

SOLUTIONS

MATHEMATICS
Paper - II

14A

151.a. We have $\alpha^2 = 5\alpha - 3$

$$\Rightarrow \alpha^2 - 5\alpha + 3 = 0 \Rightarrow \alpha = \frac{5 \pm \sqrt{13}}{2}$$

$$\text{Similarly, } \beta^2 = 5\beta - 3 \Rightarrow \beta = \frac{5 \pm \sqrt{13}}{2}$$

$$\therefore \alpha = \frac{5 + \sqrt{13}}{2} \text{ and } \beta = \frac{5 - \sqrt{13}}{2} \text{ or vice - versa}$$

$$\alpha^2 + \beta^2 = \frac{50 + 26}{4} = 19 \text{ \& } \alpha\beta = \frac{1}{4}(25 - 13) = 3$$

Thus, the equation having $\frac{\alpha}{\beta}$ & $\frac{\beta}{\alpha}$ as its roots is

$$x^2 - x \left[\frac{\alpha}{\beta} + \frac{\beta}{\alpha} \right] + \frac{\alpha\beta}{\alpha\beta} = 0 \Rightarrow x^2 - x \left(\frac{\alpha^2 + \beta^2}{\alpha\beta} \right) + 10 = 0$$

$$\text{or } 3x^2 - 19x + 1 = 0$$

152.a $y = (x + \sqrt{1+x^2})^n$

$$\frac{dy}{dx} = n(x + \sqrt{1+x^2})^{n-1} \left(1 + \frac{1}{2}(1+x^2)^{-1/2} \cdot 2x \right)$$

$$\frac{dy}{dx} = n(x + \sqrt{1+x^2})^{n-1} \frac{(\sqrt{1+x^2} + x)}{\sqrt{1+x^2}} = \frac{n(\sqrt{1+x^2} + x)^n}{\sqrt{1+x^2}}$$

$$\text{or } \sqrt{1+x^2} \frac{dy}{dx} = ny \text{ or } \sqrt{1+x^2} y_1 = ny \left(y_1 = \frac{dy}{dx} \right)$$

$$\text{Squaring, } (1+x^2)y_1^2 = n^2 y^2$$

$$\text{Differentiating, } (1+x^2)2y_1 y_2 + y_1^2 \cdot 2x = n^2 \cdot 2y y_1$$

$$\text{(Here, } y_2 = \frac{d^2 y}{dx^2} \text{) or } (1+x^2)y_2 + xy_1^2 = x^2 y_1$$

153.c.1, $\log_3(3^{1-x} + 2)$, $\log_3(4 \cdot 3^x - 1)$ are in A.P.

$$\Rightarrow 2 \log_3(3^{1-x} + 2) = 1 + \log_3(4 \cdot 3^x - 1)$$

$$\log_3(3^{1-x} + 2) = \log_3 3 + \log_3(4 \cdot 3^x - 1)$$

$$\log_3(3^{1-x} + 2) = \log_3[3(4 \cdot 3^x - 1)]$$

$$3^{1-x} + 2 = 3(4 \cdot 3^x - 1) \text{ (Put } 3^x = t)$$

$$3 \cdot 3^{-x} + 2 = 12 \cdot 3^x - 3$$

$$3 \cdot 3^{-x} + 2 = 12t - 3 \text{ or } 12t^2 - 5t - 3 = 0:$$

$$\text{Hence } t = -\frac{1}{3}, \frac{3}{4} \Rightarrow 3^x = \frac{3}{4}$$

$$\Rightarrow x = \log_3 \left(\frac{3}{4} \right) \text{ or } x = \log_3 3 - \log_3 4 \Rightarrow x = 1 - \log_3 4$$

154.a. $P(E_1) = \frac{1}{2}$, $P(E_2) = \frac{1}{3}$ and $P(E_3) = \frac{1}{4}$;

$$P(E_1 \cup E_2 \cup E_3) = 1 - P(\bar{E}_1) P(\bar{E}_2) P(\bar{E}_3)$$

$$= 1 - \left(1 - \frac{1}{2}\right) \left(1 - \frac{1}{3}\right) \left(1 - \frac{1}{4}\right) = 1 - \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{3}{4}$$

155.b. $\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$; Period = $\frac{2\pi}{2} = \pi$

156.d. $l = AR^{p-1} \Rightarrow \log l = \log A + (p-1) \log R$

$$m = AR^{q-1} \Rightarrow \log m = \log A + (q-1) \log R$$

$$n = AR^{r-1} \Rightarrow \log n = \log A + (r-1) \log R$$

Now,

$$\begin{vmatrix} \log l & p-1 & \log A + (p-1) \log R \\ \log m & q-1 & \log A + (q-1) \log R \\ \log n & r-1 & \log A + (r-1) \log R \end{vmatrix} = 0$$

157.a. $\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{\sqrt{2x}} \Rightarrow \lim_{x \rightarrow 0} \frac{\sqrt{1 - (1 - 2 \sin^2 x)}}{\sqrt{2x}}$;

$$\lim_{x \rightarrow 0} \frac{\sqrt{2 \sin^2 x}}{\sqrt{2x}} \Rightarrow \lim_{x \rightarrow 0} \frac{|\sin x|}{x}$$

the function does not exist of LHS $\neq$ RHS

158.a. $AB = \sqrt{(4+1)^2 + (0+1)^2} = \sqrt{26}$

$$BC = \sqrt{(3+1)^2 + (5+1)^2} = \sqrt{52}$$

$$CA = \sqrt{(4-3)^2 + (0-5)^2} = \sqrt{26}$$

So, In isosceles triangle side $AB = CA$

For right angled triangle. $BC^2 = AB^2 + AC^2$

So, here $BC = \sqrt{52}$ or $BC^2 = 52$ or

$$(\sqrt{26})^2 + (\sqrt{26})^2 = 52$$

So, given triangle is right angled and also isosceles

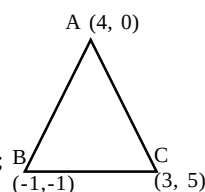
159.b. Total student = 100; for 70 stds. $75 \times 70 = 5250$

$$\Rightarrow 7200 - 5250 = 1950$$

$$\text{Average of girls} = \frac{1950}{30} = 65$$

160.a. $\cot^{-1}(\sqrt{\cos \alpha}) - \tan^{-1}(\sqrt{\cos \alpha}) = x$

$$\tan^{-1} \left(\frac{1}{\sqrt{\cos \alpha}} \right) - \tan^{-1}(\sqrt{\cos \alpha}) = x$$



$$\Rightarrow \tan^{-1} \frac{\frac{1}{\sqrt{\cos \alpha}} - \sqrt{\cos \alpha}}{1 + \frac{1}{\sqrt{\cos \alpha}} \cdot \sqrt{\cos \alpha}} = x$$

$$\Rightarrow \tan^{-1} \frac{1 - \cos \alpha}{2\sqrt{\cos \alpha}} = x \Rightarrow \tan x = \frac{1 - \cos \alpha}{2\sqrt{\cos \alpha}}$$

$$\text{OR } \cot x = \frac{2\sqrt{\cos \alpha}}{1 - \cos \alpha} \text{ or } \operatorname{cosec} x = \frac{1 + \cos \alpha}{1 - \cos \alpha}$$

$$\sin x = \frac{1 - \cos \alpha}{1 + \cos \alpha} = \frac{1 - (1 - 2 \sin^2 \alpha/2)}{1 + 2 \cos^2 \alpha/2 - 1} \text{ or } \sin x = \tan^2 \frac{\alpha}{2}$$

161.c. Order = 3, degree = 3

$$162.a. \frac{x-4}{1} = \frac{y-7}{5} = \frac{z-4}{4} \dots (i)$$

$$a(x-4) + b(y-7) + c(z-4) = 0 \dots (ii)$$

Line passing through point (3, 2, 0)

$$a + 5x + 4c + 0 \dots (iii)$$

Solving the equation we get by eqn (ii)

$$x - y + z = 1 \dots (iv)$$

$$163.b. \frac{d^2y}{dx^2} = e^{-2x}; \frac{dy}{dx} = \frac{e^{-2x}}{-2} + c; y = \frac{e^{-2x}}{4} + cx + d$$

$$164.d. \lim_{x \rightarrow \infty} \left(\frac{x^2 + 5x + 3}{x^2 + x + 3} \right)^{\frac{1}{x}} = \lim_{x \rightarrow \infty} \left(\frac{1 + \frac{5}{x} + \frac{3}{x^2}}{1 + \frac{1}{x} + \frac{3}{x^2}} \right)^{\frac{1}{x}} = 1$$

$$165.a. f(x) = \sin^{-1} \left(\log_3 \left(\frac{x}{3} \right) \right) \text{ exists if } -1 \leq \log_3 \left(\frac{x}{3} \right) \leq 1$$

$$\Leftrightarrow 3^{-1} \leq \frac{x}{3} \leq 3^1 \Leftrightarrow 1 \leq x \leq 9 \text{ or } x \in [1, 9]$$

166.b.

$$167.b. ar^4 = 2$$

$$a \times ar \times ar^2 \times ar^3 \times ar^4 \times ar^5 \times ar^6 \times ar^7 \times ar^8$$

$$= a^9 r^{36} = (ar^4)^9 = 2^9 = 512$$

$$168.d. \int_0^{10\pi} |\sin x| dx = 10 \left[\int_0^{\pi/2} \sin x dx + \int_{\pi/2}^{\pi} \sin x dx \right]$$

$$= 10 [\cos x]_0^{\pi/2} + [\cos x]_{\pi/2}^{\pi}; 10[1+1] = 10 \times 2 = 20$$

$$169.b. \int_0^{\pi/4} \tan^4 x (1 + \tan^2 x) dx =$$

$$\int_0^{\pi/4} \tan^n x \sec^2 x dx; = \int_0^1 t^n dt \text{ where } t = \tan x$$

$$I_n + I_{n+2} = \frac{1}{n+1}; \Rightarrow \lim_{x \rightarrow \infty} n[I_n + I_{n+2}]$$

$$= \lim_{x \rightarrow \infty} n \cdot \frac{1}{n+1} = \frac{n}{n+1} = \frac{n}{n \left(1 + \frac{1}{n} \right)} = 1$$

$$170.c. \int_1^0 [x^2] dx + \int_1^{\sqrt{2}} [x^2] dx = 0 + \int_1^{\sqrt{2}} dx = \sqrt{2} - 1$$

$$171.b. \int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x} dx = \int_{-\pi}^{\pi} \frac{2x}{1+\cos^2 x} + 2 \int_{-\pi}^{\pi} \frac{x \sin x}{1+\cos^2 x}$$

$$= 0 + 4 \int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx; 1 = 4 \int_0^{\pi} \frac{(\pi-x) \sin(\pi-x)}{1+\cos^2(\pi-x)}$$

$$1 = 4 \int_0^{\pi} \frac{(\pi-x) \sin x}{1+\cos^2 x} \Rightarrow 1 = 4\pi \int_0^{\pi} \frac{\sin x}{1+\cos^2 x}$$

$$-4\pi \int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx; \Rightarrow 2I = 4\pi \int_0^{\pi} \frac{\sin x}{1+\cos^2 x} dx$$

put $\cos x = t$ and solve it.

$$172.c. \text{ we have, } \lim_{x \rightarrow 2} \frac{xf(2) - 2f(x)}{x-2}$$

$$\left(\frac{0}{0} \right) = \lim_{x \rightarrow 2} \frac{f(2) - 2f(x)}{1 - 2} = f(2) - 2f(2) = 4 - 2 \times 4 = -4$$

$$173.b. \text{ Let } |z| = |\omega| = 1$$

$$\therefore z = re^{i\theta} = re^{i\phi} \text{ where } \theta + \phi = \pi$$

$$\therefore \bar{z} = re^{-i\theta}$$

$$\therefore \bar{z} = re^{i(\pi-\phi)} = re^{i\pi} \cdot e^{-i\phi} = -re^{-i\phi} = -\bar{\omega}$$

$$174.c. \text{ Given } |z-4| < |z-2| \text{ Let } z = x + iy$$

$$\Rightarrow |(x-4) + iy| < |(x-2) + iy|$$

$$\Rightarrow (x-4)^2 + y^2 < (x-2)^2 + y^2$$

$$\Rightarrow x^2 - 8x + 16 < x^2 - 4x + 4$$

$$\Rightarrow 12 < 4x \Rightarrow x > 3 \Rightarrow \operatorname{Re}(z) > 3$$

175.b.

176.b. Let a = first term of G.P.

r = common ratio of G.P. Then G.P. is a, ar, ar^2

$$\text{Given } S_{\infty} = 20 \Rightarrow \frac{a}{1-r} = 20 \Rightarrow a = 20(1-r) \dots (i)$$

$$\text{Also } a^2 + a^2 r^2 + a^2 r^4 + \dots \text{ to } \infty = 100$$

$$\Rightarrow \frac{a^2}{1-r^2} = 100 \Rightarrow a^2 = 100(1-r)(1+r) \dots (ii)$$

From (i), $a^2 = 400(1-r)^2$; From (ii) and (iii), we get

$$100(1-r)(1+r) = 400(1-r)^2$$

$$\Rightarrow 1+r = 4-4r \Rightarrow 5r-3 \Rightarrow r = 3/5$$

$$177.a. 1^3 - 2^3 + 3^3 - 4^3 + \dots + 9^3 = 1^3 + 3^3 + 5^3$$

$$+ \dots 9^3 - (2^3 + 4^3 + \dots + 8^3) = S_1 - S_2$$

$$\text{For } S_1, t_n = (2n-1)^3 = 8n^3 - 12n^2 + 6n - 1$$

$$S_1 = \sum t_n = 8 \sum n^3 - 12 \sum n^2 + 6 \sum n - \sum 1$$

$$= \frac{8n^2(n+1)^2}{4} - \frac{12n(n+1)(2n+1)}{6} + \frac{6n(n+1)}{2} - n$$

Here $n = 5$. Hence,

$$S_1 = 2 \times 25 \times 36 - 2 \times 5 \times 6 \times 11 + 3 \times 30 - 5 \\ = 1800 - 660 + 90 - 5 = 1890 - 665 = 1225.$$

$$\text{For } S_2, t_n = 8n^3; S_2 = \sum t_n = 8 \sum n^3$$

$$= \frac{8n^2(n+1)^2}{4} = 2 \times 16 \times 25 = 800. (\text{for } n = 4)$$

$$\therefore \text{Required sum} = 1225 - 800 = 425.$$

178.a. Let α, β and γ, δ are the roots of the equations.

$$x^2 + ax + b = 0 \text{ and } x^2 + bx + a = 0.$$

$$\therefore \alpha + \beta = -a, \alpha\beta = b \text{ and } \gamma + \delta = -b, \gamma\delta = a.$$

$$\text{Given } \alpha - \beta = \gamma - \delta \Rightarrow (\alpha - \beta)^2 = (\gamma - \delta)^2$$

$$\Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = (\gamma + \delta)^2 - 4\gamma\delta$$

$$\Rightarrow a^2 - 4b = b^2 - 4a \Rightarrow (a^2 - b^2) + 4(a - b) = 0$$

$$\Rightarrow a + b + 4 = 0 (\because a \neq b)$$

179.c.

$$180.a. p + q = -p \text{ and } pq = q \Rightarrow q(p - 1) = 0 \Rightarrow q = 0 \text{ or } p = 1.$$

$$\text{If } q = 0, \text{ then } p = 0. \text{ i.e., } p = q. \therefore p = 1 \text{ and } q = -2$$

$$181.a. ab + bc + ca = \frac{(a+b+c)^2 - 1}{2} < 1$$

$$182.d. \text{ Required number of numbers} \\ = 5 \times 6 \times 6 \times 4 = 36 \times 20 = 720.$$

$$183.c. \text{ Required number of numbers} = 3 \times 5 \times 5 \times 5 = 375$$

$$184.d. \text{ Required number are } 5! + 5! + \dots + 4! = 216.$$

$$185.b. \text{ Required sum} = (2 + 4 + 6 + \dots + 100) + \\ (5 + 10 + 15 + \dots + 100) - (10 + 20 + \dots + 100) \\ = 2550 + 1050 - 530 = 3050.$$

$$186.a. \text{ we have } t_{p+1} = {}^{p+q}C_p x^p \text{ and } t_{q+1} = {}^{p+q}C_q x^q$$

$${}^{p+q}C_p = {}^{p+q}C_q.$$

$$187.c. \text{ we have } 2^n = 4096 = 2^{12} \Rightarrow n = 12;$$

$$\text{so middle term} = t_7;$$

$$t_7 = t_{6+1} = {}^{12}C_6 = \frac{12!}{6!6!} = 924$$

188.c.

$$189.c. t_{r+2} = {}^{2n}C_{r+1} x^{r+1}, t_{3r} = {}^{2n}C_{3r-1} x^{3r-1}$$

$$\text{Given } {}^{2n}C_{r+1} = {}^{2n}C_{3r-1}; \Rightarrow {}^{2n}C_{2n-(r+1)}$$

$$= {}^{2n}C_{3r-1} \Rightarrow 2n - r - 1 = 3r - 1 \Rightarrow 2n = 4r$$

$$190.c. \text{ we have } \begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ ax+b & bx+c & 0 \end{vmatrix} \text{ By}$$

$$R_3 \rightarrow R_3 - (xR_1 + R_2); = \begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ 0 & 0 & -(ax^2 + 2bx + x) \end{vmatrix}$$

$$= (ax^2 + 2bx + c)(b^2 - ac) = (+)(-) = -ve.$$

$$191.b. a_1 = \sqrt{7} < 7. \text{ Let } a_m < 7 \text{ Then } a_{m+1} = \sqrt{7 + a_m}$$

$$\Rightarrow a_{m+1}^2 = 7 + a_m < 7 + 7 < 14.$$

$$\Rightarrow a_{m+1} < \sqrt{14} < 7; \therefore a_n < 7 \forall n. \therefore a_n \rightarrow 3$$

192.a.

193.d. Equation of AB is $x \cos \alpha + y \sin \alpha = p$

$$\Rightarrow \frac{x \cos \alpha}{p} + \frac{y \sin \alpha}{p} = 1; \Rightarrow \frac{1}{p/\cos \alpha} + \frac{1}{p/\sin \alpha} = 1$$

So co-ordinates of A and B are

$$\left(\frac{p}{\cos \alpha}, 0 \right) \text{ and } \left(0, \frac{p}{\sin \alpha} \right)$$

So coordinates of midpoint of AB are

$$\left(\frac{p}{2 \cos \alpha}, \frac{p}{2 \sin \alpha} \right) = (x_1, y_1) \text{ (let); } x_1$$

$$= \frac{p}{2 \cos \alpha} \text{ \& } y_1 = \frac{p}{2 \sin \alpha}$$

$$x = p/2 \cos \alpha \text{ and } y = p/2 \sin \alpha; \cos^2 \alpha + \sin^2 \alpha = 1$$

$$\Rightarrow \frac{p^2}{4} \left(\frac{1}{x_1^2} + \frac{1}{y_1^2} \right) = 1 \text{ Locus of } (x_1, y_1) \text{ is}$$

$$\frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{p^2}.$$

194.a.

$$195.a. 3a + a^2 - 2 = 0 \Rightarrow a^2 + 3a - 2 = 0;$$

$$\Rightarrow a = \frac{-3 \pm \sqrt{9+8}}{2} = \frac{-3 \pm \sqrt{17}}{2}$$

$$196.c. \text{ Equation of circles } x^2 + y^2 = 1 = (1)^2$$

$$\Rightarrow x^2 + y^2 = (y - mx)^2 \Rightarrow x^2 = m^2 x^2 - 2mxy;$$

$$\Rightarrow x^2(1 - m^2) + 2mxy = 0$$

$$\tan 45 = \pm \frac{2\sqrt{m^2 - 0}}{1 - m^2} = \frac{\pm 2m}{1 - m^2}; \Rightarrow 1 - m^2 = \pm 2m$$

$$\Rightarrow m^2 \pm 2m - 1 = 0$$

$$\Rightarrow m = \frac{-2 \pm \sqrt{4+4}}{2} = \frac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2}.$$

197.a. Let (h, k) be the centre of any such circle. Equation of such circle is $(x - h)^2 + (y - k)^2 = 3^2$ Since (h, k) lies on $x^2 + y^2 = 25$, $\therefore h^2 + k^2 = 25$.

$$x^2 + y^2 - (2xh + 2yk) + 25 = 9; \text{ Locus of } (h, k) \text{ is}$$

$$x^2 + y^2 = 16, \text{ which clearly satisfies (a).}$$

198.b.

199.c. Let ABC be an equilateral triangle, whose median is AD.

Given $AD = 3a$.

In $\triangle ABD$, $AB^2 = AD^2 + BD^2$;

$$\Rightarrow x^2 = 9a^2 + (x^2/4) \text{ where } AB = BC = AC = x.$$

$$\frac{3}{4}x^2 = 9a^2 \Rightarrow x^2 = 12a^2.$$

In $\triangle OBD$, $OB^2 = OD^2 + BD^2$

$$\Rightarrow r^2 = (3a-r)^2 + \frac{x^4}{4} \Rightarrow r^2 = 9a^2 - 6ar + r^2 + 3a^2;$$

$$\Rightarrow 6ar = 12a^2 \Rightarrow r = 2a$$

So equation of circle is $x^2 + y^2 = 4a^2$

200.b. Any tangent to the parabola $y^2 = 8ax$ is

$$y = mx + \frac{2a}{m} \dots\dots\dots(i)$$

If (i) is a tangent to the circle, $x^2 + y^2 = 2a^2$ then,

$$\sqrt{2a} = \pm \frac{2a}{m\sqrt{m^2+1}} \Rightarrow m^2(1+m^2) = 2$$

$$\Rightarrow (m^2+2)(m^2-1) = 0; \Rightarrow m = \pm 1.$$

So, from (i), $y = \pm(\lambda + 2a)$.

$$201.a. r_1 > r_2 > r_3 \Rightarrow \frac{\Delta}{s-a} > \frac{\Delta}{s-b} > \frac{\Delta}{s-c};$$

$$\Rightarrow s-a < s-b < s-c \Rightarrow -a < -b < -c; \Rightarrow a > b > c$$

202.b. The given equation is $\tan x + \sec x = 2\cos$;

$$\Rightarrow \sin x + 1 = 2\cos^2 x$$

$$\Rightarrow \sin x + 1 = 2(1 - \sin^2 x); \Rightarrow 2\sin^2 x + \sin x - 1 = 0;$$

$$\Rightarrow (2\sin x - 1)(\sin x + 1) = 0 \Rightarrow \sin x = \frac{1}{2};$$

$$\Rightarrow x = 30^\circ, 150^\circ, 270^\circ.$$

203.b.

$$204.a. \text{ we have } \lim_{n \rightarrow \infty} \frac{1^p + 2^p + \dots + n^p}{n^{p+1}}$$

$$\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{r^p}{n^p \cdot n} = \int_0^1 x^p dx = \left(\frac{x^{p+1}}{p+1} \right)_0^1 = \frac{1}{p+1}$$

205.d. Since $\lim_{x \rightarrow 0} \frac{1}{x}$ does not exist, hence the required limit does not exist.

$$206.a. \lim_{n \rightarrow \infty} \frac{\sqrt[n]{f(x)} - 1}{\sqrt[n]{x} - 1} \left(\frac{0}{0} \right) \text{ form}$$

Using L'Hospital's rule =

$$\lim_{x \rightarrow \infty} \frac{\frac{1}{2\sqrt{f(x)}} f'(x)}{\frac{1}{2\sqrt{x}}} = \frac{f'(1)}{\sqrt{f(1)}} = \frac{2}{1} = 2.$$

207.b.

208.d. $\therefore f'(x) - g''(x) = 0$ Integrating, $f'(x) - g(x) = c$;

$$\Rightarrow f'(1) - g'(1) = c \Rightarrow 4 - 2 = c \Rightarrow c = 2$$

$$\therefore f'(x) - g'(x) = 2; \text{ Integrating, } f(x) - g(x) = 2x + c_1$$

$$\Rightarrow f(2) - g(2) = 4 + c_1 \Rightarrow 9 - 3 = 4 + c_1;$$

$$\Rightarrow c_1 = 2 \therefore f(x) - g(x) = 2x + 2$$

$$\text{At } x = 3/2, f(x) - g(x) = 3 + 2 = 5.$$

209.c.f $(x+y) = f(x) \times f(y)$

Differentiate with respect to x, treating y as constant

$$f'(x+y) = f'(x) f(y)$$

Putting $x = 0$ any x , we get $f'(x) = f'(0) f(x)$;

$$\Rightarrow f'(5) = 3 f'(5) = 3 \times 2 = 6$$

210.a. Distance of origin from $(x, y) = \sqrt{x^2 + y^2}$

$$= \sqrt{a^2 + b^2 - 2ab \cos \left(t - \frac{a}{b} \right)} = \sqrt{a^2 + b^2 - 2ab}$$

$$\left(\because \max \cos \left(t - \frac{a}{b} \right) = 1 \right) = a - b$$

211.a. Let $f(x) = \frac{ax^3}{3} + \frac{bx^2}{2} + cx \Rightarrow f(0) = 0$ and

$$f(1) = \frac{a}{3} + \frac{b}{2} + c = \frac{2a+3b+6c}{6} = 0.$$

Also $f(x)$ is continuous and differentiable in $[0,1]$ and $(0,1)$. So by Rolle's theorem, $f'(x) = 0$ i.e. $ax^2 + bx + c = 0$ has at least one root in $[0,1]$.

212.d. we have $\int_0^2 f(x) dx = \frac{3}{4}$;

$$\text{Now, } \int_0^2 x f'(x) dx = x \int_0^2 f'(x) dx - \int_0^2 f(x) dx$$

$$= [x f(x)]_0^2 - \frac{3}{4} = 2f(2) - \frac{3}{4} = 0 - \frac{3}{4} (\because f(2) = 0) = -\frac{3}{4}.$$

213.a.

$$214.b. \text{ we have, } \vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \frac{\pi}{6} = 4 \times 2 \times \frac{\sqrt{3}}{2} = 4\sqrt{3}.$$

$$\text{Now, } (\vec{a} \times \vec{b})^2 + (\vec{a} \cdot \vec{b})^2 = a^2 b^2;$$

$$\Rightarrow (\vec{a} \times \vec{b})^2 + 48 = 16 \times 4 \Rightarrow (\vec{a} \times \vec{b})^2 = 16$$

215.a. we have,

$$\begin{aligned} [\vec{a} \times \vec{b} \cdot \vec{b} \times \vec{c} \cdot \vec{c} \times \vec{a}] &= (\vec{a} \times \vec{b}) \cdot \{(\vec{b} \times \vec{c}) \times (\vec{c} \times \vec{a})\} \\ &= (\vec{a} \times \vec{b}) \cdot \{(\vec{m} \cdot \vec{a}) \vec{c} - (\vec{m} \cdot \vec{c}) \vec{a}\} \quad (\text{Where } \vec{m} = \vec{b} \times \vec{c}) \\ &= \{(\vec{a} \times \vec{b}) \cdot \vec{c}\} \cdot \{(\vec{a} \cdot (\vec{b} \times \vec{c}))\} = [\vec{a} \cdot \vec{b} \cdot \vec{c}]^2 = 4^2 = 16. \end{aligned}$$

$$216.a. \vec{a} + \vec{b} + \vec{c} = 0 \Rightarrow \vec{b} + \vec{c} = -\vec{a} \Rightarrow (\vec{b} + \vec{c})^2 = (\vec{a})^2$$

$$= 5^2 + 3^2 + 2 \vec{b} \cdot \vec{c} = 7^2$$

$$\Rightarrow 2 |\vec{b}| |\vec{c}| \cos \theta = 49 - 34 = 15;$$

$$\Rightarrow 2 \times 5 \times 3 \cos \theta = 15;$$

$$\Rightarrow \cos \theta = 1/2; \Rightarrow \theta = \frac{\pi}{3} = 60^\circ$$

217.a. we have, $\vec{a} + \vec{b} + \vec{c} = \vec{0} \Rightarrow (\vec{a} + \vec{b} + \vec{c})^2 = 0$

$$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

$$\Rightarrow 25 + 16 + 9 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

$$\Rightarrow (\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = -25 \therefore |\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}| = 25$$

218.b.

219.b. we have $\vec{a} \times \vec{b} = 39 \vec{k} = \vec{c}$

Also $|\vec{a}| = \sqrt{34}, |\vec{b}| = \sqrt{45}, |\vec{c}| = 39;$

$$\therefore |\vec{a}| : |\vec{b}| : |\vec{c}| = \sqrt{34} : \sqrt{45} : 39$$

220.c.

221.a. $P(A \cup B) = P(A) + P(B) - P(A \cap B);$

$$\Rightarrow \frac{3}{4} = 1 - P(\bar{A}) + P(B) - \frac{1}{4}$$

$$\Rightarrow 1 = 1 - \frac{2}{3} + P(B) \Rightarrow P(B) = \frac{2}{3};$$

$$\text{Now, } P(\bar{A} \cap B) = P(B) - P(A \cap B) = \frac{2}{3} - \frac{1}{4} = \frac{5}{12}.$$

222.d. The event follows binomial distribution with $n = 5$, $p = 3/6 = 1/2$. $q = 1 - p = 1/2$; variance $= npq = 5/4$.

223.b. Equation of plane through $(1, 0, 0)$ is

$$a(x - 1) + by + cz = 0 \dots\dots\dots(i)$$

(i) Passes through $(0, 1, 0)$.

$$-a + b = 0 \Rightarrow b = a; \text{ Also, } \cos 45^\circ = \frac{a + a}{\sqrt{2(a^2 + a^2)}}$$

$$\Rightarrow 2a = \sqrt{2a^2 + c^2} \Rightarrow 2a^2 = c^2 \Rightarrow c = \sqrt{2}a.$$

So d.r of normal are $a, a, \sqrt{2}a$ i.e. $1, 1, \sqrt{2}$.

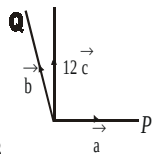
224.a. Let two forces be p and Q . Given $P + Q = 18$ and

$$P\hat{a} + Q\hat{b} = 12\hat{c}; \Rightarrow P\hat{a} - 12\hat{c} = -Q\hat{b}$$

$$\Rightarrow P^2 + 144 = Q^2 = (18 - P)^2;$$

$$\Rightarrow P^2 + 144 = 324 - 36P + P^2$$

$$\Rightarrow 36P = 180 \Rightarrow P = 5 \text{ and } Q = 13.$$



(where $\hat{a}$ and $\hat{b}$ are unit vectors along P and Q).

225.a.