<t>} = \frac{4V_0}{T}\left\{\frac{\int_0^{T/4} t^2 dt}{\int_0^{T/4} dt}\right\}^{1/2} = \frac{V_0}{\sqrt{3}}$$

22.37 8

$$I_R = \frac{V}{R} = \frac{200}{100} = 2A$$





$$I' = \frac{V}{X_L - X_C} = \frac{200}{100} = 2A$$

$$I = \sqrt{I_R^2 + I'^2} = 2\sqrt{2} \text{ Amp.}$$

22.38 200

$\tan 60 = \frac{\omega L}{R}$ or $\tan 60 = \frac{1/\omega C}{R}$ $\qquad \because \omega L = 1/\omega C$

Impendance of circuit = R

Current in the circuit

$$I_r = \frac{V_r}{Z} = \frac{V_r}{R} = 2 \text{ Amp.}$$

Average power $\overline{P} = \frac{1}{2} V_0 I_0 \cos\phi$ $\qquad$ (as $\cos\phi = 1$)

$\overline{P} = 200 \times 2 \times 1 = 400$ watt.

22.39 3

L removed

$\cos 30^\circ = \frac{100}{\sqrt{100^2 + X_C^2}} = \frac{\sqrt{3}}{2} \quad \Rightarrow \quad X_c = \frac{100}{\sqrt{3}}\,\Omega$

C removed

$\cos 60^\circ = \frac{100}{\sqrt{100^2 \times X_L^2}} = \frac{1}{2} \quad \Rightarrow \quad X_L = 100\sqrt{3}\,\Omega$

$\frac{X_L}{X_C} = \frac{\omega L}{\frac{1}{\omega C}} \qquad \omega^2 (LC) = 3$

$\omega_n = \frac{1}{\sqrt{LC}} = \frac{\omega}{\sqrt{3}} = \frac{300}{\sqrt{3}} = 100\sqrt{3} \qquad \therefore$ frequency $= \frac{100\sqrt{3}}{2\pi}$ Hz = resonant frequency

22.40 1

Given $\frac{R}{Z} = 0.8 \quad \Rightarrow \quad Z = \frac{5}{4}R \quad \Rightarrow \quad R^2 + |X_L - X_C|^2 = \frac{25}{16}R^2$

$|X_L - X_C| = \frac{3}{4}R$

Since $\omega = \frac{1}{2}\omega_0$, resonant frequency $\omega_0 = \frac{1}{\sqrt{LC}}$

$\Rightarrow X_C - X_L = \frac{3}{4}R \quad \Rightarrow \quad \frac{X_L}{X_C} = \omega^2 LC = \left(\frac{\omega}{\omega_0}\right)^2 = \frac{1}{4}$

$X_C\left(1 - \frac{1}{4}\right) = \frac{3}{4}R \quad \Rightarrow \quad X_C = R$

23. MODERN PHYSICS

23.1 (B)

We have $K_\alpha = \frac{m_y}{m_y + m_\alpha}.Q \quad \Rightarrow K_\alpha = \frac{A-4}{A}.Q \quad \Rightarrow 48 = \frac{A-4}{A}.50 \quad \Rightarrow A = 100$

23.2 (A)

After first half hrs $\quad N = N_0 \frac{1}{2}$

for $t = \frac{1}{2}$ to $t = 1\frac{1}{2}$ $\quad N = \left(N_0 \frac{1}{2}\right)\left[\frac{1}{2}\right]^4 = N_0\left(\frac{1}{2}\right)^5$

for $t = 1\frac{1}{2}$ to $t = 2$ hrs. [for both A and B $\frac{1}{t_{1/2}} = \frac{1}{1/2} + \frac{1}{1/4} = 2 + 4 = 6 \Rightarrow t_{1/2} = 1/6$ hrs.]

$$N = \left[N_0\left(\frac{1}{2}\right)^5\right]\left(\frac{1}{2}\right)^3 = N_0\left(\frac{1}{2}\right)^8$$

23.3 (B)

For α - decay : $\quad {}_xA^y \longrightarrow {}_{x-2}B^{y-4} + \alpha$

For β^- – decay : $\quad {}_xA^y \longrightarrow {}_{x+1}B^y + {}_{-1}\beta^0$

For β^+ – decay : $\quad {}_xA^y \longrightarrow {}_{x-1}B^y + {}_{+1}\beta^0$

For k - capture : there will be no change in the number of protons.
Hence, only case in which no of protons increases is β^- - decay
Hence (B).

23.4 (C)

Angular momentum (mvr) $= n.\frac{h}{2\pi} = \frac{h}{2\pi}$ $\quad (n = 1)$

23.5 (C)

$N = N_0 e^{-\lambda t}$ $\quad N_y = N_0(1 - e^{-\lambda t})$

Rate of formation of $\quad R = \frac{dN}{dt} = +\lambda N_0 e^{-\lambda t}$ $\quad X \longrightarrow Y$

At $t = 0$, $R = \lambda N_0$ $\quad \Rightarrow \quad t = \infty$, $R = 0$

23.6 (C)

Energy released = (B.E. of product – BE of reactant)
$= (80 \times 7 + 120 \times 8 - 200 \times 6.5) = 220$ MeV.

23.7 (C)

First excitation energy $= RhC\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = RhC\,\frac{3}{4}$

$\therefore \quad \frac{3}{4} RhC = V$ e.v. $\quad \Rightarrow \quad \therefore \quad RhC = \frac{4V}{3}$ e.v.

23.8 (D)

The electron ejected with maximum speed V_{max} are stopped by electric field E = 4N/C after travelling a distance d = 1m

$$\therefore \quad \frac{1}{2} mV_{max}^2 = eE\,d = 4eV$$

The energy of incident photon $= \frac{1240}{200} = 6.2$ eV

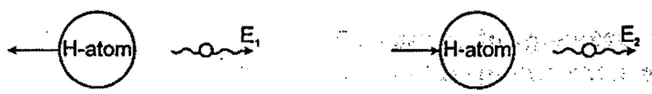
From equation of photo electric effect

$$\frac{1}{2} mv_{max}^2 = h\upsilon - \phi_0 \quad \Rightarrow \quad \therefore \quad \phi_0 = 6.2 - 4 = 2.2 \text{ eV.}$$

23.9 (B)

$$\lambda = \frac{h}{\sqrt{2mK}} \quad \Rightarrow \quad \frac{\lambda_e}{\lambda_P} = \sqrt{\frac{M}{m}} \quad \Rightarrow K = qV \text{ is same for both proton and electron.}$$

23.10 (B)



Case - I Case - II

In the first case K.E. of H-atom increases due to recoil whereas in the second case K.E. decreases due to recoil but $E_1 + KE_1 = E_2 + KE_2$.

$\therefore \quad E_2 > E_1$

स्थिति - I स्थिति - II

23.11 (A)

Linear momentum $\Rightarrow$ $mv \propto \frac{1}{n}$

angular momentum $\Rightarrow$ $mvr \propto n$

$\therefore$ product of linear momentum and angular momentum $\propto n^0$

23.12 (C)

Energy of photon is given by mc^2 now the maximum energy of photon is equal to the maximum energy of electron = eV

hence, $mc^2 = ev$ $\Rightarrow$ $m = \frac{eV}{c^2} = \frac{1.6\times10^{-19}\times18\times10^3}{(3\times10^8)^2} = 3.2 \times 10^{-32}$ kg

23.13 (D)

Using $\frac{1}{\lambda} = R(z-1)^2\left[\frac{1}{n_2^2} - \frac{1}{n_1^2}\right]$

For α particle ; $n_1 = 2$, $n_2 = 1$

For metal A ; $\frac{1875R}{4} = R(Z_1-1)^2\left(\frac{3}{4}\right)$ $\Rightarrow$ $z_1 = 26$

For metal B; $675R = R(Z_2-1)^2\left(\frac{3}{4}\right)$ $\Rightarrow$ $z_2 = 31$

Therefore, 4 elements lie between A and B, i.e. with Z= 27, 28 ,29 , 30

23.14 (D)

For 2nd line of Balmer seires in hydrogen specturm

$$\frac{1}{\lambda} = R(1)\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = \frac{3}{16}R$$

For Li^{2+} : (Li^{2+}) $\frac{1}{\lambda} = R(3)^2\left(\frac{1}{6^2} - \frac{1}{12^2}\right) = \frac{3}{16}R$

which is satisfied by only (D).

23.15 (A)

We have : K.E. = $\frac{P^2}{2m_e} = \frac{hc}{\lambda_{min}}$. $\Rightarrow$ $P = \sqrt{\frac{2hcm_e}{\lambda_{min}}}$

Also, $\lambda_{de\ broglie} = \frac{h}{p} = \sqrt{\frac{h\lambda_{min}}{2m_e C}}$

for λ_{min} = 10Å : $\lambda_{de\ broglie} \cong$ 0.3Å

23.16 (D)

Energy of n^{th} sate in Hydrogen is same as energy of $3n^{th}$ state in Li^{++}.

$\therefore$ 3 → 1 transition in H would give same energy as the 3×3 → 1× 3 transition in Li^{++}.

23.17 (D)

$$\frac{1}{2}mv^2 = \frac{hc}{\lambda} - \phi \quad \Rightarrow \quad \frac{1}{2}mv'^2 = \frac{hc}{(3\lambda/4)} - \phi = \frac{4hc}{3\lambda} - \phi$$

Clearly $v' > \sqrt{\frac{4}{3}}\,v$

23.18 (B)

$${}_{Z}X^{A} \longrightarrow {}_{Z-1}Y^{A} + e^{+} + v$$

$$\downarrow$$

$$2\alpha + {}_{Z-5}X'^{A-8}$$

Given A – 8 = 224

& Z – 5 = 89 $\Rightarrow$ A = 237, Z = 94.

23.19 (B)

no. of photoelectron emitted per second, n = $\frac{\text{Power (watt)} \times \text{Emission\%}}{\text{Energy of 1 photon (inJ)} \times 100}$

$$= \frac{1.5\times10^{-3}W\times(10^{-3})\times0.1}{\frac{1240(nm)(eV)}{400(nm)}\times e\times100} = \frac{0.48}{e}\times10^{-6}.$$ (energy of 1 photon = $\frac{1240\,nm/eV}{400\,nm\times e}$ Joule)

$\therefore$ Photo current = ne = 0.48 μA

23.20 (A)

for K_α $\frac{1}{\lambda_\alpha} = \frac{3R}{4}(Z-1)^2$ transition is from n =2 to n =1

$$\Rightarrow \quad (Z-1) = \sqrt{\frac{4}{3R\lambda_\alpha}} = \sqrt{\frac{4}{3\times1.1\times10^7\times1.8\times10^{-10}}} = \frac{200}{3}\sqrt{\frac{5}{33}} = \frac{78}{3} = 26. \Rightarrow Z = 27$$

23.21 (B)

λ_m will increase to $3\lambda_m$ due to decrease in the energy of bombarding electrons. Hence no characteristic x-rays will be visible, only continous X-ray will be produced.

23.22 (D)

$\because \quad B = \frac{\mu_0 I}{2r} \quad$ and $I = \frac{e}{T}$

$B = \frac{\mu_0 e}{2rT} \quad [r \propto n^2, T \propto n^3] \Rightarrow B \propto \frac{1}{n^5}$

23.23 (B)

$\because$ 90% of sample left undecayed after time t

$$\frac{9}{10} N_0 = N_0 e^{-\lambda t}$$

$$\lambda = \frac{1}{t} \ln\left(\frac{10}{9}\right) \qquad (1)$$

After time 2t,

$$N = N_0 e^{-\lambda(2t)} = N_0 e^{-\frac{1}{t}\left[\ln\left(\frac{10}{9}\right)\right]2t}$$

$$N = N_0 e^{\ln\left(\frac{10}{9}\right)^2} = N_0 \left(\frac{9}{10}\right)^2 \qquad (2)$$

$\therefore$ 19% of initial value will decay in time 2t.

23.24 (C)

Let N_2 be the number of atoms of X at time t = 0. Then at t = 4 hrs (two half lives)

$N_x = \frac{N_0}{4} \quad N_y = \frac{3N_0}{4}$

$\therefore \quad N_x / N_y = 1/3$

at t = 6 hrs (three half lives)

$N_x = \frac{N_0}{8} \quad$ and $\quad N_y = \frac{7N_0}{8}$

or $\quad \frac{N_x}{N_y} = \frac{1}{7}$

The given ratio $\frac{1}{4}$ lies between $\frac{1}{3}$ and $\frac{1}{7}$.

Therefore, t lies between 4 hrs and 6 hrs.

23.25 (C)

$i = \frac{q}{T}$ Now $T^2 \propto r^3 \propto n^6 \Rightarrow \quad i \propto \frac{1}{n^3} \Rightarrow T \propto n^3 \; \frac{i_1}{i_2} = \frac{H_2^3}{H_1^3} = \frac{(1)^3}{(2)^3} = i_2 = 8i_1$

23.26 (BCD)

ground state n = 1
first excited state n = 2

$$KE = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{2r} (z = 1)$$

$$\therefore \quad KE = \frac{14.4 \times 10^{-10}}{2r} eV$$

Now r = 0.53 n² A° (z = 1)

$$(KE)_1 = \frac{14.4\times10^{-10}}{2\times0.53\times10^{-10}} \text{ eV} = 13.58 \text{ eV}$$

$\therefore$ $$(KE)_2 = \frac{14.4\times10^{-10}}{2\times0.53\times10^{-10}\times4} \text{ ev} = 3.39 \text{ ev}$$

$\therefore$ KE decreases by = 10.2 ev

$$\text{Now PE} = \frac{-1}{4\pi\varepsilon_0}\frac{e^2}{r} = \frac{-14.4\times10^{-10}}{r} \text{ ev}$$

$\therefore$ $$(PE)_1 = \frac{-14.4\times10^{-10}}{0.53\times10^{-10}} \text{ ev} = -27.1 \text{ ev}$$

$$(PE)_2 = \frac{-14.4\times10^{-10}}{0.53\times10^{-10}\times4} = -6.79\text{ev}$$

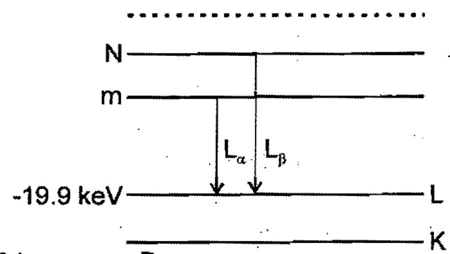
$\therefore$ PE increases by = 20.4 ev

Now Angular momentum ;

$$L = mvr = \frac{nh}{2\pi}$$

$$L_2 - L_1 = \frac{h}{2\pi} = \frac{6.6\times10^{-34}}{6.28} = 1.05\times10^{-34} \text{ J-sec.}$$

23.27 (ABC)



$$\lambda_{min.} = \frac{12400}{V_0} \text{ Å}$$

$$\frac{12400}{20{,}000} = .62 \text{ Å} \approx 62\cdot1\text{pm}$$

Any transition to L will have energy less than or equal to 19·9 kev $\Rightarrow$ so B.

23.28 (AB)

$$|F| = \frac{dU}{dr} = \frac{Ke^2}{r^4} \qquad ....(1)$$

$$\frac{Ke^2}{r^4} = \frac{mv^2}{r} \qquad ....(2)$$

$$\text{and} \quad mvr = \frac{nh}{2\pi} \qquad ....(3)$$

By (2) and (3)

$$r = \frac{Ke^2 4\pi^2}{h^2}\frac{m}{n^2} = K_1\frac{m}{n^2} \qquad ....(4)$$

$$\text{Total energy} = \frac{1}{2}\text{ (potential energy)}$$

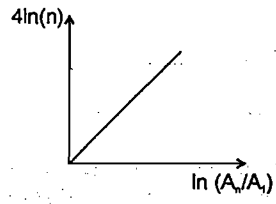
$$= \frac{Ke^2}{6r^3} = \frac{-Ke^2}{6\left(\frac{K_1 m}{n^2}\right)^3} = \frac{-Ke^2n^6}{6K_1^3m^3}$$

Total energy $\propto n^6$

Total energy $\propto m^{-3}$

$\therefore$ (A) and (B) are correct.

23.29 (AB)



$\because \quad r_n = n^2 r_1$

$$\ell n\left(\frac{A_n}{A_1}\right) = \ell n\left(\frac{\pi r_n^2}{\pi r_1^2}\right) = \ell n\ n^4 = 4\ \ell n\ (n)$$

23.30 (A)
Energy of photoelectron emitted is different because after absorbing the photon electrons within metals collide with other atom before being ejected out of metal.
Hence statement 2 is correct explanation of statement 1.

23.31 (B)
de-Broglie wavelength associated with gas molecules varies as $\lambda \propto \frac{1}{\sqrt{T}}$.

23.32 (A)
Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

23.33 (A)
Statement-2 is true by definition and correctly explains the staement-1, namely, ${}_ZX^A$ undergoes 2 α decays, 2β decays (negative β) and 2 γ decays. As a result the daughter product is ${}_{Z-2}Y^{A-8}$.

23.34 (B)
In equilibrium, rate of decay = rate of production

23.35 (D)
As Rate of decay = Rate of production

$$\Rightarrow P = \lambda N \qquad \Rightarrow N = \frac{P}{\lambda}$$

$$= \frac{Pt_{1/2}}{\ell n2} \qquad = 1.8 \times 10^{15}$$

23.36 (C)

As $\quad N = \frac{Pt_{1/2}}{\ell n2}$

it is dependent on P and $t_{1/2}$. Initial no. of ^{56}Mn nuclei will make no difference as in equilibrium rate of production equals rate of decay. Large initial no. will only make equilibrium come sooner.

23.37 (D)
For Balmer series, $n_1 = 2$, (lower) $\quad n_2 = 3, 4, \ldots\ldots$ (higher)

$\therefore$ In transition (VI), Photon of Balmer series is absorbed.

23.38 (C)
In transition II
$E_2 = -3.4$ eV, $E_4 = -0.85$ eV
$\Delta E = 2.55$ eV

$$\Delta E = \frac{hc}{\lambda} \qquad \Rightarrow \qquad \lambda = \frac{hc}{\Delta E}$$

$\lambda = 487$ nm.

23.39 (D)
Wavelength of radiation = 103 nm = 1030 Å

$$\therefore \quad \Delta E = \frac{12400}{1030\text{Å}} \simeq 12.0 \text{ eV}$$

So difference of energy should be 12.0 eV (approx)
Hence $n_1 = 1$ (–13.6)eV and $n_2 = 3$ (–1.51)eV
$\therefore$ Transition is V.

23.40 (A) p,r (B) p,r, t (C) q (D) s

Consider two equation

$$eV_s = \frac{1}{2}mv^2_{max} = h\nu - \phi_0 \quad .... (1)$$

no of photoelectrons ejected/sec. $\propto$ no. of photons/second .... (2)

(A) As frequency is increased keeping intensity constant.

$|V_s|$ will increase, $\frac{1}{2}m(v^2_{max})$ will increase and saturation current will remains same.

(B) As frequency is increased and intensity is decreased.

$|V_s|$ will increase, $\frac{1}{2}m(v^2_{max})$ will increase and saturation current will decrease.

(C) It work function is increased photo emission may stop.

(D) If intensity is increased and frequency is decreased. saturation current will increase.

23.41 (A) q,r,s, (B) q,r,s (C) q,r,s (D) p,q,r,s

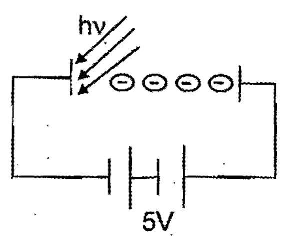
(A) In the given spontaneous radioactive decay, the number of protons remain constant and all conservation principles are obeyed.

(B) In fusion reaction of two hydrogen nuclei a proton is decreased as positron shall be emitted in the reaction. All the three conservation principles are obeyed.

(C) In the given fission reaction the number of protons remain constant and all conservation principles are obeyed.

(D) In beta negative decay, a neutron transforms into a proton within the nucleus and the electron is ejected out.

23.42 (2)



$KE_{max} = (5 - \phi)$ eV

when these electrons are accelerated through 5V,

they will reach the anode with energy = $(5 - \phi + 5)$eV

$\therefore 10 - \phi = 8$

$\phi = 2$eV Ans.

Current is less than saturation current Ans.

Because if slowest electron also reached the plate it would have 5eV energy at the anode, but there it is given that the minimum energy is 6eV.

23.43 (100)

Applying conservation of energy

$m_Ac^2 + K_A + m_Bc^2 + K_B = m_cc^2 + K_C$ + excitation energy

$(m_A + m_B - m_C)\, c^2 + K_A + K_B = K_C$ + excitation energy

$4.65 + 5 + 3 = K_c + 10$

or $K_c = 2.65$ MeV Ans. 2.65 MeV

23.44 (79)

After 4 hrs. sample will contain

30 gm .... B

12.5 gm .... A

$\left(\frac{146}{150}\times 37.5\right)$ gm .... A′

$\therefore$ Total mass = $30 + 12.5 + \frac{146}{150} \times 37.5 = 79$ gm. $\Rightarrow = 30 + 12.5 + \frac{146}{150} \times 37.5 = 79$ gm.

23.45 (700)

Let t be the time required to raise to potential by 2V. Then number of β–particles emitted in t second is 5×10^{10} t. Now the number of β–particles escaping from sphere is 40% i.e., 2×10^{10} t. So, the charge developed.

$$Q = (2 \times 10^{10}\ t)(1.6 \times 10^{-19})\ \text{Coulomb} = (3.2 \times 10^{-9}\ t)\ \text{Coulomb}$$

But $Q = (4\pi\varepsilon_0 R)V = \dfrac{10^{-2} \times 2}{9 \times 10^{9}}$ Coulomb

$\therefore \quad \dfrac{10^{-2} \times 2}{9 \times 10^{9}} = 3.2 \times 10^{-9}\ t$ or $t = 700 \times 10^{-6}$ sec. $\quad t = 700\ \mu$sec.

23.46 (120)

23.47 (34)

(23.46 to 23.47)

$N^{14} + \alpha \to O^{17} + \text{proton}$

Q value = (14.00307 + 4.00260 – 1.00783 – 16.99913) 931.5 = – 1.20 MeV

Let m and M be mass of α particle and nitrogen nucleus respectively and let minimum KE of α particle be $\frac{1}{2}$ mu^2.

$N^{14} + \alpha \to O^{17} + \text{proton}$

Q = (14.00307 + 4.00260 – 1.00783 – 16.99913) 931.5 = – 1.20 MeV

From energy equation

$$\frac{1}{2}mu^2 = |Q| + \text{minimum KE of system}$$

$$= |Q| + \frac{1}{2}(m + M)\left[\frac{mu}{(m + M)}\right]^2 \quad \Rightarrow \quad \therefore \frac{1}{2}mu^2\left(\frac{M}{m + M}\right) = |Q|$$

or $\dfrac{1}{2}mu^2 = |Q|\left(\dfrac{m + M}{M}\right)$

KE of products $= \dfrac{1}{2}(m + M)\left[\dfrac{mu}{(m + M)}\right]^2 = \dfrac{1}{2}mu^2\left(\dfrac{m}{m + M}\right) = |Q|\dfrac{m}{M}$

$= 1.2 \times \dfrac{4}{14} = 0.34$ MeV . Ans.

Q value = – 1.20 Mev, K.E. = 0.34 Mev

23.48 (48)

maximum energy of emitted photon $= \dfrac{\frac{4800}{49}Rch}{100} = \dfrac{48}{49}Rch$

23.49 (6)

Energy released if electron jumps from level n′ to level 1 $= Rch\left(\dfrac{1}{1^2} - \dfrac{1}{n'^2}\right)$

$\therefore \quad Rch\left(\dfrac{1}{1^2} - \dfrac{1}{n'^2}\right) = \dfrac{48}{49}Rch$

$\therefore \quad n' = 7$ then n excited state = n′ – 1

$\therefore \quad n = 6$

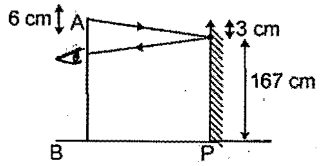
23.50 (600)

Each atom can emit a maximum of 6 photons

∵ there are 100 atoms, maximum number of photons that can be emitted = 600.

24. GEOMETRICAL OPTICS

24.1 (A)



$\Rightarrow$ Figure in self explanatory.

24.2 (C)

$$\frac{I}{O} = -\frac{V}{u}$$

If O and I are on same sides of PA. $\frac{I}{O}$ will be positive which implies v and u will be of opposite signs.

Similarly if O and I are on opposite sides, $\frac{I}{O}$ will be -ve which implies v and u will have same sign.

If O is on PA, I = $\left(-\frac{V}{u}\right)$ (O) $\Rightarrow$ I will also be on. P.A.

24.3 (A)

$\vec{V}_I m = \vec{V}_0 m$ (normal to plane mirror)

$\Rightarrow \quad \vec{V}_1 - \vec{V}_m = -(\vec{V}_0 - \vec{V}_m)$

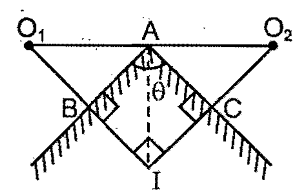
$\quad \vec{V}_I - V\sin\theta = -(0 - V\sin\theta) \quad \Rightarrow \quad V_I = 2V\sin\theta$

24.4 (B)

As AB is common and $O_1B = BI$

ΔO_1BA and ΔBAI are congruent

By symmetry AI is perpendicular to O_1 to O_2



and $\angle O_1AB = \angle BAI$

$\therefore$ BAI = 45°

and $\angle BAC = 90°$

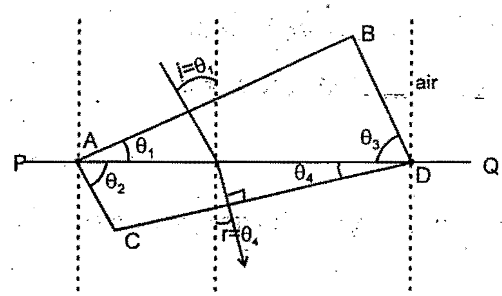
24.5 (C)

By snell's law :

$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$$

for $i = \theta_1$, $r = \theta_4$ and $\mu_1 = 1$.

$$\mu_2 = \frac{\sin\theta_1}{\sin\theta_4}.$$



24.6 (A)

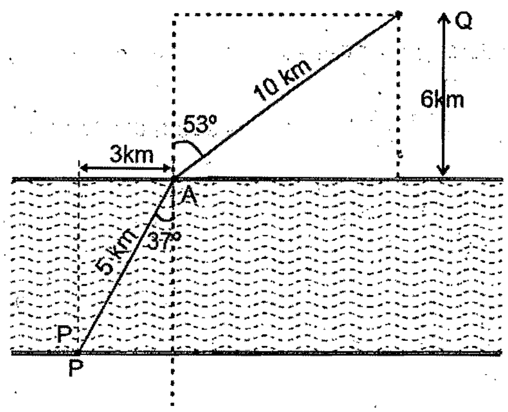
As we know that light travels in a path such as to reach from one point to another in shortest possible time.

Therefore, the man must travel along that path on which light would have travelled in moving from P to Q.

By Snell's law ;

$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} \quad \Rightarrow \quad \sin r = \frac{\mu_1}{\mu_2} \cdot \sin i$$

$$\sin r = \frac{3}{5} \cdot \frac{4}{3} = \frac{4}{5}$$



$\Rightarrow \quad r = 53°$

$\therefore \quad AQ = 10$ Km.

From P to A :

$$t_1 = \frac{5}{3}$$

From A to Q :

$$t_2 = \frac{10}{4} = \frac{5}{2}$$

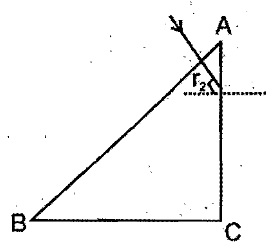
$$T = t_1 + t_2 = \frac{5}{3} + \frac{5}{2} = \frac{25}{6} \text{ hr.}$$

$$= \left(\frac{24}{6}+\frac{1}{6}\right)\text{ hr} = \left(4\text{hr} + \frac{1}{6}\text{hr}\right) = 4\text{hr} + 10 \text{ minutes} \qquad \textbf{Ans.}$$

24.7 (C)

$$\frac{x}{1} = \frac{x_{rel}}{\mu} \qquad x_{rel} = \mu x \Rightarrow \frac{d^2x_{rel}}{dt^2} = \mu\frac{d^2x}{dt^2} \qquad \Rightarrow a_{rel} = \mu g$$

24.8 (B)



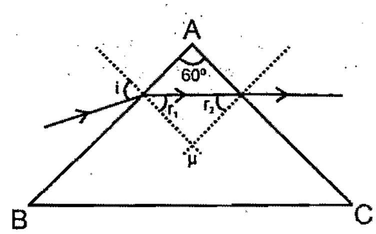
$A = 90° - \theta$

$r_2 = A = 90° - \theta > \theta_C$

$$\cos\theta > \sin\theta_C = \frac{6/5}{3/2} = \frac{4}{5} \qquad (\theta_C \text{ is critical angle })$$

$$\Rightarrow \qquad \theta < \cos^{-1}\frac{4}{5} = 37°.$$

24.9 (A)



$r_2 < \theta_C \quad ; A - r_1 < \theta_C$

$r_1 > A - \theta_C$

$\Rightarrow \quad \sin r_1 > \sin(A - \theta_C)$

$$\Rightarrow \qquad \frac{\sin i}{\mu} > \sin(A - \theta_C)$$

$\Rightarrow \sin i > \mu(\sin A \cos\theta_C - \sin\theta_C \cos A)$

$$= \qquad \sqrt{\frac{7}{3}}\left(\frac{\sqrt{3}}{2}\sqrt{1-\frac{3}{7}} - \sqrt{\frac{3}{7}}.\frac{1}{2}\right) \qquad = \quad 1-\frac{1}{2} = \frac{1}{2}$$

$$\Rightarrow \qquad \sin i > \frac{1}{2}$$

or $\quad i > 30^0$.

24.10 (A)

Given $i = 60° \quad A = \delta = e$

$\delta = i + e - A \quad \Rightarrow \quad \delta = i \qquad (\because e = A)$ and $\delta = i = e$

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\frac{A}{2}} \qquad \text{Here angle of deviation is minimum } (\because i = e)$$

$$\mu = \frac{\sin\left(\frac{60°+60°}{2}\right)}{\sin(60°/2)} = \sqrt{3} \text{ Ans.}$$

24.11 (A)

Using formula of spherical surface taking 'B' as object

$$\frac{\mu_2}{\infty}-\frac{\mu_1}{(-2R)} = \frac{\mu_2-\mu_1}{-R}$$ (R being the radius of the curved surface)

$$\frac{\mu_2}{\infty}-\frac{\mu_1}{(-2R)} = \frac{\mu_2-\mu_1}{-R}$$

$$\Rightarrow \frac{\mu_1}{\mu_2} = 2$$

24.12 (D)

For spherical surface

using $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2-n_1}{R}$

$\Rightarrow \quad \frac{n}{2R} - \frac{1}{\infty} = \frac{n-1}{R}$

$\Rightarrow \quad n = 2n - 2$

$\Rightarrow \quad n = 2.$

24.13 (D)

The image is erect hence,mirror must be between object & image. Virtual image of real object is diminished, hence mirror is convex.

24.14 (D)

Acceleration of block AB = $\frac{3mg}{3m+m} = \frac{3}{4}g$; acceleration of block CD = $\frac{2mg}{2m+m} = \frac{2g}{3}$

Acceleration of image in mirror AB

= 2 acceleration of mirror

$= \quad 2.\left(\frac{-3g}{4}\right) = \frac{-3}{2}g$

Acceleration of image in mirror CD = $2.\left(\frac{2g}{3}\right) = \frac{4g}{3}$

$\therefore$ Acceleration of the two image w.r.t. each other = $\frac{4g}{3}-\left(\frac{-3g}{2}\right) = \frac{17g}{6}$.

24.15 (C)

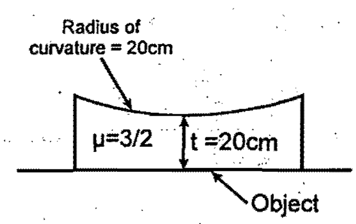
In the first case the distance travelled in the slab < distance travelled in the slab in the 2nd case.

24.16 (A)

Considering refraction at the curved surface,

$u = -21\sqrt{3}$; $\mu_2 = 1$

$\mu_1 = 3/2$; $R = +20$



applying $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2-\mu_1}{R}$

$\Rightarrow \frac{1}{v} - \frac{3/2}{-20} = \frac{1-3/2}{20} \qquad \Rightarrow v = -10$

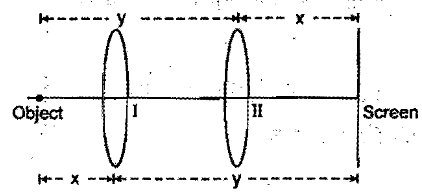
i.e. 10 cm below the curved surface or 10 cm above the actual position of flower.

24.17 (A)

$$\left(\frac{n_A-1}{1}\right)\frac{2}{R_A} = \left(\frac{n_B-1}{1}\right)\frac{2}{R_B} \quad \text{or} \quad \frac{0.63}{R_A} = \frac{n_B-1}{R_B} \quad \text{or } n_B = 1.7$$

24.18 (B)

At first position of lens, let the distance of lens from object and screen be x and y respectively.



∴ $x + y = 100$ ....(1)

∴ At second position of lens the distance of lens from object and screen shall be y and x respectively.

∴ $y - x = 40$ ....(2)

solving equation (1) and (2) we get

$$y = 70 \text{ cm} = \frac{70}{100} \text{ m} \quad \text{and } x = 30 \text{ cm} = \frac{30}{100} \text{ m}$$

∴ The power of lens is,

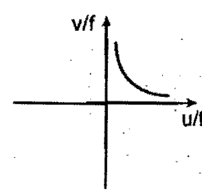
$$\frac{1}{f} = \frac{1}{y} + \frac{1}{x} = \frac{100}{70} + \left(\frac{100}{30}\right) = \frac{100}{21} \approx 5 \text{ diopters}$$

24.19 (A)

$$\frac{1}{V} - \frac{1}{u} = \frac{1}{f}$$

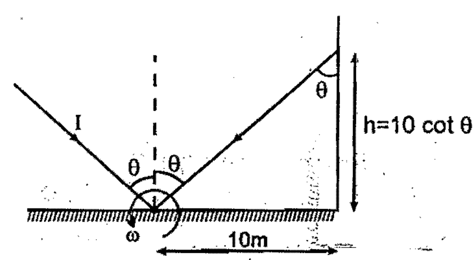
$$\frac{f}{v} - \frac{f}{u} = 1 \quad \text{or} \quad \frac{1}{y} - \frac{1}{x} = 1 \quad \text{or} \quad y = \frac{x}{x+1}$$

Hence, a virtual erect image by a **diverging** lens is represented by



24.20 (B)

When mirror is rotated with angular speed ω, then the reflected ray rotates with angular speed 2ω (= 36 rad/s)



$$\text{speed of the spot} = \left|\frac{dh}{dt}\right| = \left|\frac{d}{dt}(10\cot\theta)\right|$$

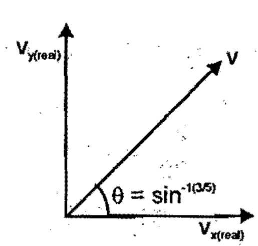
$$= \left|-10\,\text{cosec}^2\theta\,\frac{d\theta}{dt}\right| = \left|-\frac{10}{(0.6)^2}\times 36\right| = 1000 \text{ m/s.}$$

24.21 (D)

Put $A = \delta_{min}$ and $\mu = \sqrt{2}$ ⇒ The relation $\mu = \dfrac{\sin\left(\frac{A+\delta_{min}}{2}\right)}{\sin\left(\frac{A}{2}\right)}$ and solve for A

24.22 (C)

Let y-axis be vertically upwards and x-axis be horizontal.



$$V_y\,(\text{app.}) = \frac{V_y(\text{real})}{\left(\frac{1}{\mu}\right)}$$

$$V_x\,(\text{app.}) = V_x\,(\text{real})$$

$$\tan\phi = \frac{V_y(\text{app})}{V_x(\text{app})} = \frac{4}{3}\tan\theta = \frac{4}{3}\times\frac{3}{4} = 1$$

24.23 (A)

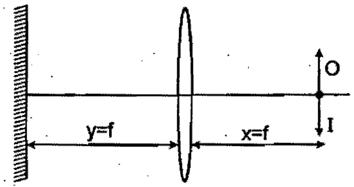
Use refraction formulae seperatelly that is for air and $\mu = 1.6$ and for air and $\mu = 2.0$ and find the positions of the two images.

24.24 (D)

Dispersion will not occur for a light of single wave length $\lambda = 4000$ Å.

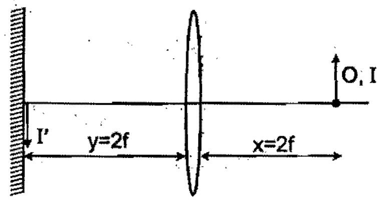
24.25 (D)

This question should be solved by directly substituting the options If x = f, y = f then final image will be formed as shown.



For option B and C the position of image will be different.

When x = 2f, y = 2f, the lens makes image I′ of object O on the surface of mirror as shown in the figure. Mirror shall make of image of I′ over I′ itself. Hence lens shall make image of I′ at the position of O. (which is I)



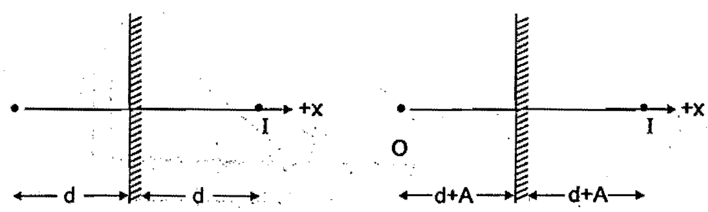
24.26 (C)

$v = -30,\ m = -\frac{v}{u} = -2 \quad \therefore A'B' = C'D' = 2 \times 1 = 2$ mm

Now $\frac{B'C'}{BC} = \frac{A'D'}{AD} = \frac{v^2}{u^2} = 4 \Rightarrow B'C' = A'D' = 4$ mm

$\therefore$ length = 2 + 2 + 4 + 4 = 12 mm Ans.

24.27 (C)



(i) (ii)

from figure (i) & (ii) it is clear that if the mirror moves distance 'A' then the image moves a distance '2A'.

24.28 (C)

for $M_1 : V = -60,\ m_1 = -2$

for $M_2 : u = +21.\ f = 10$

$\therefore \quad \frac{1}{V} + \frac{1}{20} = \frac{1}{10} \Rightarrow V = 20$

$\therefore \quad M_2 = -\frac{20}{20} = -1$

$\therefore \quad M = M_1 \times M_2 = +2$]

24.29 (A)

By mirror formula : $\frac{1}{v}+\frac{1}{-10}=\frac{1}{10}$ $\Rightarrow$ v = ± 5 cm

$\therefore$ $m = +\frac{1}{2}$

the image revolves in circle of radius $\frac{1}{2}$ cm. Image of a radius is erect $\Rightarrow$ Image will revolve in the same direction as the particle. The image will complete one revolution in the same time 2s.

velocity of image $v = \omega r = \frac{2\pi}{2} \times \frac{1}{2} = \frac{\pi}{2}$ cm/s = **1.57 cm/s Ans.**

24.30 (B)

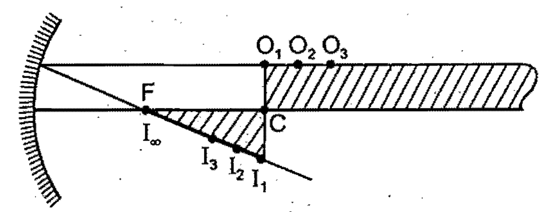
Cutting a lens in transverse direction doubles their focal length i.e. 2f.

Using the formula of equivalent focal length $\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}+\frac{1}{f_4}$

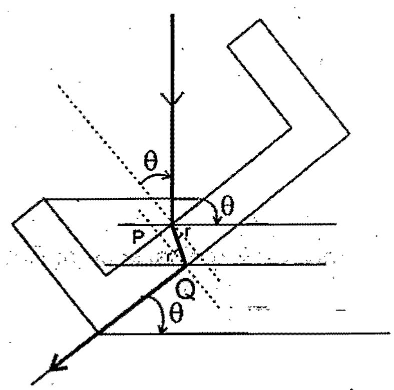
We get equivalent focal length as $\frac{f}{2}$.

24.31 (C)

(Moderate) Draw an incident ray along the top side of rectangular strip,which happens to be parallel to the principal axis. After reflection this ray passes through focus. Hence image of all points (for e.g. O_1, O_2, O_3, ...) on top side of the strip lie on this reflected ray (at I_1, I_2, I_3, ...) in between focus and centre of curvature. Thus the image of this strip is a triangle as shown in figure



24.32 (A)



For refraction at glass-air interface, ray passing through point Q,

$\frac{\sin r}{\sin 90} = \frac{1}{1.5}$ $\Rightarrow$ $\sin r = \frac{1}{1.5}$

For refraction at water-glass interface, ray passing through point P,

$\frac{\sin\theta}{\sin r} = \frac{1.5}{4/3}$ $\Rightarrow$ $\sin\theta = \frac{3}{4}$

24.33 (AC)

For convex mirror

|m| < 1 for any real object

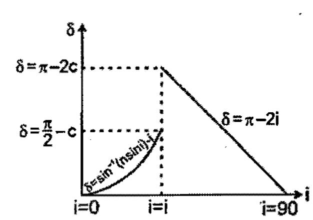
Now, $V_{image} = -m^2 V_{object}$

$\Rightarrow$ $|V_{image}| < |V_{object}|$ always.

24.34 (AC)

(Tough) The image of a point closer to the focus will be farther. As the transverse magnification of B will be more than A, the image of AB will be inclined to the optical axis.

24.35 (ABCD)

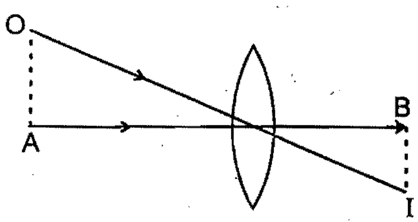


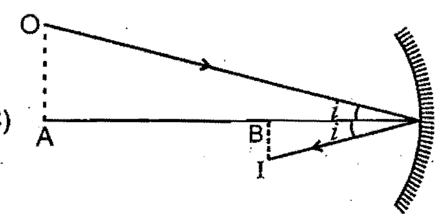
24.36 (ABC)
[Moderate] $\delta = i + e - A$ (for minimum derivation i = e)
$\therefore$ minimum deviation = $2i - A$
$60 = 2 \times 60 - A \Rightarrow \because A = 60°$

$$n = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)} = \frac{\sin\left(\frac{60+60}{2}\right)}{\sin\left(\frac{60}{2}\right)} = \sqrt{3}$$

$\delta_1 = i_1 + e - A$
$65° = i_1 + 70° - 60°$ or $i_1 = 55°$
δ versus i curve is not parabolic

24.37 (ACD)

(A) 

(C) 

(D) Image is inverted $\Rightarrow$ It should be real

24.38 (C)
The index of refraction for light at the red end of the visible spectrum is lesser than at the violet end.Hence statement -2 is false

24.39 (D)
Statement-2 is correct explanation of statement-1.

24.40 (D)
From symmetry the ray shall not suffer TIR at second interface, because the angle of incidence at first interface equals to angle of emergence at second interface. Hence statement 1 is false

24.41 (D)
If the mirror is shifted parallel to itself such that the velocity of the mirror is parallel to its surface, the image shall not shift. Hence statement 1 is false.

24.42 (D)

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

Here v = 2.5 (Distance of Retina as position of image is fixed)
u = – x

$$\frac{1}{f} = \frac{1}{2.5} + \frac{1}{x}$$ For f_{min} : x is minimum $$\frac{1}{f_{min}} = \frac{1}{2.5} + \frac{1}{25}$$

24.43 (B)

For f_{max}: x is maximum $$\frac{1}{f_{max}} = \frac{1}{2.5} + \frac{1}{\infty}$$

24.44 (B)
For near sighted man lens should make the image of the object with in 100 cm range
For lens $u = -\infty$ $v = -100$

$$\frac{1}{f_{lens}} = \frac{1}{-100} - \frac{1}{-\infty}$$

24.45 (D)
From passage, (D) is correct.

24.46 (D)

From points (2) and (3) of passage :

f and f' must be of opposite sign.

Also $\omega_C < \omega_D$ and $f_C < f_D$

which is satisfied only by (D).

24.47 (B)

$$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$$

$$\Rightarrow \quad \frac{\omega_1}{\omega_2} = -\frac{f_1}{f_2} = \frac{1}{2} \text{ ©1} \quad \text{.................(1)}$$

$$\Rightarrow \quad \frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{40} \quad \text{.................(2)}$$

After solving (1) & (2)

$f_1 = 20$ cm $\Rightarrow$ $f_2 = -40$ cm.

24.48 (B)

$$\frac{\mu_2}{v} = \frac{\mu_1}{u} + \left(\frac{\mu_2 - \mu_1}{R}\right)$$

$(\mu_2 - \mu_1)$ is +ve and R is – ve if u is –ve, v will always be –ve

i.e. for real object image is always virtual.

24.49 (D)

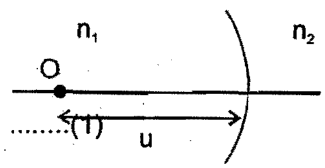
24.50 (D)

48. to 50.

Consider a object on left side of spherical surface seperating two media.

If real object is in rarer media i.e., $n_1 < n_2$

$$\text{Then } \frac{n_2}{v} = \frac{n_2 - n_1}{(-u)} + \frac{n_1}{(-R)} = -\text{ve}$$



Hence image shall be virtual for a real object lying on concave side with rarer media.

If real object is in denser media i.e., $n_1 > n_2$

$$\frac{n_2}{v} = \frac{-(n_1 - n_2)}{(-u)} + \frac{n_1}{(-R)} = \frac{n_1 - n_2}{u} - \frac{n_1}{R}$$

$\therefore$ Image is real if $\frac{n_1 - n_2}{u} > \frac{n_1}{R}$ or $u < \frac{(n_1 - n_2)R}{n_1}$ .... (2)

and image is virtual if $u > \left(\frac{n_1 - n_2}{n_1}\right)R$ .... (3)

From statements 1, 2 and 3 we can easily conclude the answers.

24.51 (A) s, (B) p, (C) s, (D) q

$$\vec{v}_A = \hat{i} + \vec{a}t = \hat{i} + (2\hat{i} + \hat{j})(2) = 5\hat{i} + 2\hat{j}$$

$$\vec{v}_{A'} = -5\hat{i} + 2\hat{j} \quad \Rightarrow \quad \vec{v}_{A',A} = \vec{v}_{A'} - \vec{v}_A = -10\hat{i}$$

$$\vec{v}_B = (-\hat{i} + 3\hat{j}),\ \vec{v}_{B'} = \hat{i} + 3\hat{j} \quad \text{so } \vec{v}_{B',B} = 2\hat{i}$$

For particle C $\frac{dv_y}{dt} = 2t \quad \Rightarrow \quad v_y - 6 = t^2 \quad \Rightarrow \quad v_y = 6 + 4 = 10$

$\vec{v}_C = 5\hat{i} + 10\hat{j}$, $\vec{v}_{C'} = -5\hat{i} + 10\hat{j}$ so $\vec{v}_{C',C} = -10\hat{i} \Rightarrow$ $\vec{v}_D = 3\hat{i} - \hat{j}$, $\vec{v}_{D'} = -3\hat{i} - \hat{j}$, $\vec{v}_{D',D} = -6\hat{i}$

24.52 (A) p,r (B) q, r (C) q, r (D) q, r

Initially the image is formed at infinity.

(A) As μ is increased the focal length decreases. Hence the object is at a distance larger than focal length. Therefore final image is real. Also final image becomes smaller is size in comparision to size of image before the change was made.

(B) If the radius of curvature is doubled, the focal length decreases. Hence the object is at a distance lesser than focal length. Therefore final image is virtual. Also final image becomes smaller is size in comparision to size of image before the change was made.

(C) Due to insertion of slab the effective object for lens shifts right wards. Hence final image is virtual. Also final image becomes smaller is size in comparision to size of image before the change was made.

(D) The object comes to centre of curvature of right spherical surface as a result. Hence the final image is virtual. Also final image becomes smaller is size in comparision to size of image before the change was made.

24.53 (A) r, (B) s (C) q (D) q

Image by convex mirror is always virtual, erect and diminished.

In case of concave mirror, see using position of object.

24.54 (A) p, (B) r, (C) q, (D) s

By snell law $n = \dfrac{\sin i}{\sin r}$

Since for 1st case angular incidence is same for all ray. So r will be less for red.

24.55 (A) p, r (B) q, s (C) q,r (D) p, s

(A) For plane incident wave fronts a divergent refracted beam can be formed only by diverging action and convergent reflected beam can be formed by converging action. Hence (A) p, r

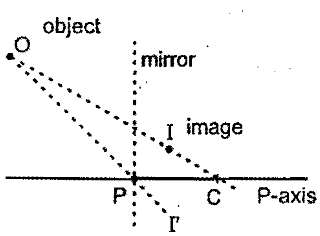
(B) For plane incident wave fronts a convergent refracted beam can be formed only by converging action and divergent reflected beam can be formed by diverging action. Hence (B) q, s

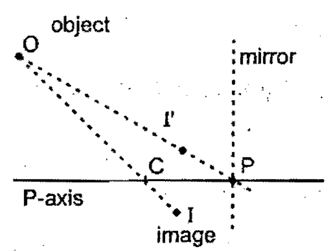
(C) For a incident divergent beam a parallel reflected or refracted beam can be formed only by converging action.Hence (C) q, r

(D) For a incident convergent beam a parallel reflected of refracted beam can be formed only by diverging action.Hence (D) q, r

24.56 (A) p, q (B) p, q (C) r, s (D) r, s

For a spherical mirror, line joining object and its image crosses principal axis at centre of curvature. The line joining object and image inverted about principal axis cuts the principal axis at the pole. Then from figure below.





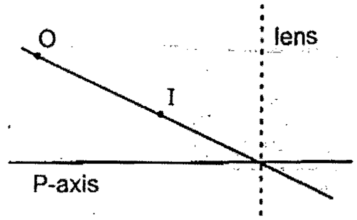
We can conclude

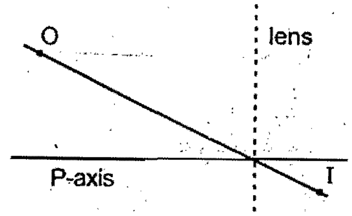
(A) If object and image are on same side of principal axis, they are on opposite side of mirror.

(B) If object and image are on opposite side of principal axis, they are on same side of mirror.

For a lens, the line joining object and image cuts the principal axis at optical centre.

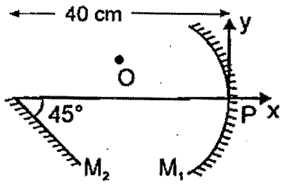
Then from figures below.





We can conclude
(C) If object and image are on same side of principal axis, they are also on same side of lens.
(D) If object and image are on opposite side of principal axis. They are also on opposite side of lens.

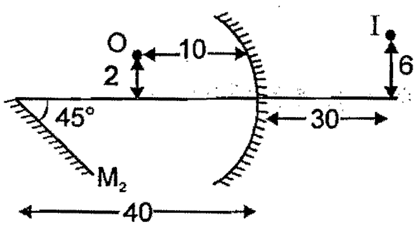
24.57 (40)



For m_1, u = – 10, f = – 15, h = 2.

Using mirror formula $\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{v}+\frac{1}{-10}=\frac{1}{-15}$

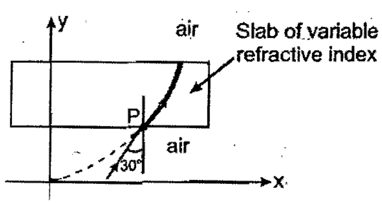
$\Rightarrow \frac{1}{v}=\frac{1}{10}-\frac{1}{15}=\frac{3-2}{30}$ = v = 30 cm & $\frac{h_2}{h_1}=-\frac{v}{u} \Rightarrow h_2 = 6$ cm

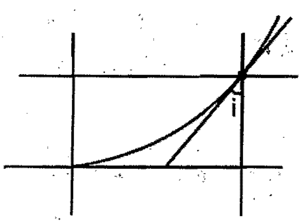


The image formed by the plane mirror is at 70 below the principal axis & 70 + 6 – 30 = 46 of the concave mirror.

∴ coordinates of I_2 w.r.t. P = (– 46, – 70) Ans.]

24.58 (10)





By snell's law.

1 × sin 30° = ... = ... = n sini

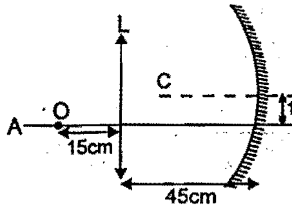
where n is R.I. at y and i is angle of incidence at y.

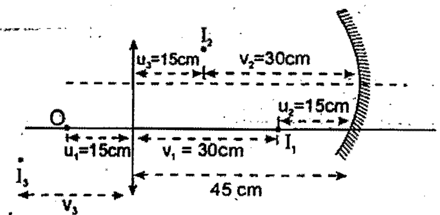
$$\tan(90-i)=\frac{dy}{dx}=8x=4\sqrt{y}$$

$$\cot i = 4\sqrt{y} = 4\sqrt{\frac{1}{2}} = 2\sqrt{2}$$

$\Rightarrow \sin i = \frac{1}{3}$ ∴ $n = \frac{\sin 30°}{\sin i} = \frac{1/2}{1/3} = 1.5$ Ans.

24.59 (26)





I_1 is the image of object O formed by the lens.

$$\frac{1}{v_1}-\frac{1}{u_1}=\frac{1}{f} \qquad u_1=-15 \qquad f_1=10$$

Solving we get

$v_1 = 30$ cm

I_1 acts as source for mirror

$\therefore \quad u_2 = -(45 - v_1) = -15$ cm

I_2 is the image formed by the mirror

$$\therefore \quad \frac{1}{v_2}=\frac{1}{f_m}-\frac{1}{u_2}=-\frac{1}{10}-\frac{1}{15} \quad \therefore v_2=-30 \text{ cm}$$

The height of I_2 above principal axis of lens is $=\frac{v_2}{u_2}\times 1+1=3$cm

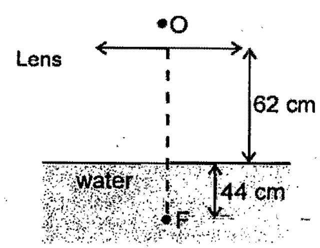
I_2 acts a source for lens $\quad \therefore u_3 = -(45 - v_2) = -15$ cm

Hence the lens forms an image I_3 at a distance $v_3 = 30$ cm to the left of lens and the image of I_3, $\frac{v_3}{u_3}\times 3=6$cm below the principal exis of lens.

The height of I_2 above principal axis of lens is $=\frac{v_2}{u_2}\times 1+1=3$cm

$\therefore \quad$ required distance $=\sqrt{30^2+6^2}=6\sqrt{26}$ cm

24.60 (91)



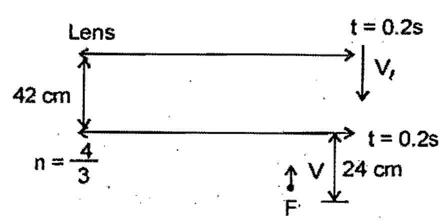
At t = 0.2 sec , velocity of lens

$V_\ell = gt = 2$m/s (downwards)

$\therefore$ for lens the fish appears to approach

with a speed of $2+\left(1\times\frac{3}{4}\right)=\frac{11}{4}$ m/s

at a distance of $\left(42+\frac{24}{\left(\frac{4}{3}\right)}\right)=60$ cm.



$\therefore \quad$ image of fish from lens , $V=\frac{-60\times 90}{-60+90}=-180$ cm.

$\therefore \quad$ Velocity of image w.r.t. lens $\quad V_i=\left(\frac{v^2}{u^2}\right)\frac{du}{dt}=\left(\frac{-180}{-60}\right)^2\times\frac{11}{4}=\frac{99}{4}$ m/s

velocity of image w.r.t. observer $= V_i - 2 = \frac{99}{4}-2=\frac{91}{4}$ m/s = 22.75 cm/s (upwards)

25. WAVE OPTICS

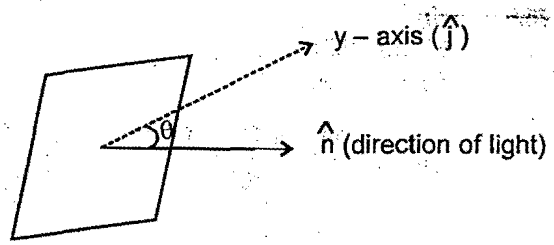
25.1 (B) $I \propto A^2 \quad \therefore \quad \frac{I_1}{I_2} = \frac{2^2}{3^2} = 4/9$

$$I \propto A^2 f^2 \quad \Rightarrow \quad \frac{I_1}{I_2} = \frac{1}{9}$$

25.2 (B) $\cos^{-1}\frac{2}{\sqrt{14}}$

$x + 2y + 3z = c$ represents a plane.

Now angle θ is given by :



$$\cos\theta = \frac{\hat{n}\cdot\hat{j}}{|\hat{n}||\hat{j}|} = \frac{b}{\sqrt{a^2+b^2+c^2}}$$

$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{2}{\sqrt{14}}\right)$$

where $\hat{n} = a\hat{i} + b\hat{j} + c\hat{k} = \hat{i} + 2\hat{j} + 3\hat{k}$.

25.3 (B)

$$\frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{\left(\sqrt{I_1}+\sqrt{I_2}\right)^2 - \left(\sqrt{I_1}-\sqrt{I_2}\right)^2}{\left(\sqrt{I_1}+\sqrt{I_2}\right)^2 + \left(\sqrt{I_2}-\sqrt{I_2}\right)^2}$$

$$= \frac{I_1}{I_1} \times \frac{\left(1+\sqrt{\frac{I_2}{I_1}}\right)^2 - \left(1-\sqrt{\frac{I_2}{I_1}}\right)^2}{\left(1+\sqrt{\frac{I_2}{I_1}}\right)^2 + \left(1-\sqrt{\frac{I_2}{I_1}}\right)^2} = \frac{(1+2)^2-(1-2)^2}{(1+2)^2+(1-2)^2} = \frac{8}{10} = \frac{4}{5}$$

25.4 (D)
in cases I, II, III, IV the path differences are respectively

$$\frac{\lambda}{2}, \lambda, \frac{\lambda}{4} \text{ and } \frac{3\lambda}{4}$$

$\Rightarrow$ phase differences are respectively
π, 2π, $\pi/2$, $3\pi/2$

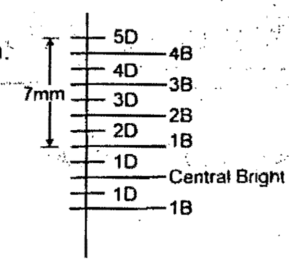
and $I = I_0 \cos^2\left(\frac{\phi}{2}\right)$

$\therefore$ the intensity in the four cases are

$0, I_0, \frac{I_0}{2}, \frac{I_0}{2}$ respetively.

25.5 (A)

There are three and a half fringes from first maxima to fifth minima as shown.



$\Rightarrow \quad \beta = \dfrac{7mm}{3.5} = 2mm$

$\Rightarrow \quad \lambda = \dfrac{\beta D}{d} = 600$ nm.

25.6 (C)

(B) For 100th max.

$d \sin\theta = 100\,\lambda$

$$\sin\theta = \frac{100 \times 5000 \times 10^{-9}}{1 \times 10^{-3}} = \frac{5 \times 10^{-4}}{10^{-3}} = 0.5 = \frac{1}{2}$$

$\therefore \quad y = D\tan\theta \quad = 1 \times \tan 30 \quad = \dfrac{1}{\sqrt{3}}$

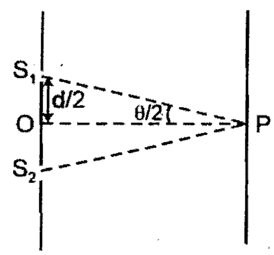
25.7 (A)

In $\Delta\, S_1PO$:

$\tan\dfrac{\theta}{2} = \dfrac{d/2}{D}$

As $D >> d$

$\therefore \quad \theta$ is very small.



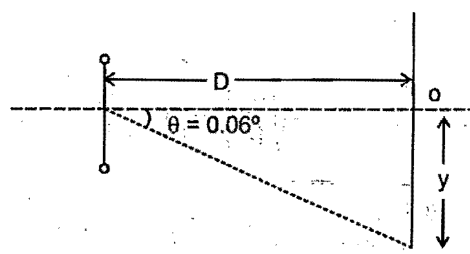
$\Rightarrow \quad \tan\dfrac{\theta}{2} \approx \dfrac{\theta}{2} \qquad \Rightarrow \quad \dfrac{\theta}{2} = \dfrac{d}{2D}$

$\Rightarrow \quad \dfrac{D}{d} = \dfrac{1}{\theta} \Rightarrow$ Fringe width $= \dfrac{\lambda D}{d} = \dfrac{\lambda}{\theta}$ Ans.

25.8 (B)

Say 'n' fringes are present in the region shown by 'y'

$\Rightarrow \quad y = n\beta = \dfrac{n.\lambda D}{d}$



$\Rightarrow \quad \dfrac{y}{D} \approx \tan(0.06°) \approx \dfrac{0.06 \times \pi}{180} = \dfrac{n\lambda}{d}$

$\Rightarrow \quad n = \dfrac{10^3 \times \pi}{180} \times 0.06 = \dfrac{\pi}{3} > 1.$

Hence; only one maxima above and below point O. So total 3 bright spots will be present (including point 'O' i.e. the central maxima).

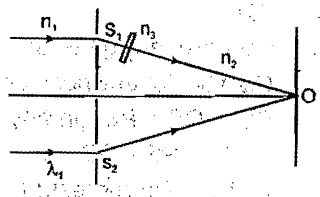
25.9 (C)

At path difference $\dfrac{\lambda}{6}$, phase difference is $\dfrac{\pi}{3}$

$I = I_0 + I_0 + 2I_0 \cos\dfrac{\pi}{3} = 3\,I_0 \qquad I_{max} = 4I_0$

So the required ratio is $\dfrac{3I_0}{4I_0} = 0.75$

25.10 (A)



$$\frac{2\pi}{n_1\lambda_1}(n_3 - n_2)\,t$$

light wavelength in medium n_1 is = λ_1

$\Rightarrow$ Wavelength in vaccum = $\lambda_0 = n_1\lambda_1$

The path difference between the lights waves reaching point O = $(n_3-n_2)\,t$ = extra path which the light from S_1 travelled compared to the path from S_2.

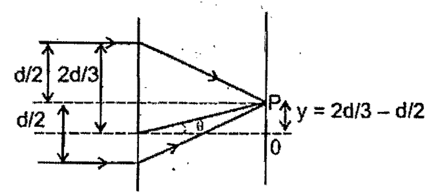
Corresponding phase difference = $\dfrac{2\pi}{\lambda_0}$ (path difference) = $\dfrac{2\pi}{n_1\lambda_1}(n_3-n_2)\,t$

25.11 (C)

Here path difference will be :

$\Delta x = (\mu_2 - \mu_1)t \;\Rightarrow\; \delta = \dfrac{2\pi}{\lambda}(\mu_2 - \mu_1)t$ Hence (C)

25.12 (D)



we know that P will be the central maxima

(at which path difference is zero)

Now OP = $\dfrac{d}{2} - \dfrac{d}{3} = \dfrac{d}{6}$

25.13 (B)

When light passes through a medium of refractive index μ, the optical path it travels is (μt).

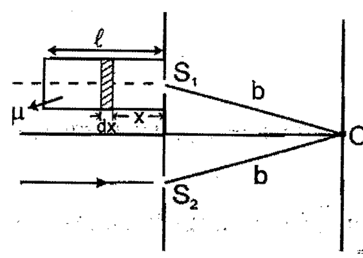
Therefore, before reaching O light through S_1 travels $(\mu l + b)$ distance while that through S_2 travels a distance $(l + b)$

i.e. : path difference = $(\mu l + b) - (l + b) = (\mu - 1)\,l$.

For a small element 'dx' path difference $\Delta x = [(1 + ax) - 1]\,dx = ax\,dx$

For the whole length ;

$$\Delta x = \int_0^{\ell} ax\,dx = \frac{a\ell^2}{2}$$



For a minima to be at 'O'.

$$\Delta x = (2n + 1)\frac{\lambda}{2}$$

i.e. : $\dfrac{a\ell^2}{2} = (2n + 1)\dfrac{\lambda}{2}$.

For minimum 'a'; n = 0

$\Rightarrow \quad \dfrac{a\ell^2}{2} = \dfrac{\lambda}{2} \quad \Rightarrow \quad a = \dfrac{\lambda}{\ell^2}$ Ans.

25.14 (D)

Shift of fringe pattern = $(\mu - 1)\dfrac{tD}{d}$

$\therefore \quad \dfrac{30\,D\,(4800\times10^{-10})}{d} = (0.6)\,t\,\dfrac{D}{d}$

$30 \times 4800 \times 10^{-10} = 0.6\,t$

$$t = \frac{30 \times 4800 \times 10^{-10}}{0.6} = \frac{1.44 \times 10^{-5}}{0.6} = 24 \times 10^{-6}$$

25.15 (C)

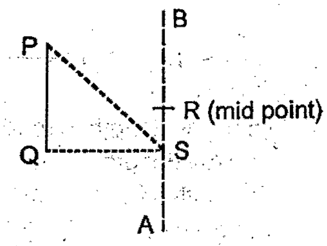
Lets take any general point S on the line AB.

Clearly: for any position of S on line AB; we have for Δ PQS:

PQ + QS > PS {in any triangle sum of 2 sides is more then the third side}

$\Rightarrow$ PS – QS < 3λ.

As PS – QS represents the path difference at any point on AB $\Rightarrow$ it can never be more than 3λ. Now minimas occur at.

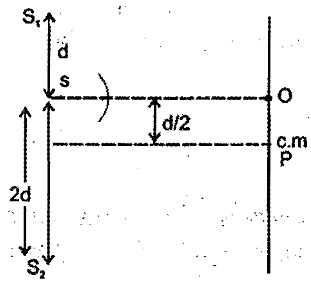


$\frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}$ only.

so 3 minimas below R (mid point of AB) and 3 also above R.

25.16 (B)

The 2 sources are.



As O is a maxima, Hence OP = β.

$\Rightarrow \quad \frac{d}{2} = \frac{\lambda.D}{(3d)}; \qquad \text{get } \lambda = \frac{3d^2}{2D}$

25.17 (B)

Ray N undergoes reflection at surface II with phase change of π

$\Rightarrow \quad n_3 > n_2$

Ray Q undergoes a phase-change of π at II, but there is no phase change when it is reflected from surface I.

25.18 (B)

Eq. of path diff. for maxima in tansmission (or weak reflection);

$\Delta P_{opt} = 2n_2L = \frac{\lambda_{vacuum}}{2}, \frac{3\lambda_{vacuum}}{2}......$

$\Rightarrow \quad 2\left(\frac{n_2}{n_1}\right)L = \frac{\lambda}{2}, \frac{3\lambda}{2}, ... \quad \Rightarrow \quad L = \frac{\lambda n_1}{4n_2}.$

(notice that λ = wavelength in medium is related to λ_{vacuum} as, $\lambda_{vacuum} = n_1\lambda$)

25.19 (B)

Constructive interference happens when $2t = (m - 1/2)\lambda$. The minimum value of m is m = 1; the maximum value is the integer portion of $\frac{2t}{\lambda} + \frac{1}{2} = \frac{2 \times 0.034 \times 10^{-3}}{680 \times 10^{-9}} + \frac{1}{2} = 100.5$

$m_{max} = 100$

25.20 (AC).

Path difference $= \sqrt{D^2 + d^2} - D = 1$ cm

Also ; $\left[\sqrt{D^2 + d^2} - D\right] = (2n - 1)\frac{\lambda}{2} \quad \Rightarrow \quad \lambda = \frac{2(1)}{2n - 1}$

For n = 1,2,3,............

$\lambda = 2\text{cm}, \frac{2}{3}\text{cm}, \frac{2}{5}\text{cm}, ...$

25.21 (ACD)

$$I_{max} = (\sqrt{I_1}+\sqrt{I_2})^2 = \left(\sqrt{I}+\sqrt{\frac{I}{2}}\right)^2 < 4I \Rightarrow I_{min} = \left(\sqrt{I}-\sqrt{\frac{I}{2}}\right)^2 > 0 \quad \Rightarrow \quad \beta = \frac{\lambda D}{d}$$

becuase λ D.d. are unchanged so β also remain unchanged.

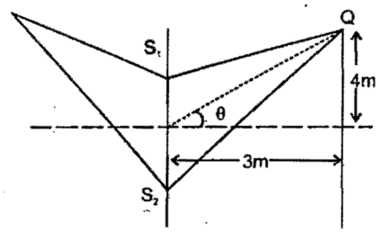
25.22 (AC)

As d << D, $\Rightarrow$ path difference = d sinθ (at 0) = 1mm × sin 30° = 0.5 mm

if it is a maxima $\Rightarrow$ $10^{-3} \times 0.5 = (5000 \times 10^{-10})$m × (n)

n must be integer. get n = 1000.

Hence O is a maxima of intensity $4I_0$



Now

Now path difference at Q = d sinθ only $QS_1 \approx QS_2$.

d sin θ = 1 × 1/2 = 0.5 mm

= integer multiple of λ . Hence maxima.

25.23 (D)

If maximum intensity is observed at P then for maximum intensity to be also observed at Q, S_1 and S_2 must have phase difference of 2mπ (where m is an integer).

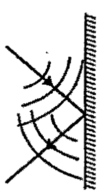
25.24 (D)

Statement 1 is false because constructive interference can be obtained if phase difference of sources is 2π, 4π , 6π, etc.

25.25 (D)

Wave fronts are spherical in shape of radius ct. Hence (D).

25.26 (C)



The wave fronts are always perpendicular to the light rays. Hence, (C).

25.27 (B)

Using snell's law ;

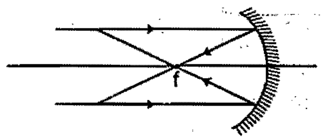
$$\frac{\sin(45^0)}{\sin r} = \frac{\sqrt{2}}{1}$$

$$\Rightarrow \quad \sin r = \frac{1}{2} \quad \Rightarrow \quad r = 30^0$$

Hence, (B) is correct. Note : The shown lines are wavefronts and not rays.

25.28 (A)

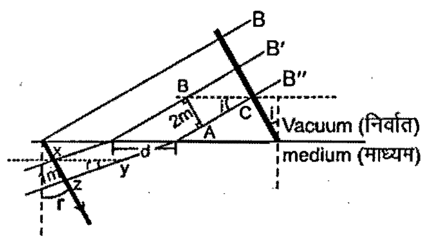
After reflection by mirror the parallel rays concentrate at the focus.



Hence the plane wave front becomes spherical concentrated at the focus. Hence, (A).

25.29 (A)

In ΔABC ; $\sin(i) = \frac{2}{d}$ In Δ xyz ; $\sin(r) = \frac{1}{d}$ $\Rightarrow \frac{\sin i}{\sin r} = 2 = \mu$.



25.30 (D)

Order of the fringe can be counted on either side of the central maximum. For example fringe no. 3 is first order bright fringe.

25.31 (C)

Since, 2nd fringe represent central bright fringe hence, 4th fringe results from a phase difference of 4π between the light waves incidenting from two slits .

25.32 (B)

$\Delta X_C = \lambda$; $\Delta X_A = \frac{\lambda}{2}$ $\Rightarrow$ $\Delta X_C - \Delta X_A = \frac{\lambda}{2} = 300$ nm .

25.33 (A) r, (B) r, (C) s, (D) p

By using $(\mu - 1)t = n\lambda$, we can find value of n, that is order of the fringe produced at P, if that particular strip has been placed over any of the slit. If two strips are used in conjuction (over each other), path difference due to each is added to get net path difference created. If two strips are used over different slits, their path differences are subtracted to get net path difference.

Now, $n_1 = \frac{(\mu_1 - 1)t_1}{\lambda} = 5$

$n_2 = 4.5$

and $n_3 = 0.5$

For (a), order of the fringe is 4.5 i.e. 5th dark.

for (b), net order is 5 – 0.5 = 4.5

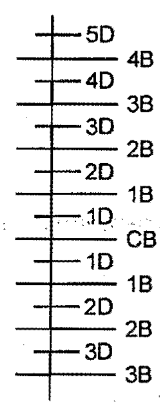
i.e. fifth dark.

for (c) net order is 5 – (0.5 + 4.5) = 0

i.e. it is central bright again at P.

for (d) net order is (5 + 0.5) – (4.5) = 1

i.e. first bright



25.34 (A) q,r,s (B) p,q,r,s, t (C) q,r,s (D) p,q,r,s

Initially at a distance x from central maxima on screen is

$$I = I_0 + 4I_0 + 2\sqrt{I_0}\sqrt{4I_0}\cos\frac{2\pi x}{\beta}, \text{ where } \beta = \frac{D\lambda}{d}$$

$I_{max} = 9I_0$ and $I_{min} = I_0$

(A) At points where intensity is $\frac{1}{9}$th of maximum intensity, minima is formed

$\therefore$ Distance between such points is β, 2β, 3β, 4β, ...

(B) At points where intensity is $\frac{3}{9}$th of maximum intensity, $\cos\frac{2\pi x}{\beta} = -\frac{1}{2}$ or $x = \frac{\beta}{3}$.

$\therefore$ Distance between such points is $\frac{\beta}{3}, \frac{2\beta}{3}, \beta, \beta + \frac{\beta}{3}, \beta + \frac{2\beta}{3}, 2\beta$, ...

(C) $\cos\frac{2\pi x}{\beta} = 0$ or $x = \frac{\beta}{4}$.

$\therefore$ Distance between such points is $\frac{\beta}{2}, \beta, \beta + \frac{\beta}{2}, 2\beta$, ...

(D) $\cos\frac{2\pi x}{\beta} = \frac{1}{2}$ or $x = \frac{\beta}{6}$.

$\therefore$ Distance between such points is $\frac{\beta}{3}, \frac{2\beta}{3}, \beta, \beta + \frac{\beta}{3}, \beta + \frac{2\beta}{3}, 2\beta$, ...

25.35 (A) p,r (B) p,s (C) q,s (D) s,t

(A) When d = 99.4 λ, 398 points of maximum intensity are formed on periphery of circle and 396points of minimum intensity are formed on periphery of circle

(B) When d = 99.6 λ, 398 points of maximum intensity are formed on periphery of circle and 400 points of minimum intensity are formed on periphery of circle

(C) When d = 100 λ, 400 points of maximum intensity are formed on periphery of circle and 400 points of minimum intensity are formed on periphery of circle

(D) When d = 100.4 λ, 402 points of maximum intensity are formed on periphery of circle and 400 points of minimum intensity are formed on periphery of circle

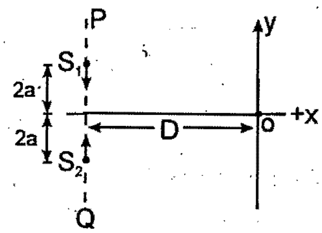
25.36 12

$$\beta = \frac{\lambda(a+b)}{2(\mu-1)\alpha} = \frac{\lambda}{\mu'}\frac{(a+b)}{\left(\frac{\mu}{\mu'}-1\right)\alpha}$$ where μ and μ' are refractive indices of prism and surrounding medium respectively.

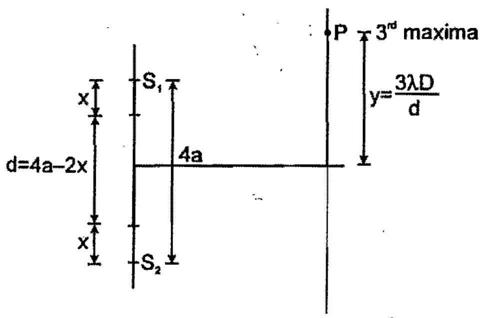
$$\beta \propto \frac{1}{\mu-\mu'} \quad \frac{\beta}{4} = \frac{1.5-1}{1.5-4/3} = \frac{0.5}{\frac{0.5}{3}} = 3$$

β = 12 mm. Ans.

25.37 3



y-coordinate of 3rd order maxima = $\frac{3\lambda D}{d}$



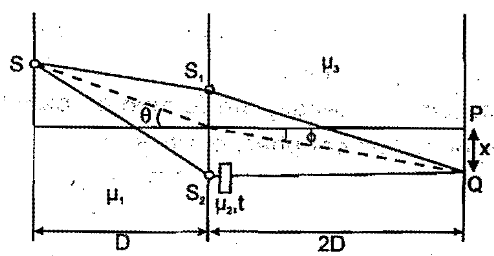
where d is the distance between both the sources at any time t

$d = 4a - 2x = 4a - 2a \sin \omega t = 2a(2 - \sin \omega t)$

$$\therefore \quad y = \frac{3\lambda D}{2a(2 - \sin\omega t)}$$

25.38 0

For the central order bright to be formed at Q



$(SS_1)\,\mu_1 + (S_1Q)\,\mu_3 = (SS_2)\,\mu_1 + (S_2Q - t)\,\mu_3 + \mu_2 t$

or $(S_1Q - S_2Q)\,\mu_3 = (SS_2 - SS_1)\,\mu_1 + (\mu_2 - \mu_3)\,t$

$d \sin \phi\, \mu_3 = (d \sin \theta)\,\mu_1 + (\mu_2 - \mu_3)\,t$ ....(1)

$$= \frac{d^2}{D}\,\mu_1 + (\mu_2 - \mu_3)\,t$$

$$= \frac{1^2}{10^3} \times \frac{4}{3} + \left(\frac{3}{2} - \frac{9}{5}\right)\frac{4}{9} \times 10^{-2} = 0$$

$\therefore \phi = 0$ or the central order bright is formed at P only.

25.39 40

In absence of slab from equation (1) ; t = 0

$d \sin\phi\, \mu_3 = (d \sin\theta)\, \mu_1$

$$\frac{x}{2D}\mu_3 = \frac{d}{D}\mu_1 \text{ or } x = \frac{2\mu_1 d}{\mu_3} = \frac{40}{27} \text{ mm.}$$

$\frac{40}{27}$ mm downwards

25.40 80

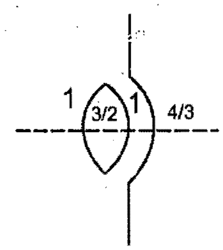
25.41 4

25.42 60

(25.40 to 25.42)

Lets find out the radius of curvature of equi. convex lens.

$$\frac{1}{f} = (\mu-1)\left(\frac{1}{R}-\frac{1}{-R}\right) \Rightarrow \frac{1}{10} = \left(\frac{3}{2}-1\right)\left(\frac{2}{R}\right) \Rightarrow R = 10 \text{ cm.}$$



Now

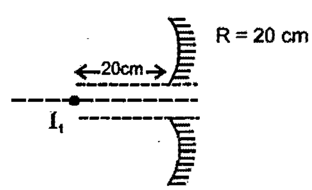
for lens :

$$\frac{1}{V} - \frac{1}{-20} = \frac{1}{10} \Rightarrow \frac{1}{V} = \frac{1}{20}$$

$\Rightarrow$ for surface of tube (of R = 10 cm.)

$$\frac{\mu_2}{V} - \frac{\mu_1}{u} = \frac{\mu_2-\mu_1}{R} \Rightarrow \frac{4/3}{V} - \frac{1}{+20} = \frac{4/3-1}{-10}$$

$\Rightarrow$ V = + 80 cm.



(b) Now for mirrors.

As the object for the mirrors is at 20 cm so the image will be at 20 cm only

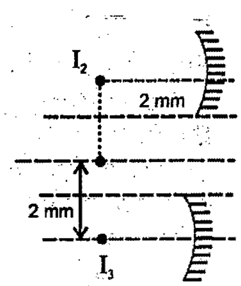
$\because$ $u = -2f$ $\Rightarrow$ $v = -2f$ also.

$$\Rightarrow \text{magnification} = m = \frac{y_I}{y_O} = \frac{-v}{u}$$

$$\Rightarrow \frac{y_I}{-(1\text{mm})} = -\left(\frac{-20}{-20}\right)$$

$\Rightarrow$ $y_I = +$ (1 mm)

so the final images are like.



so the distance between the images is 4 mm.

(c) Now, these I_2 and I_4 behave as the 2 sources for fringe pattern.

$$\Rightarrow \beta = \frac{\lambda D}{d} = \frac{vD}{fd} = \frac{(c/\mu)D}{fd}$$

$$= \left(\frac{3\times10^8}{\frac{4}{3}}\right) \times \frac{0.8}{\frac{3}{4}\times10^{15}\times(4\times10^{-3})} = 60\ \mu\text{m.}$$

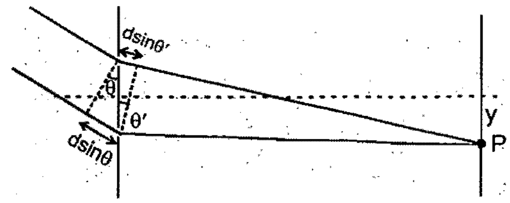
25.43 75

If phase difference at point P is zero then

$n_1 d \sin\theta = n_2 d \sin\theta' \Rightarrow \theta' = 37°$

and as $\tan\theta' = \frac{y}{D} \Rightarrow y = -\frac{3}{4}$ m

It is negative because upper path in medium n_2 is longer than lower path in the same medium.



25.44 150

As we go up from point O, path difference will increase. At O, phase difference is $3\pi + \frac{\pi}{3}$ and when it becomes 4π, there will be maximum. Extra path difference created in medium 2 must lead to $\frac{2\pi}{3}$ phase difference.

$$\frac{2\pi}{\lambda_a} \cdot d \sin\theta_1 \cdot n_2 = \frac{2\pi}{3}$$

Using values $\sin\theta_1 = \frac{3}{25} \Rightarrow \tan\theta_1 = \frac{3}{\sqrt{616}} = \frac{y}{D}$

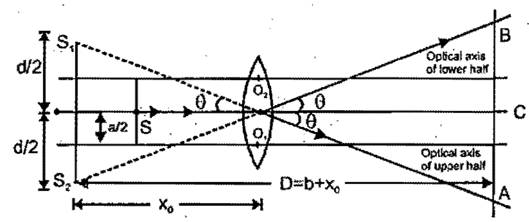
$$y = \frac{300}{2\sqrt{154}} \text{ cm} = \frac{150}{\sqrt{154}} \text{ cm}$$

25.45 13

From Fig. $\tan\theta = \theta = \frac{d}{2x_0}$

$\therefore$ $d = 2x_0\theta$

Since, the line joining S_1 and S_2 is parallel to screen.



$$\therefore \Delta x = \frac{\Delta}{d}\lambda = \left(\frac{b+x_0}{2x_0\theta}\right)\lambda = \left(\frac{b+x_0}{x_0}\right)\frac{\lambda}{2\theta} = \left(1+\frac{b}{x_0}\right)\frac{\lambda}{2\theta}$$

But source S is situated in focal plane. So, images S_1 and S_2 are situated at infinitely large distance i.e., $x_0 \to \infty$.

$$\therefore \Delta x = \left(1+\frac{b}{\infty}\right)\frac{\lambda}{2\theta} = \frac{\lambda}{2\theta} = \frac{\lambda}{2\left(\frac{a}{2f}\right)} = \frac{f\lambda}{a}$$

Here, $f = 25$ c, $\lambda = 0.6 \times 10^{-6}$ m, $a = 1$ mm

Putting the values, $\Delta x = 0.15$ mm

In Fig. the fringe pattern is observed between points A and B.

Since, the arrangement is similar to Young's experiment.

So, number of fringes $= 2[n_1] + 1$

where n_1 = number of fringes on either side of central point C of screen.

From Fig.

$$\tan\theta = \frac{BC}{OC} = \frac{BC}{b} = \frac{\ell}{b} \qquad \therefore \qquad \ell = b\tan\theta = b\theta = \frac{ba}{2f}$$

$$\therefore \qquad n_1 = \frac{\ell}{\Delta x} = \frac{ba}{2f \times \frac{f\lambda}{a}} \qquad \therefore \qquad n_1 = \frac{a^2 b}{2f^2\lambda}$$

$$n_1 = \frac{(10^{-3})^2(50\times10^{-2})}{2\times(25\times10^{-2})^2\times0.6\times10^{-6}} = 6.67 \qquad \therefore \qquad n = 2[n_1] + 1 = 2\times6+1 = 13$$

SECTION-II
PRACTICE TEST PAPERS

SOLUTIONS

Answers & Solutions

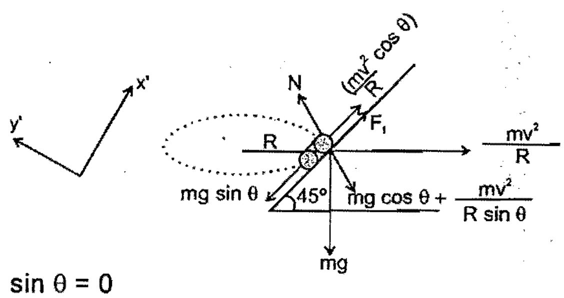
SECTION-II (PRACTICE TEST PAPERS)

PART TEST - 1 : MECHAINICS (CLASS XI)

1.	(D)	2.	(A)	3.	(B)	4.	(B)	5.	(D)	6.	(B)	7.	(C)
8.	(BD)	9.	(CD)	10.	(BC)	11.	(C)	12.	(BD)	13.	(BCD)	14.	(AB)
15.	(D)	16.	(C)	17.	(B)	18.	(C)	19.	(B)	20.	(D)	21.	(D)
22.	(B)	23.	(C)	24.	(1)	25.	(6)	26.	(2)	27.	(2)	28.	(9)
29.	2	30.	(5)										

SOLUTIONS

1. F.B.D. for minimum speed (w.r.t. automobile) :



$$\Sigma f_{y'} = N - mg\cos\theta - \frac{mv^2}{R}\sin\theta = 0.$$

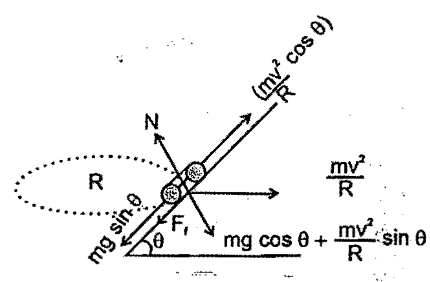
$$\Sigma f_{x'} = \frac{mv^2}{R}\cos\theta + \mu N - mg\sin\theta = 0$$

$$\Rightarrow \quad \frac{mv^2}{R}\cos\theta + \mu\left(mg\cos\theta + \frac{mv^2}{R}\sin\theta\right) - mg\sin\theta = 0$$

$$\Rightarrow \quad v^2 = \frac{(\mu Rg\cos\theta - Rg\sin\theta)}{(\cos\theta + \mu\sin\theta)}$$

for $\theta = 45^\circ$ and $\mu = 1$: $\quad v_{min} = \frac{Rg - Rg}{1+1} = 0$

F.B.D for maximum speed (w.r.t. automobile)

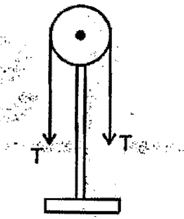


$$\Sigma f_{x'} = \frac{mv^2}{R}\cos\theta - mg\sin\theta - \mu\left(mg\cos\theta + \frac{mv^2}{R}\sin\theta\right) = 0$$

for $\theta = 45^\circ$ and $\mu = 1$ $\quad v_{max} = \infty$ (infinite)

2. $V_B = \sqrt{2\times10\times10}$; $\frac{mv_B^2}{R} \le mg$; $R \ge \frac{v_B^2}{g} \Rightarrow R \ge 20$ m

3. Reading of the weighing machine = 2T + weight of the machine.



As weight of the machine is constant $T = \frac{2m_1m_2}{m_1+m_2}$

So reading is maximum for the case m_1m_2 is maximum as $m_1 + m_2$ in all cases is same.

4. $W = T\Delta A$

$2 \times 10^{-4} = (60 - 30) \times 10^{-4}\ T \times 2$

$$T = \frac{2}{30\times2} = \frac{1}{30} = 3.3 \times 10^{-2}\ \text{N/m}$$

5. Due to surface tension, the surface area tries to minimize itself.

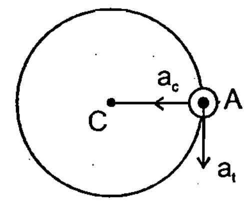
6. The COM will fall at distance $R = \frac{u^2 \sin 2\theta}{g}$ from the initial point, irrespective of explosion. Now one particle is at $\frac{R}{2}$, the other should be at $\frac{3R}{2}$.

7. Let the velocity of boat be 'V' opposite to the man. By momentum conservation, $-40V + 80(6 - V) = 0$

$\therefore$ V = 4 m/s

$\therefore$ velocity of the man = 6 – 4 = 2m/s

8. As shown in the figure, at A



$a_t = g$.

Here, a_t is maximum because at other place, $a_t = g\cos\theta$.

a_t and a_c will be equal at some point. a_c will have extreme value at lowest points and there $a_t = 0$.

9. $x = u(t-2) + a(t-2)^2$

Velocity of particle at time t

$\frac{dx}{dt} = u + 2a(t-2)$ $\Rightarrow$ Velocity at t = 0 $\frac{dx}{dt} = u - 4a$

Acceleration of particle

$\frac{d^2x}{dt^2} = 2a$ $\Rightarrow$ $x_{t=2} = 0$ So correct ans is (C) and (D).

10. The resultant force can be accelerating or decelerating, hence the momentum can increase or decrease. Hence (A) is wrong.

Since $F_{net} = M a_{cm}$

$a_{cm} \neq 0$;

hence v_{cm} must change

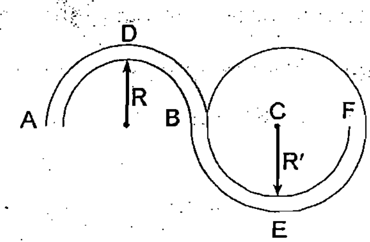
Hence (B)

In case of a circular motion of centre of mass about a point the distance of centre of mass will remain constant. Hence (C)

Kinetic energy of some particles may increase and of some particles may decrease at the same time.

12. $a_{rel} = g - g = 0 \Rightarrow v_{rel}$ = constant $\Rightarrow$ $a_{cm} = g$ = constant $\Rightarrow$ $\therefore v_{cm} \neq$ constant

13. For minimum velocity at A ;



$\frac{1}{2}mV_A^2 = mgR \Rightarrow V_A = \sqrt{2gR}$

Now ; $\frac{1}{2}mV_B^2 + mgR' = \frac{1}{2}mV_E^2$

As, $V_B = \sqrt{2gR}$

For looping the loop ; $V_E = \sqrt{5gR'}$ N = 6mg

14. $\sqrt{x} = 2t - 3 \Rightarrow x = (2t-3)^2$; Vel. $v = \frac{dx}{dt} = 2(2t-3) \times 2 = 4(2t-3)$

if v = 0 t = 3/2 ; acceleration $a = \frac{dv}{dt} = 4 \times 2 = 8$

So, correct ans is (A) and (B)

15. The time in which the planet rotates about its axis is not given for either planet.

16. For geostationary satellite, time period = 1 planet day (by def.)
Let T = 1 planet day
T_0 = 1 planet year

Now $T^2 = \frac{4\pi^2}{Gm} r_G^3 = \frac{4\pi^2}{Gm} r^3 \left(\frac{m}{M}\right) = \frac{4\pi^2}{GM} r^3 = T_0^3 \Rightarrow T = T_0$

18. (3) Eliminating 't' from both equations

$\Rightarrow \quad t = \frac{x}{\alpha} \qquad \Rightarrow \quad y = x\left(1 - \frac{\beta}{\alpha}x\right)$

(4) $\vec{r} = \alpha t\hat{i} + \alpha t(1-\beta t)\hat{j} \qquad \Rightarrow \qquad \vec{v} = \alpha\hat{i} + \alpha(1-2\beta t)\hat{j}$

$|\vec{v}| = \alpha\sqrt{1+(1-2\beta t)^2} \qquad \Rightarrow \qquad$ Speed at $t = \frac{1}{4\beta}$; $\quad |\vec{v}| = \alpha\sqrt{1+\frac{1}{4}} = \frac{\sqrt{5}}{2}\alpha$.

19. $400 = \mu_s N = \mu_s(mg) = \mu_s(100 \times 10) \quad \therefore \quad \mu_s = 0.4$

20. The frictional force is equal to the push force.

21. $P.E. = -\frac{GMm}{r} \Rightarrow K.E. = \frac{1}{2}mV^2$; Total energy $= -\frac{GMm}{r} + \frac{1}{2}mV^2$

T.E. = 0if $-\frac{GMm}{r} + \frac{1}{2}mV^2 = 0 \Rightarrow v = \sqrt{\frac{2GM}{r}}$

For $v < \sqrt{\frac{2GM}{r}}$, T.E. is – ve for $v > \sqrt{\frac{2GM}{r}}$, T.E. is + ve

If V is $\sqrt{\frac{GM}{r}}$ i.e. equal to orbital velocity, path is circular.
If T.E. is negative, path is elliptical.
If T.E. is zero, path is parabolic.
If T.E. is positive, path is hyperbolic.

22. Use equation of continuity and concept "pressure is greater at lower and broader section".

23. In all four situation of column-I, angular momentum of the disc about a point on ground is conserved. Take angular momentum out of the paper as positive

(A) Initial angular momentum about its point of contact on ground $= \frac{1}{2}mR^2\omega_0 - mR(2R\omega_0)$ = negative.
Hence final state of the disc is as shown if figure B.

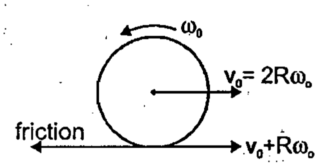


fig (a) initial state

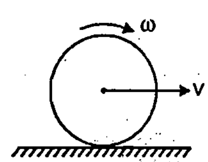


fig (b) final state

Hence angular velocity shall first decrease and then increase in opposite sense. The velocity of centre shall decrease till the disc starts rolling without slipping.

(B) The initial angular momentum about its point of contact on ground =0.

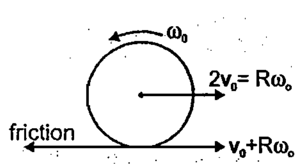


fig (a) initial state

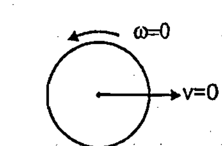


fig (b) final state

Hence angular speed and velocity of centre simultaneously reduce to zero without a change in direction.

(C) Because $v_0 > R\omega_0$, velocity of centre of mass will decrease and angular velocity will increase without a change in direction till disc starts rolling without slipping.

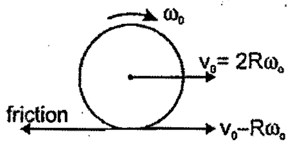


fig (a) initial state

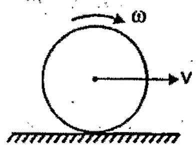


fig (b) final state

(D) Because $v_0 < R\omega_0$, velocity of centre of mass will increase and angular velocity will decrease without a change in direction till disc starts rolling without slipping.

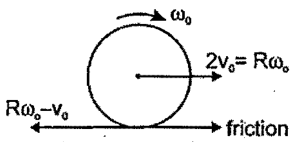


fig (a) initial state

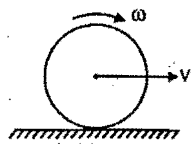


fig (b) final state

24. Work done in changing speed from 0 to V is -

$$\Delta W_1 = \frac{1}{2} mV^2$$

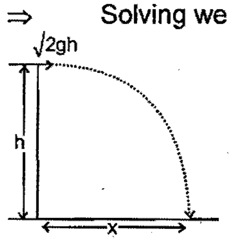
work done in changing the speed from V to 2V is

$$\Delta W_2 = \frac{1}{2} m (2V)^2 - \frac{1}{2} mV^2 = \frac{1}{2} 3\, mV^2 \qquad \therefore \qquad \frac{\Delta W_1}{\Delta W_2} = \frac{1}{3}$$

25. Let v be the speed of B at lowermost position, the speed of A at lowermost position is 2v.
From conservation of energy

$$\frac{1}{2} m (2v)^2 + \frac{1}{2} mv^2 = mg (2\ell) + mg\ell. \quad \Rightarrow \quad \text{Solving we get } v = \sqrt{\frac{6}{5} g\ell}$$

26. $x = T \times V$



$$x = \sqrt{\frac{2h}{g}} \times \sqrt{2gh} \Rightarrow x = 2h$$

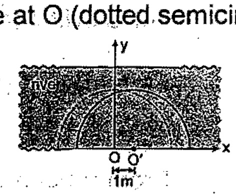
27. Velocity of I particle after time t ; $\vec{V_1} = (v \cos\theta\, \hat{i} + v \sin\theta\, \hat{j}) - (g\hat{j})\, t$

Velocity of II particle after time t ; $\vec{V_2} = (v \cos 2\theta\, \hat{i} + v \sin 2\theta\, \hat{j}) - (g\, \hat{j})\, t$

To be parallel of V_1 and V_2 ; $\Rightarrow \dfrac{v\cos\theta}{v\cos 2\theta} = \dfrac{v\sin\theta - gt}{v\sin 2\theta - gt}$

$\Rightarrow \quad t = \dfrac{v}{g}\cos\left(\dfrac{\theta}{2}\right)\operatorname{cosec}\left(\dfrac{3\theta}{2}\right)$; take $v = 20\sqrt{2}$ m/s and $\theta = 30°$ ($g = 10$ m/s²)

28. Method - 1
If the river is still, the man will be at a distance 3 metres from origin O after 1 second. The locus of all the point where man can reach at t = 1 second is a semicircle of radius 3 and centre at O (dotted semicircle shown in figure).
The river flows to right with a speed 1 m/s. Hence there shall be additional shift in position by 1 m/s × 1 sec = 1m towards right. Hence the locus of all points giving possible position after one second will be the dotted semicircle shifted to right by 1 m as shown in figure.



Hence locus all the points where the man can be at t = 1 sec. is a semicircle of radius 3 and centre at O′ (1 m, 0 m) ∴ Equation of locus of all the points is

$(x-1)^2 + (y-0)^2 = 3^2$

or $(x-1)^2 + y^2 = 9$

Method - 2

Let the relative velocity of the man make angle 'θ' with the x-axis.

Then at time 't' :

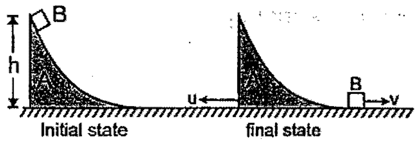
$x = (3\cos\theta + 1)\,t$

and $y = 3\sin\theta\, t$

$\Rightarrow$ $(x-t)^2 + y^2 = (3\cos\theta)^2 t^2 + (3\sin\theta)^2 t^2$

$(x-t)^2 + y^2 = 9t^2$

at $t = 1$ sec. the required equation is $(x-1)^2 + y^2 = 9$.

29. 

(figure - 1)

Let u and v be the speed of wedge A and block B at just after the block B gets off the wedge A. Applying conservation of momentum in horizontal direction, we get.

$mu = mv$ ....(1)

Applying conservation of energy between initial and final state as shown in figure (1), we get

$$mgh = \frac{1}{2}mu^2 + \frac{1}{2}mv^2 \quad ....(2)$$

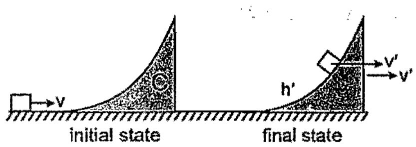
solving (1) and (2) we get

$$v = \sqrt{gh} \quad ....(3)$$

At the instant block B reaches maximum height h′ on the wedge C (figure 2), the speed of block B and wedge C are v′.

Applying conservation momentum in horizontal direction, we get

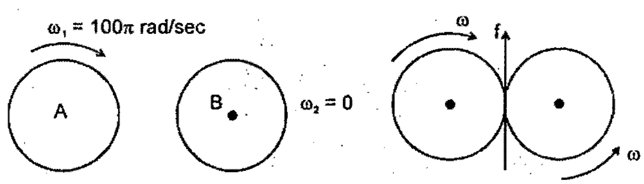
$mv = (m + m)\,v'$ ....(4)



Applying conservation of energy between initial and final state

$$\frac{1}{2}mv^2 = \frac{1}{2}(m+m)\,v'^2 + mgh' \quad ....(4)$$

Solving equations (3) (4) and (5) we get $h' = \dfrac{h}{4}$ Ans.

30. 

$(a_1 = a_2)$

$fR = I\alpha_1$ $\quad fR = I\alpha_2$

$\alpha_1 = \alpha_2 = 2\pi$ red/sec²

For A cylinder : $\omega = \omega_0 - \alpha t$ $\quad \omega = 100\pi - 2\pi t$ ....(i)

For B cylinder $\omega = \omega_0 - \alpha t$ $\quad \omega_0 = 0$

$\omega = \alpha t$ $\quad \omega = 2\pi t$ ....(ii)

From (i) and (ii) $\omega = 100\pi - \omega$

$2\omega = 100\pi$

$\omega = 50\pi$

From (ii) euqation $50\pi = 2\pi t$

$t = 25$ sec

PART TEST - 2 : HEAT & THERMODYNAMICS (CLASS XI)

1.	(B)	2	(C)	3.	(A)	4.	(D)	5.	(A)	6.	(C)	7.	(A)
8.	(BD)	9.	(ACD)	10.	(ABCD)	11.	(AD)	12.	(ABD)	13.	(ACD)	14.	(AC)
15.	(D)	16.	(A)	17	(A)	18.	(C)	19.	(A)	20.	(D)	21.	(A)
22.	(C)	23.	(B)	24.	(1)	25.	(8)	26.	(2)	27.	(9)	28.	(15)
29.	(7)	30	(3)										

SOLUTIONS

1. Higher is the temperature greater is the most probable velocity.

2 $PV = nRT$; $P = \frac{nRT}{V} = \frac{\left(\frac{12}{4}+\frac{4}{2}\right)\times 8.31\times 300}{20\times 10^{-3}} = 6.25\times 10^5$ Pa

3. Total KE = U = $\frac{f}{2}nRT$

In case of H_2 degree of freedom is greatest and number of moles n is highest
So this is the case of maximum kinetic energy.

4. $P_1 = P_2$ $T_1 = T_2$ $\Rightarrow \frac{V_1}{n_1} = \frac{V_2}{n_2}$ $\Rightarrow \frac{2\pi-\alpha}{n_1} = \frac{\alpha}{n_2}$

$\Rightarrow M_1(2\pi - \alpha) = M_2\,\alpha$ or $\alpha = \frac{2\pi M_1}{M_1+M_2} = \frac{16\pi}{15}$

5. When speed of 5 molecules which are selected randomly are recorded, then the average is most likely to be equal to the most probable speed.

$\therefore$ The average of these values is most likely equal to $\sqrt{\frac{2RT}{M}}$.

6. $\frac{T_F - 32}{180} = \frac{T_K - 273}{100}$ $= \frac{\theta-32}{180} = \frac{\theta-273}{100} \Rightarrow 5\theta - 32\times 5 = 9\theta - 273\times 9 =$ $\theta = 574.25$

7. Work done by atmosphere = $P_{atm}\Delta V$ = $P_{atm}\frac{V}{2}$ ....(i)

Initially gas in container is in thermodynamic equilibrium with its surroundings.
$\therefore$ Pressure inside cylinder = P_{atm}

& $PV = nRT$ $\Rightarrow$ $P_{atm}V = nRT$ or $V = \frac{nRT}{P_{atm}}$; Putting in (1), $W = \frac{nRT}{2}$

8. Equation of process $\Rightarrow \frac{P^2}{\rho}$ = constant = C .... (1)

Equation of State $\frac{P}{\rho} = \frac{R}{M}T$ .... (2)

From 1 and 2 PT = constant $\Rightarrow$ C is false, D is true.

As ρ-changes to $\frac{\rho}{2}$ $\Rightarrow$ P changes to $\frac{P}{\sqrt{2}}$ from equation (1) $\Rightarrow$ A is false.

Hence T changes to $\sqrt{2}T$. $\Rightarrow$ B is true.

9. $\frac{\Delta A}{A}\times 100 = 2\left(\frac{\Delta \ell}{A}\right)\times 100$ $\Rightarrow$ % increase in Area $= 2\times 0.2 = 0.4$

$\frac{\Delta V}{V} \times 100 = 3 \times 0.2 = 0.6\ \%$

Since $\Delta l = l\,\alpha\,\Delta T$

$\frac{\Delta \ell}{\ell} \times 100 = \alpha\,\Delta T \times 100 = 0.2 \Rightarrow \alpha = 0.25 \times 10^{-4}\ /°C$

10. Area under the curve is equal to number of molecules of the gas sample. Hence

$N = \frac{1}{2} . a . V_0 \Rightarrow aV_0 = 2N$

$$V_{avg} = \frac{1}{N}\int_0^{\infty} v\,N(V)dV = \frac{1}{N}\int_0^{V_0} C.\left(\frac{a}{V_0}.V\right)dV = \frac{2}{3}V_0 \Rightarrow \frac{V_{avg}}{V_0} = \frac{2}{3}$$

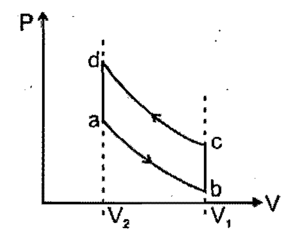
$$v_{rms}^2 = \frac{1}{N}\int_0^{\infty} V^2 N(V)dV = \frac{1}{N}\int_0^{V_0} V^2\left(\frac{a}{V_0}.V\right)dV = \frac{V_0^2}{2} \Rightarrow \frac{V_{rms}}{V_0} = \frac{1}{\sqrt{2}}$$

Area under the curve from 0.5 V_0 to V_0 is $\frac{3}{4}$ of total area.

11. $V_{r.m.s.} = \sqrt{\frac{3kT}{m}}$

Since PV = nRT therefore P and V both can change simultaneously keeping temperature constant.

12. $PV = \frac{m}{M_0}RT \quad P = \frac{\rho}{M_0}RT \quad \Rightarrow \frac{P}{\rho} = \frac{R}{M_0}T$



Equivalent PV diagram.

Slope of the curve α Temperature
Hence cd and ab are isothermal processes.

$\rho \propto \frac{1}{V}$

i.e. bc and da are constant volume process
(A) and (B) are true.
Temp. in cd process is greater than ab.

Net work done by the gas in the cycle is negative, as is clear by the PV-diagram.

13. Since , e = a = 0.2 (Since, a = (1 – r – t) = 0.2 for the body B)
E = (100) (0.2) = 20 W/m²
Power emitted = E.A = 20 × 10 = 200 Watt

14. Heat obviously flows from higher temperature to lower temperature in steady state. ⇒ A is true.

Temperature gradient $\propto \frac{1}{\text{cross section area}}$ in steady state. ⇒ B is false.

Thermal current through each cross section area is same. ⇒ C is true.
Temperature decreases along the length of the rod from higher temperature end to lower temperature end. ⇒ D is false.

15. to 16

(A)
$TP^{(1-\gamma/\gamma)}$ = K. (given)

Lapse Rate is $\frac{dT}{dy}$, therefore differentiating w.r.t. y

$$\frac{dT}{dy}.P^{\frac{1-\gamma}{\gamma}} + T.\frac{1-\gamma}{\gamma}.P^{\left(\frac{1-\gamma}{\gamma}\right)-1}.\frac{dP}{dy} = 0 \quad \Rightarrow \quad \frac{dT}{dy} = -\frac{T}{P}\left(\frac{1-\gamma}{\gamma}\right)\frac{dP}{dy}$$

Using $PV = \frac{M'}{M}RT \Rightarrow P = \frac{\rho}{M}RT \quad \Rightarrow \frac{T}{P} = \frac{M}{\rho R}$ and $\frac{dP}{dy} = -\rho g$

$$\frac{dT}{dy} = \frac{M}{\rho R}.\left(\frac{\gamma-1}{\gamma}\right)(-\rho g) = -\left(1-\frac{1}{\gamma}\right)\frac{Mg}{R}$$

17. In 40 min. temperture of water has come down by 40°C.

Therefore rate $P = \frac{mS\Delta T}{t} = \frac{0.60 \times 4200 \times 40}{40 \times 60} = 42.0$ W

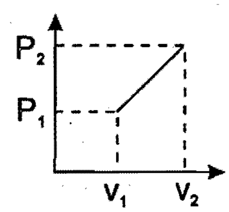
18. Sample of ice has been receiving heat at constant rate P from water. Its temperature has increased by 30°C in time 60 min.

Therefore $\frac{m_i s_i \Delta T_i}{P} = 60$ min.

$\Rightarrow \quad m_i = \frac{(60 \times 60s) \times (42\,W)}{(2100 J/kg).(30°C)} = 2.4$ kg

Alternate :
$m_w S_w \Delta T_w + m'_w L = m_i S_i\, 30°$ (m'_w = mass of water which is converted to ice)

19. W = Area under the curve = $\frac{3}{2} P_1V_1$



$V_2 = 2V_1$
$P_2 = 2P_1$

and $P_1V_1 = nRT_1$

Therefore $\frac{W}{nRT_1} = \frac{\frac{3}{2}.P_1V_1}{P_1V_1}$

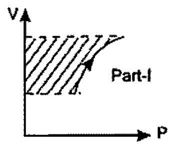
20. $Q = dU + W$
$dU = nC_v dT$
For final state $P_2V_2 = 2P_1\ 2V_1 = 4P_1V_1 = nR(4T_1)$

Hence final temp. is $4T_1$; $dU = n.\frac{3}{2}R.3T_1 = \frac{9}{2}nRT_1$

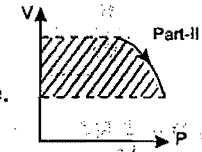
$Q = \frac{3}{2}.nRT_1 + \frac{9}{2}nRT_1 = 6nRT_1$; $\frac{Q}{nRT_1} = 6$

21. $PV = nRT$
$P = (nRT)\frac{1}{V} = (\text{constant})\frac{1}{V}$
T = constant i.e. isothermal process ∵ V increases, ΔW is positive
and $\Delta Q = \Delta U + \Delta U = \Delta W > 0$
(B) $\Delta Q = 0$ $\quad pdV = \Delta W$ = positive
(C) $PV = nRT$
As volume increases, T also increases
i.e., $\Delta U > 0$ $\quad PdV = \Delta W > 0$ So $\Delta Q > 0$
(D) For cyclic process $\Delta U = 0$
$\Delta W < 0$ (anticlockwise)
$\Delta Q < 0$.

22. In (A), V is on vertical axis.



As V is icreasing, W is positive.



As negative work in part-II is greater than positive work in part-I, net work during the process is negative.
Using PV = nRT and as $V_{remains}$ same for initial and final points of the process, it is obvious that final temp. is greater than initial temperature as pressure has increased. Therefore dU is positive. Hence option (S) is connected with (A).
Similar arguments can be applied to other graphs.

23. (A) Initially more heat will enter through section A but the metal will absorb some heat and less heat will leave from C.

(B) At steady state heat accumulatron $\Rightarrow \frac{dQ}{dt}$ is same for all sections

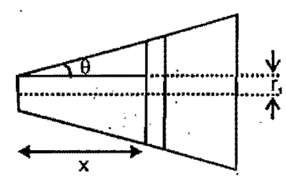
(C) At steady state $\frac{dQ}{dt} = kA\left|\frac{dT}{dx}\right|$ or $\left|\frac{dT}{dx}\right| = \frac{1}{kA}\left(\frac{dQ}{dt}\right)$

$\left|\frac{dT}{dx}\right|$ is inversely proportional to area of cross-section. Hence is maximum at B and minimum at C

(D) At steady state heat accumulation = 0

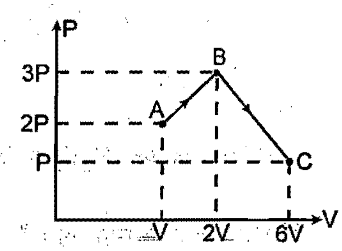
So $\frac{dT}{dt} = 0$ for any section.

24. $\int dR = \int_0^{\ell} \frac{1}{k} \frac{dx}{\pi(r_1 + x\tan\theta)^2}$



Solving $R = \frac{\ell}{k\,\pi\, r_1 r_2}$ $\quad \therefore G = \frac{1}{R} = \frac{k\,\pi\, r_1 r_2}{\ell}$ Ans 1

25. The maximum temperature of the gas will be during process BC.



Process BC can be represented by straight line, y = m x + C
So P = mV + C Putting point B & C gives
3 P = 2 mV + C — (1) P = 6 mV + C — (2)

So subtracting 2 P = – 4 mV So $m = -\frac{P}{2V}$

From (2) $P = -\frac{P}{2V}\cdot 6V + C \Rightarrow C = 4P$

Hence we get equation as, $y = \left(-\frac{P}{2V}\right)x + 4P$ — (4)

where y is pressure and x is volume of gas. Putting y from above,

now we have $xy = nRT - \frac{P}{2V}x^2 + 4Px = nRT$ — (3)

For maximum temperature $\frac{dT}{dx} = 0 \Rightarrow -\frac{2P}{2V}x + 4P = 0$

Hence x = 4V

Putting in (3) we get $nRT_{max} = 2P(4V) = 8PV$ So, $T_{max} = \frac{8PV}{nR}$

26. Before heating let the pressure of gas be P_1 from the equilibrium piston,
$PA = kx_1$

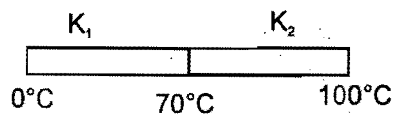
$\therefore \quad x_1 = \frac{PA}{K} = \left(\frac{nRT}{V}\right)\frac{A}{K} = \frac{1\times 8.3\times 100\times 10^{-2}}{0.83\times 200} = 0.05$ m

Since during heating process,
The spring is compressed further by 0.1 m
$\therefore \quad x_2 = 0.15$ m

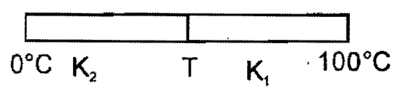
work done by gas $= \frac{1}{2}.200((0.15)^2-(0.05)^2) = \frac{1}{2}.200\,(.02) = 2$J

27. External Work = work done on both gases = – (work done by the gases)

$$= -\left[P_0V_0\ell n\frac{4/3V_0}{V_0} + P_0V_0\ell n\frac{2/3V_0}{V_0}\right] = -P_0V_0\ell n\frac{8}{9} = P_0V_0\ell n\frac{9}{8}$$

28. 

$\frac{70-0}{R_1} = \frac{100-70}{R_2} \Rightarrow \frac{R_2}{R_1} = 3/7 \Rightarrow \frac{K_1}{K_2} = 3/7$



$\frac{T-0}{R_2} = \frac{100-T}{R_1} \Rightarrow \frac{100-T}{T} = \frac{R_1}{R_2} = \frac{K_2}{K_1} = 7/3 \Rightarrow 300-3T = 7T \Rightarrow T = 30°C$

29. $n = 7v - v^2 - 10$

for most probable velocity $\frac{dn}{dv} = 0 \quad \because$ n is maximum at this velocity

$7-2v = 0 \Rightarrow v = \frac{7}{2} = 3.5$ m/sec

30 $E = \frac{3}{2}kT$

PART TEST - 3 : SHM & WAVES (CLASS XI)

1.	(B)	2.	(C)	3.	(D)	4.	(B)	5.	(B)	6.	(A)	7.	(B)
8.	(BC)	9.	(AC)	10.	(ACD)	11.	(AC)	12.	(BCD)	13.	(ABCD)	14.	(AD)
15.	(B)	16.	(C)	17.	(C)	18.	(D)	19.	(A)	20.	(C)	21.	(A)
22	(D)	23.	(C)	27.	(2)	25.	(4)	26.	(12)	27.	(2)	28.	(1)
29.	(5)	30.	(5)										

SOLUTIONS

1. Maximum velocity $v = \omega a$

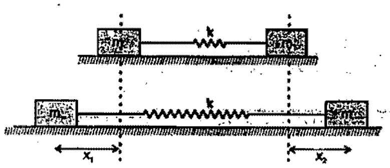
Maximum acceleration $f = \omega^2 a \Rightarrow f = \frac{v^2}{a}$

2. Both the spring are in series $\therefore \quad K_{eq} = \frac{K(2K)}{K+2K} = \frac{2K}{3}$

Time period $T = 2\pi\sqrt{\frac{\mu}{K_{eq}}}$ where $\mu = \frac{m_1 m_2}{m_1 + m_2}$

Here $\mu = \frac{m}{2}$ $\therefore T = 2\pi\sqrt{\frac{m}{2}\cdot\frac{3}{2K}} = 2\pi\sqrt{\frac{3m}{4K}}$

Method II



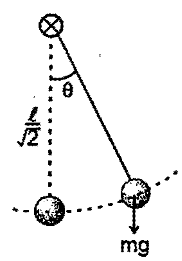
$\therefore$ $mx_1 = mx_2 \Rightarrow x_1 = x_2$

force equation for first block;

$\frac{2k}{3}(x_1 + x_2) = -m\frac{d^2x_1}{dt^2}$ Put $x_1 = x_2$ $\Rightarrow$ $\frac{d^2x_1}{dt^2} + \frac{4k}{3m} \times x_1 = 0$

$\Rightarrow \omega^2 = \frac{4k}{3m}$ $\therefore$ $T = 2\pi\sqrt{\frac{3m}{4K}}$

3. Resulting torque on the bob = $mg\frac{\ell}{\sqrt{2}}\sin\theta$



MI of bob about axis xy= $\frac{m\ell^2}{2}$

For small angle θ

$\alpha = \frac{\tau}{I} = \frac{\sqrt{2}g}{\ell}\theta$ $\omega = \sqrt{\frac{\sqrt{2}g}{\ell}}$ $\Rightarrow T = 2\pi\sqrt{\frac{\ell}{\sqrt{2}g}}$

4. $f_0 = \frac{1}{2\pi}\sqrt{\frac{mg\ell}{I}}$

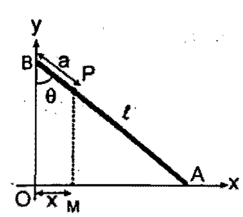
where, ℓ is distance between point of suspension and centre of mass of the body. Thus, for the stick of length L and mass m :

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{m.g.\frac{L}{2}}{(mL^2/12)}} = \frac{1}{2\pi}\sqrt{\frac{6g}{L}}$$

when bottom half of the stick is cut off $f_0' = \frac{1}{2\pi}\sqrt{\frac{\frac{m}{2}.g.\frac{L}{4}}{\frac{m}{2}\frac{(L/2)^2}{12}}} = \frac{1}{2\pi}\times\sqrt{\frac{12g}{L}} = \sqrt{2}\ f_0$

5. $\frac{d\theta}{dt} = 2$ $\therefore$ $\theta = 2t$

Let BP = a $\therefore$ x = OM = a sinθ = a sin(2t)



Hence M executes SHM within the given time period and its acceleration is opposite to 'x' that means towards left

6. Average kinetic energy with respect to position = $\frac{1}{3}kA^2$

Average potential energy with respect to position = $\frac{1}{6}kA^2$

Average kinetic energy with respect to time = $\frac{1}{4}kA^2$

Average potential energy with respect to time = $\frac{1}{4}kA^2$

7. After the system is released, m_2 moves down.
The extention in the spring becomes :

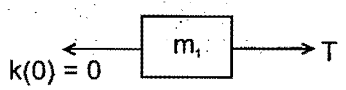
$\frac{m_2 g}{k}$ ($m_2 g = kx_0$), which is the new equilibrium position of the system.

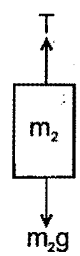
For small 'x' : restoring force on the system is

$F = kx \Rightarrow \quad a = \frac{kx}{m_1 + m_2}$ (For (m_1 + m_2 + spring) system)

$\Rightarrow \quad T = 2\pi\sqrt{\frac{x}{a}} = 2\pi\sqrt{\frac{x(m_1+m_2)}{kx}} = 2\pi\sqrt{\frac{m_1+m_2}{k}} \Rightarrow$ Angular frequency = $\omega = \frac{2\pi}{T} = \sqrt{\frac{k}{m_1+m_2}}$.

F.B.D. of m_1 and m_2 just after the system is released :





From above : $T = \frac{m_1 m_2}{m_1 + m_2} g$

Hence (C) is incorrect.

After $x = \frac{m_2 g}{k}$; m_1 moves towards right till the total kinetic energy acquired does not converted to potential energy.
Hence (D) is also incorrect.
Hence (B) is the answer.

8. At time t_1, velocity of the particle is negative i.e. going towards $-X_m$. From the graph, at time t_1, its speed is decreasing. Therefore particle lies in between $-X_m$ and 0.
At time t_2, velocity is positive and its magnitude is less than maximum i.e. it has yet not crossed O.
It lies in between $-X_m$ and 0.
Phase of particle at time t_1 is $(180 + \theta_1)$.
Phase of particle at time t_2 is $(270 + \theta_2)$
Phase difference is $90 + (\theta_2 - \theta_1)$
$\theta_2 - \theta_1$ can be negative making $\Delta\phi < 90°$ but can not be more than 90°.

9. $y = 2A \sin kx. \sin\omega t$; $V_y = \frac{dy}{dt} = 2A \sin kx. \cos\omega t$

$V_y = 0 \Rightarrow t = T/4, 3T/4 \left(T = \frac{2\pi}{\omega}\right)$; (2 times in one time period)

10. $\frac{\lambda}{4} = 0.1 \quad \Rightarrow \quad \lambda = 0.4$ m

from graph $\Rightarrow$ T = 0.2 sec. and amplitude of standing wave is 2A = 4 cm.
Equation of the standing wave

$$y(x, t) = -2A\cos\left(\frac{2\pi}{0.4}x\right).\sin\left(\frac{2\pi}{0.2}t\right) \text{ cm}$$

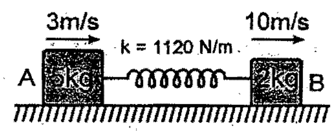
$y(x = 0.05,\ t = 0.05) = -2\sqrt{2}$ cm $\quad \Rightarrow y(x = 0.04, t = 0.025) = -2\sqrt{2}\cos 36°$

speed $= \frac{\lambda}{T} = 2$ m/sec. $\quad \Rightarrow V_t = \frac{\partial y}{\partial t} = -2A\ 10\pi\cos\left(\frac{2\pi}{0.4}\frac{1}{15}\right).\cos\left(\frac{2\pi}{0.2}0.1\right) = 20\pi$ cm/s

11. The speed of the particle (x, t) is $\frac{\partial y}{\partial t} = -\frac{12(x-3t)}{[(x-3t)^2+1]^2}$. Therefore at x = 3 and t = 1, $\frac{\partial y}{\partial t} = 0$.

The speed of the pulse $= \frac{\text{coefficient of t}}{\text{coefficient of x}} = 3.0$ cm/s

12. At max. extension both should move with equal velocity.



$\therefore$ By momentum conservation,
(5 × 3) + (2 × 10) = (5 + 2)V
V = 5 m/sec.
Now, by energy conservation

$$\frac{1}{2}5 \times 3^2 + \frac{1}{2} \times 2 \times 10^2 = \frac{1}{2}(5+2)V^2 + \frac{1}{2}kx^2$$

Put V and k $\quad \therefore \quad x_{max} = \frac{1}{4}m = 25$ cm.

Also first maximum compression occurs at ;

$$t = \frac{3T}{4} = \frac{3}{4}2\pi\sqrt{\frac{\mu}{k}} = \frac{3}{4}2\pi\sqrt{\frac{10}{7\times1120}} = \frac{3\pi}{56} \text{ sec.}$$ (where $\mu \Rightarrow$ reduced mass, $\mu = \frac{m_1 m_2}{m_1+m_2}$).

15. When speed of block is maximum, net force on block is zero. Hence at that instant spring exerts a force of magnitude 'mg' on block.

16. At the instant block is in equilibrium position, its speed is maximum and compression in spring is x given by kx = mg .... (1)
From conservation of energy

$$mg(L + x) = \frac{1}{2}kx^2 + \frac{1}{2}mv^2_{max} \quad .... (2)$$

from (1) and (2) we get $v_{max} = \frac{3}{2}\sqrt{gL}$.

17. Difference in path travelled by the sound $= \frac{hd}{D}$. Hence, time gap between the recordings $= \frac{hd}{Dv}$

18. For resultant pressure amplitude to be maximum, path difference. $\Delta D = m\lambda$

$$\frac{hd}{D} = m\frac{v}{f} \qquad \left[f = \frac{mvD}{hd}\right]$$

For minimum frequency, m = 1 $\left[f = \frac{vD}{hd}\right]$

19. The equation of wave moving in negative x-direction, assuming origin of position at x = 2 and origin of time (i.e. initial time) at t = 1 sec.

$y = 0.1 \sin(4\pi t + 8x)$

Shifting the origin of position to left by 2m, that is, to x = 0. Also shifting the origin of time backwards by 1 sec, that is to t = 0 sec.

$y = 0.1 \sin[(4\pi(t-1) + 8(x-2)]$

20. As given the particle at x = 2 is at mean position at t = 1 sec.

$\therefore$ its velocity $v = \omega A = 4\pi \times 0.1 = 0.4\,\pi$ m/s.

21. (A) In frame of lift effective acceleration due to gravity is $g + \frac{g}{2} = \frac{3g}{2}$ downwards

$\therefore T = 2\pi\sqrt{\frac{2\ell}{3g}}$

(B) $K\ell = mg \quad \therefore \frac{k}{m} = \frac{g}{L}$

constant acceleration of lift has no effect in time period of oscillation.

$\therefore T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{\ell}{g}}$

(C) $T = 2\pi\sqrt{\frac{I}{mgd}} = 2\pi\sqrt{\frac{\frac{m\ell^2}{3}}{mg\frac{\ell}{2}}} = 2\pi\sqrt{\frac{2\ell}{3g}}$ (D) $T = 2\pi\sqrt{\frac{m}{\rho Ag}} = 2\pi\sqrt{\frac{(\rho/2)A\ell}{\rho Ag}} = 2\pi\sqrt{\frac{\ell}{2g}}$

22 (A) Number of loops (of length $\lambda/2$) will be even or odd and node or antinode will respectively be formed at the middle.

Phase of difference between two particle in same loop will be zero and that between two particles in adjacent loops will be π.

(B) and (D) Number of loops will not be integral. Hence neither a node nor an antinode will be formed in in the middle.

Phase of difference between two particle in same loop will be zero and that between two particles in adjacent loops will be π.

(C) Number of loops (of length $\lambda/2$) will be even or odd and antinode or node will respectively be formed at the middle.

Phase of difference between two particle in same loop will be zero and that between two particles in adjacent loops will be π.

23. The velocity-displacement relation is $x^2 + \frac{v^2}{\omega^2} = A^2$

which may be a circle if $\omega = 1$. and ellipse of $\omega \neq 1$.

The acceleration – velocity relation is $v^2 + \frac{a^2}{\omega^2} = A^2$ which maybe a circle if $\omega = 1$ and ellipse of $\omega \neq 1$.

Acceleration-displacement graph is straight and acceleration time graph is sinusoidal.

27. The coordinates of the particles are

$x_1 = A_1 \cos \omega t,\ x_2 = A_2 \cos \omega t$

separation $= x_1 - x_2 = (A_1 - A_2)\cos \omega t = 12 \cos \omega t$

Now $x_1 - x_2 = 6 = 12\cos\omega t \Rightarrow \omega t = \frac{\pi}{3} \Rightarrow \frac{2\pi}{12}.t = \frac{\pi}{3} \Rightarrow t = 2s$

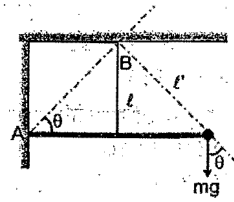
25. The bob will execute SHM about a stationary axis passing through AB. If its effective length is ℓ' then

$T = 2\pi\sqrt{\frac{\ell'}{g'}}$

$\ell' = \ell / \sin\theta = \sqrt{2}\,\ell$ (because θ = 45)

$g' = g\cos\theta = g/\sqrt{2}$

$T = 2\pi\sqrt{\frac{2\ell}{g}} = 2\pi\sqrt{\frac{2\times0.2}{10}} = \frac{2\pi}{5}$ s. X = 4



26. Imagine a cylinder of radius 7m and length 10m. Intensity of sound at the surface of cylinder is same everywhere. Therefore $I = \frac{P}{2\pi rL}$
(As sound is propagating radially out only, sound energy does not flow out through the ends) ∴ I = 50 W/m² ; Energy intercepted by the detector= I × A = 12 mW

27. ∵ Acceleration of the wave pulse remains constant in this case. ∴ distance = <V>.Δt

$\therefore\ CB = \ell = \frac{V_C + V_B}{2} t_{CB}$ and $BA = \ell = \frac{V_{B'} + V_A}{2} t_{BA}$.

$V_C = 0$, $V_B = \sqrt{\frac{2mg}{2m/\ell}} = \sqrt{g\ell}$, $V_{B'} = \sqrt{\frac{2mg}{m/\ell}} = \sqrt{2g\ell}$, $V_A = \sqrt{\frac{3mg}{m/\ell}} = \sqrt{3g\ell}$,

$\ell = \frac{\sqrt{g\ell}}{2} t_{CB}$, $\ell = \frac{\sqrt{2g\ell}+\sqrt{3g\ell}}{2} t_{BA}$; $t = t_{CB} + t_{BA} = \frac{2\ell}{\sqrt{g\ell}} + \frac{2\ell}{\sqrt{2g\ell}+\sqrt{3g\ell}} = 2\sqrt{\frac{\ell}{g}}\left[1+\frac{1}{\sqrt{2}+\sqrt{3}}\right]$

$= 2\sqrt{\frac{9610}{1681\times10}}\left[1+\frac{1}{1.4+1.7}\right] = 2\sqrt{\frac{961}{1681}}\left[\frac{4.1}{3.1}\right] = 2\sqrt{\frac{961\times41^2}{1681\times31^2}} = 2$ Ans.

28. In first case, $T = 2\pi\sqrt{\frac{m}{K}}$ and $A = \frac{mg}{K}$ $\therefore A.T. = 2\pi\sqrt{\frac{m}{K}}.\frac{mg}{K} = g$ ....(i)

In second case,

$T_1 = 2\pi\sqrt{\frac{m}{K_1+K_2}}$ ⇒ $A_1 = \frac{mg}{K_1+K_2}$

$\therefore A_1T_1 = 2\pi\sqrt{\frac{m}{K_1+K_2}}\cdot\frac{mg}{K_1+K_2} = 2\pi\sqrt{\frac{m}{(1+n)^2}\cdot\frac{n}{K}}\cdot\frac{mg.n}{(1+n)^2K} = 2\pi\sqrt{\frac{m}{K}}\cdot\frac{mg}{K}\cdot\frac{n^{3/2}}{(1+n)^3}$

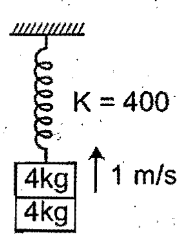
If n = 1 ⇒ $A_1T_1 = 8.\frac{1}{2^3} = 1$.

29. $\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{400}{8}} = \sqrt{50}$

$\omega^2 = 50$

$V = \omega\sqrt{A^2 - x^2}$

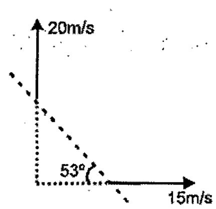
$V^2 = \omega^2(A^2 - x^2)$



$1 = 50\left(A^2 - \frac{1}{100}\right)$; $A^2 = \frac{1}{50}+\frac{1}{100}$; $A^2 = \frac{3}{100}$; $A = \frac{1}{5}\sqrt{\frac{3}{4}}$; x = 5

30. $f' = \left(\frac{334-9}{334+16}\right)700$

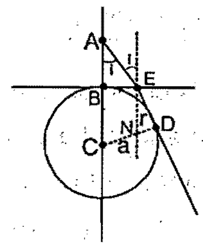
$= \frac{325}{350} \times 700 = 650$ Hz.



PART TEST - 4 : OPTICS (CLASS XII)

1.	(D)	2.	(B)	3.	(C)	4.	(D)	5.	(A)	6.	(D)	7.	(B)
8.	(AC)	9.	(BC)	10.	(ABD)	11.	(ABCD)	12.	(AD)	13.	(AC)	14.	(BD)
15.	(D)	16.	(D)	17.	(C)	18.	(D)	19.	(A)	20.	(B)	21.	(B)
22.	(B)	23.	(D)	24.	8	25.	4	26.	8	27.	5	28.	3
29.	2	30.	5										

SOLUTIONS



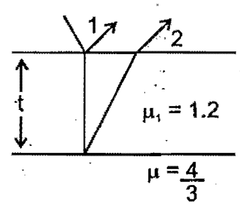
1. Given $\angle NED = 30^\circ$ $\therefore$ $\angle BED = 120^\circ$

BCDE is cyclic quadrilateral $\therefore$ $\angle BCD = 60^\circ$

The line CE will be angle bisector of $\angle BCD$

$\therefore$ $BE = a \tan 30^\circ = \frac{a}{\sqrt{3}}$ $\therefore$ now $\tan i = \frac{BE}{AB} = \frac{a\sqrt{3}}{a/2} = \frac{2}{\sqrt{3}}$

$\therefore$ $\sin i = \frac{2}{\sqrt{7}}$ now by snell's law $1 \times \sin i = n \sin r \Rightarrow \frac{2}{\sqrt{7}} = n \times \frac{1}{2}$ $\Rightarrow n = \frac{4}{\sqrt{7}}$

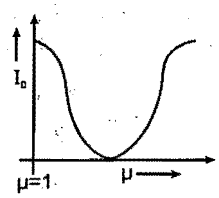


2. For normal incidence path difference between ray 1 and ray 2 is $2\mu_1 t$

For minimum thickness increment $2\mu_1 \Delta t = \frac{\lambda}{2}$

$\Rightarrow$ $(t_2 - t_1) = \frac{\lambda}{4\mu_1} = \frac{9.6\times10^{-7}}{4\times1.2} = 2 \times 10^{-7}$ m

3. In absence of film or for μ=0 intensity is maximum at screen. As the value of μ is increased, intensity shall decrease and then increase alternately. Hence the correct variation is



4. Acceleration of block AB = $\frac{3mg}{3m+m} = \frac{3}{4}g$; acceleration of block CD

$= \frac{2mg}{2m+m} = \frac{2g}{3}$. Acceleration of image in mirror AB ($\because$ ViG = –VoG + 2VmG)

= 2 acceleration of mirror = $2.\left(\frac{-3g}{4}\right) = \frac{-3}{2}g$; Acceleration of image in mirror CD = $2.\left(\frac{2g}{3}\right) = \frac{4g}{3}$

$\therefore$ Acceleration of the two image w.r.t. each other = $\frac{4g}{3} - \left(\frac{-3g}{2}\right) = \frac{17g}{6}$.

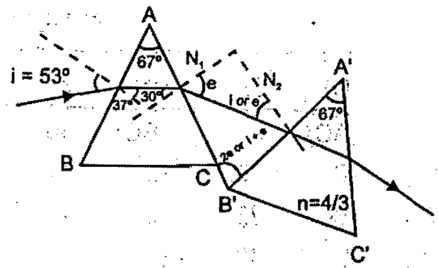
5. Let be the angle of emergence from the first prism be 'e'

Snell's law on surface AB

$1 \times \sin 53^\circ = \frac{4}{3} \sin r$; $r = 37^\circ$ $\because A = r_1 + r_2$

$r_2 = 67^\circ - 37^\circ = 30^\circ$

Snell's law on AC $\frac{4}{3}$ sin 30° = 1 × sin e

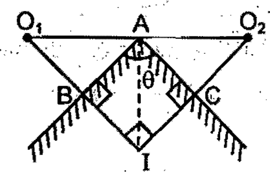


$e = \sin^{-1}\frac{2}{3}$

Then for net deviation to be double, the incident ray on side A' B' of second prism should make angles i or e with normal. Hence the angle between the given then will be 2e or i + e.

6. Dispersion will not occur for a light of single wave length λ = 4000Å.

7. As AB is common and O_1B = BI



Δ O_1BA and BAI are congruent

By symmetry AI is perpendicular to O_1 to O_2 and $\angle$ O_1BA = $\angle$BAI

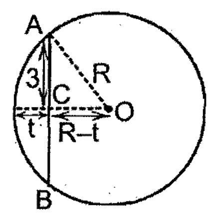
$\therefore$ BAI = 45° and $\angle$BAC = 90°

8. For convex mirror

|m| < 1 for any real object

Now, Vimage = – m² Vobject ⇒ | Vimage | < | Vobject | always.

9. Now $\mu = \frac{C_{air}}{C_{med}}$ $\mu = \frac{3\times10^8}{2\times10^8} = 1.5$



As f = $\frac{R}{\mu-1}$ (Plano convex)

where R = Radius of curvature i.e. the radius of that sphere of which the given lens is the part.

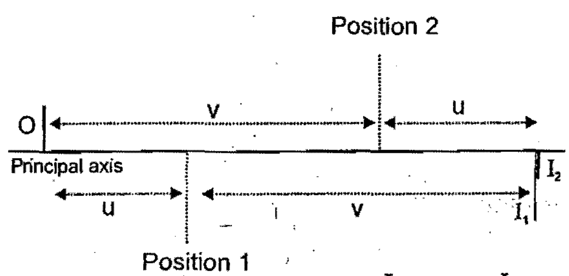
$\therefore$ In Δ OAC ⇒ $R^2 = 3^2 + (R-t)^2$ $R^2 = 3^2 + R^2 - 2Rt + t^2$

$$R = \frac{3^2+t^2}{2t} = \frac{9+0.09}{2\times0.3}$$ (t = 3 mm)

$$R = \frac{9.09}{0.6} = \frac{90.9}{6} = 15.16$$

$\therefore$ R = 15.2 cm (approx) $\therefore$ $f = \frac{15.16}{1.5-1} = 30.32$ cm or f = 30.4 cm (approx)

10.



For first & second position $\frac{v}{u} = \frac{I_1}{O}$, $\frac{u}{v} = \frac{I_2}{O}$ ⇒ $\frac{v^2}{u^2} = \frac{I_1}{I_2} = 4.84$

⇒ $\frac{v}{u} = 2.2$ and v + u = 96 ⇒ v = 66 , u = 30

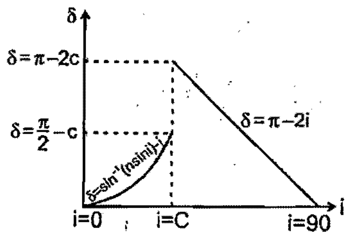
$\frac{O}{I_2} = \frac{v}{u} = 2.2 = \frac{11}{5}$ ⇒ A is True

Distance between two position of lens = v - u = 36 cm ⇒ B is True

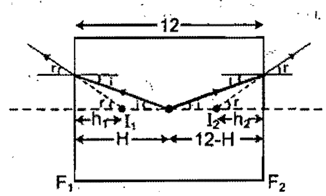
Focal length of lens $f = \dfrac{uv}{u+v} = \dfrac{66 \times 30}{66+30} = 20.63$ ⇒ C is False

Distance of lens from shorter image = u = 30 cm ⇒ D is True

11.



12. Let the bubble B is at distance H from the face F1 of the cube.



$n_c \sin i = n_a \sin r$

$h_1 = \dfrac{n_a}{n_c} H = 5$ cm

Similarly when looking from opposite face F2,

$n_c \sin i = n_a \sin r$

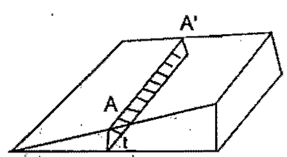
$h_2 = \dfrac{n_a}{n_c}(12 - H) = 3$ cm. Solving H = 7.5 cm and nC = 1.5

13. Path difference at point O is d sin α = 0.5 mm corresponding phase difference, $\Delta\phi = \dfrac{2\pi}{\lambda} \times \Delta p$

$= \dfrac{2\pi(0.5 \times 10^{-3})}{5000 \times 10^{-10}} = 2000\,\pi = 2\pi \times 1000$ ⇒ O is a point correesponding – 1000th maxima.

The point at 1 m below O corresponds to central maxima. So, 4 m from O will be maxima position.

14. In case x no fringes are formed because light passes the slab normaly & in case y fringes are obtained.



Suppose for point A 't' is such that it satisfies the condition for bright interference. The same 't' will be present throughout the line A A' & therefore the line AA' will be bright & a bright line will be seen. The same applies for dark lines.Hence fringes are straight line

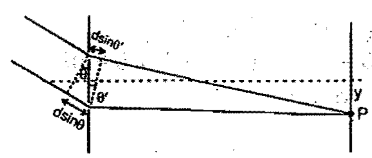
15. No interference is observed above y = 0. Therefore y coordinate of second order bright below y = 0 is

$$y = -2\beta = -2\left(\frac{\frac{\lambda}{\mu}D}{2d}\right) = 2\times\left(\frac{\frac{5000\sqrt{2}}{\sqrt{2}} \times 10^{-10} \times 1}{2 \times 10^{-3}}\right) = -500\ \mu\text{m}.$$

16. The region in which inteference pattern can be observed is OP.

OP = AP ≈ 1 metre

17. If phase difference at point P is zero then



$n_1\, d \sin\theta = n_2\, d \sin\theta'$ ⇒ θ' = 37°

and as $\tan\theta' = \dfrac{y}{D}$ ⇒ $y = -\dfrac{3}{4}$ m

It is negative because upper path in medium n_2 is longer than lower path in the same medium.

18. Path lengths in medium 2 are equal for point O. Therefore path difference = d sinθ

$$\lambda_{n_1} = 0.3\text{mm}, \quad \lambda_{n_2} = \frac{(0.3)\left(\frac{4}{3}\right)}{\frac{10}{9}} = 0.36\,\text{mm}\ ; \qquad \Delta p = \frac{d\sin\theta\,\frac{4}{3}}{\frac{10}{9}}, \qquad \therefore \quad \Delta\phi = \frac{2\pi}{\lambda}\Delta p$$

$$\Delta\phi = \frac{2\pi}{0.3\left(\frac{4/3}{10/9}\right)} \times \left(1\times\frac{1}{2}\right)\left(\frac{4/3}{10/9}\right) = \frac{10\pi}{3} \Rightarrow \qquad I = I_0 + I_0 + 2I_0\cos\left(2\pi + \frac{4\pi}{3}\right) = I_0$$

19. $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

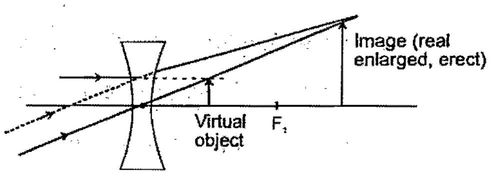
Here v = 2.5 (Distance of Retina as position of image is fixed) u = – x

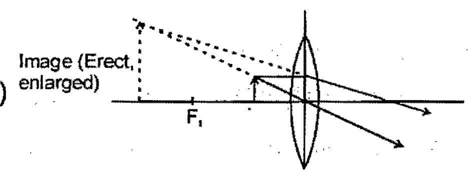
$\frac{1}{f} = \frac{1}{2.5} + \frac{1}{x}$ For f_{min} : x is minimum $\frac{1}{f_{min}} = \frac{1}{2.5} + \frac{1}{25}$

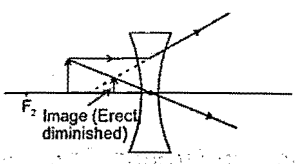
20. For f_{max} : x is maximum $\frac{1}{f_{max}} = \frac{1}{2.5} + \frac{1}{\infty}$

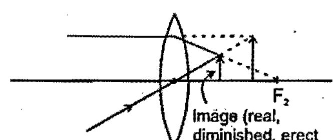
21. By snell's law $n = \frac{\sin i}{\sin r}$ and $n = a + \frac{b}{\lambda^2}$

So, at inclined face angular incidence is same for all ray and refraction is from denser to rarer. So emergent angle will be less for red.

22. (A) 

(B) 

(C) 

(D) 

23. By using $(\mu - 1)t = n\lambda$, we can find value of n, that is order of the fringe produced at P, if that particular strip has been placed over any of the slit. If two strips are used in conjuction (over each other), path difference due to each is added to get net path difference created. If two strips are used over different slits, their path differences are subtracted to get net path difference.

Now, $n_1 = \frac{(\mu_1 - 1)t_1}{\lambda} = 5$ $\qquad n_2 = 4.5$

and $\quad n_3 = 0.5$

For (A), $\Delta x = (\mu_B - 1)t_B$

$= (1.5 - 1)\,2.5 = n\lambda$

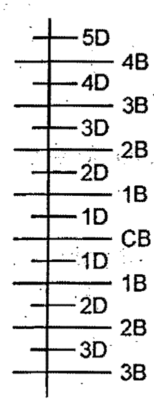
order of the fringe is 4.5 i.e. 5th dark.

for (B), $\Delta x = |(\mu_A - 1)t_A - (\mu_B - 1)t_B|$

$\Delta x = (1.5 - 1)5 - (2.5 - 1)1.5 = n\lambda$

net order is 5 – 0.5 = 4.5

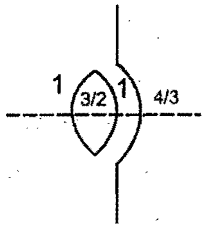
i.e. fifth dark.



for (C), $\Delta x = |(\mu_A - 1)t_A - [(\mu_B - 1)t_B + (\mu_C - 1)t_C]|$
net order is $5 - (0.5 + 4.5) = 0$
i.e. it is central bright again at P.
for (D), $\Delta x = |(\mu_A - 1)t_A + (\mu_B - 1)t_B - (\mu_C - 1)t_C|$
net order is $(5 + 0.5) - (4.5) = 1$
i.e. first bright

24. Lets find out the radius of curvature of equi. convex lens.

$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R} - \frac{1}{-R}\right) \Rightarrow \frac{1}{10} = \left(\frac{3}{2} - 1\right)\left(\frac{2}{R}\right) \Rightarrow R = 10$ cm.

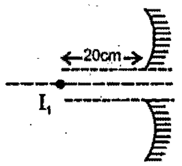


Now
for lens :

$\frac{1}{V} - \frac{1}{-20} = \frac{1}{10} \Rightarrow \frac{1}{V} = \frac{1}{20}$

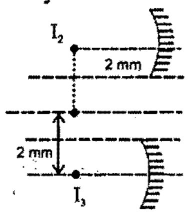
$\Rightarrow$ for surface of tube (of R = 10 cm.)

$\frac{\mu_2}{V} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \Rightarrow \frac{4/3}{V} - \frac{1}{+20} = \frac{4/3 - 1}{-10}$

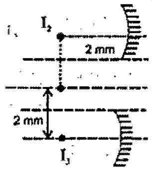


$\Rightarrow$ V = + 80 cm.
Now for mirrors.
As the object for the mirrors is at 20 cm so the image will be at 20 cm only
$\because$ u = – 2 f $\Rightarrow$ v = – 2f also.

$\Rightarrow$ magnification $= m = \frac{y_I}{y_O} = \frac{-v}{u}$



$\Rightarrow \frac{y_I}{-(1mm)} = -\left(\frac{-20}{-20}\right)$

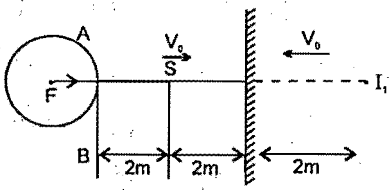


$\Rightarrow$ y_I = + (1 mm). So, the final images are like.
so the distance between the images is 4 mm.

25. Velocity and acceleration of central maximum = velocity and acceleration of screen
[$\because$ it does not move to the left or right on the screen.]
vscreen = 0 + gt = 100 m/s

$\therefore \vec{v}_s = 100\,\hat{j}$ m/s (in vector form) and $\vec{a}s = 10\,\hat{j}$ Ans. 4

26. Solutions : Given, vf = 10 m/s (⟶)
$v_i + v_f = 13$ $v_i = 13 - 10 = 3$ m/s
Refraction from surface AB (object I1)

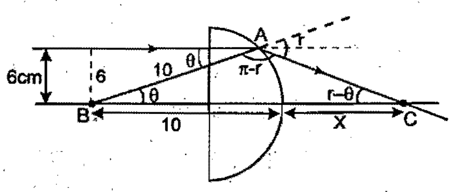


$\frac{3/2}{v} - \frac{1}{-6} = \frac{(3/2) - 1}{1} \Rightarrow \frac{3}{2v} = \frac{1}{2} - \frac{1}{6} = \frac{2}{6} \Rightarrow v = \frac{9}{2}$;

By diff. of the image formula, $\frac{dv}{dt} = \frac{n_1}{n_2}\frac{v^2}{u^2}\frac{du}{dt} \Rightarrow 3 = \frac{1}{3/2} \times \frac{(9/2)^2}{(6)^2} v_0 \Rightarrow v_0 = 8$ m/s. Ans.

27. R = 10cm

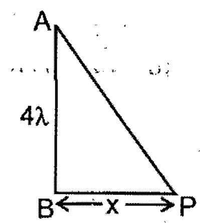
Applying snell's law $\frac{\sin\theta}{\sin r} = \frac{3}{4} \Rightarrow r = 53°$



By sine law in Δ ABC $\frac{\sin(r-\theta)}{10} = \frac{\sin(\pi - r)}{(10+x)}$; $\frac{10+x}{10} = \frac{4}{5\,(\sin r\cos\theta - \cos r\sin\theta)}$

$= \frac{4}{5\left(\frac{4}{5}\times\frac{4}{5} - \frac{3}{5}\times\frac{3}{5}\right)}$; $10 + x = \frac{200}{7}$ $\Rightarrow$ $x = \frac{200-70}{7} = \frac{130}{7}$ = X/ 7 cm. So X = 130 **Ans.**

28. Path difference at P
$\Delta x = AP - BP$
for maxima $\Delta x = n\lambda$



$n\lambda = \sqrt{x^2 + (4\lambda)^2} - x$

(n λ +x)2 = x2 + 16λ2
n2λ2 x2 + 2nλx = x2 + 16λ2

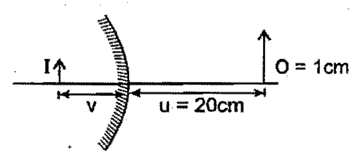
$x = \frac{16\lambda^2 - n^2\lambda^2}{2n\lambda}$

n = 1, 2, 3 are possible sol.

29. Using formula of spherical surface taking 'B' as object

$\frac{\mu_2}{\infty} - \frac{\mu_1}{(-2R)} = \frac{\mu_2 - \mu_1}{-R}$ (R being the radius of the curved surface) $\Rightarrow$ $\frac{\mu_1}{\mu_2} = 2$

30. $\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$ $\Rightarrow$ $\frac{1}{v} = \frac{1}{10} - \frac{1}{-20}$ $\Rightarrow$ $v = \frac{20}{3}$ cm



$I = -\frac{v}{u} \times O = \frac{\frac{20}{3}}{20} \times 1 = \frac{1}{3}$

∴ The distance between tip of the object and image is

$S = \sqrt{\left(20+\frac{20}{3}\right)^2 + \left(1-\frac{1}{3}\right)^2} = \sqrt{\frac{6404}{9}}$ cm

PART TEST - 5 : ELECTRODYNAMICS (CLASS XII)

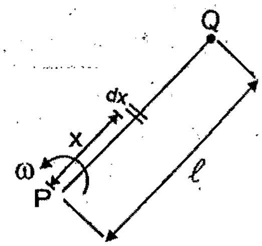
1.	(D)	2.	(B)	3.	(C)	4.	(D)	5.	(B)	6.	(C)	7.	(B)
8.	(AC)	9.	(BD)	10.	(ABCD)	11.	(BD)	12.	(ABCD)	13.	(BD)	14.	(ABC)
15.	(C)	16.	(C)	17.	(B)	18.	(B)	19.	(C)	20.	(A)	21.	(A)
22.	(A)	23.	(B)	24.	(9)	25.	(2)	26.	(8)	27.	(5)	28.	(6)
29.	(8)	30.	(3)										

SOLUTIONS

1. $U = \frac{1}{2}C_{eq}V^2$; $C_1 = \frac{k\varepsilon_0 A}{d/2} = \frac{2\varepsilon_0 A}{(d/2)}$; $C_2 = \frac{\varepsilon_0 A}{d/2}$; $C_{eq} = \frac{C_1C_2}{C_1 + C_2}$

$C_{eq} = \frac{\left(2\frac{\varepsilon_0 A}{d/2}\right)\times\frac{\varepsilon_0 A}{d/2}}{3\frac{\varepsilon_0 A}{d/2}} = \frac{4}{3}\frac{\varepsilon_0 A}{d}$; $U = \frac{1}{2}\left(\frac{4}{3}\frac{\varepsilon_0 A}{d}\right)V^2 = \frac{2}{3}\left(\frac{\varepsilon_0 A}{d}\right)V^2$

2. Charge on the differential element dx, $dq = \frac{Q}{\ell}.dx$



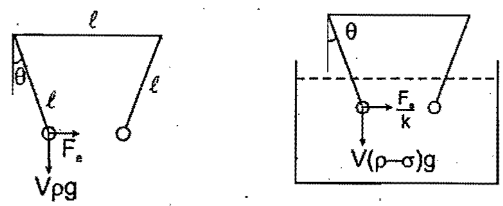
equivalent current di = f dq

$\therefore$ magnetic moment of this element

$d\mu = (di)\, NA \qquad (N = 1)$

$$= \left(\pi x^2\right) f \frac{Q}{\ell} dx \Rightarrow \mu = \int_0^{\mu} d\mu = \frac{\pi f Q}{\ell}\int_0^{\ell} x^2\, dx \ ; \ \mu = \frac{1}{3}\pi f Q \ell^2$$

3.

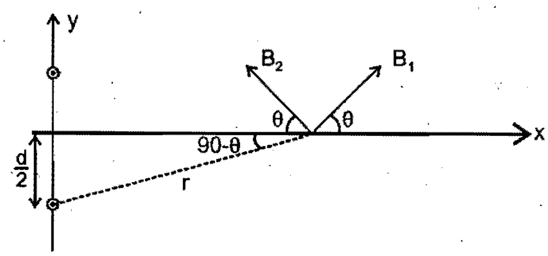


$$\tan\theta = \frac{F_e}{V\rho g} = \frac{F_e/k}{V(\rho-\sigma)g} \quad \Rightarrow \quad k = \frac{\rho}{\rho-\sigma} = \frac{2.4}{2.4-0.8} = 1.5$$

4. The magnetic field at point P is $\vec{B}_1 + \vec{B}_2$ where $\vec{B}_1$ and $\vec{B}_2$ are magnetic field at P due to wire 1 and 2.

$$\vec{B}_1 = \frac{\mu_0}{4\pi}\frac{2I}{r}(\cos\theta\,\hat{i} + \sin\theta\,\hat{j})$$

$$\vec{B}_2 = \frac{\mu_0}{4\pi}\frac{2I}{r}(-\cos\theta\,\hat{i} + \sin\theta\,\hat{j})$$

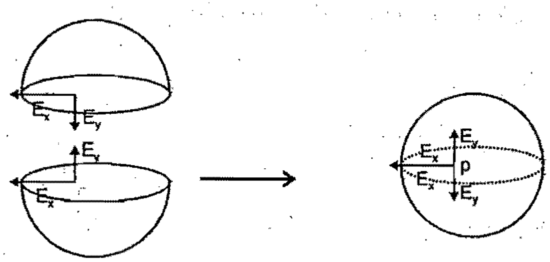


where $r^2 = x^2 + (d/2)^2$

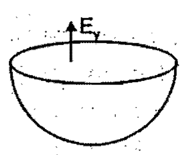
$\therefore$ field is along +y direction at point P and its magnitude is

$$B = \frac{\mu_0}{4\pi}\frac{4I}{r}\sin\theta = \frac{\mu_0}{4\pi}\frac{4Ix}{[x^2+(d/2)^2]} \quad ; \quad \text{B is max when } \frac{dB}{dx} = 0 \text{ i.e. at } x = d/2.$$

5. Let electric field at point . 'p' has both x and y component.
So similar electric field will be, for other hemisphere (upper half).
Now lets overlap both.



$(E_{net})_p = 2E_x$ and it should be zero (as E inside a full shell = 0).
So $E_x = 0$, So electric field at 'p' is purely in y direction.

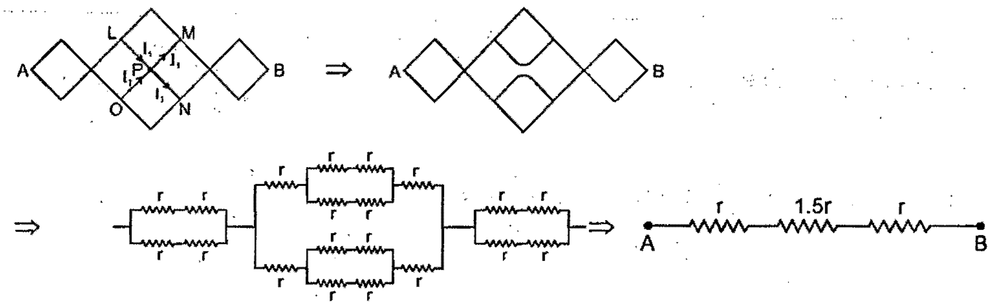


6. When the rod rotates, there will be an induced current in the rod. The given situation can be treated as if a rod 'A' of length '3ℓ' rotating in the clockwise direction, while an other rod 'B' of length '2ℓ' rotating in the anticlockwise direction with same angular speed 'ω'.

As, $e = \frac{1}{2}B\omega\ell^2$; For 'A' : $e_A = \frac{1}{2}B\omega(3\ell)^2 \quad \Rightarrow \quad$ For 'B' : $e_B = \frac{1}{2}B(-\omega)(2\ell)^2$

Resultant induced emf will be ; $\qquad e = e_A + e_B = \frac{1}{2}B\omega\ell^2(9-4) \quad \Rightarrow \quad e = \frac{5}{2}B\omega\ell^2$

7. From symmetry, the current distribution in branches LP, MP, NP and OP are as shown in figure 1. Therefore junction at P can be broken as shown in figure 2



Hence equivalent resistance is 3.5 r.

8. Net charge on both the capacitors is = $C_1V - C_2V$

The effective capacitance of system is $C_1 + C_2$ because both are in parallel.

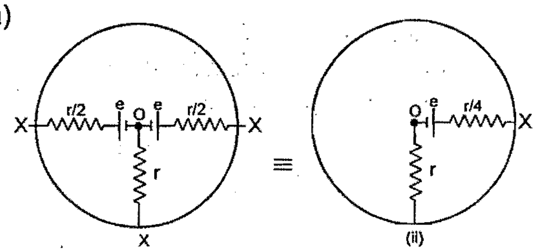
Therefore p.d across the system is $\dfrac{Q_{eff}}{C_1+C_2} = \dfrac{(C_1-C_2)V}{C_1+C_2}$

Initial energy$=\dfrac{1}{2}(C_1+C_2)V^2$; Final energy $=\dfrac{1}{2}(C_1+C_2)\left(\dfrac{(C_1-C_2)V}{C_1+C_2}\right)^2$

Therefore ratio of final to initial energy is $=\left(\dfrac{(C_1-C_2)}{(C_1+C_2)}\right)^2$

9. Equivalent circuit :

Induced emf e = $\left(\dfrac{B\omega a^2}{2}\right)$ ($\because$ Radius = a)



By nodal equation :

nodal : $\left(\dfrac{X-e}{r/4}\right) + \left(\dfrac{X-0}{r}\right) = 0$

5X = 4e

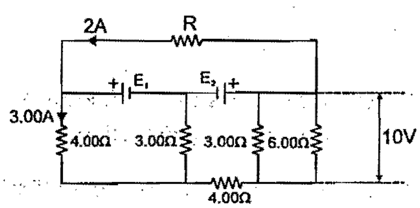
$\Rightarrow \quad X = \dfrac{4e}{5} = \dfrac{2B\omega a^2}{5}$ and $I = \dfrac{X}{r} = \dfrac{2B\omega a^2}{5r}$

also direction of current in 'r' will be towards negative terminal i.e. from rim to origin.

Alternately; by equivalent of cells (figure (ii)) :

$$I = \dfrac{e}{r+\dfrac{r}{4}} = \dfrac{4e}{5r}$$

10. after redrawing the circuit



(a) $I_4 = 5A$,

(b), (c) From loop (1)

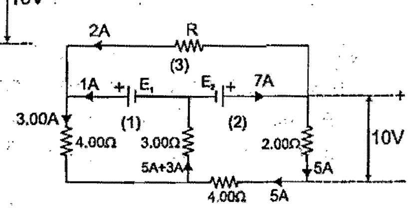
$-8(3) + E_1 - 4(3) = 0$

$\Rightarrow$ $E_1 = 36$ volt

from loop (2)

$+4(5) + 5(2) - E_2 + 8(3) = 0$

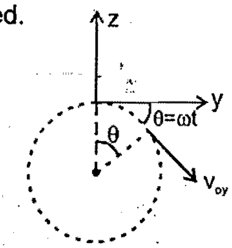
$E_2 = 54$ volt



(d) from loop (3)

$-2R - E_1 + E_2 = 0$; $R = \dfrac{E_2 - E_1}{2} = \dfrac{54-36}{2} = 9\,\Omega$

11. The x-component of velocity, being parallel to magnetic field, shall remain unchanged. The component of velocity perpendicular to x-axis will always have magnitude V_{oy}, and at any time t it shall make an angle $\theta = \omega t$ with y-axis as shown.



$\therefore$ y-component of velocity is $V_{oy}\cos\omega t$ and z-component of velocity along negative z-direction at any time t is $V_{oy}\sin\omega t$. Where $\omega = \frac{qB}{m}$

12. $V_0 = I_0 R = 10 \times 10 = 100$ volts (since, $I_0 = 10$amp from figure). Also $I = I_0 e^{-t/RC}$

Taking ℓn ; $\ell n\left(\frac{I_0}{I}\right) = \frac{t}{RC} \Rightarrow C = \frac{t}{R\ln(I_0/I)}$

At ; t = 2 sec, I = 2.5 A $\Rightarrow C = \frac{2}{10\ln\left(\frac{10}{2.5}\right)} \Rightarrow C = \frac{2}{10\ln 4} = \frac{2}{10\times 2\ln 2} = \frac{1}{10\ln 2}$

$C = \frac{1}{10\ell n2}$. Heat produced $= \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{1}{10\ell n2}\right)(100^2) = \frac{500}{\ell n2}$ joules. Hence (C) is correct.

$P = I^2 R = I_0^2 R e^{-2t/RC} = P_0 e^{-2t/RC} \Rightarrow$ Time constant $= \frac{RC}{2} = \frac{10}{2} \times \frac{1}{10\ell n2} = \frac{1}{2\ell n2}$

13. For given condition :
Magnitude of $B_{solenoid}$ = Magnitude of B_{loop}

$\mu_0 ni = \frac{\mu_0 I}{2R}$ here $n = \frac{\text{Total no. of turn}}{\text{Total length}} = \frac{1300}{0.65}$; $i = \frac{I}{2R}\times\frac{1}{n} = \frac{8\times 0.65}{2\times 0.02\times 1300} = 100$ mA.

For given condition :
Total magnetic field at the centre of loop

$= |B_{loop}| + |B_{solenoid}| \quad \because |B_{loop}| = |B_{solenoid}|$

$= 2|B_{loop}| = 2\times\frac{\mu_0 I}{2R} = \frac{2\times 4\pi\times 10^{-7}\times 8}{2\times 0.02} = 16\pi\times 10^{-5}$ T.

14. Charge is distributed over the surface of conductor in such a way that net field due to this charge and outside charge q is zero inside the conductor. Field due to only q is non-zero.

15. 15 to 16
For t = 0 to t_0 = RC seconds, the circuit is of charging type. The charging equation for this time is

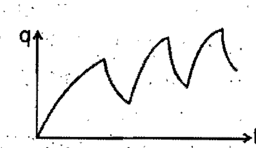
$q = CE(1 - e^{-\frac{t}{RC}})$

Therefore the charge on capacitor at time t_0 = RC is $q_0 = CE(1 - \frac{1}{e})$

For t = RC to t = 2RC seconds, the circuit is of discharging type. The charge and current equation for this time are

$q = q_0 e^{-\frac{t-t_0}{RC}}$ and $i = \frac{q_0}{RC}e^{-\frac{t-t_0}{RC}}$. Hence charge at t = 2 RC and current at t = 1.5 RC are

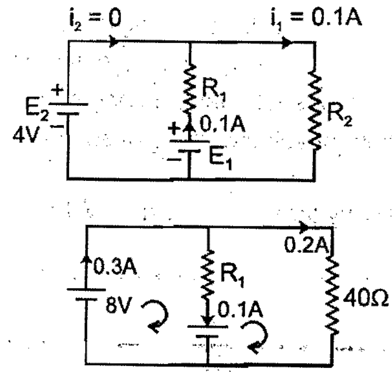
$q = q_0 e^{-\frac{2RC-RC}{RC}} = \frac{q_0}{e} = \frac{1}{e}CE(1-\frac{1}{e})$



and $i = \frac{q_0}{RC}e^{-\frac{1.5RC-RC}{RC}} = \frac{q_0}{\sqrt{e}RC} = \frac{E}{\sqrt{e}R}(1-\frac{1}{e})$ respectively

Since the capacitor gets more charged up from t = 2RC to t= 3RC than in the interval t=0 to t=RC, the graph representing the charge variation is as shown in figure

18. 17 to 18



$i_1 = 0.1A, E_2 = 4V, i_2 = 0$

As ; $0.1\ R_1 + 0.1\ R_2 - E_1 = 0$

$0.1\ R_2 - 4\ V = 0$

$R_2 = 40\ \Omega$

Now ; $i_2 = 0.3A, i_1 = 0.1\ A,\ \ E_2 = 8A$

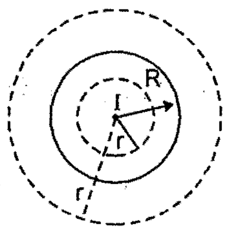
Now ; $0.1\ R_1 + E_1 - 8 = 0$

$0.1R_1 + 4 - E_1 = 0$

$0.2R_1 - 4\ = 0$

$\Rightarrow \quad R_1 = \frac{4}{0.2} = 20\ \Omega \quad \Rightarrow \quad E_1 = 2 + 4 = 6\ V$

19. Inside the cylinder



$B.2\pi r = \mu_0 . \frac{I}{\pi R^2} \pi r^2$

$B = \frac{\mu_0 I}{2\pi R^2} . r$ ....(1)

outside the cylinder

$B.2\pi r = \mu_0 I$

$\therefore\ B = \frac{\mu_0 I}{2\pi r}$ ....(2)

Inside cylinder $B \propto r$ and outside $B \propto \frac{1}{r}$

So at the surface equation of magnetic field changes.

Hence clear from graph, wire 'c' has greatest radius.

20. Magnitude of magnetic field is maximum at the surface of wire 'a'.

21. P-1, Q-1, R-1,S-3

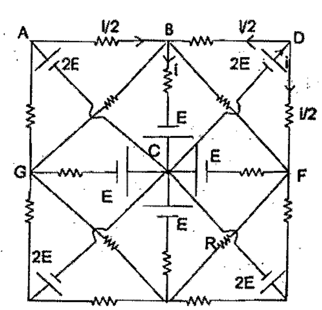
(A) Uniform electric field exerts constant force on the charged particle, hence the particle may move in straight line or a parabolic path.

(B) Under action of uniform magnetic field, the charged particle may move in straight line when projected along or opposite to direction of magnetic field. The charged particle moves in circle when it is projected perpendicular to the magnetic field. If the initial velocity of the charged particle makes an angle between 0° and 180° (except 90°) with magnetic field, the particle moves along a helical path of uniform pitch.

(C) If charged particle is shot parallel to both fields it moves along a straight line. If the charged particle is shot at any angle with both the field (except 0° and 180°), the particle moves along a helix with non-uniform pitch.

(D) from results of A and B all the given paths are possible.

22. (P) 3 (Q) – 4 ; (R) – 4 ; (S) – 3



By symmetry $V_B = V_F$

No current in the resistor joining B and F and in similar resistors like between G and B etc.

The currents are assumed as shown in the figure using symmetry.

Applying KVL along DBCD, we have $V_D - i/2$

$R - iR + E + 2E = V_D$

$i = 2E/R$

23. (P) 3, 4 (Q) 3, 4 (R) 2, 3 (S) 1, 3

(A) Because the magnetic field is parallel to x-axis, the force on wire parallel to x-axis is zero. The force on each wire parallel to y-axis is $B_0\frac{i}{2}\ell$. Hence net force on loop is $B_0 i\ell$. Torque of both the forces is canceling each other.

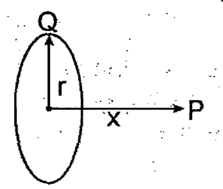
(B) Because the magnetic field is parallel to y-axis, the force on wire parallel to y-axis is zero. The force on each wire parallel to x-axis is $B_0\frac{i}{2}\ell$. Hence net force on loop is $B_0 i\ell$. Torque of both the forces is canceling each other.

(C) Since net displacement of current from entry point in the loop to exit point in the loop is along the diagonal of the loop. The direction of external uniform magnetic field is also along the same diagonal. Hence net force on the loop is zero. Torque of both the forces is canceling each other.

(D) The net displacement of current from entry point in the loop to exit point in the loop is along the diagonal (of length $\sqrt{2}\ell$.) of the loop. The direction of external uniform magnetic field is also perpendicular to the same diagonal. Hence magnitude of net force on the loop is $B_0 i(\sqrt{2}\ell)$. Torque of both the forces is canceling each other.

24. Electric field at P is

$$E = \frac{Q\,x}{4\pi\epsilon_0 (x^2+r^2)^{3/2}}$$



Magnetic field at P is $B = \frac{\mu_0}{4\pi}\,\frac{2\pi i r^2}{(x^2+r^2)^{3/2}} = \frac{\mu_0}{4\pi}\,\frac{2\pi Q f r^2}{(x^2+r^2)^{3/2}}$

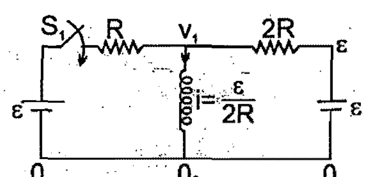
f = frequency of revolution.

Electric energy density = $\frac{1}{2}\epsilon_0 E^2$; Magnetic energy density $\frac{B^2}{2\mu_0}$

$$\frac{\text{Electric energy density}}{\text{magnetic energy density}} = \frac{\frac{1}{2}\epsilon_0 E^2}{\frac{B^2}{2\mu_0}} = \frac{x^2}{4\pi^2\epsilon_0\mu_0 f^2 r^4} = \frac{x^2 c^2}{4\pi^2 f^2 r^4} = \frac{9}{\pi^2}\times 10^{10} = 9\times 10^9$$

$= X \times 10^9$ So X = 9 Ans.

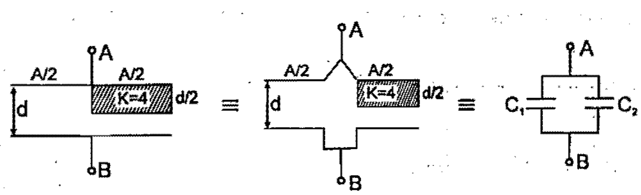
25. When S_1 is closed current in inductor remains, $i = \frac{\varepsilon}{2R}$



$$\therefore \quad \frac{\varepsilon - V_1}{R} + \frac{\varepsilon - V_1}{2R} = \frac{\varepsilon}{2R} \quad \Rightarrow \quad \left(V_1 = \frac{2\varepsilon}{3}\right)$$

$\therefore$ Potential difference $(V_1) = \frac{2\varepsilon}{3}$

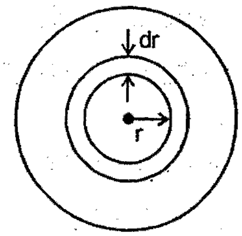
And $L\frac{di}{dt} = \frac{2\varepsilon}{3}$ $\qquad \frac{di}{dt} = +\frac{2\varepsilon}{3L} = \frac{2\times 3}{3\times 10^{-3}} = 2000$ A/s

26. 

$C_1 = \frac{\varepsilon_0 A/2}{d}$, $C_2 = \frac{\varepsilon_0 A/2}{\frac{d/2}{k}+\frac{d}{2}} = \frac{4\varepsilon_0 A}{5d}$ $\qquad C = C_1 + C_2 = \frac{13}{10}\frac{\varepsilon_0 A}{d}$

27. Current in the element = $J(2\pi r \, . \, dr)$

Current enclosed by Amperian loop of radius $\frac{a}{2}$



$$I = \int_0^{a/2} \frac{J_0 r}{a} \, . \, 2\pi r \, . \, dr = \frac{2\pi J_0}{3a}\left(\frac{a}{2}\right)^3 = \frac{\pi J_0 a^2}{12}$$

Applying Ampere's law

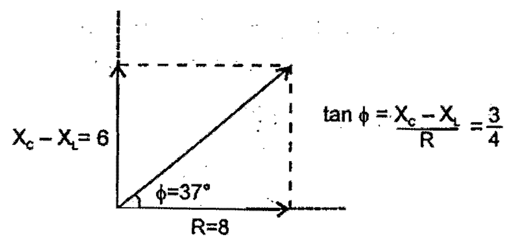
$$B \, . \, 2\pi \, . \, \frac{a}{2} = \mu_0 \, . \, \frac{\pi J_0 a^2}{12} \Rightarrow B = \frac{\mu_0 J_0 a}{12}$$

On putting values

$B = 10\ \mu T$

28. The current leads in phase by $(\because X_C > X_L)$

$\phi = 37°$



$\therefore \quad i = \frac{10 \cos(100\pi t + 37°)}{Z} = \cos(100\pi t + 37°)$

The instantaneous potential difference across A B is

$= I_m (X_C - X_L) \cos(100\pi t + 37° - 90°)$

$= 6 \cos(100\pi t - 53°)$

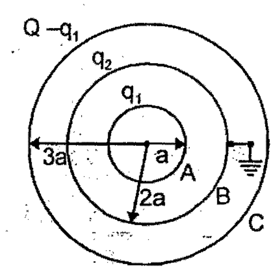
The instantaneous potential difference across A B is half of source voltage.

$\Rightarrow \quad 6 \cos(100\pi t - 53°) = 5 \cos 100\pi t$

solving we get $\cos 100\pi t = \frac{1}{\sqrt{1+(7/24)^2}} = \frac{24}{25}$

$\therefore$ instantaneous potential difference $= 5 \times \frac{24}{25} = \frac{24}{5}$ volts

29. From given conditions,



$V_A = V_C$ and $V_B = 0 \quad \Rightarrow V_B = \frac{K(Q-q_1)}{3a} + \frac{Kq_2}{2a} + \frac{kq_1}{2a} = 0$

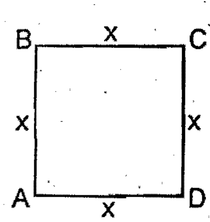
$\Rightarrow \quad 2Q + q_1 + 3q_2 = 0 \quad .... (1)$

Using $V_C = V_A$

$$\frac{K(Q-q_2)}{3a} + \frac{Kq_2}{3a} + \frac{Kq_1}{3a} = \frac{Kq_1}{a} + \frac{K(Q-q_1)}{3a} + \frac{Kq_2}{2a} \Rightarrow \quad q_1 = -\frac{q_2}{4} \quad .... (2)$$

Using it in (1), $q_2 = -\frac{8}{11}Q$

30. Let the equivalent resistance of one infinite ladder be x. Then the complete network reduces to



$\therefore \quad R_{AB} = \frac{x \times 3x}{x + 3x} = \frac{3}{4}x$

and $\quad R_{AC} = \frac{2x \times 2x}{2x + 2x} = x$

Hence $\frac{R_{AB}}{R_{AC}} = \frac{3}{4}$

PART TEST - 6 : MODERN PHYSICS (CLASS XII)

1.	(B)	2.	(B)	3.	(A)	4.	(B)	5.	(C)	6.	(B)	7.	(B)
8.	(ACD)	9.	(ABC)	10.	(BCD)	11.	(AD)	12.	(D)	13.	(BC)	14.	(BC)
15.	(C)	16.	(D)	17.	(D)	18.	(D)	19.	(B)	20.	(C)	21.	(A)
22.	(D)	23.	(C)	24.	3	25.	28	26.	6	27.	55	28.	4
29.	8	30.	8										

SOLUTIONS

1. Rate of decay of A keeps on decreasing continuously because concentration of A decreases with time ⇒ A is false
Initial rate of production of B is $\lambda_1 N_0$ and rate of decay is zero. With time, as the number of B atoms increase, the rate of its production decreases and its rate of decay increases. Thus the number of nuclei of B will first increase and then decrease ⇒ B is the correct choice
The initial activity of B is zero whereas initial activity of A is $\lambda_1 N_0$ ⇒ C is false.
As time $t \to \infty$: $N_A = 0$, $N_B = 0$ and $N_C = N_0$ ⇒ D is false

2. $E = \frac{3}{2}kT$ & $P = \sqrt{2mE}$ $\Rightarrow \lambda_{de-Broglie} = \frac{h}{p} = \frac{h}{\sqrt{2m\left(\frac{3}{2}kT\right)}}$ $\Rightarrow$ $\lambda_{de-Broglie} = \frac{h}{\sqrt{3mkT}}$

Substituting values : $\lambda_{de-Broglie} = 0.63$ Å

3. $\frac{hc}{\lambda} = 5\,eV_0 + \phi$ $\Rightarrow$ $\frac{hc}{3\lambda} = eV_0 + \phi$ $\Rightarrow$ $\frac{2hc}{3\lambda} = 4eV_0$ $\Rightarrow$ $\phi = \frac{hc}{6\lambda}$

4. Change in momentum due to photon $= \frac{h}{\lambda}$

F = rate of change of momentum ; $F = n\frac{h}{\lambda} = ma$ $\Rightarrow a = \frac{nh}{\lambda m}$

5. $$\frac{\left(\frac{dN}{dt}\right)_A}{\left(\frac{dN}{dt}\right)_B} = \frac{(\lambda N)_A}{(\lambda N)_B} = \frac{\frac{\ell n2}{1}.N_0\, e^{-\frac{\ell n2}{1}.2}}{\frac{\ell n2}{2}.N_0\, e^{-\frac{\ell n2}{2}.2}} = \frac{2\times 2^{-2}}{2^{-1}} = 1$$

6. Mass defect $=(238.05079 - 234.04363 - 4.00260)\ u = 4.56 \times 10^{-3}\ u$
$= 4.56 \times 10^{-3} \times 1.66 \times 10^{-27} = 7.57 \times 10^{-30}$ kg ; $mc^2 = 7.57 \times 10^{-30} \times 9 \times 10^{16} = 6.8 \times 10^{-13}$ J

7. The correct statement in nuclear force is not a central force.

8. Given, $\lambda = 0.173$

$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{0.173} \cong 4$

Also, $N_0 - N = N_0\, e^{-\lambda t}$ $\Rightarrow$ for $t = \frac{1}{0.173}$ year : $\Rightarrow$ $N_0 - N = \frac{N_0}{e} = 0.37\, N_0$

9. $\lambda_{min} = \frac{hc}{eV} = 62.1$ pm

10. In ground state n = 1 and for first excited state n = 2

$KE = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{2r}(z=1) = \frac{14.4\times10^{-10}}{2r}eV$ $(\because r = 0.53 n^2 A^o (z = 1))$

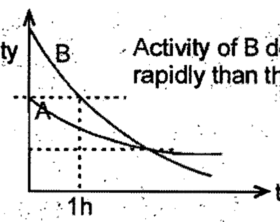
$(KE)_1 = \frac{14.4\times10^{-10}}{2\times0.53\times10^{-10}} eV = 13.58\ eV$ and $(KE)_2 = \frac{14.4\times10^{-10}}{2\times0.53\times10^{-10}\times4} ev = 3.39\ ev$

∴ KE decreases by = 10.2 ev

∴ PE increases by = Excitation energy + Loss in kinetic energy = 10.2 + 10.2 = 20.4 ev

Now Angular momentum ; $L = mvr = \frac{nh}{2\pi}$ $\Rightarrow L_2 - L_1 = \frac{h}{2\pi} = \frac{6.6\times10^{-34}}{6.28} = 1.05\times10^{-34}$ J–sec.

11. As $\frac{1}{\lambda} = \frac{t_{1/2}}{\ell n2}$ $\Rightarrow$ $t_{(1/2)A} > t_{(1/2)B}$



at t = 60 min.

$\Rightarrow$ $\lambda_A N_A = \lambda_B N_B$ $\Rightarrow$ $N_A > N_B$

Activity curves are $(T_{1/2(A)} > T_{1/2(B)})$

12. Ionization energy = 13.6 Z^2 eV

∴ (Ionization energy)$_A$ > (Ionization energy)$_B$

$\Rightarrow$ $Z_A > Z_B$

As $u \propto Z$ $\Rightarrow$ $u_A > u_B$

13. K x-ray is emitted when electron jumps from outer shell to K shell. This is equivalent to hole jumping from K shell to outer shell.

$\lambda_K < \lambda_L$

14. Magnetic field at centre (site of nucleus)

$B = \frac{\mu_0 I}{2r} = \frac{\mu_0 qf}{2r} = \frac{\mu_0 qv}{2r\times2\pi r}]$ $\Rightarrow B\alpha\frac{1}{r^2}$ and $B \alpha v$

∴ $B \alpha \frac{1}{n^5}$; $\frac{B_1}{B_2} = \frac{(2)^5}{(1)^5}$ (Since, $n_1 = 1$ to $n_2 = 2$) ∴ $B_1 = 32 B_2$

Also, $mvr = n.\frac{h}{2\pi}$,therefore angular momentum is decreased by $\frac{h}{2\pi}$.

15. Q = CV $\Rightarrow$ $ne = \frac{\varepsilon_0 A}{d}V$; $n = \frac{2.85\times10^{-12}\times10}{0.5\times10^{-3}\times1.6\times10^{-19}}\times16$ $n = 8.85\times10^9$

16. $P = \frac{nhC}{\lambda}$ where n = no. of photons incident per unit time.

Also, $I = ne$ $\Rightarrow$ $P = \frac{IhC}{e\lambda}$; $\lambda = \frac{(2\times10^{-6})(6.6\times10^{-34})(3\times10^8)}{(4\times10^{-6})(1.6\times10^{-19})}$

$= \frac{9.9}{1.6}\times10^{-7} m = \frac{9900}{1.6} Å = 6187\ Å$. Which came in the range of orange light.

17. Total energy released from $Au^{198} \rightarrow Hg^{198}$ in ground state
$= (\Delta m_{loss}) c^2 = (197.9682 - 197.9662)(930) = 1.86$ MeV
Energy released from ^{198}Hg in first excited state $\rightarrow$ Hg in ground state
$= (-1.6) - (-2)$ MeV $= 0.4$ MeV
$\Rightarrow$ Energy released from $Ag^{198} \rightarrow Hg^{198}$ second excited state
$= 1.86 - 0.4 = 1.46$ MeV = max. K.E. of β_2 particle

18. Similarly maximum kinetic energy of β_1 particle $= 1.86 - 1 = 0.86$ MeV

19. $F_z = \mu_z \cdot \frac{dB_z}{dz} = (9.3 \times 10^{-24}\ J/T).(16 \times 10^{-3}\ T/m) = 1.5 \times 10^{-25}$ N

20. Energy change is $\mu B \times 2$ as the spin is completely flipped.
$\Delta U = 2\mu B = 2 \times (5.8 \times 10^{-5}\ eV/T) \times (620 \times 10^{-3}\ T) = 7.2 \times 10^{-5}$ eV

21. **(P) 1,3 (Q)2,4 (R)2 (S)2,4**

P. $B = \frac{\mu_0 i}{2\pi r}$ where $i_{eq} = \frac{q}{T} = \frac{e}{2\pi r/v} \Rightarrow B = \frac{\mu_0 e}{4\pi^2}\frac{v}{r^2} \propto \frac{v}{r^2} \propto \frac{(z/n)}{(n^2/z)^2} \propto \frac{z^3}{n^5}$

Q. Magnetic moment $M = iA = \left(\frac{q}{T}\right)(\pi r^2) = \frac{e}{2\pi r/v}\pi r^2 \propto rv \propto \left(\frac{n^2}{z}\right)\left(\frac{z}{n}\right)$

R. $\lambda = \frac{h}{mv} \propto \frac{1}{(z/n)} \propto \frac{n}{z}$

S. Areal velocity $= \frac{L}{2m} = \frac{nh/2\pi}{2\pi} \propto n$

22. **(P) 1 (Q) 2 (R) 1 (S) 4**

23. **(P) 2,3,4 (Q) 2,3,4 (R) 2,3,4 (S) 1,2,3,4,**

P. In the given spontaneous radioactive decay, the number of protons remain constant and all conservation principles are obeyed.

Q. In fusion reaction of two hydrogen nuclei, a proton is decreased as positron shall be emitted in the reaction. All the three conservation principles are obeyed.

R. In the given fission reaction the number of protons remain constant and all conservation principles are obeyed.

S. In beta negative decay, a neutron is converted into a proton and the electron is ejected out.

24. In one half-life the number of active nuclei reduces to half the original number. Thus, in two half-lives the number is reduced to $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$ of the original number. The number of remaining active nuclei is, therefore,

$$12 \times 10^{18} \times \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = 3 \times 10^{18} = n \times 10^{18} \text{ So } n = 3$$ **Ans.**

25. The volume of liquid in beaker at any instant of time t is
$V = 100 + 8t$
The volume of liquid ejected in t seconds is 2t
$\therefore$ Number of active atoms being taken out is

$$-dN = \frac{N}{V} 2dt \quad \text{or} \quad -\frac{dN}{dt} = \frac{2N}{V} = \frac{2N}{100 + 8t}$$

multiplying both sides with disintegration constant.

$$-\lambda\, dN = \lambda N \frac{2dt}{V} \quad \text{or} \quad -dA = A.\frac{2dt}{V}$$

where A is activity of the solution. The time taken for 10 ml solution to come out is 5 second.

or $$\int_{A_0}^{A}\frac{dA}{A} = \int_0^5 \frac{-2t}{100+8t}\,dt \quad \text{or} \quad A = A_0\left(\frac{5}{7}\right)^{1/4}$$

$\therefore$ required activity of the ejected solution is $A - A_0 = A_0\left[1-\left(\frac{5}{7}\right)^{1/4}\right]$ where $A_0 = 50$ dps

26. Energy of incident photons = $\frac{12400}{2000}$ eV.

maximum kinetic energy of ejected electrons = $\frac{12400}{2000} - \phi$

maximum kinetic energy of electrons striking plate B = $\frac{12400}{2000} - \phi + 10$

minimum wavelength of photons emitted from B = $\frac{12400}{1000}$

$\therefore \quad \frac{12400}{2000} - \phi + 10 = \frac{12400}{1000} \qquad \therefore \quad \phi = 3.8\text{eV}$

27. $\lambda_{min} = \frac{hc}{20000e}, \lambda'_{min} = \frac{hC}{10000e}$ given $4(\lambda_\alpha - \lambda_{min}) = (\lambda_\alpha - \lambda'_{min})$

$\Rightarrow \quad 3\lambda\alpha = 4\lambda_{min} - \lambda'_{min} \quad \Rightarrow \quad \lambda_\alpha = \frac{hC}{30000e} \quad \Rightarrow \quad \frac{hC}{\lambda_\alpha} = 30000\ e$

By using the formula of energy of electrons according to Bohr's model and considering shielding effect

$30000 = (13.6)\left(\frac{1}{1^2} - \frac{1}{2^2}\right)(z-1)^2 \quad \Rightarrow \quad z - 1 = 100\sqrt{\frac{5}{17}} = \frac{100}{\sqrt{3.4}} = 54 \quad \Rightarrow \quad z = 55$

28. Using $\frac{1}{\lambda} = R(z-1)^2\left[\frac{1}{n_2^2} - \frac{1}{n_1^2}\right]$; For α particle ; $n_1 = 2$, $n_2 = 1$

For metal A ; $\frac{1875R}{4} = R(Z_1-1)^2\left(\frac{3}{4}\right) \Rightarrow z_1 = 26$

For metal B; $675R = R(Z_2-1)^2\left(\frac{3}{4}\right) \Rightarrow z_2 = 31$. Therefore, 4 elements lie between A and B.

29. Using Mosely's law for both cobalt and impurity

$\sqrt{f} = K(Z-1) \quad \Rightarrow \quad \sqrt{\frac{c}{\lambda}} = K(Z-1)$

$\Rightarrow \quad \sqrt{\frac{c}{\lambda_{co}}} = K(Z_{co}-1)$ and $\sqrt{\frac{c}{\lambda_x}} = K(Z_x - 1) \quad \Rightarrow \quad \sqrt{\frac{\lambda_{co}}{\lambda_x}} = \frac{Z_x - 1}{Z_{co}-1} = Z_x = 40$

30. $R \,\alpha\, A^{1/3}$ and here $A = 2Z$ $\qquad \frac{R_1}{R_2} = \left(\frac{z_1}{z_2}\right)^{1/3} \Rightarrow \left(\frac{z_1}{z_2}\right) = \frac{27}{8}$

Radius of Bohr's orbit $r \,\alpha\, \frac{1}{z} \qquad \frac{r_1}{r_2} = \frac{z_2}{z_1} = \frac{8}{27}$

FULL SYLLABUS TEST - 1 (XI)

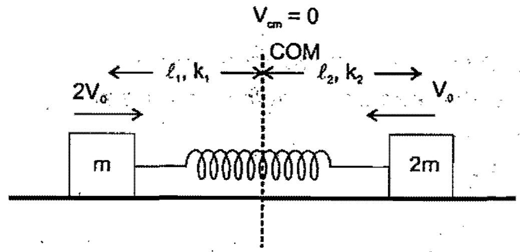
1.	(A)	2.	(D)	3.	(D)	4.	(C)	5.	(C)	6.	(AC)	7.	(ABCD)
8.	(ABCD)	9.	(ABC)	10.	(C)	11.	(A)	12.	(B)	13.	(A)	14.	(C)
15.	(B)	16.	5	17.	9	18.	2	19.	8	20.	4		

SOLUTIONS

1. $f = f_0\left(1+\dfrac{V_{ob}}{V_{sound}}\right) \Rightarrow \dfrac{f}{f_0} = 1+\dfrac{V_{ob}}{V_{sound}}$ (straight line) ; when $\dfrac{V_{ob}}{V_{sound}} = 0$; $\dfrac{f}{f_0} = 1$.

and as $\dfrac{V_{ob}}{V_{sound}} \to 1 \Rightarrow \dfrac{f}{f_0} \to 2$

2. $\ell_1 = \dfrac{2m\ell}{2m+m} = \dfrac{2\ell}{3}$



$K_1\ell_1 = K\ell \Rightarrow K_1\left(\dfrac{2\ell}{3}\right) = K\ell \Rightarrow K_1 = \dfrac{3K}{2}$

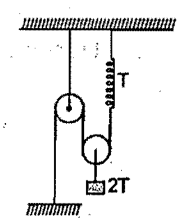
$|x_{1\,max}| = \sqrt{\dfrac{m(2V_0)^2}{\frac{3K}{2}}} = \sqrt{\dfrac{8mV_0^2}{3K}}$

$|d_{1\,max}| = \ell_1 + x_{1\,max} = \dfrac{2\ell}{3} + \sqrt{\dfrac{8mV_0^2}{3K}}$

3. average velocity $\bar{v} = \dfrac{\int_0^t \frac{dx}{dt}.dt}{t} = \dfrac{\int_0^t dx}{t} = \dfrac{x(t)-x(0)}{t} = \dfrac{A(\cos\pi/6-1)}{\pi/6\omega} = \dfrac{3A\omega}{\pi}\left(\sqrt{3}-2\right)$

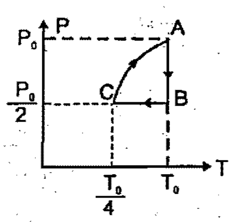
since particle does not change it's direction in the given interval , average speed = $|\bar{v}| = \dfrac{3A\omega}{\pi}\left(2-\sqrt{3}\right)$

4. Initially the block is at rest under action of force 2T upward and mg downwards. When the block is pulled downwards by x, the spring extends by 2x. Hence tension T increases by 2kx. Thus the net unbalanced force on block of mass m is 4kx.



$\therefore$ acceleration of the block is = $\dfrac{4kx}{m}$

5. Process AB is isothermal expansion,
BC is isobaric compression and in process CA



$P \propto \dfrac{nRT}{P} \Rightarrow P^2 \propto T.$

6. Slope of graph is greater in the liquid state i.e., temperature is rising faster, hence lower heat capacity. The transition from solid to liquid state takes lesser time, hence latent heat is smaller.

7. $y = 4 \sin\left(\pi \frac{x}{15}\right) \cos 96 \pi t$

At x = 5 cm, $y = 4 \sin \frac{\pi}{3} \cos (96 \pi t)$ and $y_{max} = 2\sqrt{3}$ cm

Positions of nodes is given by equation

$$\sin\left(\frac{\pi x}{15}\right) = 0 \quad \Rightarrow \quad \frac{\pi x}{15} = n\pi$$

$$\Rightarrow \quad x = 15n$$

At x = 7.5 cm and t = 0.25 sec.

Velocity of the particle = $\frac{\partial y}{\partial t} = -344\, \pi \sin\left(\frac{\pi x}{15}\right) \sin (96 \pi t) = 0$

8. In the equilibrium position the net force on the partion will be zero.
Hence pressure on both sides are same.
Hence, (A) is correct.
Initially, PV = nRT

$$n_1 = \frac{P_1V_1}{RT_1} = \frac{PV}{RT} \quad \text{and} \quad n_2 = \frac{(2P)(2V)}{RT} = 4\frac{PV}{RT} \Rightarrow n_2 = 4n_1$$

Moles remains conserved.
Finally, pressure becomes equal in both parts.
Using, $P_1V_1 = n_1RT_1$
$P_2V_2 = n_2RT_2$
$\because \quad P_1 = P_2 \ \& \ T_1 = T_2$

$$\therefore \quad \frac{V_1}{V_2} = \frac{n_1}{n_2} = \frac{1}{4} \quad \Rightarrow \quad V_2 = 4V_1$$

Also $V_1 + V_2 = 3V \quad \Rightarrow \quad V_1 + 4V_1 = 3V \quad \Rightarrow \quad V_1 = \frac{3}{5}V$

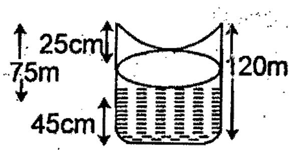
And $V_2 = \frac{12}{5}V$

Hence (B) and (C) are correct. In compartment (I) :

$$P_1' V_1 = n_1RT_1 \ ; \qquad P_1'\left(\frac{3V}{5}\right) = \left(\frac{PV}{RT}\right) RT$$

$$P_1' = \frac{5Pv}{3v} = \frac{5}{3}P \quad ; \qquad \text{Hence (D) is also correct.}$$

9. As $V = n\lambda$



$\lambda = \frac{V}{\nu} = \frac{340}{340} = 1$m

first Resonance length

$R_1 = \frac{\lambda}{4} = \frac{1}{4}$ m = 25 cm

∴ $R_2 = \frac{3\lambda}{4} = \frac{3}{4}m = 75$ cm ∴ $R_3 = \frac{5\lambda}{4} = \frac{5}{4}m = 125$ cm

i.e. third resonance does not establish. Now H_2O is poured,

∴ Minimum length of H_2O Column to have the resonance = 45 cm

∴ Distance between two succensive nodes = $\frac{\lambda}{2} = \frac{1}{2}$ m = 50 cm

& maximum length of H_2O column to create resonance

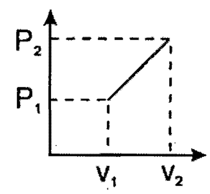
i.e. 120 – 25 = 95 cm.

10. $f = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$

If radius is doubled and length is doubled, mass per unit length will become four times. Hence

$f' = \frac{1}{2 \times 2\ell}\sqrt{\frac{2T}{4\mu}} = \frac{f}{2\sqrt{2}}$

11. W = Area under the curve = $\frac{3}{2}P_1V_1$



and $P_1V_1 = nRT_1$

Therefore : $\frac{w}{nRT_1} = \frac{\frac{3}{2}.P_1V_1}{P_1V_1}$

12. $nC\,\Delta T = Q \quad \Rightarrow nC\Delta T = 6n\,RT_1$

$dT = 4T_1 - T_1 = 3T_1$

$n\,.\,C\,.\,3T_1 = 6nRT_1$

$\frac{C}{R} = 2$

13. $S_\ell = \frac{\frac{(100.0)(10.0)(700.0)}{80.0-10.0} - (1.0)(3.0\times10^3)}{3.0-1.0}$

$= 3.5 \times 10^3$ J/kg°C (According to addn. and multiplication rule of S.F.)

14. If $m_1 \to 0$, $S_c \to 0$ $\quad S_\ell = \frac{VIt}{m_2(\theta_f - \theta_0)}$

$\frac{dS_\ell}{S_\ell} = \frac{\Delta V}{V} + \frac{\Delta I}{I} + \frac{\Delta t}{t} + \frac{\Delta m_2}{m_2} + \frac{\Delta\theta_f + \Delta\theta_0}{\theta_f - \theta_0} = \frac{0.1}{10.0} + \frac{0.1}{10.0} + \frac{0.01\times10^2}{1.00\times10^2} + \frac{0.01}{1.00} + \frac{1+1}{50} = 8\%$

15. (P) 1, (Q) 2, (R) 2, (S) 4

P. If resultant force is zero, $\vec{P}_{system}$ will be constant.

Q. If resultant torque is zero, $\vec{L}_{system}$ will be constant.

R. If external forces are absent, both $\vec{P}_{system}$ and $\vec{L}_{system}$ will be constant.

S. If no non conservative force acts, total mechanical energy of system will be constant.

16. Since $e = \frac{1}{5}$

∴ Final normal component of velocity = $\frac{v\cos37°}{5}$.

As the angle of rebound is equal to the angle before impact.

Therefore, both normal & tangential components of velocities must change by the same factor.

$\therefore$ Tangential velocity after impact becomes $\dfrac{v\sin 37^\circ}{5}$.

Let the time of impact be Δt. $N = \dfrac{m\left(v\cos 37^\circ + \dfrac{v\cos 37^\circ}{5}\right)}{\Delta t} = \dfrac{6mv\cos 37^\circ}{5\Delta t}$

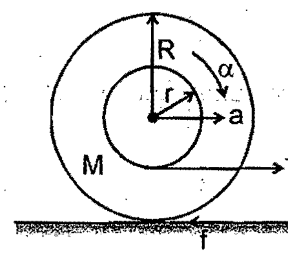
where N is the normal force imparted on the ball by the wall.

Frictional force = $\mu N = \dfrac{6}{5}\dfrac{\mu mv\cos 37^\circ}{\Delta t}$

Also frictional force = $\dfrac{m\left[v\sin 37^\circ - \dfrac{v\sin 37^\circ}{5}\right]}{\Delta t}$ $\Rightarrow$ $\dfrac{m\left[v\sin 37^\circ - \dfrac{v\sin 37^\circ}{5}\right]}{\Delta t} = \dfrac{6}{5}\dfrac{\mu mv\cos 37^\circ}{\Delta t}$

$\mu = \dfrac{2}{3}\tan 370$ $\Rightarrow$ $\mu = \dfrac{2}{3}\cdot\dfrac{3}{4} = \dfrac{1}{2}$ $\Rightarrow$ $\dfrac{1}{2} = \dfrac{x}{10}$ $\Rightarrow$ $x = 5$

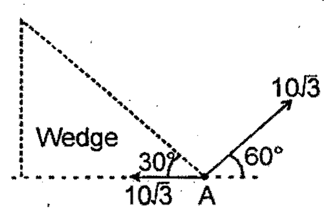
17. Since the spool rolls without slipping $a = R\alpha$ .... (1)
The F.B.D. of spool is shown in figure.
for linear motion
$T - f = Ma$
and for rotational motion



$f.R - T.r = \dfrac{MR^2}{2}\left(\dfrac{a}{R}\right)$. For maximum value of T, $f = \mu Mg$

On solving $T = \dfrac{3\mu MgR}{R+2r}$. Using values $T = \dfrac{3\times(0.4)\times 3\times 10\times 0.2}{0.2+0.2} = 18N$

18. From the frame of wedge
The component of velocity of ball normal
to wedge is $u_\perp = 10\sqrt{3} - 10\sqrt{3}\,\sin 30^\circ = 5\sqrt{3}$ m/s.



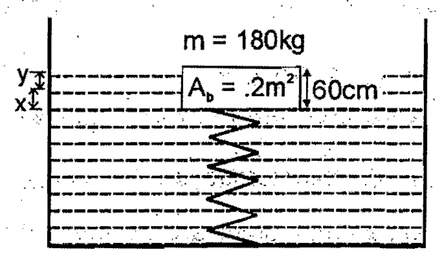
$\therefore$ Time of flight = $\dfrac{2u_\perp}{g\cos 30} = 2$ sec.

Hence the particle strikes the wedge after t = 2 sec.

19. As block goes down by distance x, water comes up by distance y. As both are measured from initial level of water, compression in the spring is x but the block is in depth (x + y) in water. Applying conservation of volume

$0.2x = (1m^2 - 0.2\ m^2).y$

$x = 4y \Rightarrow y = \dfrac{x}{4}$



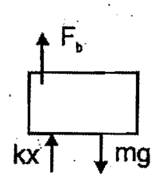
Thus total depth of block in water = $\dfrac{x}{4} + x = \dfrac{5x}{4}$

Free body diagram in equilibrium :

$F_b = (0.2)(5x/4)(1000)(10)$

For equilibrium :

$mg = kx + f_B$



$1800 = 2000\,x + (0.2)\left(\dfrac{5x}{4}\right)(1000)\,(10)$

$\Rightarrow 18 = 20x + 25x$ $\Rightarrow x = \dfrac{18}{45}m = 40$ cm

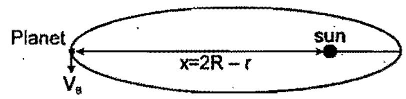
20. Area covered by line joining planet and sun in time dt is

$dS = \frac{1}{2}x^2 d\theta$; Areal velocity = $dS/dt = \frac{1}{2}x^2 d\theta/dt = \frac{1}{2}x^2\omega$

where x = distance between planet and sun

and ω = angular speed of planet about sun.

From Kepler's second law areal velocity of planet is constant.



At farthest position

$$A = dS/dt = \frac{1}{2}(2R-r)^2\omega = \frac{1}{2}(2R-r)[(2R-r)\omega] = \frac{1}{2}(2R-r)V_B$$

or $V_B = \frac{2A}{2R-r}$ (least speed). (Using values)

$V_B = 40$ km/s.

FULL SYLLABUS TEST - 2 (XII)

1.	(D)	2.	(D)	3.	(A)	4.	(A)	5.	(C)	6.	(BCD)	7.	(ACD)
8.	(ACD)	9.	(ABD)	10.	(BC)	11.	(B)	12.	(A)	13.	(C)	14.	(C)
15.	(D)	16.	9	17.	5	18.	5	19.	1	20.	4		

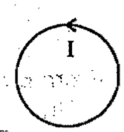
SOLUTIONS

1. $V_B - V_A = -\int E_x dx = -$ [Area under $E_x - x$ curve]

$V_B - 10 = -\frac{1}{2}.2.(-20) = 20$

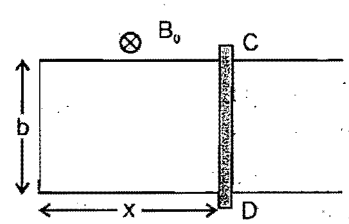
$V_B = 30$ V.

2. As soon as the field changes, current is induced in the anticlockwise direction.



Now direction of $\vec{M}$ and $\vec{B}$ are parallel thats why torque on coil is zero.

3. The magnetic flux must remain constant



$\therefore \phi_m = B_0 ab = \frac{B_0}{1+kt}bx$

where x is as shown

$\therefore$ $x = a(1+kt)$ or $v = \frac{dx}{dt} = ak$

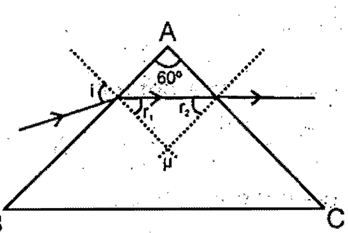
4. $r_2 < \theta_C$; $A - r_1 < \theta_C$

$r_1 > A - \theta_C$

$\Rightarrow$ $\sin r_1 > \sin(A - \theta_C)$

$\Rightarrow$ $\frac{\sin i}{\mu} > \sin(A - \theta_C)$

$\Rightarrow \sin i > \mu(\sin A \cos\theta_C - \sin\theta_C \cos A)$



$$= \sqrt{\frac{7}{3}}\left(\frac{\sqrt{3}}{2}\sqrt{1-\frac{3}{7}}-\sqrt{\frac{3}{7}}\cdot\frac{1}{2}\right)$$

$$= 1-\frac{1}{2}=\frac{1}{2}$$

$$\Rightarrow \quad \sin i > \frac{1}{2} \quad \text{or} \quad i > 30^0.$$

5. Initially the potential at centre of sphere is

$$V_C = \frac{1}{4\pi\epsilon_0}\frac{Q}{x} + \frac{1}{4\pi\epsilon_0}\frac{2Q}{x} = \frac{1}{4\pi\epsilon_0}\frac{3Q}{x}$$

After the sphere is grounded, potential at centre becomes zero. Let the net charge on sphere finally be q.

$\therefore \frac{1}{4\pi\epsilon_0}\frac{q}{r}+\frac{1}{4\pi\epsilon_0}\frac{3Q}{x} = 0$ or $q = -\frac{3Q}{x}r$ $\therefore$ The charge flowing out of sphere is $\frac{3Qr}{x}$.

6. Potential at each point on y-z plane is zero. The electric field will be zero on y-z plane at a distance $\sqrt{2}a$ from origin.

7. (A) $E = \frac{1}{2}CV^2$

As potential difference source between the plates is connected, p.d. remains constant. But capacitance C becomes KC hence energy stored is increased by factor K.

(B) Electric field $\frac{V}{d}$ is not changed.

(C) Charge on each plate is increased by factor K hence force between them increases by factor K^2. For effect of the medium, they must completely lie in the medium.

(D) Q = CV

Hence charge becomes KQ as C becomes KC and V remainin unchanged.

8. The potential at surface, 5 cm from surface and 10 cm from surface outwards is

$V_s = \frac{KQ}{R}$ .... (1) $\frac{KQ}{R+5} = 100$ .... (2) $\frac{KQ}{R+10} = 75$ .... (3)

From Equation 2 and 3 $\Rightarrow R = 10$ cm

$\therefore$ From equation 2

$$Q = \frac{100\times 15\times 10^{-2}}{9\times 10^{9}} = \frac{5}{3}\times 10^{-9}\,C \qquad \Rightarrow \text{B is false}$$

$$V_{surface} = \frac{KQ}{R} = \frac{100\times(R+5)}{R} = \frac{100\times 15}{10} = 150\,V \Rightarrow \text{A is true}$$

$$V_{centre} = \frac{3}{2}\frac{KQ}{R} = 225 \text{ volts} \qquad \Rightarrow \text{D is true}$$

$$E_{surface} = \frac{KQ}{R^2} = \frac{150}{10\times 10^{-2}} = 1500\text{ V/m.} \qquad \Rightarrow \text{C is true}$$

9. Rate of work done by external agent is :

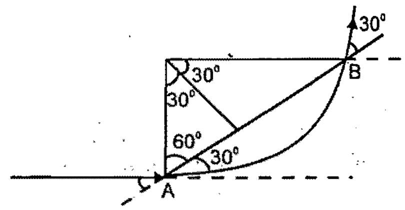
$\frac{dw}{dt} = \frac{BIL.dx}{dt} = BILv$ & thermal power dissipated in the resistor = eI = (BvL) I

clearly both are equal

If applied external force is doubled, the rod will experience a net force and hence acceleration. As a result velocity increases

Since ; $I = \frac{e}{R}$

On doubling 'R', current and hence required power becomes half. Since power is halved hence velocity is also halved as $P = BILv$



10. Arc $AB = \frac{\pi}{3} r = \frac{\pi m V}{3qB}$

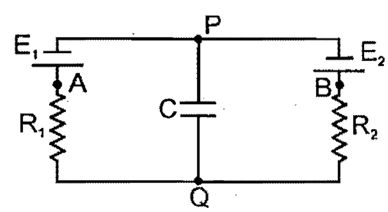
Time 't' $= \frac{\theta}{\omega} = \left(\frac{T}{2\pi}\right)\left(\frac{\pi}{3}\right) = \frac{T}{6} = \frac{\pi m}{3qB}$

11. From the passage: $b \propto \cot\theta/2$
therefore $\cot\theta/2 = 0. \Rightarrow \theta/2 = \pi/2 \Rightarrow \theta = \pi.$

12. β^+ emission:
$x \to y + \beta^+ + \upsilon \Rightarrow n \to P + \beta^+ + \upsilon$
$Q_1 = (M_x - M_y - 2me)C^2$
Electron capture:
$x \to y + \bar{\upsilon}$ + x–rays $\Rightarrow p + e^- \to n + \bar{\upsilon}$
$Q_2 = (M_x - M_y)C^2 \Rightarrow Q_1 > 0$ implies $Q_2 > Q$ but $Q_2 > 0$ does not necessarily mean $Q_1 > 0$.

13. & 14. Let potential of point P be zero
∴ Just after switch closed $V_Q = +3$ volts.
Also $V_A = +1$ volt
∴ Current through R_1 just after switch K

is closed is $\frac{V_Q - V_A}{R_1} = \frac{3-1}{2} = 1$ ampere



Any time after switch K is closed
$V_A = 1$ volt, $V_B = 3$ volt
∴ $V_B - V_A$ remains constant.
After the system has finally achived steady state, potential difference across capacitor.

$$V_P - V_Q = \frac{R_1R_2}{R_1+R_2}\left(\frac{E_1}{R_1}+\frac{E_2}{R_2}\right) = \frac{5}{3} \text{ volts} \quad \therefore \text{ Charge on capacitor} = C \times \frac{5}{3} = 50\ \mu C$$

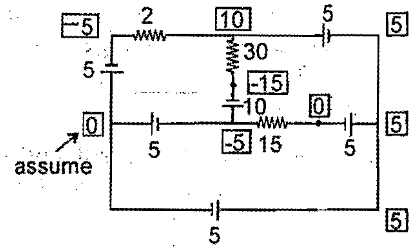
15. (P) Electrostatic potential energy $= \frac{1}{4\pi\epsilon_0}\frac{(-Q)^2}{2a} = \frac{Q^2}{8\pi\epsilon_0 a}$

(Q) Electrostatic potential energy $= \frac{1}{4\pi\epsilon_0}\left[\frac{(-Q)\times(-Q)}{5a/2} + \frac{(-Q)^2}{2(5a/2)}\right] = \frac{3}{20}\frac{Q^2}{\pi\epsilon_0 a}$

(R) Electrostatic potential energy $= \frac{1}{4\pi\epsilon_0}\frac{3Q^2}{5a} = \frac{3}{20}\frac{Q^2}{\pi\epsilon_0 a}$

(S) Electrostatic potential energy $= \frac{1}{4\pi\epsilon_0}\left[\frac{3Q^2}{5a} + \frac{(-Q)^2}{2(2a)} + \frac{(-Q)\times(-Q)}{2a}\right] = \frac{27Q^2}{80\pi\epsilon_0 a}$

16. Potentials are indicated in figure

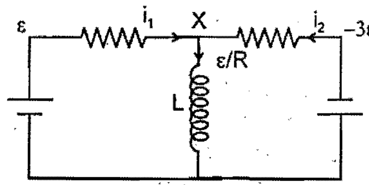


Current in 2Ω $= \frac{10-(-5)}{2} = \frac{15}{2} = 7.5$ A, leftwards

Current in 30 Ω $= \frac{10-(-15)}{30} = \frac{25}{30} = \frac{5}{6}$ A, downwards

$\frac{i_1}{i_2} = 9$

17. Just before S_2 is closed the current in the inductor is ε/R. It will not change just after S_2 is closed due to property of inductor. At the moment S_2 is closed



Let X be the potential of the junction then

$$i_1 + i_2 = \frac{\varepsilon}{R}$$

$$\frac{\varepsilon - X}{R} + \frac{-3\varepsilon - X}{2R} = \frac{\varepsilon}{R}$$

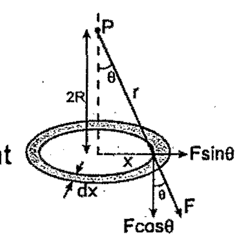
$\Rightarrow$ $X = -\varepsilon$

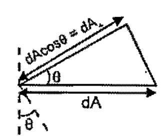
Now using Kirchoff's law ; $3\varepsilon + L\frac{di}{dt} - 2\varepsilon = 0$ $\Rightarrow \frac{di}{dt} = -\frac{\varepsilon}{L} = -10$ A/s

18. Number of photons striking per second = $N = \frac{IA}{h\nu}$

Area A here is the area perpendicular to the direction of intensity or direction of energy flow.

Consider a ring of radius x and width dx on the disc. Intensity I on the ring due to source is $I = \frac{P}{4\pi r^2}$





now, $dA_\perp = dA \cos\theta$
$dA = 2\pi x dx$

Gives, $dA_\perp = (2\pi x dx)\cos\theta$

A photon will exert force F as shown, only the Fcos θ component will remain and F sin θ will cancel out

as we integrate on the ring. $\frac{\Delta P}{\Delta t} = \frac{Nh}{\lambda} = \left(\frac{IdA_\perp}{h\nu}\right)\frac{h}{\lambda}$

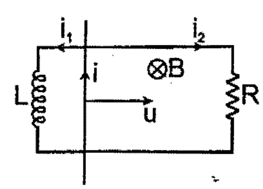
as only Fcosθ component of force remains

$$dF = \left(\frac{IdA_\perp}{h\nu}\right) \times \frac{h}{\lambda}\cos\theta$$

$$\therefore \int_0^F dF = \int_0^R \left[\frac{P}{4\pi(4R^2+x^2)}\right] \times \frac{(2\pi dx\cos\theta)}{\left(\frac{hC}{\lambda}\right)} \times \frac{h}{\lambda}\cos\theta \qquad [\because r = \sqrt{4R^2+x^2}\,]$$

solving we get $F = \frac{P}{20c}$

19. Let i_1 and i_2 be the current through L and R at any time t



$\therefore i = i_1 + i_2 \quad \Rightarrow \frac{B\ell v}{R} = i_2 \quad$ and $\quad B\ell v = L\frac{di_1}{dt}$

Force on conducting rod = $m\frac{dv}{dt} = -i\ell B = -\left(i_1 + \frac{B\ell v}{R}\right)\ell B$

$$\Rightarrow \quad m\,dv = -\ell B i_1 dt - \frac{B^2\ell^2}{R} v\,dt$$

$$\Rightarrow \quad m\int dv = -\ell B \int i_1 dt - \frac{B^2\ell^2}{R}\int v\,dt$$

$$\Rightarrow \quad m(v_f - u) = -\ell B Q - \frac{B^2\ell^2}{R} x$$

(v_f = velocity, when it has moved a distance 'x') $\Rightarrow$ $Q = \dfrac{-\frac{B^2\ell^2}{R}x - m(v_f-u)}{B\ell} = 1C.$

20. Let nth minima of 400 nm coincides with m th minima of 560 nm, then

$$(2n-1)\left(\frac{400}{2}\right) = (2m-1)\left(\frac{560}{2}\right) \quad \text{or} \quad \frac{2n-1}{2m-1} = \frac{7}{5} = \frac{14}{10} = \ldots\ldots$$

i.e. 4th minima of 400 nm coincides with 3rd minima of 560 nm.

Location of this minima is, $Y_1 = \dfrac{(2\times4-1)(1000)(400\times10^{-6})}{2\times0.1} = 14$ mm

Next 11th minima of 400 nm will coincide with 8th minima of 560 nm.
Location of this minima is,

$$+Y_2 = \frac{(2\times11-1)(1000)(400\times10^{-6})}{2\times0.1} = 42 \text{ mm}$$

$\therefore$ Required distance = $Y_2 - Y_1$ = 28 mm = 7X
Hence X = 4

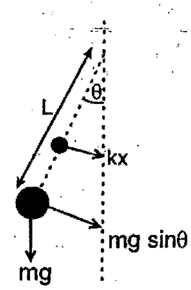
FULL SYLLABUS TEST - 3 (XI & XII)

1.	(D)	2.	(B)	3.	(B)	4.	(A)	5.	(C)	6.	(C)	7.	(A)
8.	(D)	9.	(A)	10.	(B)	11.	(ABCD)	12.	(ABC)	13.	(BD)	14.	(AB)
15.	(ABCD)	16.	(BD)	17.	(ABC)	18.	(AC)	19.	(AC)	20.	(BC)	21.	(C)
22.	(B)	23.	(A)	24.	(B)	25.	(C)	26.	(C)	27.	(C)	28.	(A)
29	(C)	30.	(B)	31.	153	32.	6	33.	3	34.	2	35.	7
36.	8	37.	20	38.	5	39.	6	40.	2				

SOLUTIONS

1. The extension developed in the string due to small values of 'θ' is :

$x = h\sin\theta \cong h\theta$



Torque about 'O' :

$\tau_0 = (Mg\sin\theta)L + (kx)h$

or, $\tau_0 \cong mg\,\theta L + kh^2\theta = (mgL + kh^2)\theta$ ... (1)

Also; $\tau_0 = I_0\alpha = mL^2\alpha$ ... (2)

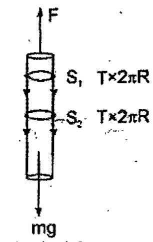
From (1) and (2) :

$mL^2\alpha = (mgL + kh^2)\theta$

or $\alpha = \dfrac{1}{L^2}\left(gL + \dfrac{kh^2}{m}\right)\theta$

Now : $T = 2\pi\sqrt{\dfrac{\theta}{\alpha}} = 2\pi\sqrt{\dfrac{\theta}{\frac{1}{L^2}\left(gL+\frac{kh^2}{m}\right)\theta}} \Rightarrow \upsilon = \dfrac{1}{T} = \dfrac{1}{2\pi L}\sqrt{\left(gL + \dfrac{kh^2}{m}\right)}$

2. The free body diagram of the capillary tube is as shown in the figure.
Net force F
required to hold tube is F = force due to surface tension at cross-section $(S_1 + S_2)$ + weight of tube.
$= (2\pi RT + 2\pi RT) + mg = 4\pi RT + mg$



Free body daigram of capillary tube

4. $V = V_1 + V_2 + V_3 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R} + \frac{1}{4\pi\varepsilon_0}\left(\frac{-2Q}{R}\right) + \frac{1}{4\pi\varepsilon_0}\left(\frac{3Q}{R}\right) = \frac{1}{4\pi\varepsilon_0}\cdot\left(\frac{2Q}{R}\right)$

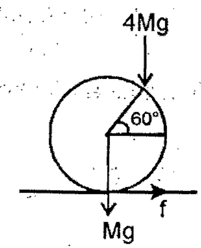
5. Just before opening the switch, the current in the inductor is ε/R. Energy stored in it $= \frac{1}{2}L\left(\frac{\varepsilon}{R}\right)^2$.

This energy will dissipate in the resistors R_1 and R_2 in the ratio $\frac{1}{R_1}$ & $\frac{1}{R_2}$.

6. $4Mg \times \frac{R}{2} - f \times R = I\alpha$

f = Ma and तथा N = 5 Mg

$\therefore \quad \mu_{min} = \frac{2}{7}$

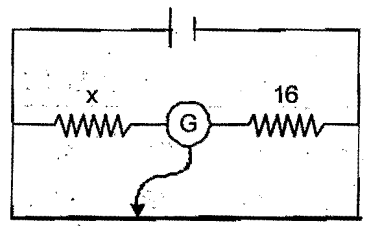


7. $\frac{x}{16} = \frac{36}{64} = 9\Omega$

$\frac{dx}{x} = \frac{d\ell}{\ell(1-\ell)} = \frac{10^{-1}\times 100}{36\times 64}$

$dx = \frac{9\times 10}{36\times 64} = \frac{5}{128}\Omega$

$x = 9 \pm \frac{5}{128}$



8. $\vec{E} = E_x\hat{i} + E_y\hat{j}$, $\Delta V = -E_x\,\Delta x - E_y\,\Delta y$
for A and B
$16 - 4 = -E_x(-2-2) - E_y(2-2)$
$E_x = 3$ V/m
for B and C
$12 - 16 = -E_x\{2-(-2)\} - E_y(4-2)$
$E_y = -4$ V/m. $\quad\therefore\quad \vec{E} = (3\hat{i} - 4\hat{j})$ V/m

9. Focal length of silvered mirror

$\frac{1}{f} = \frac{1}{f_m} - \frac{2}{f_\ell} \quad\Rightarrow\quad \frac{1}{f} = \frac{1}{\infty} - \frac{2}{f_\ell} \quad\Rightarrow\quad f = -\frac{f_\ell}{2}$

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \quad\Rightarrow\quad \frac{1}{-60} + \frac{1}{-30} = \frac{1}{f} \quad\Rightarrow\quad f = -20$ cm

Focal length of unsilvered lens will be 40 cm

So, $\frac{1}{v} + \frac{1}{30} = \frac{1}{40}$; $\quad \frac{1}{v} = \frac{1}{40} - \frac{1}{60} = \frac{3-2}{120} = +\frac{1}{120} \quad\Rightarrow\quad v = 120$ cm.

10. $\lambda_{air} = 2\Delta\ell \quad \Rightarrow \quad \lambda_{gas} = 2\Delta\ell'$

$$\frac{V_{gas}}{V_{air}} = \frac{f\lambda_{gas}}{f\lambda_{air}} = \frac{\Delta\ell'}{\Delta\ell} \quad \Rightarrow \quad V_{gas} = \frac{1000}{3} \times \frac{3}{2}\frac{\Delta\ell}{\Delta\ell} = 500\ ms^{-1}$$

11.

A	B
Molecular wt. = 16 M_0	Molecular wt. = M_0
mass = 2 m_0	mass = m_0
$n_A = \frac{n_0}{8}$	$n_B = n_0$

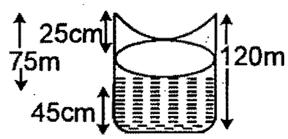
(A) K.E./atom $= \frac{f}{2}$k.T. $= \frac{f}{2}$ k.T. for both the gases.

(B) $C_{rms_A} = \sqrt{\frac{3RT}{16M_0}}$, $\quad C_{rms_B} = \sqrt{\frac{3RT}{M_0}}$, $\quad (C_{rms})_B = 4\,(C_{rms})_A$

(C) $(P)_A = \frac{(n_0/8)RT}{V}$, $\quad (P_B) = \frac{n_0 RT}{V}$, $\quad (P_B) = 8\,(P_A)$

(D) $n_B = 8\, n_A$

12. As $\quad V = n\lambda$



$\lambda = \frac{V}{\nu} = \frac{340}{340} = 1$m

first Resonance length

$R_1 = \frac{\lambda}{4} = \frac{1}{4}$ m = 25 cm

$\therefore \quad R_2 = \frac{3\lambda}{4} = \frac{3}{4}$m = 75 cm $\qquad \therefore \quad R_3 = \frac{5\lambda}{4} = \frac{5}{4}$m = 125 cm

i.e. third resonance does not establish. Now H_2O is poured,

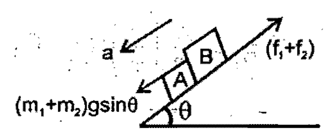
$\therefore$ Minimum length of H_2O Column to have the resonance = 45 cm

$\therefore$ Distance between two succensive nodes $= \frac{\lambda}{2} = \frac{1}{2}$ m = 50 cm

& maximum length of H_2O column to create resonance. i.e. 120 – 25 = 95 cm.

13. In standing waves, particles may have phase differences only 0 or π.

14. Independent acceleration of A and B would be



$a_1 = g\,(\sin\theta - \mu_1 \cos\theta)$

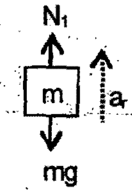
and $\quad a_2 = g\,(\sin\theta - \mu_2 \cos\theta)$

if $\mu_1 > \mu_2$, $a_1 < a_2$, so blocks will be in contact and will have common acceleration

$$a_1 = a_2 = \frac{(m_1+m_2)g\sin\theta - \mu_1 m_1 g\cos\theta - \mu_2 m_2 g\cos\theta}{(m_1+m_2)} = g[\sin\theta - (\mu_1+\mu_2)\cos\theta]$$

15. Charge on capacitor before insertion of dielectric slab = 100 μC
Charge on capacitor after insertion of dielectric slab = 300 μC
Increase in charge on the capacitor = 300 – 100 = 200 μC
charge increases very slowly that's why heat will not be produced
Energy supplied by the cell = increase in stored potential energy + work done on the person who is filling the dielectric slab. $\Rightarrow$ Heat produced is zero so (D) option is also correct.

17. At point A

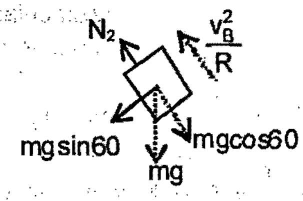


$$mgR = \frac{1}{2}mv_A^2 \Rightarrow V_A = \sqrt{2gR}$$

$$N_1 - mg = ma_r = \frac{mV_A^2}{R}$$

$\Rightarrow N_1 = mg + 2mg = 3mg$

At point B



$$mg(R\cos 60) = \frac{1}{2}mV_B^2 \Rightarrow V_B = \sqrt{gR}$$

$$N_2 - mg\cos 60 = \frac{mV_B^2}{R}$$

$$N_2 = \frac{3mg}{2} \quad ; \quad F_{net} = \frac{mV_B^2}{R} = mg$$

18. $\vec{F} = -\frac{\partial u}{\partial x}\hat{i} - \frac{\partial u}{\partial y}\hat{j}$

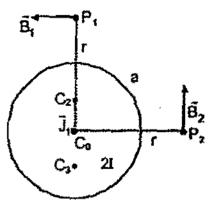
$\vec{F} = -(6xy^2 + 6)\,\hat{i} - (6x^2 y)\,\hat{j}$

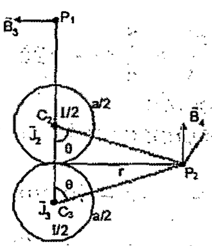
At (1,1)

$$\vec{F} = -12\hat{i} - 6\hat{j} \quad ; \quad |\vec{a}| = \frac{|\vec{F}|}{m}$$

Also, $w_{ext} = \Delta u$.

19.





Let $J_1 = J_0$ $\qquad J_2 = J_3 = -J_0$

$$\vec{B}_1 = \frac{\mu_0}{2}\left(\frac{a}{r}\right)^2(\vec{J}_1 \times \overline{C_0P_1}) \Rightarrow \vec{B}_2 = \frac{\mu_0}{2}\left(\frac{a}{r}\right)^2(\vec{J}_1 \times \overline{C_0P_2})$$

$$\vec{B}_3 = \frac{\mu_0}{2}\left(\frac{\frac{a}{2}}{r-\frac{a}{2}}\right)^2(\vec{J}_2 \times \overline{C_1P_2}) + \frac{\mu_0}{2}\left(\frac{\frac{a}{2}}{r+\frac{a}{2}}\right)^2(\vec{J}_3 \times \overline{C_2P_1})$$

$$\vec{B}_4 = \left|\frac{\mu_0}{2}\left(\frac{\frac{a^2}{4}}{r^2+\frac{a^2}{4}}\right)(\vec{J}_2 \times \overline{C_1P_2})\right| \sin q + \left|\frac{\mu_0}{2}\left(\frac{\frac{a^2}{4}}{r^2+\frac{a^2}{4}}\right)^2(\vec{J}_3 \times \overline{C_3P_2})\right| \sin q$$

$$\vec{B}_{P_1} = \vec{B}_1 + \vec{B}_3 = \frac{\mu_0 I}{\pi r}\left(\frac{2r^2 - a^2}{4r^2 - a^2}\right) \text{ to the left}$$

$$\vec{B}_{P_2} = \vec{B}_2 + \vec{B}_4 = \frac{\mu_0 I}{\pi r}\left(\frac{2r^2 + a^2}{4r^2 + a^2}\right) \text{ towards the top of the page}$$

20. $W_B = U_C + U_L$
when q is maximum $i = 0$ þ $U_L = 0$

$qE = \frac{q^2}{2C}$ þ $q_{max} = 2CE$

Charge on the capacitor will oscillates between 0 & 2CE
By KVL

$E = \frac{q}{C} + V_L$

Minimum value of q is zero so maximum value of $V_L = E$

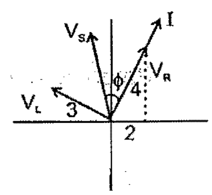
21. $V_T = \frac{mg}{b}$

It is greatest for the object with greatest value of $\frac{m}{b}$.

22. $\therefore \quad V_T = \frac{mg}{b} \quad \Rightarrow \quad \frac{m}{b} = 10^{-2}$ s

In this time sphere would achieve 0.63 fraction of its terminal speed 10 cm/s.

23. Let at an instant $v_R = (V_R)_m \cos\omega t \Rightarrow \therefore 2 = 4\cos\omega t \Rightarrow \cos\omega t = ½ \Rightarrow \therefore \cos\omega t = 60°$.
Since V_L is 90° ahead of $V_R \Rightarrow v_L = (V_L)_m \cos(\omega t + 90)$
$= -3\sin\omega t \Rightarrow = -3\sin 60° = -3\cos 30° \Rightarrow \therefore |(V_L)_m| = 3\cos 30°$



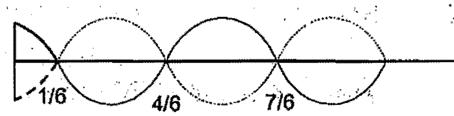
24. From phasor diagram $(V_S)_m = \sqrt{(V_R)_m^{\ 2} + (V_L)_m^{\ 2}} = 5$ volt. $\Rightarrow \tan\phi = \frac{V_L)_m}{V_R)_m} = \frac{3}{4}$

$\therefore \quad \phi = 37° \Rightarrow \quad \therefore \quad |v_S| = |(V_S)_m \cos(\omega t + 37°)|$
$= 5|\cos(60° + 37°)| \Rightarrow = 5|\cos 97°| \Rightarrow = 5\cos 83°$

26. $y_{net} = 4\text{ mm}\,[\sin(4\pi(\text{sec}^{-1})t + \frac{\pi}{6}]\cos\{2\pi(\text{m}^{-1})x + \frac{\pi}{6}\} = (4\text{mm})\cos(2\pi x + \frac{\pi}{6})\sin\{4\pi t + \frac{\pi}{6}\}$

position of antinode

$2\pi x + \frac{\pi}{6} = 2n\frac{\pi}{2} \Rightarrow x = (6n-1)\frac{1}{12} = \frac{5}{12}, \frac{11}{12}, \frac{17}{12}$ ....



27. $f_{max} = 0.2 \times 60 \times 10 = 120$ N
relative motion about to start when acceleration = 2 m/s² and
120t = 180 × 2
t = 3 sec.
For velocity of block at t = 3 sec.
t = 3 sec

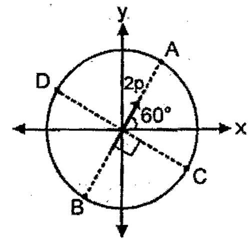
$a = \frac{120t}{180} = \frac{2t}{3}$

$v = \left.\frac{t^2}{3}\right|_0^3 = 3$ m/s. So, velocity of ball $= 3\hat{i} + 10\hat{j} + 5\hat{k}$

28. Time of flight $= \dfrac{2\times 10}{10} = 2$ sec

29 (P) Activity of the sample II becomes half in minimum time. Hence it has maximum disintegration constant.
(Q) Activity of the sample III takes maximum life to become half therefore it has maximum half - life.
(R) Parent nuclei will be left maximum in the sample, for which half life is maximum i.e. minimum decay.
(S) It can not be compared without information about atomic weight as energy radiated will depend upon no. of atoms, not upon amount of substance.

30. The resultant dipole moment has magnitude



$\sqrt{(\sqrt{3}\,P)^2 + P^2} = 2P$ at an angle $\theta = \tan^{-1}\dfrac{\sqrt{3}\,P}{P} = 60°$ with positive x direction.

Diameter AB is along net dipole moment and diameter CD is normal to net dipole moment.

Potential at A $\left(\dfrac{R}{2}, \dfrac{\sqrt{3}R}{2}\right)$ is maximum

Potential is zero at C $\left(\dfrac{\sqrt{3}R}{2}, -\dfrac{R}{2}\right)$ and D $\left(-\dfrac{\sqrt{3}R}{2}, \dfrac{R}{2}\right)$

Magnitude of electric field is $\dfrac{1}{4\pi\varepsilon_0}\dfrac{4p}{R^3}$ at A $\left(\dfrac{R}{2}, \dfrac{\sqrt{3}R}{2}\right)$ and B $\left(-\dfrac{R}{2}, -\dfrac{\sqrt{3}R}{2}\right)$

Magnitude of electric field is $\dfrac{1}{4\pi\varepsilon_0}\dfrac{2p}{R^3}$ at C $\left(\dfrac{\sqrt{3}R}{2}, -\dfrac{R}{2}\right)$ and D $\left(-\dfrac{\sqrt{3}R}{2}, \dfrac{R}{2}\right)$

32. Heat released by 5 kg of water when its temperature falls from 20°C to 0°C is,

$$Q_1 = ms\Delta\theta = (5)(10^3)(20 - 0) = 10^5 \text{ cal}$$

when 2 kg ice at –20°C comes to a temperature of 0°C, it takes an energy

$$Q_2 = ms\Delta\theta = (2)(500)(20) = 0.2 \times 10^5 \text{ cal}$$

The remaining heat

$Q = Q_1 - Q_2 = 0.8 \times 10^5$ cal will melt mass m of the ice, where

$$m = \frac{0.8\times 10^5}{80\times 10^3} = 1 \text{ kg}$$

So, the temperature of the mixture will be 0°C,
mass of water in it is 5 + 1 = 6 kg and mass of ice is 2 – 1 = 1 kg.

33. $(10 \times 11 - 10 \times 6) \times 10^{-4} \times 2T = \Delta w$

$$50 \times 10^{-4} \times 2T = 3 \times 10^{-4}$$

$$\Rightarrow \quad T = \frac{3}{100} = 3 \times 10^{-2} \text{ N/m.}$$

34. $a_t = \frac{F}{m}\sin\theta$

$$\frac{Rd^2(2\theta)}{dt^2} = \frac{F}{m}\sin\theta$$

$$\frac{d^2\theta}{dt^2} = \frac{F\sin\theta}{2mR} \qquad ...(1)$$

$$a_c = \frac{F}{m}\cos\theta$$

$$R\left[\frac{d}{dt}(2\theta)\right]^2 = \frac{F}{m}\cos\theta \qquad ...(ii)$$

$$\frac{\left(\frac{d^2\theta}{dt^2}\right)}{\left(\frac{d\theta}{dt}\right)^2} = 2\tan\theta = 2$$

35. $\Delta t = \frac{10\text{km}}{2\times10^8} - \frac{10\text{km}}{2.1\times10^8} = \frac{10\times10^3}{10^8}\left[\frac{2.1-2}{4.2}\right] = \frac{1}{10^4}\times\frac{1}{(42)}$

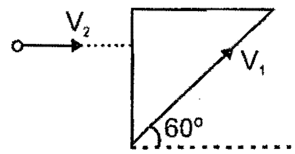
$f = 42 \times 10^4 = 420 \text{ Khz} = 60 \times \text{Khz} \quad \Rightarrow \quad X = 7$

36. $v\frac{dv}{dx} = 8 - 2x$

$$\Rightarrow \int_0^v v\,dv = \int_0^x (8-2x)dx \qquad \Rightarrow \frac{v^2}{2} = 8x - x^2 \qquad \Rightarrow v^2 = 16x - 2x^2$$

At B, $v = 0$ so, $x = 8$ Hence, $AB = 8$

37. $V_2 = V_1 \cos 60°$

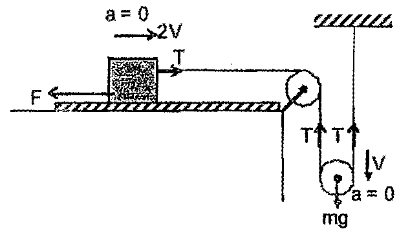


$V_2 = \frac{V_1}{2}$

$\frac{1}{2} \times 90 \times \frac{1}{2} = 1V_1 + \frac{1}{2} V_2 \frac{1}{2}$

$\frac{90}{4} = V_1 + \frac{V_2}{4}$; $V_1 = 20$ m/s.

38. By constraint relation, velocity of block is 2V. As acceleration for Block and disc is zero, for disc 2T = mg



$\Rightarrow T = \frac{mg}{2}$

and for block T = f

$\Rightarrow \quad \frac{mg}{2} = \eta A \frac{2V}{d}$

$\Rightarrow \quad \eta = \frac{mgd}{4AV}$

$= \frac{20\times10^{-3}\times10\times0.2\times10^{-3}}{4\times10^{3}\times10^{-4}\times2\times10^{-2}}$

$= 5 \times 10^{-3}$ NSm^{-2}

39. To enter the patrol in the tube

$P_b \le P_0 - \rho_\ell g\,(0.1)$

Applying Bernoulli equation from section A section B

a is area of cross-section A and b is area of cross-section B

$P_0 + \frac{1}{2}\rho_{air}V^2 = (P_0 - \rho_\ell g(0.1)) + \frac{1}{2}\rho_{air}\left(V\frac{a}{b}\right)^2$. Solving $\frac{a}{b} = \sqrt{11}$.

40. By the diagram, we can C

Current in R_2 = 6A

Current in R_1 = 3A

$\frac{I_2}{I_1} = 2A$