தமிழ்நாடு அரசு
வாய்மையே வெல்லும்

EDUCATION DEPARTMENT
VILLUPURAM DISTRICT
X STANDARD

MATHEMATICS

REVISED SYLLABUS MATERIAL – Q&A

2020 – 2021

BEST WISHES

Mrs.K.Krishnapriya, B.Sc., M.A., B.Ed.,
Chief Educational Officer
Villupuram District

தன்னம்பிக்கை + விடாமுயற்சி + கடின உழைப்பு = வெற்றி

“The Struggle you’re in Today will definitely develop the strength you need for Tomorrow””

MESSAGE TO TEACHERS

First and Foremost I would like to express my hearty gratitude to all the teachers who are taking much effort to attain the outstanding performance in Tenth Public examination in this academic year.

Congratulations to all the teachers who are taking atmost care to improve the level of gifted students and as well as the slow learners even in the period of COVID – 19 with precautious measures.

To enhance the percentage of X standard result in Villupuram District in the State level should be our Motto.

"In my point of view a dedicated and service – minded teacher is blessed by the God ever"

Hence it is my appeal to all the Tenth handling teachers to devote more time for the welfare and upliftment of the poor, the destitute, the down trodden and the rural pupils fruitfully.

With Best Wishes

Mrs.K.Krishnapriya, B.Sc., M.A., B.Ed.,
Chief Educational Officer
Villupuram District

பள்ளிக்கல்வித்துறை விழுப்புரம் மாவட்டம்
10ம் வகுப்பு குறைக்கப்பட்ட கணித பாடத்திட்டம்
2020 – 2021

இயல்/ Chapter	எடுத்துக்காட்டு (Example)	பயிற்சி (Exercise)	One Marks
1	Eg. 1.1 – 1.5	Ex.1.1,1.2	Ex. 1.6 Q.No. 1 – 7
2	Eg. 2.1 – 2.10 & 2.19 – 2.30	Ex. 2.1, 2.2, 2.4, 2.5	Ex. 2.10 Q.No.1 – 5 & 7-10
3	Eg. 3.1 – 3.46 & 3.51 – 3.55	Ex.3.1 – 3.14 & 3.16	Ex. 3.20 Q.No. 1 – 13
4	Eg. 4.1 – 4.34 (அடிப்படை விகிதசம தேற்றத்தின் மறுதலை, கோண இருசமவெட்டித் தேற்றத்தின் மறுதலை, பிதாகரஸ் தேற்றத்தின் மறுதலை, மாற்று வட்டத்துண்டு தேற்றம் ஆகியவற்றின் கூற்றுகள் மட்டும் (நிரூபணமின்றி) Converse of BPT, Converse of ABT, Converse of Phythagorous, Tangent Chord Theorem.(Only Statement, Without Proof)	Ex.4.1 – 4.4	Ex. 4.5 Q.No. 1 – 15
5	Eg.5.1 – 5.29	Ex. 5.1 - 5.3	Ex. 5.5 Q.No.1 – 9 & 12 - 15
6	Eg. 6.18 – 6.33	Ex. 6.2 - 6.4	Ex. 6.5 Q.No.10 - 15
7	Eg. 7.1 – 7.28	Ex. 7.1 – 7.3	Ex. 7.5 Q.No. 1 – 15
8	Eg. 8.17 – 8.25 & Eg. 8.27, 8.31	Ex. 8.3 Ex. 8.4 Q.No. : 2(i), (ii), 3, 5 & 7	Ex. 8.5 Q.No. 9 - 15

அலகுத்தேர்வு பாடத்திட்டம் (Unit Test)

அலகுத்தேர்வு **1** / Unit Test 1	Chapter 1 & 2 வடிவியல் – வடிவொத்த முக்கோணங்களை வரைதல் மற்றும் சிறப்பு முக்கோணங்கள் வரைதல்/ Geometry – Construction of Similar Triangles and Special Triangles
அலகுத்தேர்வு **2**/ Unit Test 2	Chapter 3 & 4 வரைபடம்/Graph Ex.3.16, Eg.3.51 – 3.55
அலகுத்தேர்வு **3**/ Unit Test 3	Chapter 5 & 6 வடிவியல் தொடுகோடு வரைதல் / Geometry – Construction of Tangents to a Circle
அலகுத்தேர்வு **4**/ Unit Test 4	Chapter 7 & 8 வரைபடம்/ Graph Ex.3.16

STAGE - 1

S.No.	CHAPTERS	1	2	5	8	Total
1	GEOMETRY	1	-	1	1	14
2	GRAPH	-	-	-	1	8
3	RELATIONS AND FUNCTIONS	2	2	1	-	11
4	NUMBERS AND SEQUENCES	2	1	-	-	4
5	ALGEBRA	2	2	1	-	11
6	CO-ORDINATE GEOMETRY	2	1	1	-	9
7	TRIGNOMETRY	1	1	-	-	3
8	MENSURATION	2	-	-	-	2
9	PROBABILITY	2	2	2	-	16
	TOTAL	14	18	30	16	78

STAGE - 2

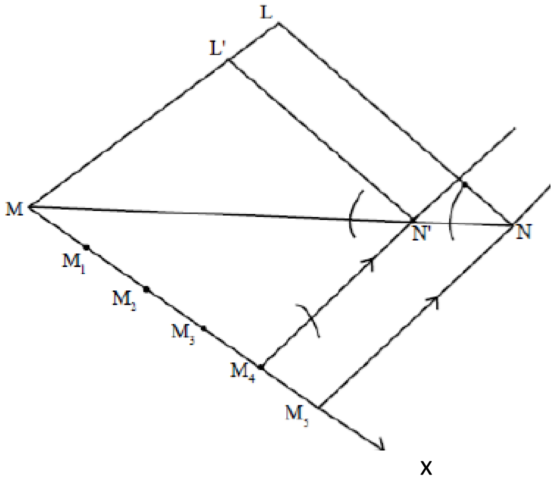
S.No.	CHAPTERS	1	2	5	8	Total
1	GEOMETRY	1	1	1	1	16
2	GRAPH	-	-	-	1	8
3	RELATIONS AND FUNCTIONS	2	2	1	-	11
4	NUMBERS AND SEQUENCES	2	1	1	-	9
5	ALGEBRA	2	2	2	-	16
6	CO-ORDINATE GEOMETRY	2	1	2	-	14
7	TRIGNOMETRY	1	1	1	-	8
8	MENSURATION	2	1	2	-	14
9	PROBABILITY	2	2	2	-	16
	TOTAL	14	22	60	16	112

- ✓ If Students Practice Stage – 1 only, sure they get minimum of 70marks.
- ✓ If Students Practice Stage – 2 with Stage - 1, sure he / she get minimum of 90 marks

PREPARED BY
A. SIVAMOORTHY, M.Sc, M.Phil., B.Ed.,
B.T. Asst., Government High School,
Perumpakkam / Villupuram District
9080961984, 9750827997

STAGE – 1 GEOMETRY

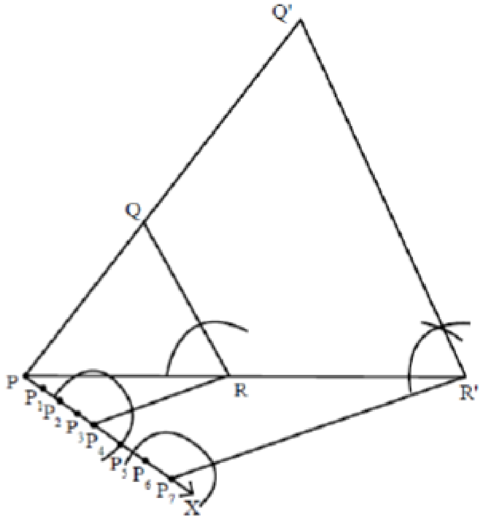
1. Construct a triangle similar to a given triangle LMN with its sides equal to $\frac{4}{5}$.



CONSTRUCTION

1. Construct a ΔLMN with any measurement.
2. Draw a ray MX making an acute angle with MN on the side opposite to vertex L. Locate 5 points(greater of 4 and 5 in $\frac{4}{5}$) points. M_1, M_2, M_3, M_4, and M_5 so that $MM_1 = M_1M_2 = M_2M_3 = M_3M_4 = M_4 M_5$
3. Join M_5 to N and draw a line through M_4 parallel to M_5N to intersect MN at N'.
4. Draw line through N’ parallel to the line LN intersecting line segment ML to L’.
5. Then$\Delta L'MN'$ is the required Δ.

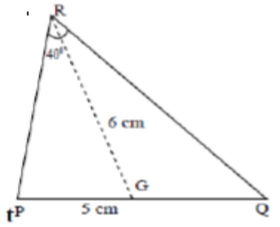
2. Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{7}{3}$ of the corresponding sides of the triangle PQR (Scale Factor $\frac{7}{3}$)

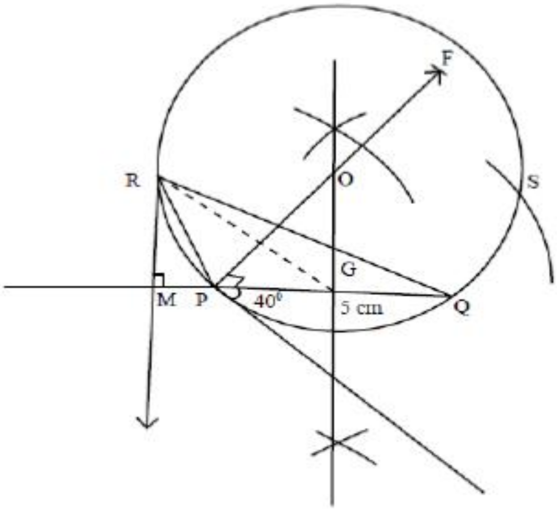


CONSTRUCTION

1. Construct a ΔPQR With any measurement.
2. Draw a ray PX making an acute angle with PR on the side opposite to vertex Q. Locate 7 points (Greater of 3 and 7 in $\frac{7}{3}$) points. $P_1, P_2, P_3, P_4, P_5, P_6, P_7$ on PX so sthat $PP_1 = P_1P_2 = P_2P_3 \ldots\ldots = P_6P_7$.
3. Join P_3R and draw a line through P_7 parallel to to intersecting the extended line segment PR at R’. Draw line through R' parallel to QR intersect the extended line segment PQ to Q'.
4. Then $\Delta PQ'R'$ is the required Δ .

3. Construct a ΔPQR in which PQ = 5 cm ∠P = 40°and the median PG from P to QR is 4.4 cm. Find the length of the altitude from P to QR.

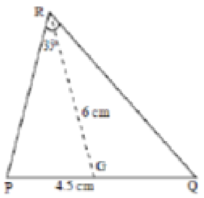


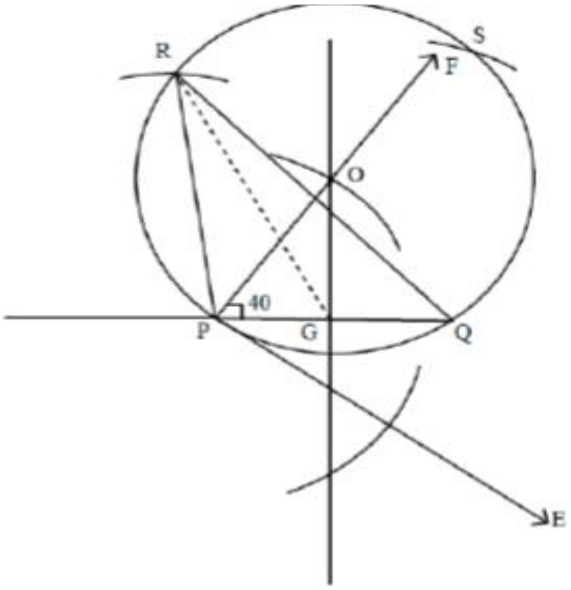


CONSTRUCTION

1. Draw a line segment QR = 5cm. At P, draw QE such that $\angle RQE = 40°$. At Q, dra QF such that ∠EQF = 90°.
2. Draw the perpendicular bisector to QR, meets QF at O and QR at G.
3. With O as centre and OQ as radius draw a circle.
4. From G mark arcs of radius 4.4 cm on the circle.
5. Join PR,PQ.Then ΔPQR is the required Δ.
6. Length of altitude is PM = 3 cm.

4. Construct a ΔPQR which the base PQ = 4.5cm, $\angle R = 35°$ and the median from R to PQ is 6 cm.

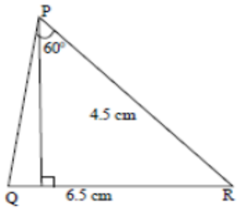


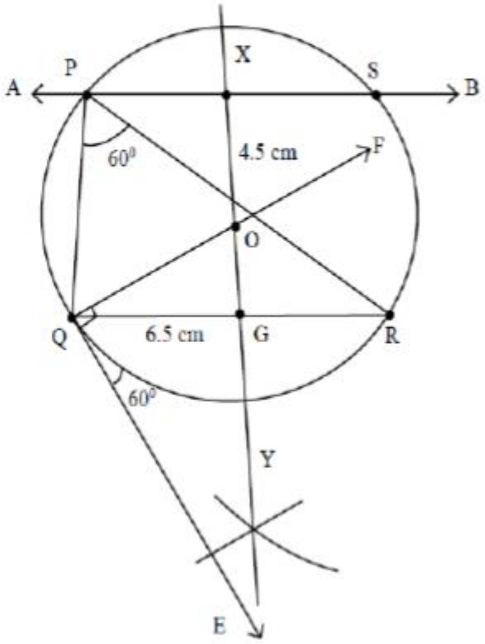


CONSTRUCTION

1. Draw a line segment PQ = 4.5cm. At P, draw PE such that $\angle QPE = 35°$. At P, draw PF such that ∠EPF = 90°.
2. Draw the perpendicular bisector to PQ, meets PF at O and PQ at G.
3. With O as centre and OP as radius draw a circle.
4. From G mark arcs of 6 cm on the circle at RAS.
5. Join PR,RQ.Then ΔPQR is the required Δ.
6. Join RG, which is the median.

5. Construct a ΔPQR such that QR = 6.5cm, ∠P = 60°and the altitude from P to QR is of length 4.5 cm.

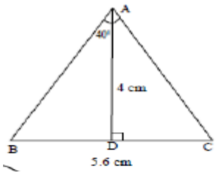


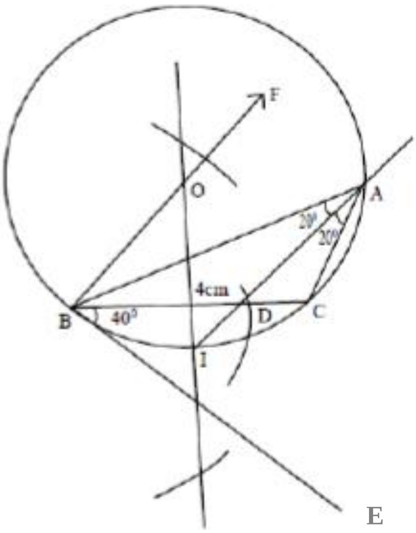


CONSTRUCTION

1. Draw a line segment QR = 6.5cm. At Q, draw QE such that $\angle RQE$ = 60°. At Q, draw QF such that ∠EQF = 90°.
2. Draw the perpendicular bisector to XY to QR intersects QF at O & QR at G.
3. With O as centre and OQ as radius draw a circle.
4. XY intersects QR at G. On XY, from G, mark arc M such that GM = 4.5cm.
5. Draw AB, through M which is parallel to QR. AB meets the circle at P and S.
6. Join QP, RP. Then ΔPQR is the required Δ.

6. Draw a triangle ABC of base BC=5.6cm, ∠A=40° and the bisector of ∠Ameets BC at D such that BD = 4cm.

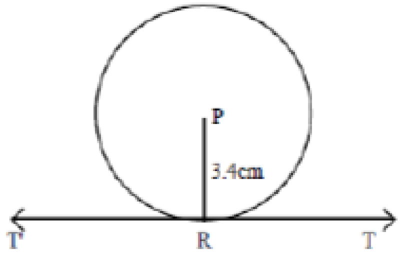


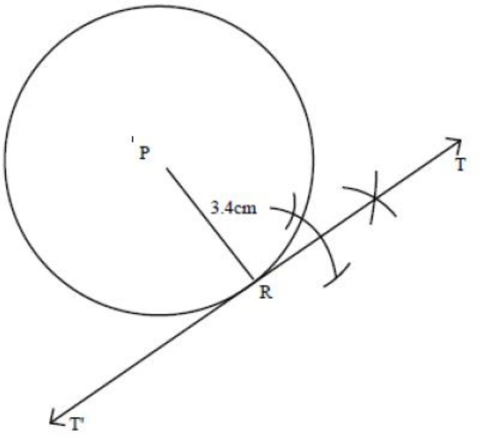


CONSTRUCTION

1. Draw a line segment BC = 5.6cm. At B, draw BE such that $\angle CBE$ = 40°. At B, draw BF such that $\angle EB$F = 90°.
2. Draw the perpendicular bisector to BC meets BF at O & BC at G.
3. With O as centre and OB as radius draw a circle.
4. From B, mark an arc of 4cm on BC at D. The perpendicular meets the circle at I & Join ID.
5. ID producted meets the circle at A. Join AB & AC.
6. Then ΔABC is the required Δ.

7. Draw a tangent at any point R on the circle of radius 3.4cm and centre at P.

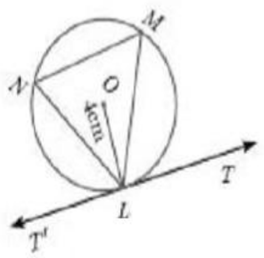


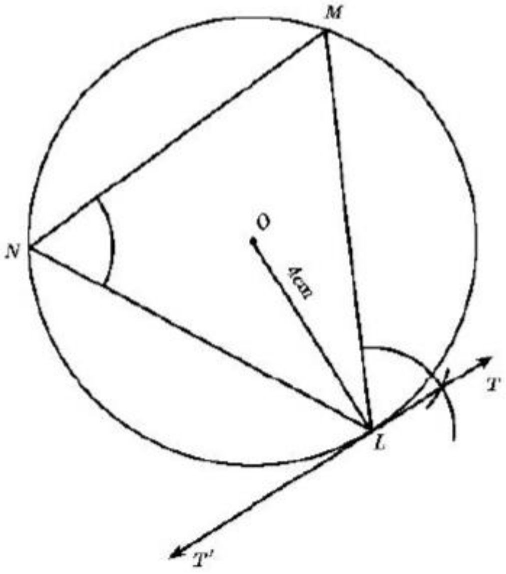


CONSTRUCTION

1. Draw a circle with centre at P of radius 3.4 cm.
2. Take a point R on the circle and Join PR.
3. Draw perpendicular line TT' to PR which passes through R.
4. TT' is the required tangent.

8. Draw a circle of radius 4cm. At a point L on it draw a tangent to the circle using the alternate segment.

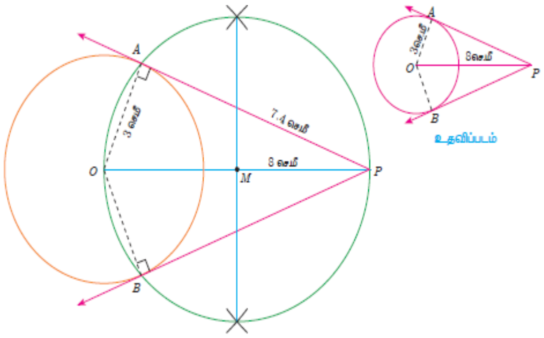




CONSTRUCTION

1. With O as the centre, draw a circle of radius 4 cm.
2. Take a point L on the circle. Through L draw any chord LM.
3. Take a point M distinct from L and N on the circle, so that L, M and N are in anticlockwise direction. Join LN and NM.
4. Through L draw a tangent TT’ such that $\angle TLM = \angle MNL$.
5. TT' is the required tangent.

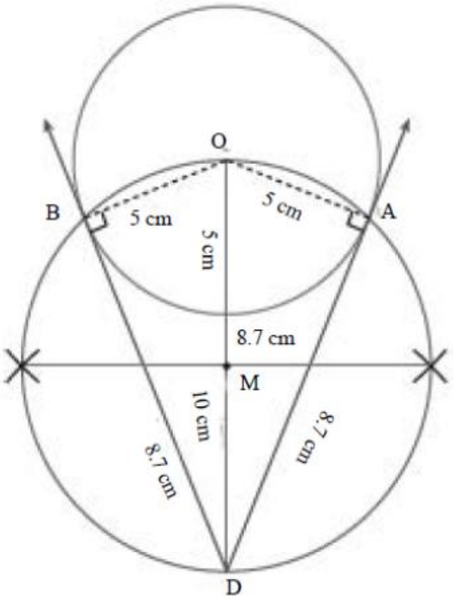
9. Draw a circle of diameter 6cm from a point P, which is 8cm away from its centre. Draw the two tangents PA and PB to the circle and measure their lengths.



CONSTRUCTION

1. With centre at O, draw a circle of radius 3cm.
2. Draw a line OP of length 8cm.
3. Draw a perpendicular bisector of OP which cuts OP at M.
4. With M as centre and MO as radius, draw a circle which cuts previous circle at A and B.
5. Join AP and BP, AP and BP are the required tangents. Thus lengthof the tangents are PA = PB = 7.4cm.

10. Draw the two tangents from a point which is 10cm away from the centre of a circle of radius 5cm. Also, measure the lengths of the tangents.



CONSTRUCTION

1. With centre at O, draw a circle of radius 5cm.
2. Draw a line OP = 10 cm
3. Draw a perpendicular bisector of OP, which cuts OP at M.
4. With M as centre and MO as radius, draw a circle which cuts previous circle at A and B.
5. Join AP and BP. AP and BO are the required tangents. Thus length of the tangents are PA = PB = 8.7 cm/

Proof:

In ΔOPA

$$PA^2 = OP^2 - OA^2 = 10^2 - 5^2$$

$$= 100 - 25 = 75$$

$$PA = \sqrt{75} = 8.6\text{cm. (approx)}$$

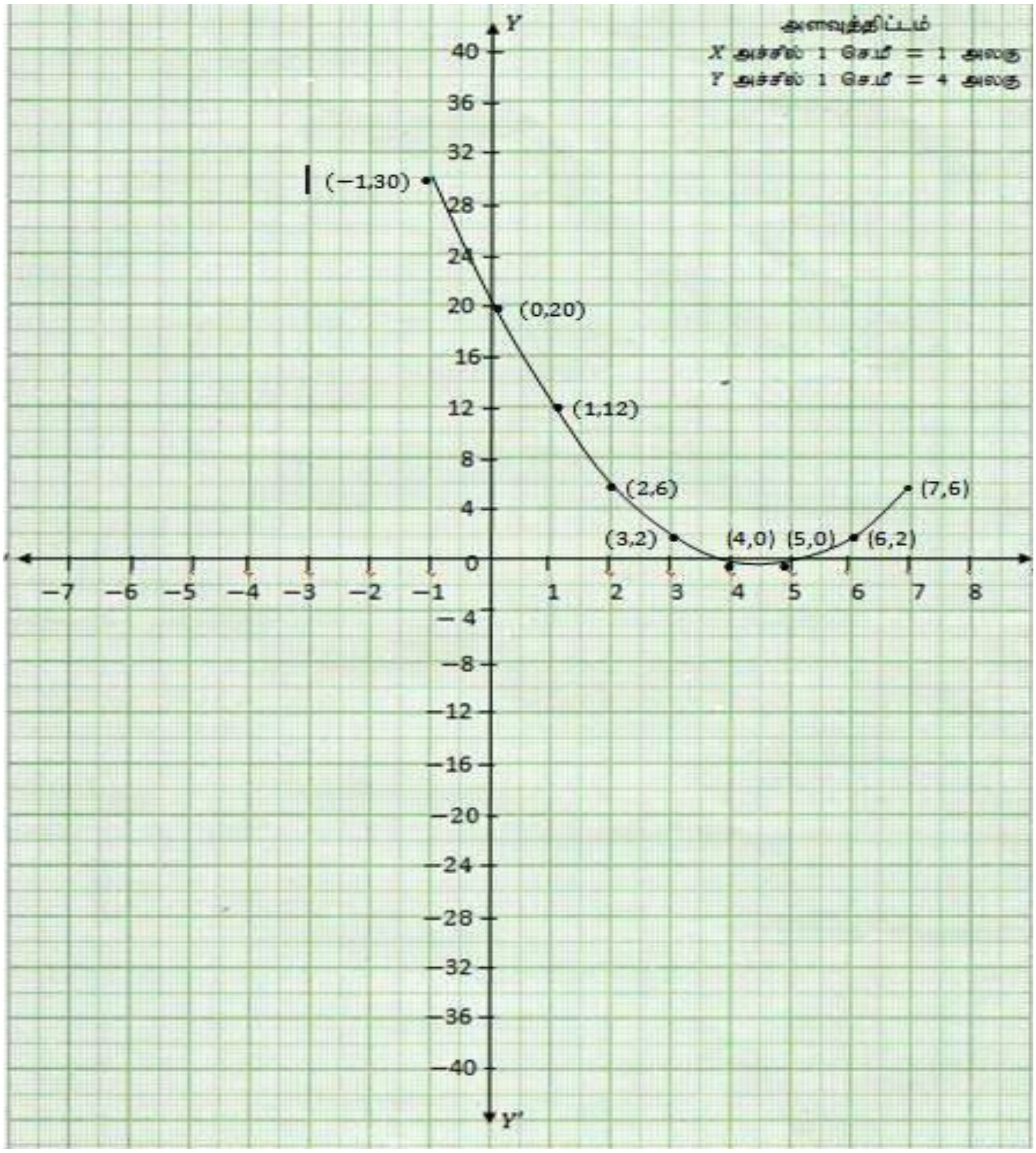
GRAPHS

1. Discuss the nature of solution of the following quadratic equation. x^2 - 9x + 20 = 0.

Table:

x	-4	-3	-2	-1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
-9x	36	27	18	9	0	-9	-18	-27	-36
+20	20	20	20	20	20	20	20	20	20
y	72	56	42	30	20	12	6	2	0

Points (-4, 72), (-3, 56), (-2, 42), (-1, 30), (0, 20), (1,12), (2, 6), (3,2), (4, 0)



Solution

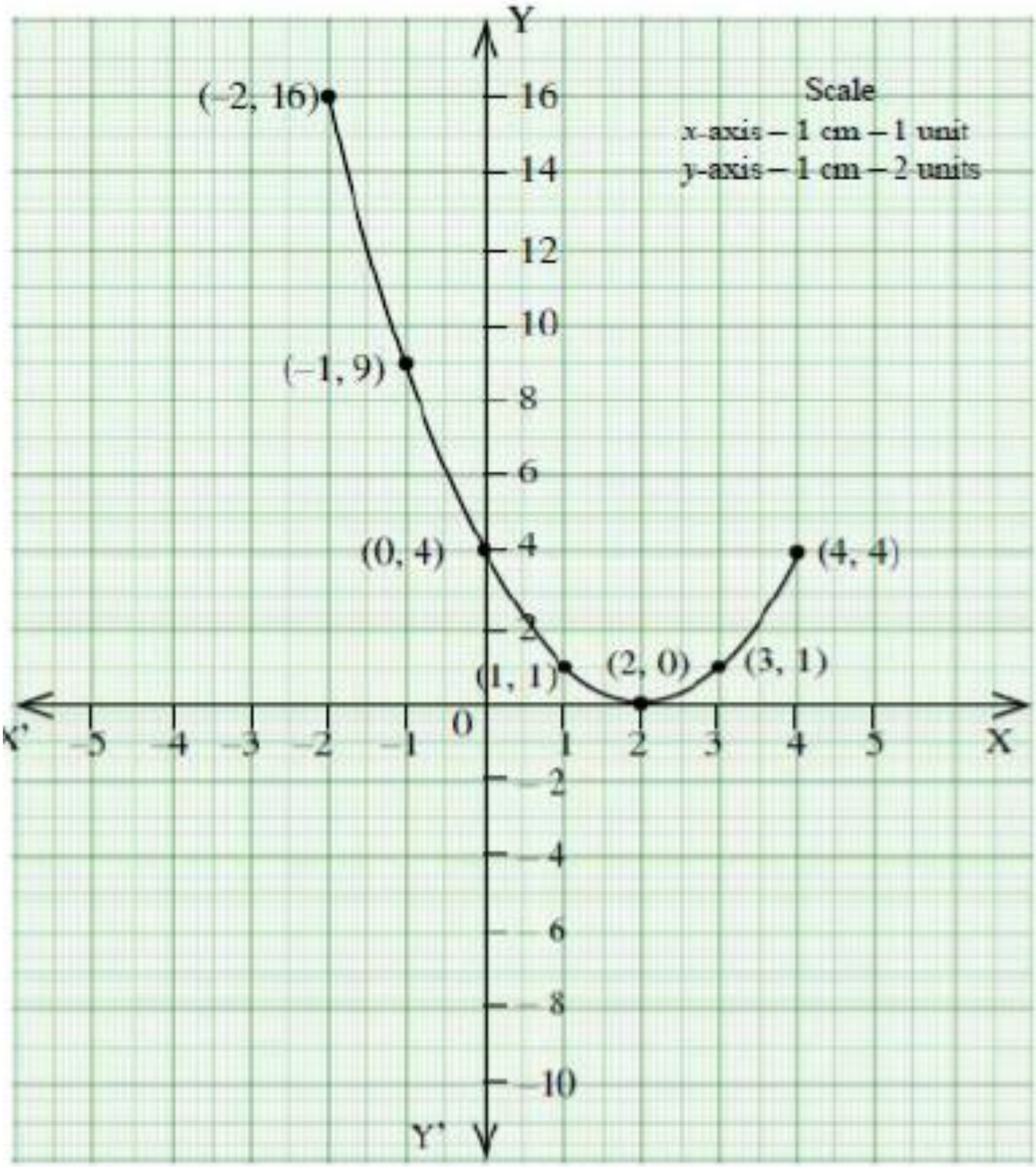
Since there are two points of intersection with the X-axis. Therefore the roots are real and unequal

2. Discuss the nature of solution of the following quadratic equation.: x^2 - 4x + 4 = 0.

Table

x	-4	-3	-2	-1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
-4x	16	12	8	4	0	-4	-8	-12	-16
+4	4	4	4	4	4	4	4	4	4
y	36	25	16	9	4	1	0	1	4

Points (-4, 36), (-3, 25), (-2, 16), (-1, 9), (0, 4), (1,1), (2, 0), (3,1), (4, 4)



Solution

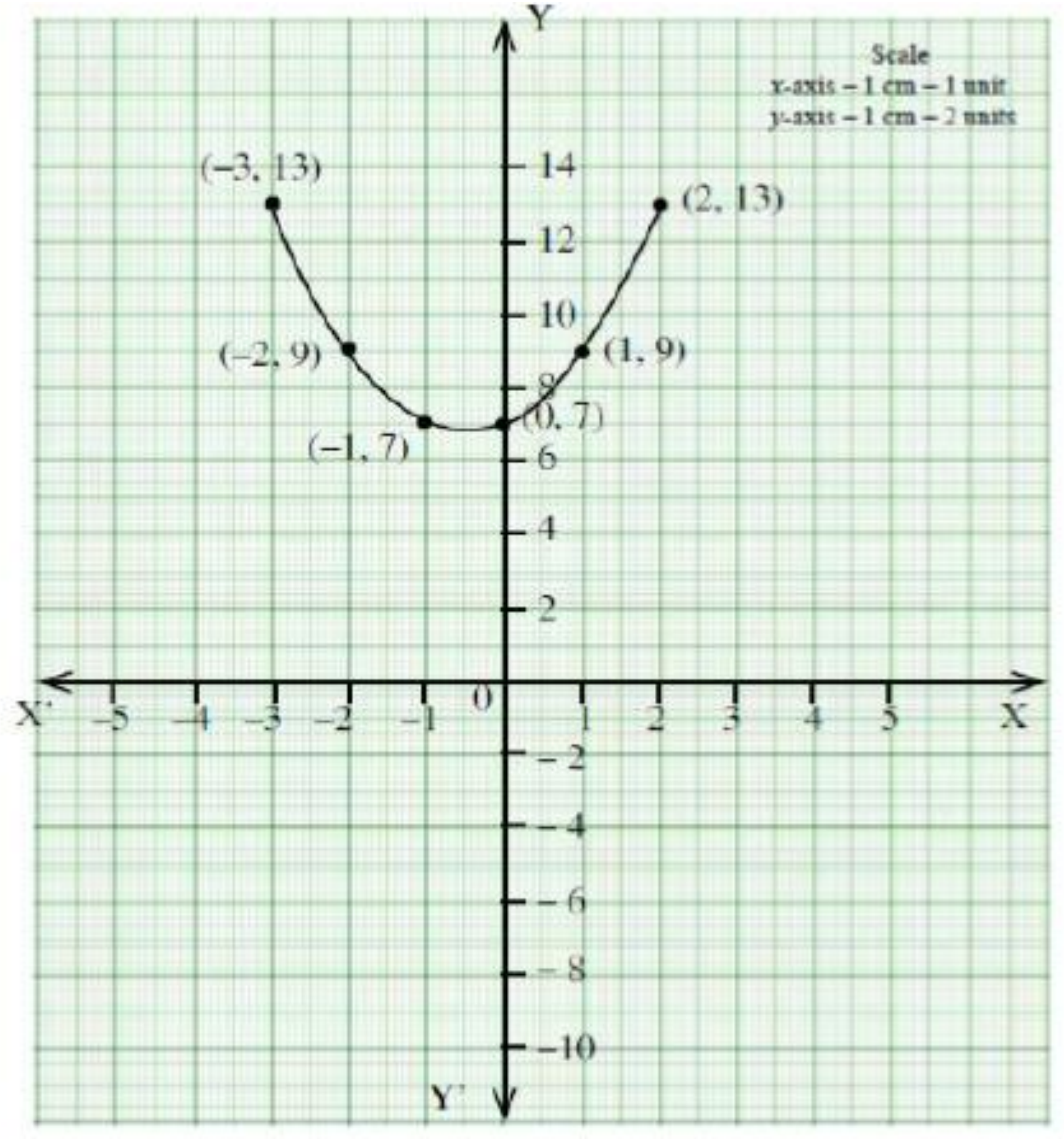
Since there is only one point of intersection with the X-axis. Therefore the roots are real and equal.

3. Discuss the nature of solution of the following quadratic equation. : $x^2 + x + 7 = 0$.

<u>Table</u>

x	-4	-3	-2	-1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
+x	-4	-3	-2	-1	0	1	2	3	4
+7	7	7	7	7	7	7	7	7	7
y	19	13	9	7	7	9	13	19	27

<u>Points</u> (-4, 19), (-3, 13), (-2, 9), (-1, 7), (0, 7), (1,9), (2, 13), (3,9), (4, 27)



<u>Solution</u>

Here the parabola does not intersect or touch the x-axis. So there is no real root.

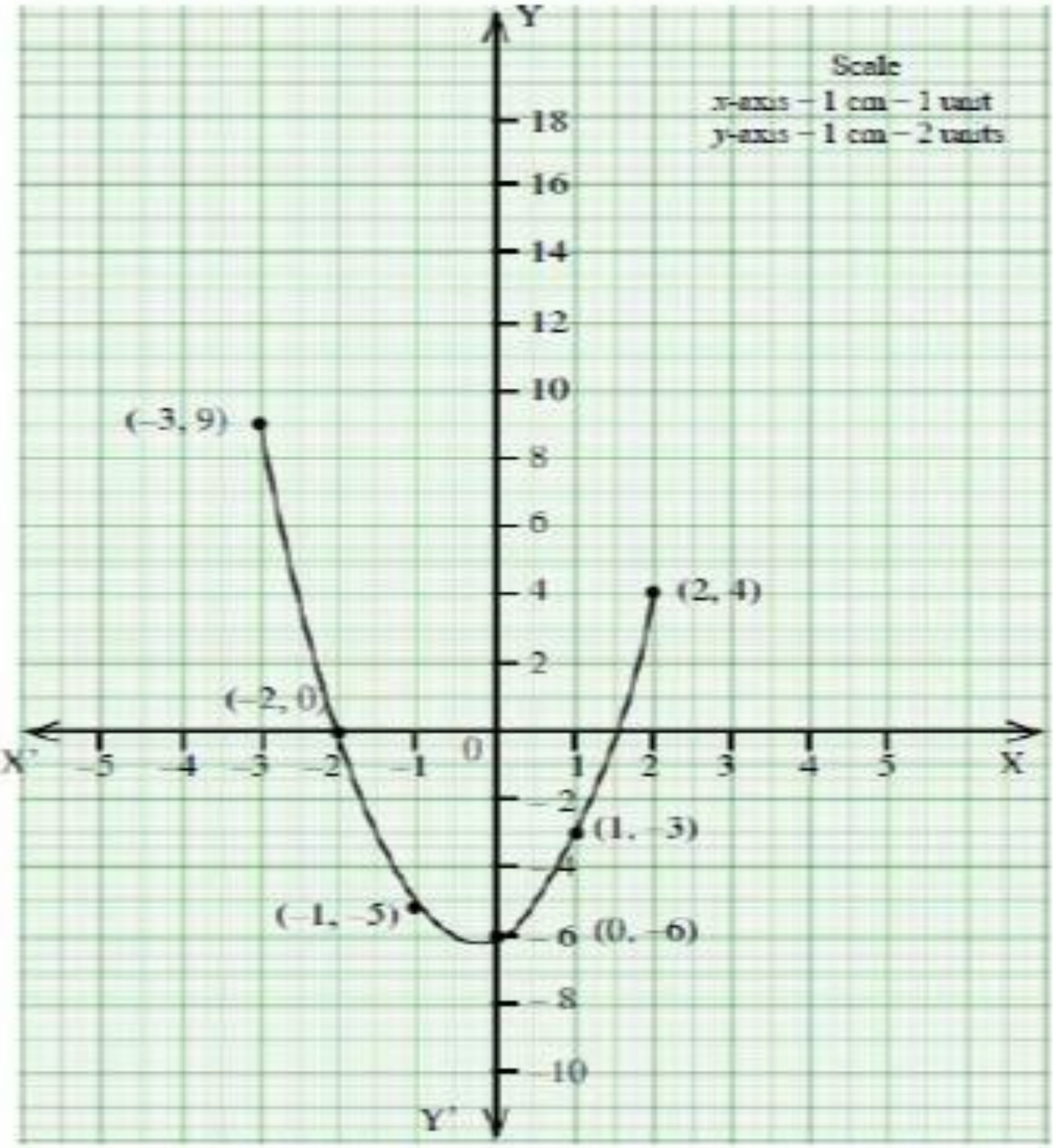
4. Discuss the nature of solution of the following quadratic equation. : (2x – 3) (x + 2) = 0.

$y = (2x - 3)(x + 2) = 2x^2 + 4x - 3x - 6 = 2x^2 + x - 6$

Table

x	-4	-3	-2	-1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$2x^2$	32	18	8	2	0	2	8	18	32
+x	-4	-3	-2	-1	0	1	2	3	4
-6	-6	-6	-6	-6	-6	-6	-6	-6	-6
y	22	9	0	-5	-6	-3	4	15	30

Points (-4, 22), (-3,9), (-2, 0), (-1,-5), (0,-6), (1,-3), (2, 4), (3,15), (4, 30)



Solution

Since there are two points of intersection with the X-axis. Therefore the roots are real and unequal

5. Draw the graph of $y = x^2 + x - 2$ and hence solve $x^2 + x - 2 = 0$

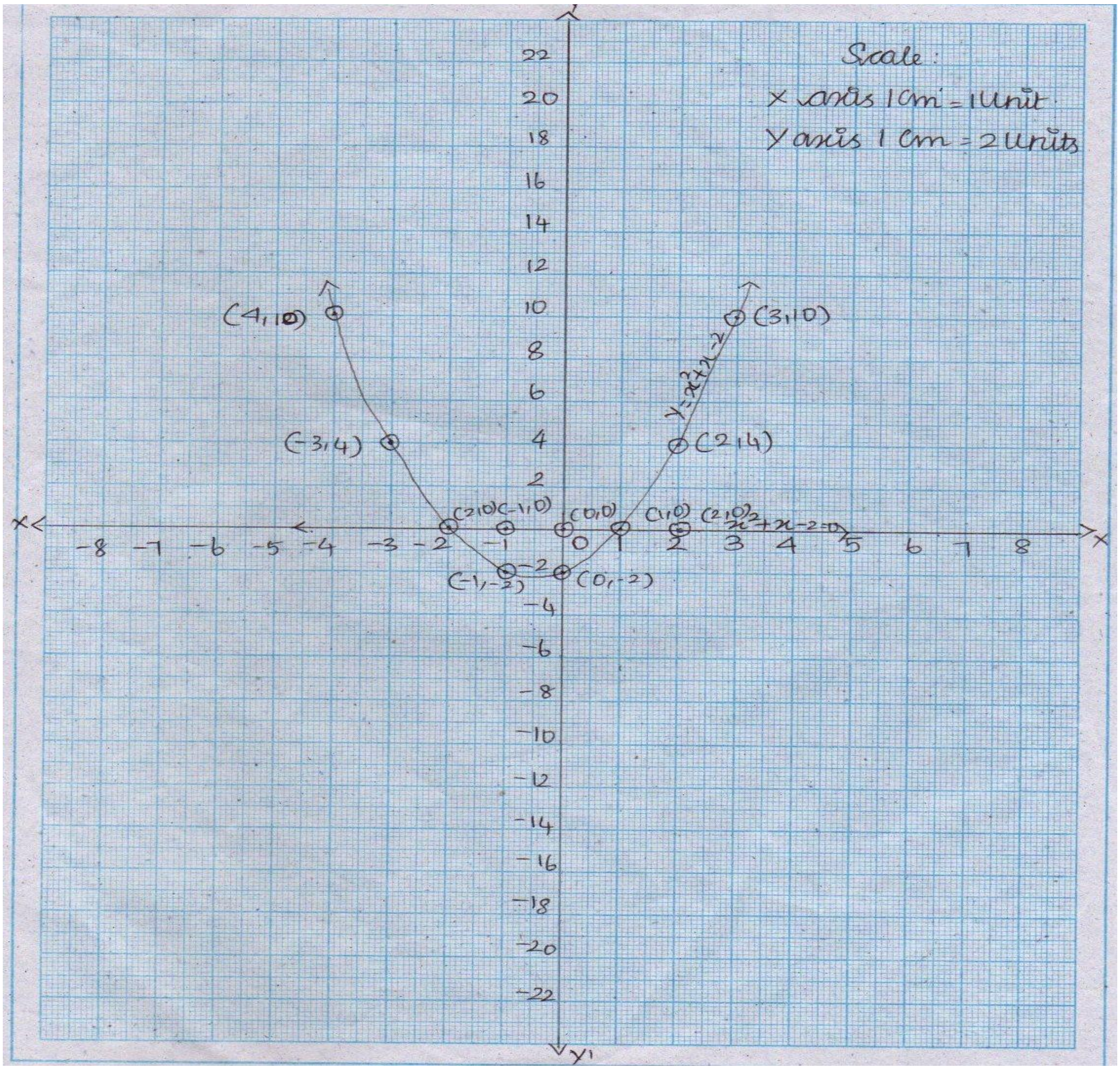
Table

x	-4	-3	-2	-1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
+x	-4	-3	-2	-1	0	1	2	3	4
-2	-2	-2	-2	-2	-2	-2	-2	-2	-2
y	10	4	0	-2	-2	0	4	10	18

Points (-4, 10), (-3,4), (-2, 0), (-1,-2), (0,-2), (1,0), (2, 4), (3,10), (4, 18)

Subtraction

$$\begin{array}{rrrr} y = & x^2 + & x & -\ 2 \\ 0 = & x^2 + & x & -\ 2 \\ (-) & (-) & (-) & (-) \\ \hline & y = 0 & & \\ \hline \end{array}$$



Solution **-2 and 1**

6. Draw the graph of $y = x^2 - 5x - 6$ and hence use it to solve $x^2 - 5x - 14 = 0$

Table

x	-4	-3	-2	-1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
-5x	20	15	10	5	0	-5	-10	-15	-20
-6	-6	-6	-6	-6	-6	-6	-6	-6	-6
y	30	18	8	0	-6	-10	-12	-12	-10

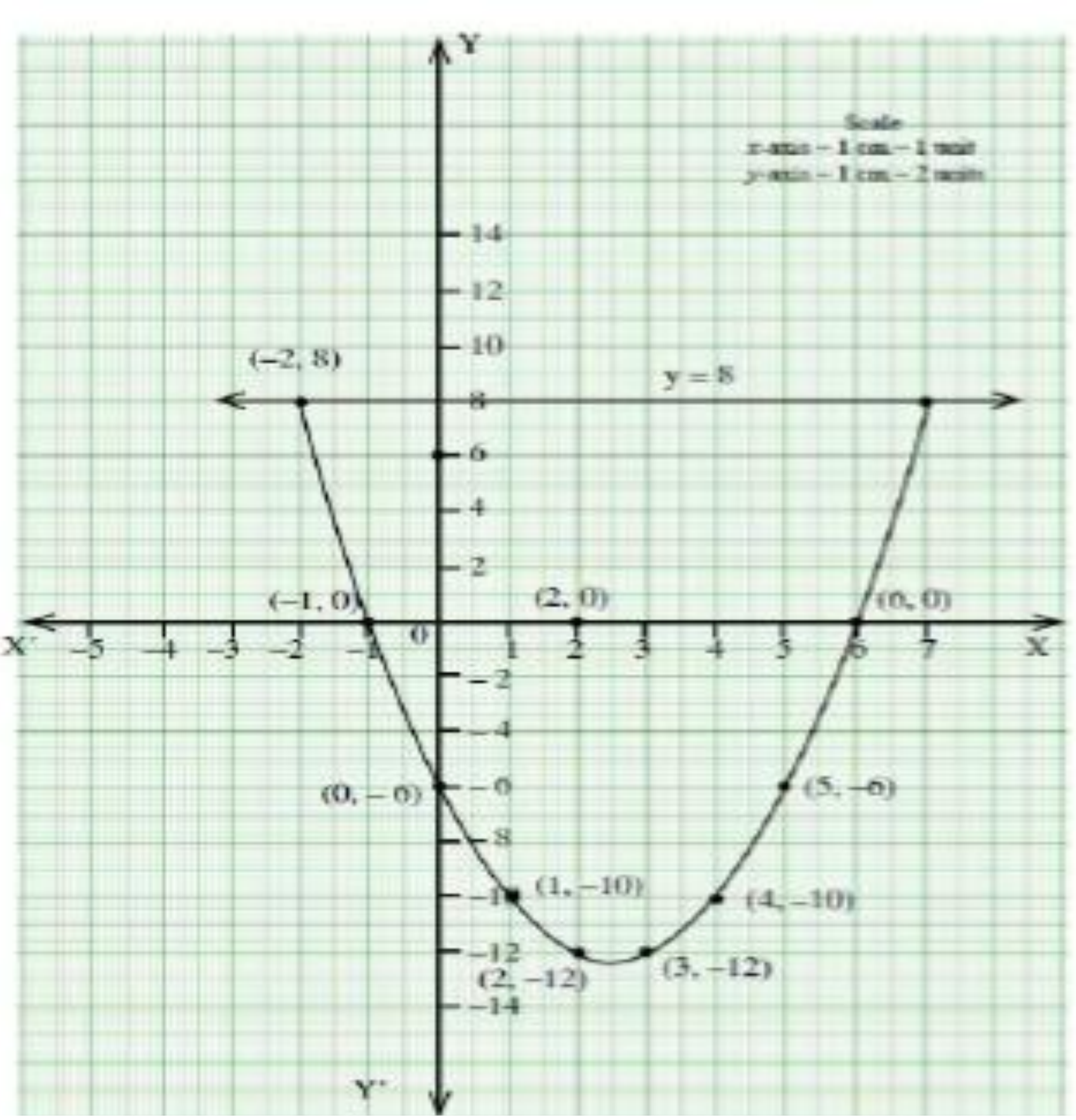
Points (-4, 30), (-3,4),(-2, 8), (-1,0),(0,-6),(1,-10),(2, -12),(3,-12),(4, 10)

Subtraction $y = x^2 - 5x - 6$

$0 = x^2 - 5x - 14$

(-) (-) (+) (+)

y = 8



Solution **-2 and 7**

7. **Draw the graph of $y = 2x^2 - 3x - 5$ and hence use it to solve $2x^2 - 4x - 6 = 0$**

Table

x	-4	-3	-2	-1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$2x^2$	32	18	8	2	0	2	8	18	32
-3x	12	9	6	3	0	-3	-6	-9	-12
-5	-5	-5	-5	-5	-5	-5	-5	-5	-5
y	39	22	9	0	-5	-6	-3	4	15

Points (–4, 39), (–3,22), (–2, 9), (–1,0), (0,–5), (1,–6), (2, –3), (3,4), (4, 15)

Subtraction $y = 2x^2 - 3x - 5$

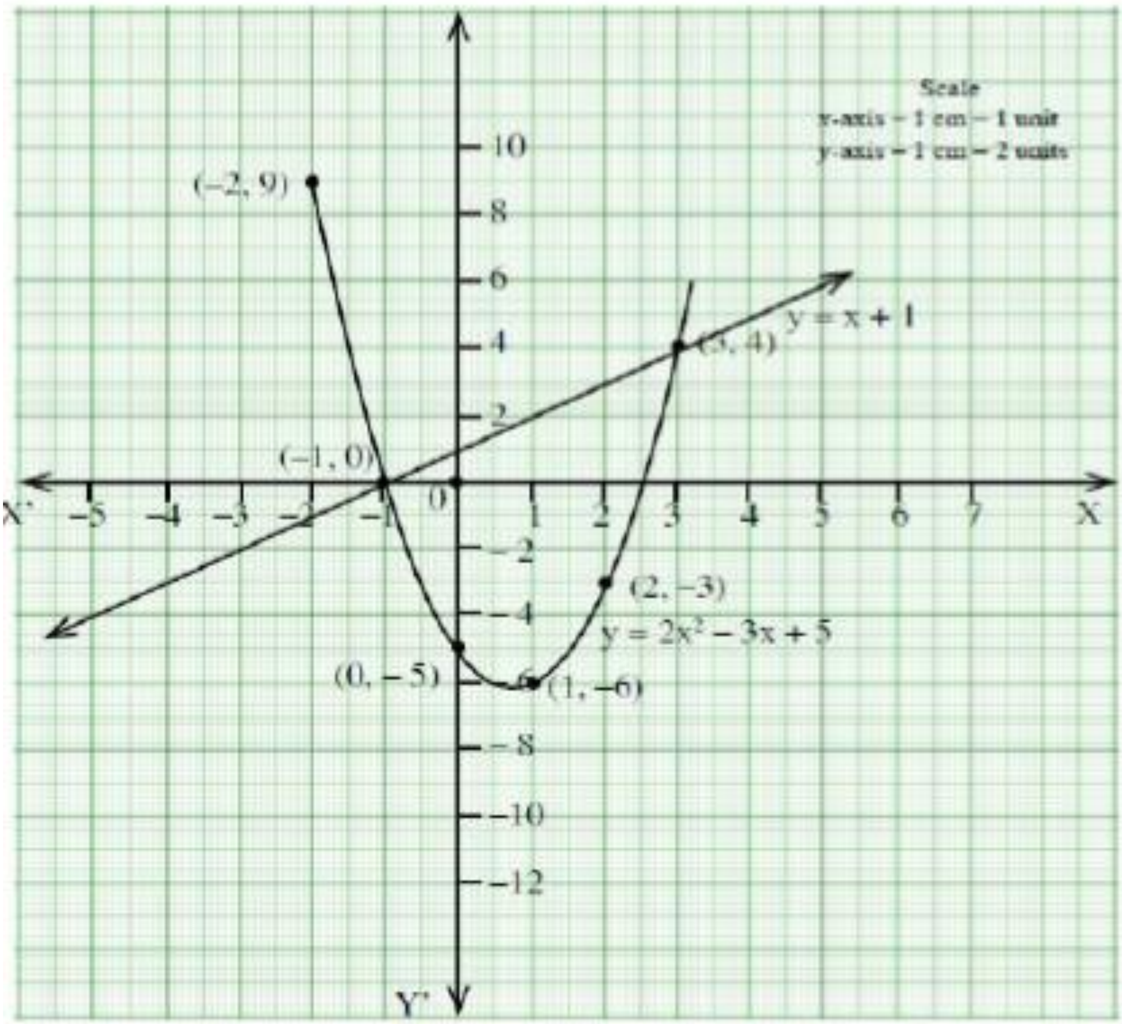
$0 = 2x^2 - 4x - 6$

(-) (-) (+) (+)

$y = x + 1$

Table

x	-4	-3	-2	-1	0	1	2	3	4
x	-4	-3	-2	-1	0	1	2	3	4
+1	1	1	1	1	1	1	1	1	1
y	-3	-2	-1	0	1	2	3	4	5



Solution **-1 and 3**

8. **Draw the graph of $y = 2x^2$ and hence use it to solve $2x^2 - x - 6 = 0$**

Table

x	-4	-3	-2	-1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$2x^2$	32	18	8	2	0	2	8	18	32
y	32	18	8	2	0	2	8	18	32

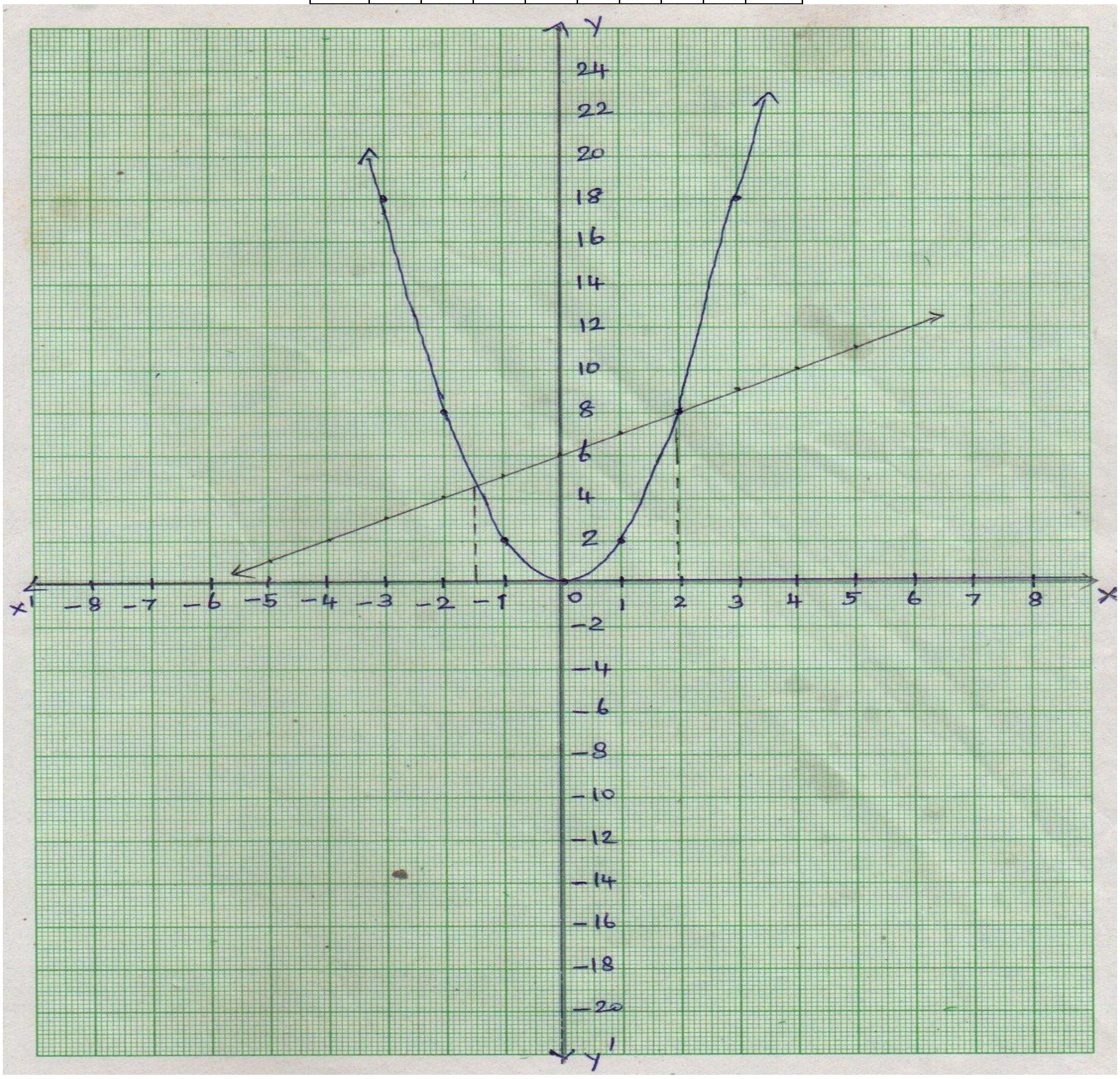
Points (–4, 32), (–3,18), (–2, 8), (–1,2), (0,0), (1,2), (2, 8), (3,18), (4, 32)

Subtraction

$$\begin{array}{rllll} y = & 2x^2 & & \\ 0 = & 2x^2 & -x & -6 \\ (-) & (-) & (+) & (+) \\ \hline & \mathbf{y =} & \mathbf{x + 6} & \\ \hline \end{array}$$

Table

x	-4	-3	-2	-1	0	1	2	3	4
x	-4	-3	-2	-1	0	1	2	3	4
+6	6	6	6	6	6	6	6	6	6
y	2	3	4	5	6	7	8	9	10



Solution **-1.5 and 2**

9. **Draw the graph of $y = x^2 - 4x + 3$ and hence use it to solve $x^2 - 6x + 9 = 0$**

Table

x	-4	-3	-2	-1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
-4x	16	12	8	4	0	-4	-8	-12	-16
+3	3	3	3	3	3	3	3	3	3
y	35	24	15	8	3	0	-1	0	3

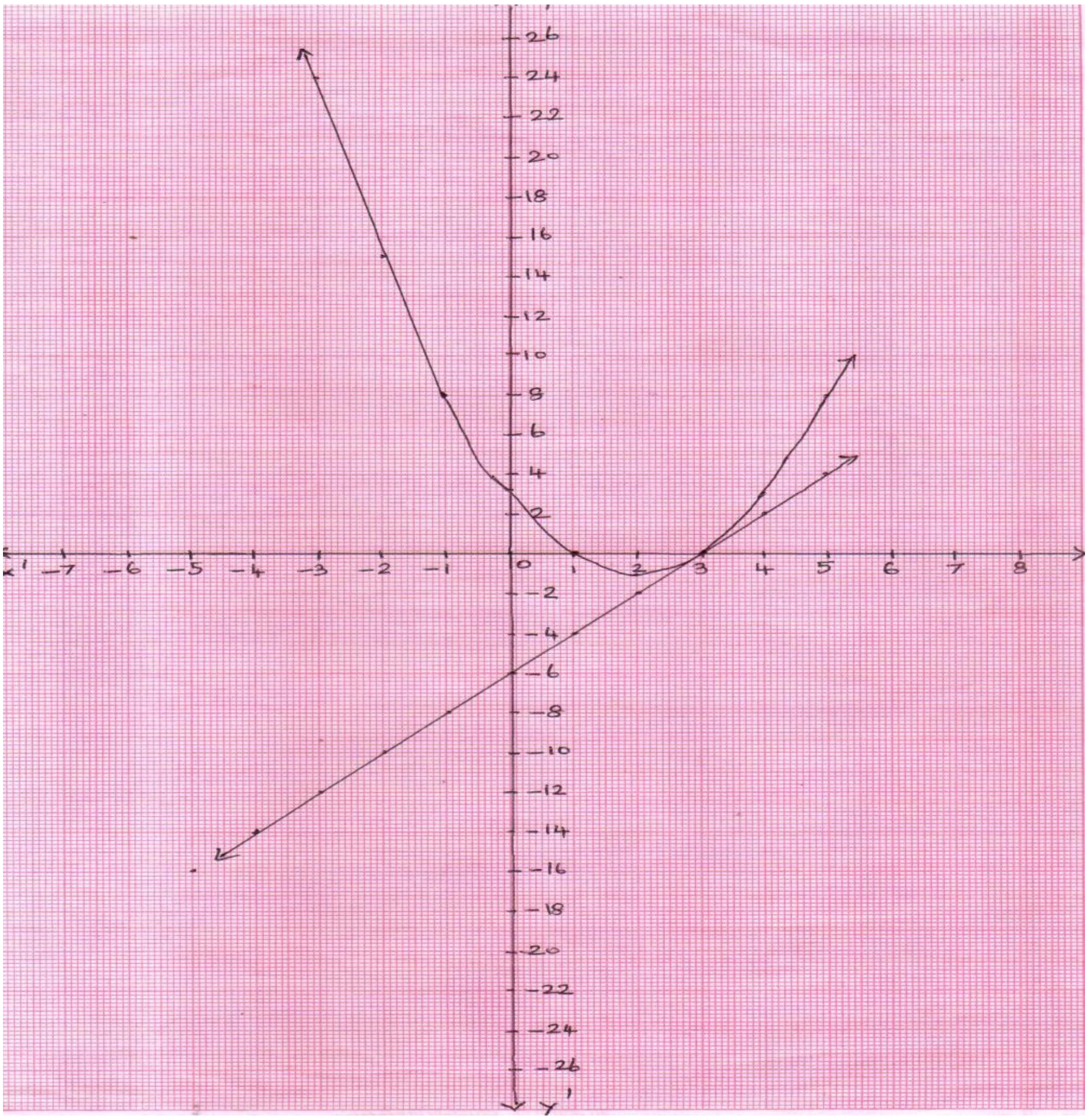
Points (−4, 35), (−3,24), (−2, 15), (−1,8), (0,3), (1,0), (2, −1), (3,0), (4, 3)

Subtraction

$$\begin{array}{rcccc} y = & x^2 & -\,4x & +\,3 \\ 0 = & x^2 & -6x & +\,9 \\ (-) & (-) & (+) & (-) \\ \hline & \mathbf{y =} & \mathbf{2x} & \mathbf{-\,6} \\ \hline \end{array}$$

Table

x	-4	-3	-2	-1	0	1	2	3	4
2x	-8	-6	-4	-2	0	2	4	6	8
-6	-6	-6	-6	-6	-6	-6	-6	-6	-6
y	-14	-12	-10	-8	-6	-4	-2	0	2



Solution **3**

10. Draw the graph of y = (x-1) (x + 3) and hence use it to solve $x^2 - x - 6 = 0$

$y = (x-1)(x + 3) \qquad = x^2 + 3x - x - 3 \qquad = x^2 + 2x - 3$

Table

x	-4	-3	-2	-1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
+2x	-8	-6	-4	-2	0	2	4	6	8
-3	-3	-3	-3	-3	-3	-3	-3	-3	-3
y	5	0	-3	-4	-3	0	5	12	24

Points (–4,5), (–3,0), (–2, –3), (–1, –4), (0,–3), (1,0), (2, 5), (3,12), (4, 24)

Subtraction y = x^2 +2x - 3

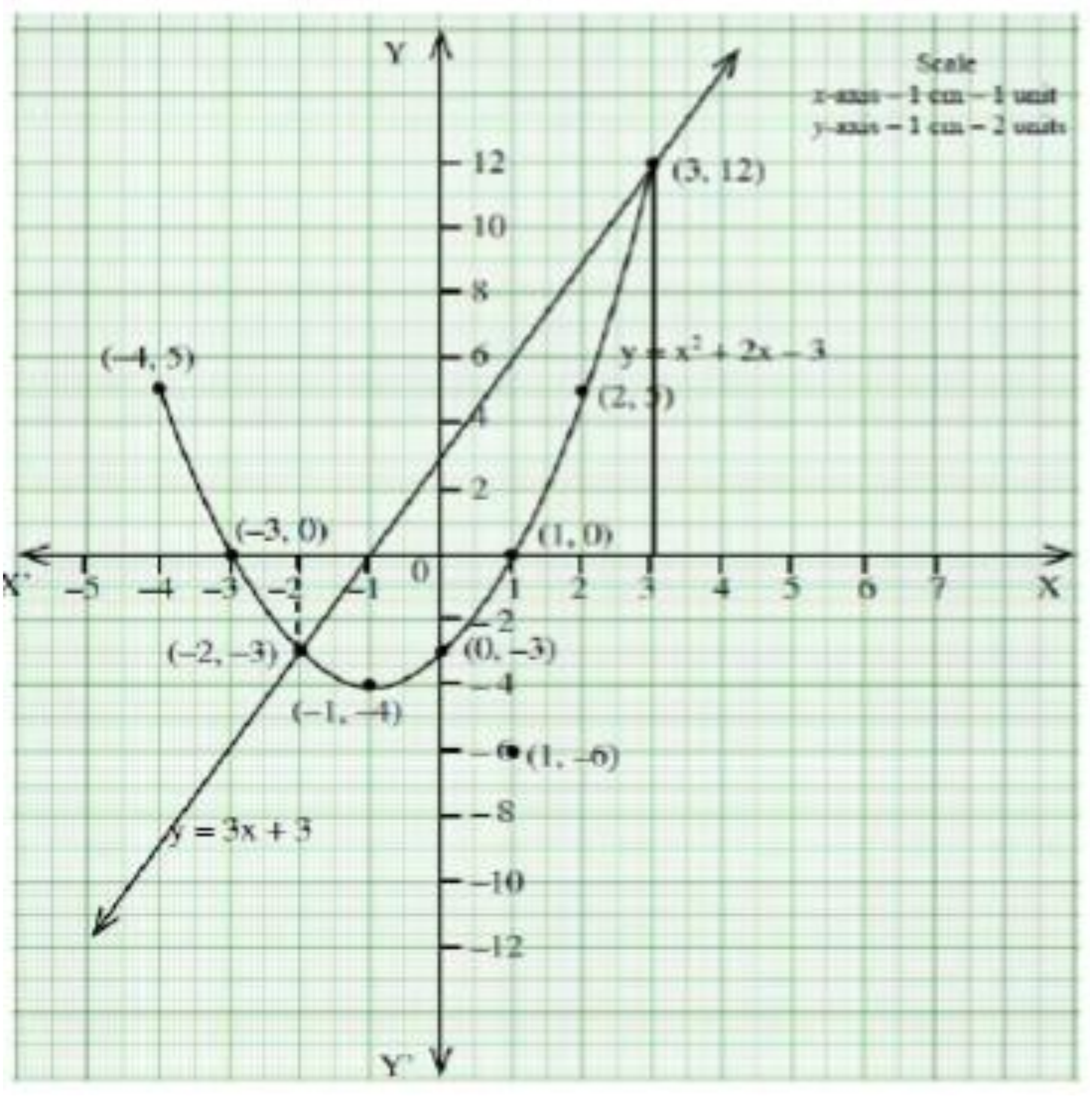
0 = x^2 - x - 6

(-) (-) (+) (-)

y = 3x + 3

Table

x	-4	-3	-2	-1	0	1	2	3	4
3x	-12	-9	-6	-3	0	3	6	9	12
3	3	3	3	3	3	3	3	3	3
y	-9	-6	-3	0	3	6	9	12	15



Solution **-2 and 3**

PRACTICE

GEOMETRY

1. Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{3}{5}$ of the corresponding sides of the ΔPQR .(Scale Factor $\frac{3}{5} < 1$).
2. Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{2}{3}$ of the corresponding sides of the ΔPQR . (Scale Factor $\frac{2}{3} < 1$).
3. Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{7}{4}$ of the corresponding sides of the ΔPQR .(Scale Factor $\frac{7}{4} > 1$).
4. Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{6}{5}$ of the corresponding sides of the ΔPQR (Scale Factor $\frac{6}{5} > 1$).
5. Construct a Δ PQR such that QR=5 cm, ∠P=30° and the altitude from P to QR is of length 4.2 cm.
6. Construct a ΔABC such that AB = 5.5 cm, ∠C = 25° and the altitude from C to AB is 4 cm .
7. Construct a ΔPQR such that PQ = 6.8 செ.மீ, $\angle R$ = 55° and the median RG from R to PQ = 6 cm.
8. Construct a Δ PQR in which PQ = 8 cm, $\angle R$ = 60°,and the median RG from R to PQ is 5.8cm. Find the length of the altitude from R to PQ.
9. Draw a ΔABC of base BC=8cm, ∠A =60° and the bisector of ∠A meets BC at D such that BD = 6cm.
10. Draw the two tangents from a point which is 5cm away from the centre of the circle of diameter 6cm. Also measure the length of the tangents.
11. Draw the two tangents from a point which is 11cm away from the centre of the circle of diameter 4cm. Also measure the length of the tangents.
12. Construct a Δ PQR in which PQ = 5 cm, $\angle P$ = 40°,and the median PG from P to QR is 4.4cm. Find the length of the altitude from P to QR.
13. Draw a ΔPQR such that PQ=6.8cm, vertical angle is 50° and the bisector of the vertical angle meets the base at D where PD = 5.2cm.

GRAPHS

1. Draw the graph for the quadratic equation $x^2 + x – 12 = 0$ and state their nature of solutions
2. Draw the graph for the quadratic equation $x^2 - 8x +16 = 0$ and state their nature of solutions
3. Draw the graph for the quadratic equation $x^2 + 2x + 5 = 0$ and state their nature of solutions
4. Draw the graph for the quadratic equation $x^2 - 9 = 0$ and state their nature of solutions
5. Draw the graph for the quadratic equation $x^2 - 6x + 9 = 0$ and state their nature of solutions
6. Draw the graph of $y = x^2 – 4$ and hence solve $x^2 - x - 12 = 0$.
7. Draw the graph of $y = x^2 + 4x +3$ and hence solve $x^2 + x + 1 = 0$.
8. Draw the graph of $y = x^2 +x$ and hence solve $x^2 + 1 = 0$.
9. Draw the graph of $y = x^2 + 3x +2$ and hence solve $x^2 + 2x + 1 = 0$.
10. Draw the graph of $y = x^2 + 3x -4$ and hence solve $x^2 +3x - 4 = 0$.
11. Draw the graph of $y = x^2 - 4$ and hence solve $x^2 - x - 12 = 0$.

UNIT - 1

RELATIONS AND FUNCTIONS

1 MARKS

1. If n(AxB) = 6 and A = {1,3} then n(B) is -----------

   (1) 6 **(2) 3** (3) 2 (4) 1

2. A ={a,b,p}, B ={2, 3}, C ={p,q,r,s}, then n[(A∪C)xB] is -----------

   (1) 8 **(2) 12** (3) 16 (4) 20

3. A={1,2}, B = {1,2,3,4}, C ={5,6} and D={5,6,7,8} then state which of the following statement is true. (1)(B x D) ⊂(A x C) **(2) (A x C) ⊂(B x D)** (3) (Ax B) ⊂(A x D) (4) (DxA) ⊂(B x A)

4. If there are 1024 relations from a set A ={1, 2,3,4,5} to a set B, then the number of elements in B is **(1) 2** (2) 3 (3) 4 (4) **8**

5. The range of R={(x, x^2) | x is a prime number less than 13} is

   (1) {4, 9, 25,49,121} (2) {1, 4, 9, 25, 49,121} (3) { 2, 3, 4, 5,7} (4) {2, 3, 5, 7, 11}

6. If the ordered pairs (a+2, 4) and (**5**, **2**a + b) are equal then (a,b) is

   (1) (2,-2) (2) (5, 1) (3) (2, 3) **(4) (3, -2)**

7. Let n(A) = m and n(B) = n then the total number of non-empty relations that can be defined from A to B is (1) m^n (2) n^m (3) 2^{mn} **(4) 2^{mn} - 1**

2 MARKS

1. **If A = {1,3,5}, B = {2,3}, then find A X B and B X A.**

   Solution

   Given that A = {1, 3, 5} and B = {2, 3}

   A X B = {1, 3, 5} x {2x 3} = {(1,2), (1,3), (3,2), (3,3), (5,2), (5,3)}

   B X A = {2, 3} x {1, 3, 5} = {(2,1), (2,3), (2,5), (3,1), (3,3), (3,5)}

2. **If A = {1,3,5}, B = {2,3}, then A X B = B X A?**

   Solution

   Given that A = {1, 3, 5} and B = {2, 3}

   A X B = {1, 3, 5} x {2x 3} = {(1,2), (1,3), (3,2), (3,3), (5,2), (5,3)} ………(1)

   B X A = {2x 3} x {1, 3, 5} = {(2,1), (2,3), (2,5), (3,1), (3,3), (3,5)} ………(2)

   From (1) and (2) we conclude that A x B ≠ B x A and (1,2) ≠ (2,1) and (1,3) ≠ (3,1)…

3. **If A = {1,3,5}, B = {2,3}, then show that n(A×B) = n(B×A) = n(A)× n(B)**

   Solution

   A X B = {1, 3, 5} x {2x 3} = {(1,2), (1,3), (3,2), (3,3), (5,2), (5,3)} ………(1)

   B X A = {2x 3} x {1, 3, 5} = {(2,1), (2,3), (2,5), (3,1), (3,3), (3,5)} ………(2)

   n(A) = 3 ; n(B) = 2

   From (1) and (2) We conclude that n(A x B) = n(B x A) = 6, we see that n(A) x n(B) = 3 x 2 = 6 மற்றும் n(B) x n(A) = 2 x 3 = 6

Hence, n(A x B) = n(B x A) = n(A) x n(B) = 6.

Thus, n(A x B) = n(B x A) = n(A) x n(B).

4. **If A×B = {(3,2), (3,4), (5,2), (5,4)} then find A and B.**

Solution A×B = {(3,2), (3,4), (5,2), (5,4)} then

A = {3, 5} and B = {2, 4}

5. **Find A x B, A x A and B x A if A = {2, -2, 3} and B={1, -4}**

Solution

A x B = {2, -2, 3} x{1, -4} = {(2,1), (2, -4), (-2,1), (-2, -4), (3, 1), (3,-4)}

A x A = {2,-2,3}x{2,-2,3}={(2, 2),(2, -2),(2,3),(-2, 2),(-2, -2),(-2, 3),(3, 2),(3,-2), (3,3)}

B x A = {1, -4} x {2, -2, 3} = {(1,2), (1,-2), (1,3), (-4,2), (-4,-2), (-4,3)}

6. **Find A x B, A x A and B x A if A = B = {p, q}**

Solution

A = B = {p, q}

A x B = {(p, p), (p, q), (q, p),(q ,q)}

A x A = {(p, p), (p, q), (q, p),(q ,q)}

B x A = {(p, p), (p, q), (q, p),(q ,q)}

7. **Find A x B, A x A and B x A if A = {m, n} ; B = ϕ**

Solution

A = {m, n}, B = ϕ A x B = {m, n} x { } = { }

A x A = {m, n} x {m, n} = {(m, m), (m, n), (n, m), (n,n)}

B x A = { } x {m, n} = { }

8. **Let A = {1, 2, 3} and B = { x |x is a prime number less than 10}. Find A x B and B x A.**

Solution

A = {1, 2, 3} B = {2, 3, 5, 7}

A x B={1,2,3}x{2,3,5,7}={(1,2),(1,3),(1,5),(1,7),(2,2),(2,3),(2,5),(2,7),(3,2),(3,3),(3,5),(3,7)}

B x A ={2,3,5,7}x{1,2,3}={(2,1),(2,2),(2,3),(3,1),(3,2),(3,3),(5,1),(5,2),(5,3),(7,1),(7,2),(7,3)}

9. **If B x A={(-2, 3), (-2, 4), (0,3), (0,4), (3,3), (3,4)}, findA and B.**

Solution

A = {3,4} B = {-2, 0,3}

10. **Let A={3,4,7,8} and B = {1,7,10}. Which of the following sets are relations from A to B.**

(i) R_1 = {(3,7), (4,7), (7,10), (8,1)} (ii) R_2= {(3,1), (4,12)}

(iii) R_3 = {(3,7), (4,10), (7, 7), (7, 8), (8, 11), (8, 7), (8,10)}

Solution

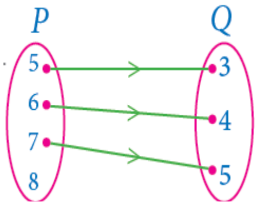
A x B = {(3,1),(3,7),(3,10),(4,1),(4,7),(4,10),(7,1), (7,7), (7,10), (8,1), (8,7), (8,10) }

(i) We note that, $R_1 \subseteq A \times B$. Thus R_1 is a relation from A and B.

(ii) Here $(4, 12) \epsilon R_2$, but $(4,12) \notin A \times B$. So R_2 is not a relation from A to B.

(iii) Here $(7, 8) \epsilon R_3$, but $(7,8) \notin A \times B$. So R_3 is not a relation from A to B.

11. The arrow diagram shows (Fig. 1.10) a relationship between the sets P and Q. Write the relation in (i) Set builder form (ii) Roster form (iii) What is the domain and range of R.



Solution Set builder form of R = $\{(x,y) \mid y = x - 2, x \in P, y \in Q\}$

Roster form R = {(5,3), (6,4), (7,5)}

Domain of R = {5,6,7} and range of R = {3,4,5}

12. Let A = {1,2,3,7} and B = {3,0,-1.7}, which of the following are rela

(i) R_1={(2,1),(7,1)}(ii)R_2={(-1,1)}(iii)R_3={(2,-1),(7,7),(1,3)}(iv)R_4={(7,-1),(0,3), (3,3),(0,7)}

Solution Given A = {1,2,3,7} and B = {3,0,-1,7}

$\therefore$ A x B ={(1,3),(1,0), (1,-1), (1,7), (2,3), (2,0), (2,-1), (2,7), (3,3), (3,0), (3,-1), (3,7), (7,3), (7,0), (7,-1), (7,7)}

i) $R_1 = \{(2,1), (7,1)\}$

$(2,1) \in R_1$ but $(2, 1) \notin A \times B$

$\therefore R_1$ is not a relation from A to B.

ii) $R_2 = \{(-1,1)\}$

$(-1,1) \in R_2$ but $(-1, 1) \notin A \times B$

$\therefore R_2$ is not a relation from A to B.

iii) $R_3 = \{(2,-1), (7,7), (1,3)\}$

We note that $R_3 \subseteq A \times B$

$\therefore R_3$ is a relation.

iv) $R_4 = \{(7,-1), (0,3), (3,3), (0,7)\}$

$(0,3), (0,7) \in R_4$ but not in A x B

$\therefore R_4$ is not a relation from A to B.

13. **Let A={1,2,3,4,...,45} and R be the relation defined as " is square of" on A. Write R as a subset of A x A. Also, find the domain and the range of R.**

Solution A = {1,2,3,….,45}

R = {(1,1), (2,4), (3,9), (4,16), (5,25), (6,36)}

$R \subset (A \times A)$ $\therefore$ Domain of R = {1,2,3,4,5,6} Range of R = {1,4,9,16,25,36}

14. A relation ℝ is given by the set {(x,y)/y= x + 3, x ∈{0,1, 2,3,4,5}. Determine its domain and range.

Solution x = {0,1,2,3,4,5}

f(x) = y = x +3 f(0) = 3; f(1) = 4 ; f(2) = 5 ; f(3) = 6 ; f(4) = 7 ; f(5) = 8

$\therefore$R = {(0,3),(1,4),(2,5),(3,6),(4,7),(5,8)}

Domain of R = {0,1,2,3,4,5} Range of R = {3,4,5,6,7,8}

5 MARKS

1. **Let A = { $\boldsymbol{x \epsilon \mathbb{N} \mid 1 < x < 4}$}, B = {$\boldsymbol{x \epsilon \mathbb{W} \mid 0 \leq x < 2}$} and C = { $\boldsymbol{x \epsilon \mathbb{N} \mid x < 3}$}, then verify that A X (B ∪ $\boldsymbol{C) = (A\,X\,B) \cup (A\,X\,C)}$.**

   Solution

   Given A = { $x \epsilon \mathbb{N} \mid 1 < x < 4$} = {2, 3},

   B = {$x \epsilon \mathbb{W} \mid 0 \leq x < 2$} = {0, 1},

   C= { $x \epsilon \mathbb{N} \mid x < 3$} = {1, 2}

   A X (B ∪ $C) = (A\,X\,B) \cup (A\,X\,C)$

   B∪C = {0,1 } ∪{ 1 ,2} = { 0, 1, 2} → (1)

   A x (B ∪ C) = {2, 3} x {0, 1, 2} = {(2,0), (2,1), (2,2), (3,0), (3,1), (3,2)} → (2)

   A x B = {2, 3} x {0, 1} = {(2,0), (2,1), (3, 0), (3, 1)} → (3)

   A x C = {2, 3} x {1, 2} = {(2,1), (2, 2), (3, 1), (3, 2)} → (4)

   (A x B) ∪ (A x C) = { (2,0), (2,1), (3, 0), (3,1) } ∪ {(2,1), (2, 2), (3, 1), (3, 2)}

   = { (2,0), (2,1), (2, 2), (3,0) , (3, 1), (3, 2)} → (5)

   (2) = (5) ∴ A X (B ∪ $C) = (A\,X\,B) \cup (A\,X\,C)$. Hence Proved

2. **Let A = { $\boldsymbol{x \epsilon \mathbb{N} \mid 1 < x < 4}$}, B = {$\boldsymbol{x \epsilon \mathbb{W} \mid 0 \leq x < 2}$} and C = { $\boldsymbol{x \epsilon \mathbb{N} \mid x < 3}$}, then verify that A X (B ∩ $\boldsymbol{C) = (A\,X\,B) \cap (A\,X\,C)}$.**

   Solution

   Given A = { $x \epsilon \mathbb{N} \mid 1 < x < 4$} = {2, 3},

   B = {$x \epsilon \mathbb{W} \mid 0 \leq x < 2$} = {0, 1},

   C= { $x \epsilon \mathbb{N} \mid x < 3$} = {1, 2}

   A X (B ∩ $C) = (A\,X\,B) \cap (A\,X\,C)$

   B ∩ C = {0,1 } ∩ {1, 2} = {1} → (1)

   A x (B ∩ C) = {2,3 } ∩{1} = {(2,1), (3,1)} → (2)

   $A\,X\,B$ = {2, 3} x {0,1} = {(2,0), (2,1), (3,0), (3,1)} → (3)

   $A\,X\,C$ = {2, 3} x {1,2} = {(2,1), (2,2), (3,1), (3,2)} → (4)

   $(A\,X\,B) \cap (A\,X\,C)$ = {(2,0), (2,1), (3,0), (3,1)} ∩{(2,1), (2,2), (3,1), (3,2)}

   = {(2,1), (3,1)} → (5)

   (2) = (5) ∴ A X (B ∩ $C) = (A\,X\,B) \cap (A\,X\,C)$. Hence Proved.

3. **If A = {5,6}, B={4,5,6}, C={5,6,7}. Show that A x A = (B x B) ∩ (C x C).**

   Solution

   Given A = {5,6}, B = {4,5,6}, C = {5,6,7}

   LHS : A x A = {5,6} x {5,6}

   = {(5,5), (5,6), (6,5), (6,6)} ---------- (1)

   RHS : B x B = {4,5,6} x {4,5,6}

$= \{(4,4), (4,5), (4,6), (5,4), (5,5), (5,6), (6,4), (6,5), (6,6)\}$

$C \times C = \{5,6,7\} \times \{5,6,7\}$

$= \{(5,5), (5,6), (5,7), (6,5), (6,6), (6,7), (7,5), (7,6), (7,7)\}$

$\therefore (B \times B) \cap (C \times C) = \{ (5,5), (5,6), (6,5), (6,6)\}$ ---------- (2)

$\therefore$ From (1) and (2). LHS = RHS.

4. Given $A = \{1, 2, 3\}$, $B = \{2, 3, 5\}$, $C=\{3,4\}$ and $D = \{1, 3, 5\}$, check if $(A \cap C) \times (B \cap D) = (A \times B) \cap (C \times D)$ is true?

<u>Solution</u> $A \cap C = \{ 3\}$, $B \cap D = \{3, 5\}$

$(A \cap C) \times (B \cap D) = \{3\} \times \{3, 5\} = \{(3, 3), (3, 5)\}$ → (1)

$A \times B = \{1, 2, 3\} \times \{2, 3, 5\} =\{(1, 2),(1, 3), (1,5), (2, 2), (2, 3),(2, 5),(3, 2), (3, 3),(3, 5)\}$

$C \times D = \{3, 4\} \times \{1, 3, 5\} = \{(3, 1), (3, 3), (3, 5), (4, 1), (4, 3), (4, 5)\}$

$(A \times B) \cap (C \times D) = \{(3, 3), (3, 5)\}$ → (2)

(1), (2) are equal **.** $\therefore (A \cap C) \times (B \cap D) = (A \times B) \cap (C \times D)$ **.** Hence it is verified.

5. Let $A = \{x \in W / x < 2\}$, $B = \{x \in N / 1< x \leq 4\}$ and $C = \{3,5\}$ Verify that

(i) $A \times (B\cup C)= (A \times B) \cup (A \times C)$ (ii) $A \times (B\cap C) = (A \times B) \cap (A \times C)$

(iii) **$(A\cup B) \times C = (A \times C) \cup (B \times C)$**

<u>Solution</u>

(i) $A \times (B\cup C)= (A \times B) \cup (A \times C)$

$A = \{x \in W / x < 2\} \Rightarrow A = \{0,1\}$

$B = \{x \in N / 1< x \leq 4\} \Rightarrow B = \{2, 3, 4\}$

$C = \{ 3, 5\}$

<u>To Prove</u> $A \times (B\cup C)= (A \times B) \cup (A \times C)$.

$B\cup C = \{2, 3, 4, 5\}$

$A \times (B\cup C) = \{(0,2), (0,3), (0,4), (0,5), (1,2), (1,3), (1,4), (1,5)\}$ ------- (1)

$A \times B = \{(0,2), (0,3), (0,4), (1,2), (1,3), (1,4)\}$

$A \times C = \{(0,3), (0,5), (1,3), (1,5)\}$

$\therefore (A \times B) \cup (A \times C) = \{(0,2), (0,3), (0,4), (0,5), (1,2), (1,3), (1,4), (1,5)\}$ ------- (2)

$\therefore$ (1) = (2). Hence Proved

(ii) $A \times (B\cap C) = (A \times B) \cap (A \times C)$

$A = \{x \in W / x < 2\} \Rightarrow A = \{0,1\}$

$B = \{x \in N / 1< x \leq 4\} \Rightarrow B = \{2, 3, 4\}$

$C = \{ 3, 5\}$

<u>To Prove</u> $A \times (B\cap C) = (A \times B) \cap (A \times C)$

$B\cap C = \{3\}$

$A \times (B\cap C) = \{(0,3), (1,3)\}$ ------- (1)

$A \times B = \{(0,2), (0,3), (0,4), (1,2), (1,3), (1,4)\}$

$A \times C = \{(0,3), (0,5), (1,3), (1,5)\}$

$\therefore (A \times B) \cap (A \times C) = \{(0,3), (1,3)\}$ ------- (2)

$\therefore$ (1) = (2). Hence Proved.

(iii) **$(A \cup B) \times C = (A \times C) \cup (B \times C)$**

$A = \{x \in W / x < 2\} \quad \Rightarrow \; A = \{0,1\}$

$B = \{x \in N / 1 \leq x \leq 4\} \quad \Rightarrow \; B = \{2, 3, 4\}$

$C = \{ 3, 5\}$

<u>To Prove</u> $(A \cup B) \times C = (A \times C) \cup (B \times C)$

$A \cup B = \{ 0, 1, 2, 3, 4\}$

$\therefore (A \cup B) \times C = \{ 0, 1, 2, 3, 4\} \times \{3,5\}$

$= \{ (0,3), (0,5), (1,3), (1,5), (2,3), (2,5), (3,3), (3,5) , (4,3), (4,5)\}$….(1)

$A \times C = \{(0,3), (0,5), (1,3), (1,5)\}$

$B \times C = \{(2,3), (2,5), (3,3), (3,5), (4,3), (4,5)\}$

$\therefore (A \times C) \cup (B \times C) = \{(0,3), (0,5), (1,3), (1,5), (2,3), (2,5), (3,3), (3,5), (4,3), (4,5)\}$….(2)

$\therefore$ From (1) and (2) LHS = RHS.

6. **Let A = The set of all natural numbers less than 8, B = The set of all prime numbers less than 8, C= The set of even prime number. Verify that $(A \cap B) \times C = (A \times C) \cap (B \times C)$**

<u>Solution</u>

Given $A = \{1, 2, 3, 4, 5, 6, 7\}$ $B = \{2, 3, 5, 7\}$ $C = \{2\}$

To verify $(A \cap B) \times C = (A \times C) \cap (B \times C)$

$A \cap B = \{2, 3, 5, 7\}$

$\therefore (A \cap B) \times C = \{(2,2), (3,2), (5,2), (7,2)\}$ ……………(1)

$A \times C = \{(1,2), (2,2), (3,2), (4,2), (5,2), (6,2), (7,2)\}$

$B \times C = \{(2,2), (3,2), (5,2), (7,2)\}$ ……… (2)

$\therefore$ From (1) and (2), LHS = RHS.

7. **Let A = The set of all natural numbers less than 8, B = The set of all prime numbers less than 8, C= The set of even prime number. Verify that $A \times (B - C) = (A \times B) - (A \times C)$**

<u>Solution</u>

Given $A = \{1, 2, 3, 4, 5, 6, 7\}$ $B = \{2, 3, 5, 7\}$ $C = \{2\}$

To verify $A \times (B - C) = (A \times B) - (A \times C)$

$A = \{1, 2, 3, 4, 5, 6, 7\}$ $B = \{2, 3, 5, 7\}$ $C = \{2\}$

$B - C = \{2, 3, 5, 7\} - \{2\} = \{3, 5, 7\}$ → (1)

$A \times (B - C) = \{1, 2, 3, 4, 5, 6, 7\} \times \{3, 5, 7\}$

$=\{(1,3),(1,5),(1,7),(2,3),(2,5),(2,7),(3,3),(3,5),(3,-),(4,3),(4,5), (4,7),$
$(5,3), (5,5), (5,7), (6,3), (6,5), (6,7), (7,3), (7,5), (7,7)\}$ → (2)

$A \times B = \{1,2,3,4,5,6,7\} \times \{2,3,5,7\}$

= {(1,2),(1,3),(1,5),(1,7), (2,2),(2,3),(2,5),(2,7),(3,2),(3,3),(3,5),(3,7),
(4,2),(4,3),(4,5),(4,7),(5,2),(5,3),(5,5),(5,7),(6,2),(6,3),(6,5),(6,7),(7,2)
(7,3),(7,5),(7,7)} → (3)

A x C = {1, 2, 3, 4, 5, 6, 7}x{2} = {(1,2),(2,2),(3,2),(4,2),(5,2),(6,2),(7,2)} → (4)

(A x B) – (A x C) = {(1,3),(1,5),(1,7),(2,3),(2,5),(2,7),(3,3),(3,5),(3,7),(4,3),
(4,5),(4,7),(5,3),(5,5), (5,7),(6,3),(6,5),(6,7),(7,3),(7,5),(7,7)} → (5)

(2), (5) are equal **.** ∴ A X (B – C) = (A X B) – (A X C). Hence it is verified.

8. Represent each of the given relation by (a) an arrow diagram (b) a graph and (c) a set in roster form, wherever possible.

(i) {(x,y)|x = 2y, x ∈ {2,3,4,5}, y ∈ {1,2,3,4} (ii) {(x,y)|y=x+3, x,y are natural numbers< 10}

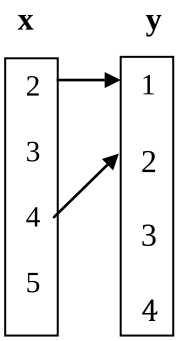
<u>Solution</u>

(i) {(x,y)|x = 2y, x ∈ {2,3,4,5}, y ∈ {1,2,3,4}

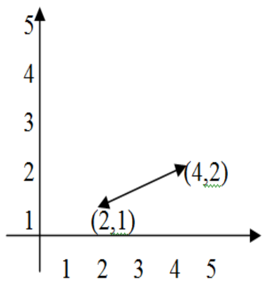
x = 2y

f(x) = y = $\frac{x}{2}$; f(2) = $\frac{2}{2}$ = 1 ; f(3) = $\frac{3}{2}$; f(4) = $\frac{4}{2}$ = 2 ; f(5) = $\frac{5}{2}$

(a) An Arrow diagram



(b) Graph



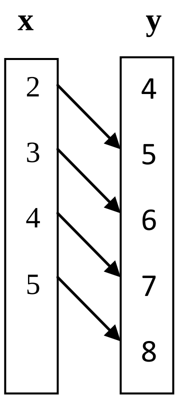
(c)Roaster Form

{(2,1), (4,2)}

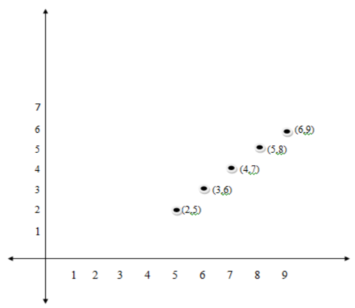
(ii) {(x,y)| y = x + 3, x, y are natural numbers < 10}

f(x) = y = x + 3 ; f(2) = 5 ; f(3) = 6 ; f(4) = 7 ; f(5) = 8

(a) An Arrow diagram



(b) Graph



(c) Roaster Form

{(2,5),(3,6),(4,7), (5,8)}

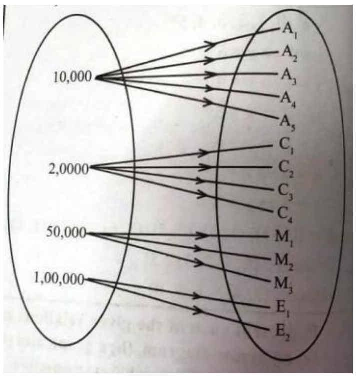
9. **A company has four categories of employees given by Assistants (A), Clerks (C), Managers (M) and an Executive Officer (E). The company provide ₹10,000, ₹ 25,000, ₹50,000 and ₹1,00,000 as salaries to the people who work in the categories A, C, M and E respectively. If A_1, A_2, A_3, A_4 and A_5 were Assistants; C_1, C_2, C_3, C_4 were Clerks; M_1, M_2, M_3 were managers and E_1, E_2 were Executive Officers and if the relation R is defined by xRy, where x is the salary given to person y, express the relation R through an ordered pair and an arrow diagram.**

<u>Solution</u>

a) Ordered Pair:

{(10000, A_1), (10000, A_2), (10000, A_3), (10000, A_4), (10000, A_5)
(25000, C_1), (25000, C_2), (25000, C_3), (25000, C_4), (50000, M_1), (50000, M_2), (50000, C_3)
(100000, E_1), (100000, E_2)}

b) Arrow Diagram



UNIT - 2

NUMBER AND SEQUENCES

1 MARKS

1. Euclid’s division lemma states that for positive integers α and $b, then$ $exist$ $unique$integers q and r such that a = bq + r wher r must satisfy.

   (1) $0 < r < b$ (2) $1 < r < b$ (3) $0 < r \leq b$ **(4) $0 \leq r < b$**

2. Using Euclids’ division lemma, if the cube of any positive integer is divided by 9 then the possible remainders are

   (1) 1,3,5 (2) 1,4,8 (3) 0,1,3 **(4) 0,1,8**

3. If the HCF of t65and 117is expressible in the form of 65m-117 than ,the value of m is

   (1) 1 (2) 3 **(3) 2** (4) 4

4. The sum of the exponents of prime factors in the prime factorization of 1729 is

   (1) 4 **(2)3** (3) 2 (4) 1

5. The least number that is divisible by all the numbers form 1 to 10 (both inclusive) is

   (1) 2025 **(2)2520** (3) 5025 (4) 5220

6. Given $F_1 = 1$, $F_2 = 3$and $F_n = F_{n-1} + F_{n-2}$ then F_5

   (1) 8 **(2) 11** (3) 3 (4) 5

7. The first term of an arithmetic progression is unity and the common differenc is 4. which of the following will be aterm of this A.P.? **(1) 7881** (2) 10091 (3) 4551 (4) 13531

8. If 6times of 6th term of an A.P., is equal to 7 times the 7th term ,then the 13th term of the A.P. is 120?

   (1) 0 (2) 6 (3) 7 (4) 13

9. An A.P., consists of 31 terms. if its 16th trem is m, then the sum of all the terms of this A.P. is

   (1)16m (2) 62m (3) $\frac{31}{2}$ m **(4) 31 m**

2 MARKS

1. **‘a’ and ‘b’ are two positive integers such that a^b x b^a = 800. Find ‘a’ and ‘b’.**

   Solution

   $800 = a^b \text{ x } b^a$

   $800 = 2 \text{ x } 2 \text{ x } 2 \text{ x } 2 \text{ x } 2 \text{ x } 5 \text{ x } 5$

   $= 2^5 \text{ x} 5^2$

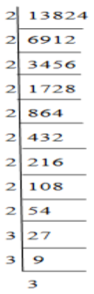
   $\therefore a = 2$, $b = 5$ (or) $a = 5$, $b = 2$

2. If $13824 = 2^a \text{ x } 3^b$ then find a and b.

   Solution

   $\Rightarrow 13824 = 2^9 \text{ x } 3^3$

   $\therefore a = 9, b = 3$



2	13824
2	6912
2	3456
2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
	3

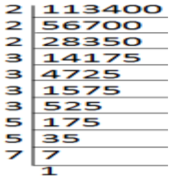
3. **If $p_1^{x_1} \times p_2^{x_2} \times p_3^{x_{3+}} \times p_4^{x_4}$ = 113400 where, P_1, P_2 , P_3 ,P_4 are primes in ascending order and x_1, x_2 , x_3 ,x_4 are integers, find the value of P_1, P_2 , P_3 ,P_4 and x_1, x_2 , x_3 ,x_4**

<u>Solution</u>

11340 = 2^3 x3^4x5^2x7^1

∴ P_1 =2, P_2 =3, P_3 = 5, P_4 =7

x_1 =3, x_2 =4, x_3 =2, x_4 =1



2	113400
2	56700
2	28350
3	14175
3	4725
3	1575
3	525
5	175
5	35
7	7
	1

4. **Write an A.P. whose first term is 20 and common difference is 8**

<u>Solution</u>

First term, a = 20; Common Difference, d = 8

Arithmetic Progression is a, a+d, a+3d,…

In case, we get 20, 20 + 8, 20+ 2(8), 20 + 3(8),...

So, the required A.P is 20, 28, 36, 44, ...

5. **Find the number of terms in the A.P. 3,6,9,12...111.**

<u>Solution</u>

First Term, a = 3, Common Difference, d = 6 – 3 = 3, Last term l = 111

We know that n = $\left(\frac{l-a}{d}\right) + 1$

n = $\left(\frac{111-3}{3}\right) + 1 = 37$. Thus, the A.P. contains 37 terms.

6. **First term "a" and common difference "d" are given below. Find the corresponding A.P.**

(i) a = 5, d= 6 (ii) a =7, d = -5 (iii) a = $\frac{3}{4}$, d= $\frac{1}{2}$

<u>Solution</u>

(i) Given a = 5, d = 6. Genereal form of A.P⇒ a, a + d, a + 2d, … ⇒5, 11, 17, 23,…

(ii) Given a = 7, d = -5 . Genereal form of A.P ⇒ a, a + d, a + 2d, …⇒7, 2, -3, -8,…

(iii) Given a = $\frac{3}{4}, d = \frac{1}{2}$. Genereal form of A.P ⇒ a, a + d, a + 2d, … ⇒ $\frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \frac{9}{4}$, ...

7. **Find the 19th term of an A.P. -11,-15,-19,...**

<u>Solution</u>

Given a = -11; d = -15+11 = -4; n=19

t_n = a+(n-1)d

The 19th term is

t_{19} = -11 + 18(-4)

= -11-72

t_{19} = -83

8. **Which term of an A.P. 16, 11, 6, 1... is -54?**

<u>Solution</u>

n = $\frac{l-a}{d} + 1$

a = 16; d = 11-16= -5;l = -54

$n = \frac{-54-16}{-5} + 1$

$n = \frac{-70}{-5} + 1$

n = 14 + 1

n = 15

9. **If 3 + k, 18 – k, 5k + 1are in A.P, then find k.**

Solution 3 + k, 18 – k, 5k + 1 is a A.P

$\mathbf{t_2 - t_1 = t_3 - t_2}$

(18 – k) – (3 + k) = (5k + 1) – (18 – k)

15 – 2k = 6k – 17

-2k – 6k = -17 -15

-8k = -32

k = 4

10. **In a theatre, there are 20 seats in the front row and 30 rows were allotted. Each successive row contains two additional seats than its front row. How many seats are there in the last row?**

தீர்வு

Here a = 20, d = 2

Number of seats in the last row $= t_n = a + (n-1)d$

$t_{30} = a + 29d$

= 20 + 29(2)

= 20 + 58 = 78

UNIT – 3

ALGEBRA

1 MARKS

1. A system of three linear equations in three variables is inconsistent if their planes

   (1) intersect only at a point (2) intersect in a line

   (2) coincides with each other **(4) do not intersect**

2. The solution of the system x + y – 3z = -6, -7y + 7z = 7, 3z=9 is

   (1) x=-1, y=2, z=3 (2)x=1, y=-2, z=3 **(3)x=1, y=2, z = 3** (4) x = -1, y = -2, z = 3

3. If (x -6) is the HCF of x^2 - 2x – 24 and x^2 - kx - 6 then the value of k is

   (1) 8 (2) 6 **(3) 5** (4) 3

4. $\frac{3y-3}{y} \div \frac{7y-7}{3y^2} is$

   (1) $\frac{9y^3}{(21y-21)}$ **(2) $\mathbf{\frac{9y}{7}}$** (3) $\frac{21y^2-42y+21}{3y^3}$ (4) $\frac{7(y^2-2y+1)}{y^2}$

5. $y^2+\frac{1}{y^2}$ is not equal to

   (1) $\left[y-\frac{1}{y}\right]^2+2$ (2) $\left[y+\frac{1}{y}\right]^2-2$ **(3) $\mathbf{\left[y+\frac{1}{y}\right]^2}$** (4) $\frac{y^4+1}{y^2}$

6. $\frac{x}{x^2-25}$ - $\frac{8}{x^2+6x+5}$ gives

   (1) $\frac{x^2-7x+40}{(x-5)(x+5)}$ (2) $\frac{x^2+7x+40}{(x-5)(x+5)(x+1)}$ (3) $\frac{x^2+10}{(x^2-25)(x+1)}$ **(4) $\mathbf{\frac{x^2-7x+40}{(x^2-25)(x+1)}}$**

7. The square root of $\frac{256\,x^8y^4z^{10}}{25\,x^6y^6z^6}$ is equal to

   (1) $\frac{16}{5}\left|\frac{x^2z^4}{y^2}\right|$ **(2)16 $\mathbf{\left|\frac{xz^2}{y}\right|}$** (3) $\frac{16}{5}\left|\frac{y^2}{x^2z^4}\right|$ (4) $\frac{16}{5}\left|\frac{y}{xz^2}\right|$

8. W hich of the following should be added to make x^2 + 64a parfect square?

   (1) $4x^2$ (2) $8x^2$ (3) $-8x^2$ **(4) $16x^2$**

9. The solution of $(2x-1)^2 = 9$ is equal to

   (1) -1, 2 (2) -1 (3) 2 (4)None of therse

10. The values of a and b if $4x^4 - 24x^3 + 76x^2 + ax + b$ is a perfect square are

    (1) -120, 100 (2) 100,120 (3) 10, 12 (4) 12, 10

11. $q^2x^2 + p^2x + r^2 = 0$ are the squares of the roots of the equation $qx^2 + px + r = 0$,then q,p,r are in

    (1) A.P **(2) G.P** (3) Both A.P And G.P (4) none of these

12. Graph of a linear polynomial is a

    (1) **straight line** (2)circle (3) parabola (4)hyperbola

13. The number of points of intersection of the quadratic polynomial $x^2 + 4x + 4$ With the X axis is

    (1) 0 or 1 (2) 0 **(3) 1** (4) 2

2 MARKS

1. **Find the LCM of the given polynomial: $4x^2y$, $8x^3y^2$**

Solution

$4x^2y = 2^2x^2y$

$8x^3y^2 = 2^3x^3y^2$

$\therefore$ LCM = $2^3x^3y^2 = 8x^3y^2$

2. **Find the LCM of the given polynomial:-$9a^3b^2$, $12a^2b^2c$**

Solution

$- 9a^3b^2 = (-1)(3)^2\ a^3b^2$

$12a^2b^2c = 2^2 \times 3 \times a^2 \times b^2\ \times c$

$\therefore$ LCM = $(-1) \times 2^2 \times 3^2 \times a^3 \times b^2\ \times c = -36a^3b^2\ c$

3. **Find the LCM of the given polynomial: $16m$, $12m^2n^2$, $8n^2$**

Solution

$16m = 2^4$ x m

$12m^2n^2 = 2^2$ x 3x m^2 xn^2

$8n^2\ = 2^3$ x n^2

$\therefore$LCM = 2^4 x 3 x m^2 x n^2 = 48 $m^2\ n^2$

4. **Find the LCM of the given polynomial: $p^2 – 3p + 2$, $p^2 – 4$**

Solution

$p^2 – 3p + 2 = (p -1)\ (p – 2)$

$p^2 – 4 = (p + 2)\ (p – 2)$

$\therefore$ LCM = $(p -1)\ (p + 2)\ \ (p – 2)$

5. **Find the LCM of the given polynomial: $2x^2 – 5x – 3$, $4x^2 – 36$**

Solution

$2x^2 – 5x – 3 = (x - 3)\ (2x + 1)$

$4x^2 – 36\ = 4(x + 3)\ (\ x – 3)$

$\therefore$ LCM = $4(x - 3)\ (\ x + 3)\ (2x + 1)$

6. **Find the LCM of the given polynomial: $(2x^2 – 3xy)^2$, $(4x – 6y)^3$, $8x^3 – 27y^3$**

Solution

$(2x^2 – 3xy)^2 = \{x(2x – 3y)\}^2 =x^2(2x – 3y)^2$

$(4x – 6y)^3\ \ \ = 2^3\,(2x – 3y)^3$

$8x^3 – 27y^3\ = (2x)^3 – (3y)^3 = (2x – 3y)\ (4x^2 + 6xy + 9y^2)$

$\therefore$ LCM = $2^3 \times x^2\times(2x – 3y)^3\ \ (4x^2 + 6xy + 9y^2)$

$= 8x^2(2x – 3y)^3\,(4x^2 + 6xy + 9y^2)$

7. Find the LCM of the given polynomial: $8x^4y^2$, $48x^2y^4$

<u>Solution</u>

Let us find the LCM of the numerical coefficients. That is LCM (8, 48) = 2 x 2 x 2 x 6 = 48

Then find the LCM of the terms involving variables. That is LCM (x^4y^2, x^2y^4) = x^4y^4

Finally find the LCM of the given expression. We conclude that the LCM of the given expression is the product of the LCM of the numerical coefficient and the LCM of the terms with variables. Therefore, LCM ($8x^4y^2$, $48x^2y^4$) = 48 x^4y^4

8. Find the LCM of the given polynomial: $5x – 10$, $5x^2 – 20$

<u>Solution</u>

$5x – 10 = 5(x – 2)$

$5x^2 – 20 = 5(x^2 – 4) = 5(x + 2) (x – 2)$

Therefore, LCM [(5x – 10), ($5x^2$ – 20)] = 5(x + 2) (x – 2)

9. Find the LCM of the given polynomial: $x^4 – 1$, $x^2 – 2x +1$

<u>Solution</u>

$x^4 – 1 = (x^2)^2 – 1 = (x^2 + 1) (x^2 - 1) = (x^2 + 1) (x + 1) (x – 1)$

$x^2 – 2x +1 = (x - 1)^2$

Therefore, LCM [x^4 – 1, x^2 – 2x +1] = $(x^2 + 1) (x + 1) (x – 1)^2$

10. Find the LCM of the given polynomial: $x^3 – 27$, $(x -3)^2$, $(x^2 - 9)$

<u>Solution</u>

$x^3 – 27 = x^3 – 3^3 = (x – 3) (x^2+3x+3^2) = (x – 3) (x^2 + 3x + 9)$

$(x – 3)^2 = (x – 3)^2$

$(x^2 – 9) = x^2 - 3^2 = (x + 3) (x – 3)$

Therefore, LCM = $(x – 3)^2 (x + 3) (x^2 + 3x + 9)$

11. Reduce the rational expressions to its lowest form : $\frac{x-3}{x^2-9}$

<u>Solution</u>

$$\frac{x-3}{x^2-9} = \frac{x-3}{(x+3)(x-3)} = \frac{1}{(x+3)}$$

12. Reduce the rational expressions to its lowest form : $\frac{x^2-16}{x^2+8x+16}$

<u>Solution</u>

$$\frac{x^2-16}{x^2+8x+16} = \frac{x^2-4^2}{(x+4)(x+4)} = \frac{(x+4)(x-4)}{(x+4)(x+4)} = \frac{x-4}{(x+4)}$$

13. Find the excluded values of the following expressions (if any) : $\frac{x+10}{8x}$

<u>Solution</u>

The expression $\frac{x+10}{8x}$ is undefined when 8x = 0 or x = 0. Hence the excluded value is 0.

14. Find the excluded values of the following expressions (if any) : $\frac{7p+2}{8p^2+13p+5}$

Solution

The expression $\frac{7p+2}{8p^2+13p+5}$ is undefined when $8p^2+13p+5$ = 0 that is $(8\,p+5)(p+1)=0,$

$p = \frac{-5}{8}, p = -1.$ The excluded values are $\frac{-5}{8}$ and -1

15. Find the excluded values of the following expressions (if any) : $\boldsymbol{\frac{x}{x^2+1}}$

Solution

Here $x^2 \geq 0$ for all x. Therefore, $x^2+1 \geq 0+1=1.$ Hence, $x^2+1 \neq 0$ for any x. Therefore, there can be no real excluded values for the given rational expression $\boldsymbol{\frac{x}{x^2+1}}$.

16. Reduce each of the following rational expressions to its lowest form. $\boldsymbol{\frac{x^2-1}{x^2+x}}$

Solution

$$\frac{x^2-1}{x^2+x} = \frac{x^2-1^2}{x(x+1)} = \frac{(x+1)(x-1)}{x(x+1)} = \frac{(x-1)}{x}$$

17. Reduce each of the following rational expressions to its lowest form $\boldsymbol{\frac{x^2-11x+18}{x^2-4x+4}}$

Solution

$$\frac{x^2-11x+18}{x^2-4x+4} = \frac{(x-9)(x-2)}{(x-2)(x-2)} = \frac{x-9}{x-2}$$

18. Find the excluded values of $\boldsymbol{\frac{y}{y^2-25}}$

Solution $\frac{y}{y^2-25}$ is not defined when of $y^2 - 25 = 0$

$y^2 - 5^2 = 0$

(y + 5) (y – 5) = 0

y + 5 = 0 , y – 5 = 0

y = -5, y = 5

The expression is undefined if y = -5, y = 5

∴ The excluded values are 5, - 5

19. Find the excluded values of $\boldsymbol{\frac{t}{t^2-5t+6}}$

Solution $\frac{t}{t^2-5t+6}$ is not a valid expression if $t^2 - 5t + 6 = 0$

$t^2 - 5t + 6 = 0$

(t – 2) (t – 3) = 0

t – 2 = 0, t – 3 = 0

t = 2, t =3

(t – 2) (t – 3) = 0 $\Longrightarrow$ t = 2, t =3

The expression is undefined if t = 2, t = 3

∴ The excluded values are 2, 3

20. Find the excluded values of $\frac{x^2+6x+8}{x^2+x-2}$

Solution

$$\frac{x^2+6x+8}{x^2+x-2} = \frac{(x+4)(x+2)}{(x+2)(x-1)} = \frac{x+4}{x-1}$$

The expression $\frac{x+4}{x-1}$ is undefined when x – 1 = 0. Hence the excluded values is 1

21. Find the excluded values of. $\frac{x^3-27}{x^3+x^2-6x}$

Solution

$$\frac{x^3-27}{x^3+x^2-6x} = \frac{(x-3)(x^2+3x+9)}{x(x^2+x-6)} = \frac{(x-3)(x^2+3x+9)}{(x)(x+3)(x-2)}$$

The expression $\frac{x^3-27}{x^3+x^2-6x}$ is undefined when $x^3 + x^2 - 6x = 0$

$\Longrightarrow$ (x) (x+3) (x - 2) = 0 $\Longrightarrow x = 0$ or x = -3 or x = 2

Hence the excluded values are 0, -3, 2

22. Multiply $\frac{x^3}{9y^2}$ by $\frac{27y}{x^5}$

Solution

$$\frac{x^3}{9y^2} \times \frac{27y}{x^5} = \frac{27}{9yx^2} = \frac{3}{x^2y}$$

23. Multiply $\frac{x^4b^2}{x-1}$ by $\frac{x^2-1}{a^4b^3}$

Solution

$$\frac{x^4b^2}{x-1} \times \frac{x^2-1}{a^4b^3} = \frac{x^4}{x-1} \times \frac{(x+1)(x-1)}{a^4b} = \frac{x^4(x+1)}{a^4b}$$

24. Solve $\frac{14x^4}{y} \div \frac{7x}{3y^4}$

Solution

$$\frac{14x^4}{y} \div \frac{7x}{3y^4} = \frac{14x^4}{y} \times \frac{3y^4}{7x} = 6x^3y^3$$

25. Solve $\frac{x^2-16}{x+4} \div \frac{x-4}{x+4}$

Solution

$$\frac{x^2-16}{x+4} \times \frac{x+4}{x-4} = \frac{(x+4)(x-4)}{x+4} \times \frac{x+4}{x-4} = x+4$$

26. Solve $\frac{16x^2-2x-3}{3x^2-2x-1} \div \frac{8x^2+11x+3}{3x^2-11x-4}$

Solution

$$\frac{16x^2-2x-3}{3x^2-2x-1} \div \frac{8x^2+11x+3}{3x^2-11x-4} = \frac{16x^2-2x-3}{3x^2-2x-1} \times \frac{3x^2-11x-4}{8x^2+11x+3}$$

$$= \frac{(8x+3)(2x-1)}{(3x+1)(x-1)} \times \frac{(3x+1)(x-4)}{(8x+3)(x+1)}$$

$$= \frac{(2x-1)(x-4)}{(x-1)(x+1)} = \frac{2x^2-9x+4}{x^2-1}$$

27. Simplify $\frac{4x^2y}{2z^2} \times \frac{6xz^3}{20y^4}$

Solution

$$\frac{4x^2y}{2z^2} \times \frac{6xz^3}{20y^4} \qquad = \frac{3x^3z}{5y^3}$$

28. Simplify $\frac{p^2-10p+21}{p-7} \times \frac{p^2+p-12}{(p-3)^2}$

Solution

$$\frac{p^2-10p+21}{p-7} \times \frac{p^2+p-12}{(p-3)^2} = \frac{(p-7)(p-3)}{(p-7)} \times \frac{(p+4)(p-3)}{(p-3)^2} \quad = (p+4)$$

29. Simplify $\frac{5t^3}{4t-8} \times \frac{6t-12}{10t}$

Solution

$$\frac{5t^3}{4t-8} \times \frac{6t-12}{10t} = \frac{5t^3}{4(t-2)} \times \frac{6(t-2)}{10t} = \frac{3t^2}{4}$$

30. Simplify $\frac{x(x+1)}{x-2} + \frac{x\,(1-x)}{x-2}$

Solution

$$\frac{x(x+1)}{x-2} + \frac{x\,(1-x)}{x-2} = \frac{x(x+1+1-x)}{x-2} = \frac{2x}{x-2}$$

31. Simplify $\frac{x^3}{x-y} + \frac{y^3}{y-x}$

Solution

$$\frac{x^3}{x-y} + \frac{y^3}{y-x} = \frac{x^3-y^3}{x-y} = \frac{(x^2+xy+y^2)(x-y)}{(x-y)} = x^2+xy+y^2$$

32. Find the square root of 256 $(x\text{-}a)^8$ $(x\text{-}b)^4(x\text{-}c)^{16}$ $(x\text{-}d)^{20}$

Solution $= \sqrt{256(x-a)^8(x-b)^4(x-c)^{16}(x-d)^{20}}$

$$= 16\,|(x-a)^4\ (x-b)^2\ (x-c)^8\ (x-d)^{10}|$$

33. Find the square root of $\frac{144\,a^8b^{12}c^{16}}{81f^{12}g^4h^{14}}$

Solution $\sqrt{\frac{144\,a^8b^{12}c^{16}}{81f^{12}g^4h^{14}}} \quad = \frac{12}{9}\left|\frac{a^4b^6c^8}{f^6g^2h^7}\right|$

34. Find the square root of $\frac{400x^4\,y^{12}\,z^{16}}{100x^8y^4\,z^4}$

Solution

$$\sqrt{\frac{400x^4\,y^{12}\,z^{16}}{100x^8y^4\,z^4}} \quad = \sqrt{\frac{4y^8z^{12}}{x^4}} = 2\left|\frac{y^4z^6}{x^2}\right|$$

35. Find the square root of $\frac{121(a+b)^8(x+y)^8(b-c)^8}{81(b-c)^4(a-b)^{12}\ (b-c)^4}$

Solution $\sqrt{\frac{121(a+b)^8(x+y)^8(b-c)^8}{81(b-c)^4(a-b)^{12}\ (b-c)^4}} = \frac{11}{9}\ \left|\frac{(a+b)^4(x+y)^4}{(a-b)^6}\right|$

36. Find the square root of $4x^2$+20x+25

Solution

$\sqrt{4x^2 + 20x + 25}$ = $\sqrt{(2x)^2 + (2)(2)(5)x + 5^2}$ = $\sqrt{(2x+5)^2}$ = $|2x + 5|$

37. Find the square root of 9x^2 -24xy+30xz–40yz + 25z^2 +16y^2

Solution

9x^2 -24xy+30xz–40yz + 25z^2 +16y^2

$$=\sqrt{(3x)^2 + (2)(3x)(-4y) + (2)(3x)(5z) + (2)(-4y)(5z) + (5z)^2 + (-4y)^2}$$

$\because$ $(a+b+c)^2$ = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca

$$= \sqrt{(3x - 4y + 5z)^2} = |3x - 4y + 5z|$$

38. Find the zeroes of the quadratic expression x^2+8x+12 .

Solution

Let p(x) = x^2 + 8x + 12 =(x + 2) (x + 6)

p(-2) = 4 -16 + 12 = 0

p(-6) = 36 -48 + 12 = 0

Therefore -2 and -6 are the zeroes of p(x) = x^2 + 8x + 12

39. Write down the quadratic equation in general form for which sum and product of the roots: 9,14

Solution

General form of the quadratic equation when the roots are given is

x^2 –(Sum of the roots) x + (Product of the roots) = **0**

x^2 – 9x + 14 = 0

40. Write down the quadratic equation in general form for which sum and product of the roots: -$\frac{7}{2}$, $\frac{5}{2}$

Solution x^2 – $\left(-\frac{7}{2}\right)x + \frac{5}{2}$ gives , $2x^2 + 7x + 5 = 0$

41. Write down the quadratic equation in general form for which sum and product of the roots: -$\frac{3}{5}, -\frac{1}{2}$

Solution

x^2 – $\left(-\frac{3}{5}\right)x + \left(-\frac{1}{2}\right) = 0 \Rightarrow \frac{10x^2 + 6x - 5}{10} = 0$

$10x^2 + 6x - 5 = 0$

42. Find the sum and product of the roots for quadratic equations:x^2+8x-65= 0

Solution

Let α and βbe the roots of the given quadratic equation

x^2 + 8x - 65 = 0

$a = 1, b = 8, c = -65$

$\alpha + \beta = -\frac{b}{a} = -8$ and$\alpha\beta = \frac{c}{a} = -65$

43. Find the sum and product of the roots for quadratic equations: $2x^2+5x+7 = 0$

Solution

Let α and βbe the roots of the given quadratic equation

$2x^2+5x+7 = 0$

$a = 2, b = 5, c = 7$

$\alpha + \beta = -\frac{b}{a} = -\frac{5}{2}$ and $\alpha\beta = \frac{c}{a} = \frac{7}{2}$

44. Find the sum and product of the roots for quadratic equations: $kx^2 – k^2x – 2k^3 = 0$

Solution

Let α and βbe the roots of the given quadratic equation

$kx^2 – k^2x – 2k^3 = 0$

$a = k, b = -k^2, c = -2k^3$

$\alpha + \beta = -\frac{b}{a} = -\frac{(-k)^2}{k} =$ k and $\alpha\beta = \frac{c}{a} = \frac{-2k^3}{k} = -2k^2$

45. Determine the quadratic equations , whose sum and product of roots are -9, 20.

Solution

Required Equation is

$x^2 - [\propto + \beta]x + \alpha\beta = 0$

$x^2 - [-9]x + 20 = 0 \Longrightarrow x^2 + 9x + 20 = 0$

46. Determine the quadratic equations , whose sum and product of roots are $\frac{5}{3}$, 4

Solution

Required Equation is

$x^2 -$ (sum of the roots) x + product of the roots = 0

$x^2 - \frac{5}{3}x + 4 = 0$

Multiply both sides by 3

$3x^2 - 5x + 12 = 0$

47. Determine the quadratic equations , whose sum and product of roots are $\frac{-3}{2}$, -1

Solution

Required Equation is

$x^2 - [\propto + \beta]x + \alpha\beta = 0$

$x^2 - \left[-\frac{3}{2}\right]x - 1 = 0 \qquad \Longrightarrow x^2 + \frac{3}{2}x - 1 = 0$

Multiply both sides by 2

$2x^2 + 3x - 2 = 0$

48. Determine the nature of roots for the quadratic equation $x^2 – x – 20 = 0$

Solution

$x^2 - x - 20 = 0$

Here, a = 1, b = -1, c = -20

Now, $\Delta = b^2 - 4ac$; $\Delta = (-1)^2 - 4(1)(-20) = 81$

Here $\Delta = \mathbf{81} > 0$.

So, the equation will have real and unequal roots.

49. Determine the nature of roots for the quadratic equation $9x^2 - 24x + 16 = 0$

Solution

$9x^2 - 24x + 16 = 0$

Here, a = 9, b = -24, c = 16

Now, $\Delta = b^2 - 4ac$; $\Delta = (-24)^2 - 4(9)(16) = 0$

Here $\Delta = 0$.

So, the equation will have real and equal roots.

50. Determine the nature of roots for the quadratic equation $2x^2 - 2x + 9 = 0$

Solution

$2x^2 - 2x + 9 = 0$

Here, a = 2, b = -2, c = 9

Now, $\Delta = b^2 - 4ac$; $\Delta = (-2)^2 - 4(2)(9) = -68$

Here $\Delta = \mathbf{-68} < 0$.

So, the equation will have no real roots.

51. Find the values of 'k', for which the quadratic equation kx^2–(8k+4) + 81 = 0 has real and equal roots?

Solution

kx^2–(8k+4) + 81 = 0

Since the equation has real and equal roots, $\Delta = 0$.

That is $b^2 - 4ac = 0$ Here a = k, b = -(8k+4), c = 81

That is, $[-(8k+4)]^2 - 4(k)(81) = 0$

$64k^2 + 64k + 16 - 324k = 0$

$64k^2 - 260k + 16 = 0$

Dividing by 4 we get, $16k^2 - 65k + 4 = 0$

$(16k - 1)(k - 4) = 0$

$k = \frac{1}{16}$ or k = 4.

52. Find the value of 'k' such that quadratic equation (k + 9) x^2 + (k +1) x +1 = 0 has no real roots?

Solution

(k + 9) x^2 + (k +1) x +1 = 0. Since the equation has no real roots, $\Delta < 0$

That is $b^2 - 4ac = 0$. Here a = k+9, b = k +1, c = 1

That is, $(k + 1)^2 - 4(k + 9)(1) < 0$

$k^2 + 2k + 1 - 4k - 36 < 0$

$k^2 - 2k - 35 < 0$

$(k + 5)(k - 7) < 0$

Therefore, -5 < k < 7. { If $\alpha < \beta$ $and\ if$ $(x - \alpha)(x - \beta) < 0$ then,$\alpha < x < \beta$ }.

53. Find the values of k for which the roots of the following equations are real and equal $(5k - 6)x^2 + 2kx + 1 = 0$

Solution $(5K - 6)x^2 + 2kx + 1 = 0$

a = 5k – 6, b = 2k, c = 1

The roots are real and equal. So

$b^2 - 4ac = 0$

$(2k)^2 - 4(5k-6)1 = 0 \Longrightarrow 4k^2 - 20k + 24 = 0 \quad \div 4$

$k^2 - 5k + 6 = 0 \quad \Longrightarrow (k - 2)(k - 3) = 0$

k =2 or k =3

54. Find the values of k for which the roots of the following equations are real and equal $kx^2 + (6k+2)x + 16 = 0$

Solution $kx^2 + (6k+2)x + 16 = 0$

a = k, b = 6k+2, c = 16

The roots are real and equal. So

$b^2 - 4ac = 0$

$\Longrightarrow (6k+2)^2 - 4(k)16 = 0$

$\Longrightarrow 36k^2 + 24k + 4 - 64k = 0$

$\Longrightarrow 36k^2 - 40k + 4 = 0 \ (\div 4)$

$\Longrightarrow 9k^2 - 10k + 1 = 0$

$\Longrightarrow (k - 1)(9k - 1) = 0$

$\therefore k = 1$ (or) k = $\frac{1}{9}$

5 marks

1. **Find the square root of $64x^4 – 16x^3 + 17x^2 – 2x +1$.**

Solution

	8		-1	1		
8	64	-16	17	-2	1	
	(-)64					
16 -1		-16	17			$-\frac{16}{16}$ = -1
	(+)	-16	(-) 1			
16 – 2 1			16	-2	1	$\frac{16}{16}$ = 1
		(-)	16	(+) -2	(-) 1	
			0			

Therefore, $\sqrt{64x^4 - 16x^3 + 17x^2 - 2x + 1} = \boldsymbol{|8x^2 - x + 1|}$

2. **If $9x^4 + 12x^3 + 28x^2+ax+b$ is a perfect square, find the values of a and b**

Solution

	3	2	4			
3	9	12	28	a	b	
	(-) 9					
6 2		12	28			$\frac{12}{6}$ =2
		(-) 12	(-) 4			
6 4 4			24	a	b	
			(-) 24	(-)16	(-) 16	$\frac{24}{6}$ = 4
			a = 16 , b = 16			

3. **Find the square root of the following polynomials by division method $x^4 – 12x^3 + 42x^2 –36x +9$**

Solution $x^4 – 12x^3+42x^2 –36x+9$

	1	-6	3				
1	1	-12	42	-36	9	.	
	(-)1						
2 - 6		-12	42				$\frac{-12}{2}$ = -6
		(+)-12	(-) 36				
2 – 12 3			6	-36	9		$\frac{6}{2}$ = 3
			(-)6	(+) -36	(-) 9		
			0				

Solution is $|x^2 - 6x + 3|$

4. **Find the square root of the following polynomials by division method $37x^2–28x^3 +4x^4 + 42x +9$**

Solution

	2	-7	- 3			
2	4	-28	37	42	9	
	(-) 4					
4 – 7		-28	37			$\frac{-28}{4}$ = -7
		(+) -28	(-) 49			
4 – 14 -3			- 12	42	9	$\frac{-12}{4}$ = -3
		(+)	- 12	(-)4 2	(-) 9	
			0			

Solution is $|2x^2 - 7x - 3|$

5. **Find the square root of the following polynomials by division method 16x^4 + 8x^2 + 1.**

Solution

	4	0	1		
4	16	0	8	0	1
	(-) 16				
8 0		0	8		
		(-) 0	(-)0		
8 0 1			8	0	1
			(-) 8	(-)0	(-)1
			0		

Required square root = $|4x^2 + 1|$

6. **Find the square root of the following polynomials by division method**
121 x^{4-} 198x^3 - 183x^2 + 216x +144

Solution

	11	- 9	-12			
11	121	-198	-183	216	144	
	(-) 121					
		- 198	-183			$-\frac{198}{22}$ = -9
22 –9		(+) - 198	(-)81			
			-264	216	144	
22 -18 -12			(+)-264	(-)216	(-)144	- $\frac{264}{22}$ = -12
			0			

Solution is = $|11x^2 - 9x - 12|$

7. **Find the values of a and b if following polynomials are perfect squares4x^4 –12x^3+ 37x^2+bx +a**
Solution

	2	-3	7			
	4	-12	37	b	a	
2	(-) 4					
		-12	37			$-\frac{12}{4}$ = -3
4 –3		(+) -12	(-) 9			
			28	b	a	$\frac{28}{4}$ = 7
4 -6 7			(-) 28	(+)-42	(-) 49	
				0		

a = 49 , b = -42

8. **Find the values of a and b if the following polynomials are perfect squares $ax^4 + bx^3 + 361x^2 + 220x + 100$**

Solution

	10	11	12			
10	100	220	361	b	a	
	(-) 100					
		220	361			$\frac{220}{20}$ = 11
20 11		(-)220	(-)121			
			240	b	a	$\frac{240}{20}$ = 12
20 22 12			(-) 240	(-) 264	(-)144	
				0		

a = 144 , b = 264

9. **Find the values of m and n if the following expressions are perfect squares**

$36x^4 - 60x^3 + 61x^2 - mx + n$.

Solution

	6	-5	3			
6	36	-60	61	-m	n	
	(-) 36					
12 - 5		- 60	61			
		- 60	25			$-\frac{60}{12} = -5$
		(+)	(-)			
12 – 10 + 3			36	-m	n	
			36	-30	9	$\frac{36}{12} = 3$
			(-)	(+)	(-)	
			0			

$-m = -30 \Rightarrow m = 30$

and $n = 9$

10. Find the values of m and n if the following expressions are perfect squares

$x^4 - 8x^3 + mx^2 + nx + 16$

<u>Solution</u>

	1	-4	4			
1	1	-8	m	n	16	
	1					
		-8	m			$-\frac{8}{2}$ = -4
2 –4		(+)-8	(-)16			
			m-16	n	16	$\frac{m-16}{2}$
2 -8 4			8	-32	16	
				0		

m-16 = 8 , n = -32

m = 8 + 16

m = 24

CHAPTER - 4
GEOMETRY

1 MARKS

1. If in triangles ABC and EDF ,$\frac{AB}{DE} = \frac{BC}{FD}$ then they will be similar, when

   (1)$\angle B = \angle D$ (2) $\angle A = \angle D$ (3) $\angle B = \angle E$ (4) $\angle A = \angle F$

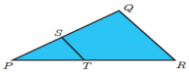
2. In ΔLMN - ∠L=60° , ∠M=50°, If ΔLMN ~ ΔPQR then the value of, ∠R is

   (1) 30° (2) 40° **(3) 70°** (4) 110°

3. If ΔABC-is an isosceles triangle with ∠C=90°and AC=5cm,then AB is

   (1) $5\sqrt{2}cm$ (2) 10 cm (3) 2.5cm (4) 5cm

4. In a given figure ST∥QR, PS = 2cmand SQ =3cm and, Then the ratio of the area of ΔPQR to the area of ΔPST is 1) 25 : 7 **(2) 25 : 4** (3) 25 : 13 (4) 25 : 11



5. The perimeters of two similar triangles ΔABCand ΔPQR are 36cm and 24cm respectively. PQ = 10cm,then length of AB is

   (1) $6\frac{2}{3}$ cm (2) $66\frac{2}{3}$ cm (3)$\frac{10\sqrt{6}}{3}$ cm **(4) 15cm**

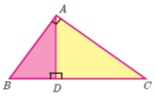
6. If in ΔABC, DE ∥ BC, AB = 3.6cm, AC=2.4cm and AD=2.1cm then the length of AE is

   (1) 1.05 cm (2) 1.2 cm **(3) 1.4 cm** (4) 1.8 cm

7. In a ΔABC,AD is the bisector of, $\angle BAC$ If AB= 8CM BD = 6 cm and DC = 3 cm The length of the side AC is (1) 3cm **(2) 4cm** (3) 6 cm (4) 8 cm

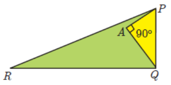
8. In the adjacent figure $\angle BAC = 90°$ and AD ⊥BC then,

   (1) BD.CD = BC^2 (2) AB.AC = BC^2 **(3) BD.CD = AD^2** (4) AB.AC = AD^2



9. Two poles of heights 6m and 11m stand vertically on a plane ground.If the distance between their feet is 12m What is the distance between their tops ?(1)12.8m **(2) 13m** (3) 14m (4) 15m

10. In the given figure , PR = 26 cm , QR = 24 cm ∠PAQ = 90°, PA = 6cm , QA = 8cm Find ∠PQR **(1) 90°** (2) 85° (3) 80° (4) 75°



11. A Tangent is perpendicular to the radius at the

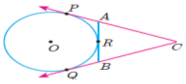
    (1) Centre (2)infinity **(3)point of contact** (4) chord

12. How many tangents can be drawn to the circle from an exterior point?

    (1) Zero (2)one **(3) two** (4)infinite

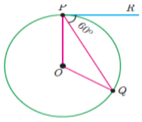
13. The two tangents from an external points P to a circle With centre at O Are PA and PB.IF $\angle APB = 70°$ *then the value of* $\angle AOB$ is (1)120° (2) 130° (3) 100° **(4) 110°**

14. If figure CP and CQ are tangents to a circle With centre at O. ARB is another tangent touching the circle at R. If. CP = 11cm and BC = 7cm then the length of BR is (1) 8cm (2) 6cm (3) 5cm **(4) 4cm**



15. In figure if PR is tangent to the circle at P and O is the centre of the circle,then ∠POQ is

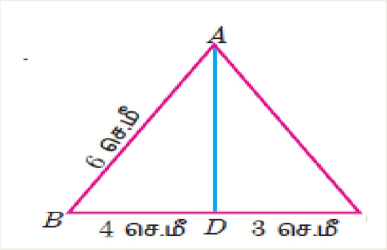
    (1) 90° **(2) 120°** (3) 100° (4) 110°



2 MARKS

1. **From the figure, AD is the bisector of ∠A. If BD=4 cm, DC=3 cm and AB= 6 cm,find AC.**

Solution In ΔABC, AD is the bisector of ∠A.



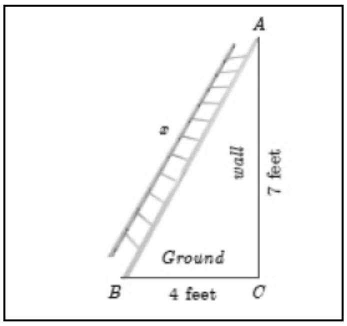
Therefore by Angle Bisector Theorem

$$\frac{BD}{DC}=\frac{AB}{AC}.$$

$\frac{4}{3}=\frac{6}{AC}$ gives 4AC = 18.Hence AC = $\frac{9}{2}$ = 4.5 cm

2. **What length of ladder is needed to reach a height of 7 ft along the wall when the base of the ladder is 4ft from the wall? Round off your answer to the next tenth place?**

Solution



Let x be the length of the ladder, BC = 4ft, AC = 7ft

By Pythagoras theorem we have,$AB^2 = AC^2 + BC^2$

$$x^2 = 7^2 + 4^2 \text{ gives } x^2 = 49 + 16$$

$$x^2 = 65. \text{ Hence } x = \sqrt{65}$$

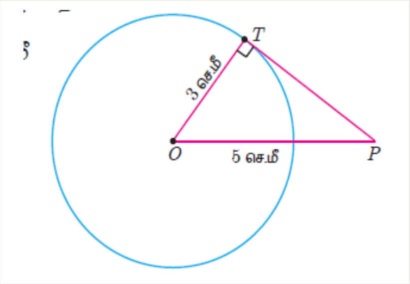
The number $\sqrt{65}$ is between 8 and 8.1.

$$8^2 = 64 < 65 < 65.61 = 8.1^2$$

Therefore, the length of the ladder is approximately 8.1 ft.

3. **Find the length of the tangent drawn from a point whose distance from the centre of a circle is 5cm and radius of the circle is 3 cm.**

Solution Given OP = 5cm, radius r= **3 செ.மீ**



To find the length of tangent PT.

In right angled Triangle OTP

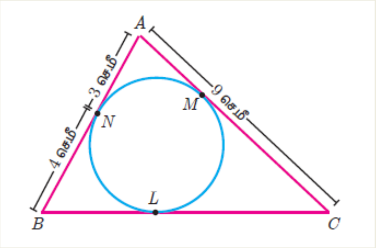
$OP^2 = OT^2 + PT^2$ (By Phythagorous Theorem)

$5^2 = 3^2 + PT^2 \Rightarrow PT^2 = 25 – 9 = 16$

Length of the tangent PT = 4 cm.

4. **In the figure Δ ABC is circumscribing a circle. Find the length of BC.**

Solution AN = AM = 3 cm(Tangents drawn from same external point are equal)



BN = BL = 4cm

CL = CM = AC – AM

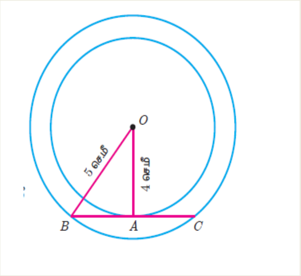
= 9 – 3 = 6cm

BC = BL + CL

= 4 + 6

= 10cm

5. **If radii of two concentric circles are 4cm and 5cm then find the length of the chord of one circle which is a tangent to the other circle.**



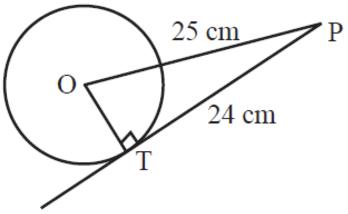
<u>Solution</u> OA = 4 cm, OB = 5cm, also OA ⊥ BC

$OB^2 = OA^2 + AB^2 \quad \Rightarrow 5^2 = 4^2 + AB^2 \quad \Rightarrow AB^2 = 25 - 16 = 9$

Therefore AB = 3cm, BC = 2AB hence, BC = 2 X 3 = 6cm

6. **The length of the tangent to a circle from a point P, which is 25cm away from the centre is 24cm. What is the radius of the circle?**

<u>Solution</u>



From the figure, OP = 25 cm, PT = 24 cm

$r = \sqrt{OP^2 - PT^2}$

$= \sqrt{25^2 - 24^2}$

$= \sqrt{625 - 576} = \sqrt{49}$

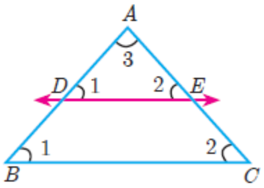
r = 7 cm

5 MARKS

1. State and Prove Basic Proportionality Theorem (BPT) or Thales Theorem

Statement

A straight line drawn parallel to a side of triangle intersecting the other two sides, divides the sides in the same ratio.



Proof **Given :** In ΔABC, D is a point on AB and E is a point on AC

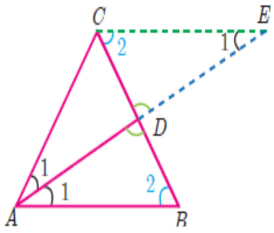
To Prove : $\frac{AD}{DB} = \frac{AE}{EC}$. Construction : Draw a line **DE ∥BC**

No.	Statement	Reason
1.	∠ABC = ∠ADE = ∠1	Corresponding angles are equal because DE ∥BC
2.	∠ACB = ∠AED = ∠2	Corresponding angles are equal because DE ∥BC
3.	∠DAE = ∠BAC = ∠3	Both triangles have a common angle.
4.	ΔABC ~ ΔADE $\frac{AB}{AD} = \frac{AC}{AE}$ $\frac{AD+DB}{AD} = \frac{AE+EC}{AE}$ $1 + \frac{DB}{AD} = 1 + \frac{EC}{AE}$ $\frac{DB}{AD} = \frac{EC}{AE}$ $\frac{AD}{DB} = \frac{AE}{EC}$	By AAA similarity Corresponding sides are proportional Split AB and AC using the points D and E On Simplication Cancelling 1 on both sides Taking reciprocals
Hence Proved		

2. State and Prove Angle Bisector Theorem.

Statement

The internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the corresponding sides containing the angle



Proof Given : In ΔABC, AD is the internal bisector

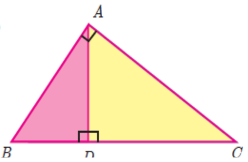
To Prove : $\frac{AB}{AC}=\frac{BD}{CD}$.

Construction : Draw a line through C parallel to AB. Extend AD to meet line through C at E

No.	Statement	Reason
1.	∠AEC = ∠BAE = ∠1	Two parallel lines cut by a transversal make alternate angles equal
2.	ΔACE is isosceles AC = CE …. (1)	In ΔACE ∠CAE = ∠CEA.
3.	ΔABD ~ ΔECD $\frac{AB}{CE}=\frac{BD}{CD}$	By AA Similarity
4.	$\frac{AB}{AC}=\frac{BD}{CD}$	From (1) AC = CE. Hence Proved.

3. State and Prove Pythagoras Theorem

Statement In a right angle triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides



Proof Given : In ΔABC, ∠A = 90°

To Prove : $AB^2 + AC^2 = BC^2$

Construction : Draw AD ⊥BC

No.	Statement	Reason
1.	Compare ΔABC and ΔABD ∠B is common ∠BAC = ∠BDA = 90° Therefore, ΔABC ~ ΔABD $\frac{AB}{BD}=\frac{BC}{AB}$ AB^2 = BC X BD ... (1)	Given ∠BAC = 90° and by construction ∠BDA = 90° By AA similarity
2.	Compare ΔABC and ΔADC ∠C is common ∠BAC = ∠ADC = 90° Therefore, ΔABC ~ ΔADC $\frac{BC}{AC}=\frac{AC}{DC}$ AC^2 = BC X DC ... (2)	Given ∠BAC = 90° and by construction ∠CDA = 90° By AA similarity

Adding (1) and (2) we get

$AB^2 + AC^2$ = BC X BD + BC X DC

= BC(BD + DC)

$AB^2 + AC^2$ = BC X BC = $BC^{2.}$ Hence the theorem is proved.

4. **Show that in a triangle, the medians are concurrent.**

Solution:

Medians are line segments joining each vertex to the midpoint of the corresponding opposite sides.

Thus medians are the cevians where D, E, F are midpoints of BC, CA and AB respectively.

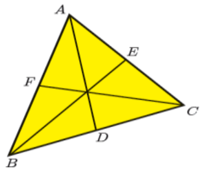
Since D is midpoint of BC, BD = DC. So $\frac{BD}{DC} = 1$ ----- (1)

Since E is midpoint of CA, CE = EA. So $\frac{CE}{EA} = 1$ ----- (2)

Since F is midpoint of AB, AF = FB. So $\frac{AF}{FB} = 1$ ----- (3)

Thus, multiplying (1), (2), (3) we get $\frac{BD}{DC}$ x $\frac{CE}{EA}$ x $\frac{AF}{FB}$ = 1 x 1 x 1 = 1

And so, Ceva's theorem is satisfied. Hence the Medians are concurrent



5. **A circle is inscribed in ΔABC having sides 8 cm, 10cm and 12cm as shown in figure. Find AD, BE and CF.**

Solution

By result for tangents from external point

AD = AF = x , DB = BE = y, EC = CF = z

AB + BC + AC = 30

⇒ AD + BD + BE + CE+ CF + FA = 30

⇒ x + y + y + z + z + x = 30

⇒ 2(x + y + z) = 30

x + y + z = 15 …….(1)

AB = AD + BD = 12

⇒ $x + y = 12$ …….(2)

BC = 8 ⇒ $y + z = 8$ …….(3)

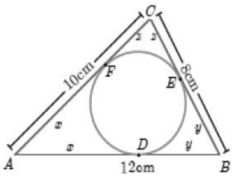
AC = 10 ⇒ $x + z = 10$ …….(4)

(3) – (4) ⇒ $y - x = -2$ …….(5)

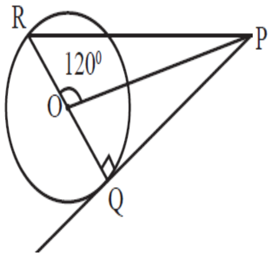
(5) + (2) ⇒ $2y = 10$ ⇒ $y = 5$

Substituting y = 5 in (2) ⇒ $x = 7$ and y = 5 in(1) ⇒ z = 3

Hence AD = 7cm , BE = 5cm, CF = 3cm.



6. **PQ is a tangent drawn from a point P to a circle with centre O and QOR is a diameter of the circle such that ∠PQR = 120° . Find ∠OPQ.**



Solution From the figure, $\angle ROQ = 180°$

$\angle ROP = 120°$

∴ $\angle POQ = 60°$ (∵ $\angle ROQ = \angle ROP + \angle POQ$)

$\angle POQ + \angle OQP + \angle QPO = 180°$ (from triangle property)

$60° + 90° + \angle QPO = 180°$ ($\angle OQP = 90°$ from tangents property)

$\angle QPO = 30°$. $\angle OPQ = 30°$

CHAPTER - 5
COORDINATE GEOMETRY
1 MARKS

1. The area of triangle formed by the points(-5,0), (0,-5)and (5,0) is

   (1) 0 sq.units (2) 5 Sq .units **(3) 25 sq.units** (4) non of these

2. A man walks near a wall ,such that the distance between him and the wall is 10 units. consider the wall to be the Y axis .The path travelled by the man is

   (1) x = 0 **(2) x = 10** (3) y = 0 (4) y = 10

3. The straight line given by the equation x = 11

   (1) Passing through the origin (2)Passing through the point (0,11)

   (3)Parallel to X –axis **(4) Parallel to Y - axis**

4. I f (5,7), (3,p) and (6,6) are collinear, then the value of , p is

   (1) 9 (2) 12 (3) 3 (4) 6

5. The point of intersection of 3x – y = 4 and x + y = 8 is

   (1) (3,5) (2) (2,4) (3) (5,3) (4) (4,4)

6. The slope of the line joining (12,3), (4,a) is $\frac{1}{8}$. The value of a is

   (1) 1 **(2) 2** (3) 4 (4) -5

7. The slope of the line which is perpendicular to a line joining the points (0,0)and (-8,8) is

   (1) -1 **(2) 1** (3) $\frac{1}{3}$ (4) -8

8. If slope of the line PQ is $\frac{1}{\sqrt{3}}$, *then slope of the perpendicular bisector of* PQ is

   (1) 0 (2) $\sqrt{3}$ **(3) - $\sqrt{3}$** (4) $\frac{1}{\sqrt{3}}$

9. If A is a point on the Yaxis whose ordinate is 8 and B is a point on the X axis whose abscissae is 5 then the equation of the line, AB is

   (1) 8x – 5y = 40 **(2) 8x + 5y = 40** (3) y = 5 (4) x = 8

10. A Straight line has equation 8y= 4x + 21,Which of the following is true?

    (1) The slope is 0.5and the y intercept is 1.6 **(2)The slope is 0.5and the yintercept is 2.6**

    (3)The slope is 5 and the yintercept is 2.6 (4) The slop is 5and y intercept is1.6

11. When proving that a quadrilateral is a trapezium ,it is necessary to show

    (1) **Two parallel and two non-parallel sides** (2) Two sides are parallel.

    (3) Opposite side are parallel (4)All sides are of equal length

12. When proving that a quadrilateral is a parallelogram by using slopes you must find .

    (1) The lengths of all sides **(2)The slopes of two sides**

    (3)The slopes of two pair of opposite sides (4) Both the lengths and slopes of two sides

13. (2,1) is the point of intersection of two lines.

    (1) x + 3y – 3 = 0; x – y – 7 =0 (2) 3x + y = 3; x + y =7

    (2) **x + y = 3; 3x + y = 7** (4) x – y – 3 = 0; 3x – y – 7 = 0

2 MARKS

1. **Find the area of the triangle whose vertices are (-3,5), (5,6) and (5,-2).**

Solution Area of Δ $= \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix}$

$$= \frac{1}{2}\begin{vmatrix} -3 & 5 \\ 5 & 6 \\ 5 & -2 \\ -3 & 5 \end{vmatrix} = \frac{1}{2}\ [(-18-10+25) - (25 + 30 + 6)]$$

$$= \frac{1}{2}\ [-3-61] \quad = \left|\frac{-64}{2}\ \right| \quad = 32 \text{ sq.units.}$$

2. **Show that the points P(-1.5, 3), Q (6,-2) and R(-3, 4) are collinear.**

Solution To Prove Area of Δ PQR = 0

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} -1.5 & 3 \\ 6 & -2 \\ -3 & 4 \\ -1.5 & 3 \end{vmatrix}$$

$$= \frac{1}{2}\,[(3+24-9) - (18+ 6 - 6)\ ] \quad = \frac{1}{2}\,[18 - 18] \ = 0$$

∴ Therefore, the given points are collinear.

3. **If the area of the triangle formed by the vertices A (-1, 2), B(k,-2) and C (7, 4) (taken in order) is 22 sq.units, find the value of k.**

Solution

The vertices are A (–1, 2), B(k,–2) and C (7, 4)

Area of ΔABC is 22 sq.units.

$$\frac{1}{2}\begin{vmatrix} -1 & 2 \\ k & -2 \\ 7 & 4 \\ -1 & 2 \end{vmatrix} = 22$$

$$\begin{vmatrix} -1 & 2 \\ k & -2 \\ 7 & 4 \\ -1 & 2 \end{vmatrix} = 44$$

{(2 + 4k + 14) – (2k – 14 -4)} = 44

4k + 16 -2k +18 =44

2k + 34 = 44

2k = 10. Therefore K = 5.

4. **The floor of a hall is covered with identical tiles which are in the shapes of triangles. One such triangle has the vertices at (-3,2), (-1,-1) and (1,2). If the floor of the hall is completely covered by 110 tiles, find the area of the floor.**

Solution

Vertices of one triangular tile are at (-3,2), (-1,-1) and (1,2)

Area of this tile $= \frac{1}{2}\begin{vmatrix} -3 & 2 \\ -1 & -1 \\ 1 & 2 \\ -3 & 2 \end{vmatrix}$ sq.units

$= \frac{1}{2}$ (12) = 6 sq.units

Since the floor is covered by 110 triangle shaped identical tiles,

Area of floor = 110 × 6 = 660 sq.units

5. **Find the area of the triangle formed by the points (1, -1), (-4, 6) and (-3, -5) .**

Solution Area of Δ $= \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 1 & -1 \\ -4 & 6 \\ -3 & -5 \\ 1 & -1 \end{vmatrix}$

$= \frac{1}{2}$ [(6 + 20 + 3) – (4 – 18 – 5)]

$= \frac{1}{2}$ [29 + 19] $= \frac{1}{2}$ [48]= 24 sq.units.

6. **Find the area of the triangle formed by the points (-10,- 4), (-8, -1) and (-3, -5)**

Solution Area of Δ$= \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} -10 & -4 \\ -8 & -1 \\ -3 & -5 \\ -10 & -4 \end{vmatrix}$

$= \frac{1}{2}$ [(10 + 40 + 12) – (32 + 3 + 50)]

$= \frac{1}{2}$ [62 - 85] $= \frac{1}{2}$ [-23]= -11.5 = 11 sq.units.

7. **Determine whether the sets of points are collinear.$\left(-\frac{1}{2}, 3\right)$, (-5, 6) and (-8, 8)**

Solution Area of Δ $= \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} -\frac{1}{2} & 3 \\ -5 & 6 \\ -8 & 8 \\ -\frac{1}{2} & 3 \end{vmatrix}$

$= \frac{1}{2}$ [(−3 - 40 -24) – (-15 –48 – 4)]

$= \frac{1}{2}$ [(-67) – (-67)] = 0 sq.units.

∴ The given points are collinear.

8. **If the Vertices of a triangle taken in order are (0,0), (p,8), (6,2) and the area of the triangle formed by the vertices is 20 sq.units. Find the value of 'p'.**

Solution Area of Δ = 20 sq.units.

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = 20 \quad \Rightarrow \frac{1}{2}\begin{vmatrix} 0 & 0 \\ p & 8 \\ 6 & 2 \\ 0 & 0 \end{vmatrix} = 20$$

$(0 + 2p + 0) - (0 + 48 + 0) = 40$

2p – 48 = 40

2p = 88

p = 44

9. If the vertices of a triangle taken in order are (p, p), (5,6), (5,-2) and the area of the triangle formed by the vertices is 32 sq.units. Find the value of 'p'.

Solution Area of Δ = 32

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = 32 \qquad \Rightarrow \frac{1}{2}\begin{vmatrix} p & p \\ 5 & 6 \\ 5 & -2 \\ p & p \end{vmatrix} = 32$$

$$\begin{vmatrix} p & p \\ 5 & 6 \\ 5 & -2 \\ p & p \end{vmatrix} = 64$$

$$(6p - 10 + 5p) - (5p + 30 - 2p) = 64$$

11p – 10 – (3p +30) = 64 ⇒ 11p–10–3p – 30 = 64

8p – 40 = 64 ⇒ 8p = 64 + 40

8p = 104 ⇒ p $=\frac{104}{8}$ ⇒ p = 13

10. **Find the value of 'a' for which the given points are collinear.(2,3), (4,a) (6,-3)**

Solution A(2,3), B(4,a) C(6,-3)are collinear

Area of ΔABC = 0

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = 0 \Rightarrow \frac{1}{2}\begin{vmatrix} 2 & 3 \\ 4 & a \\ 6 & -3 \\ 2 & 3 \end{vmatrix} = 0$$

$\frac{1}{2}$ [(2a – 12 + 18) – (12 + 6a - 6)] = 0

$(2a + 6) - (6 + 6a) = 0 \times \frac{2}{1}$ ⟹ 2a + 6 – 6 – 6a = 0

-4a=0 ⟹a=0

11. Find the slope of a line joining the given points (-6,1) and (-3,2)

Solution A(-6,1), B(-3,2)

Slope of AB, m = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{2-1}{-3-(-6)} = \frac{2-1}{-3+6}$

∴Slope m = $\frac{1}{3}$

12. Find the slope of a line joining the given points$\left(-\frac{1}{3}, \frac{1}{2}\right)$and $\left(\frac{2}{7}, \frac{3}{7}\right)$

Solution Slope, m = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{\frac{3}{7} - \frac{1}{2}}{\frac{2}{7} - (-\frac{1}{3})} = \frac{\frac{6-7}{14}}{\frac{6+7}{21}} = -\frac{1}{14} \times \frac{21}{13} = -\frac{3}{26}$

∴ Slope, m = - $\frac{3}{26}$

13. Find the slope of a line joining the given points (14, 10) and (14,- 6)

Solution Slope,, m = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{-6-10}{14-14} = \frac{-16}{0}$

∴ Slope, m $=\frac{-16}{0}$

14. Show that the points (-2,5), (6,-1) and (2,2) are collinear.

Solution

Let the given points be A, B and C

Slope of AB = $\frac{-1-5}{6+2} = \frac{-6}{8} = \frac{-3}{4}$

Slope of BC $= \frac{2+1}{2-6} = \frac{3}{-4} = \frac{-3}{4}$

Slope of AB = Slope of BC

Therefore, the points A, B, C all lie in a same straight line.

Hence A, B and C are collinear.

15. Show that the given points are collinear: (-3, -4), (7, 2) and (12, 5) .

Solution

Let the given points be A, B and C

Slope of AB = $\frac{2-(-4)}{7-(-3)} = \frac{6}{10} = \frac{3}{5}$ ....(1)

Slope of BC = $\frac{5-2}{12-7} = \frac{3}{5}$ ....(2)

Slope of AC = $\frac{5-(-4)}{12-(-3)} = \frac{9}{15} = \frac{3}{5}$ ....(3)

(1), (2), (3) ⇒ A, B, and C are Collinear.

16. **If the three points (3, -1), (a, 3) and (1, -3) are collinear, find the value of a.**

Solution

Let the given points be A (3, -1), B (a, 3) and C(1, -3) and given A, B and C are collinear.

∴ Slope of AB = Slope of BC

$\Rightarrow \frac{3-(-1)}{a-3} = \frac{-3-3}{1-a}$

$\Rightarrow \frac{4}{a-3} = \frac{-6}{1-a}$

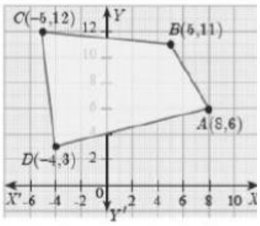
⇒ 4 – 4a = - 6a + 18

⇒ 2a = 14 ⇒ $a = 7$

5 MARKS

1. Find the Area of the quadrilateral formed by the points (8, 6), (5, 11), (-5, 12) and (-4, 3)

Solution Before determining the area of the quadrilateral, plot the vertices in a graph A(8, 6), B (5, 11), C(-5, 12)and D(-4, 3)



Therefore, area of the quadrilateral ABCD

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_4 & y_4 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 8 & 6 \\ 5 & 11 \\ -5 & 12 \\ -4 & 3 \\ 8 & 6 \end{vmatrix}$$

$= \frac{1}{2}$ [(88+ 60–15 – 24)–(30 –55– 48 + 24)] = $\frac{1}{2}$ [(88+60-15-24-30+55+48-24)]

$= \frac{1}{2}$ [88+60-15-24-30+55+48-24]

$= \frac{1}{2}$ [88+60+55+48-15-24-30-24]

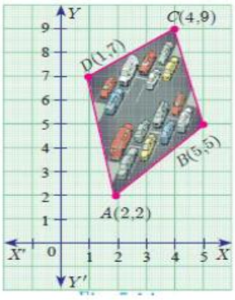
$= \frac{1}{2} [251 - 93]$

$= \frac{1}{2}$ [158] = 79 sq.units.

2. The given diagarm shows a plan for constructing a new parking lot at a campus. It is estimated that such construction would cost ₹1300 per square feet. What will be the total cost for making the parking lot?

Solution The parking lot is a quadrilateral whose vertices

A(2, 2), B (5, 5), C(4, 9) and D(1, 7).



Therefore, Area of parking lot is

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_4 & y_4 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 2 & 2 \\ 5 & 5 \\ 4 & 9 \\ 4 & 7 \\ 2 & 2 \end{vmatrix} = \frac{1}{2}[(10+ 45 +28 + 2) – (10 + 20 + 9 + 14)]$$

$= \frac{1}{2}$ [85 – 53]

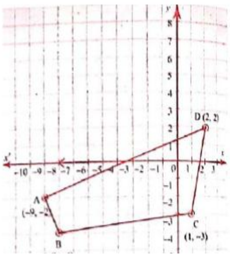
$= \frac{1}{2}$ [32] = 16 sq.units.

So, Area of parking lot = 16 sq.feets. Construction rate per square fee = ₹1300

Therefore, total cost for constructing the parking lot = 16 x 1300 = ₹ 20800

3. Find the area of the quadrilateral whose vertices are at (-9, - 2), (-8, -4), (2, 2) and (1, -3) (ii) (-9, 0) , (-8, 6), (-1, -2) and (-6, -3)

Solution **(i) Let** A(-9,-2), B(-8,-4), C(1, −3), D(2,2)



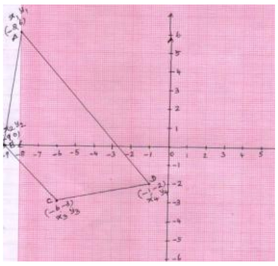
Area of quadrilateral $= \frac{1}{2}\begin{vmatrix} -9 & -2 \\ -8 & -4 \\ 1 & -3 \\ 2 & 2 \\ -9 & -2 \end{vmatrix}$

$= \frac{1}{2}$ [(36 +24 +2 - 4) – (16 – 4 -6 - 18)]

$= \frac{1}{2}$ [58 + 12] $= \frac{1}{2}$ [70] = 35 sq.units

(ii) (-9, 0), (-8, 6), (-1, -2) and (-6, -3)

Solution A(-8,6), B(-9,0), C(-6, -3), D(-1, -2)



Area of quadrilateral $= \frac{1}{2}\begin{vmatrix} -8 & 6 \\ -9 & 0 \\ -6 & -3 \\ -1 & -2 \\ -8 & 6 \end{vmatrix}$

$= \frac{1}{2}$ [(0 +27+12 - 6) – (-54 + 0 + 3 + 16)]

$= \frac{1}{2}$ [33 + 35] $= \frac{1}{2}$ [68] = 34 sq.units

4. **Find the value of k, if the area of quadrilateral is 28 sq.units, whose vertices are (-4,-2), (-3, k), (3, -2) and(2, 3)**

Solution Area of quadrilateral, $\frac{1}{2}\begin{pmatrix}-4 & -3 & 3 & 2 & -4\\ -2 & k & -2 & 3 & -2\end{pmatrix}$ = 28

⇒ (-4k + 6 + 9 - 4) – (6 +3k - 4 -12) = 56

⇒ (11 – 4k) – (3k – 10) = 56

⇒ 21 – 7k = 56 ⇒ 7k = -35 ⇒ k = -5

(or)

$$\frac{1}{2}\begin{vmatrix}-4 & -2\\ -3 & k\\ 3 & -2\\ 2 & 3\\ -4 & -2\end{vmatrix} = \mathbf{28}$$

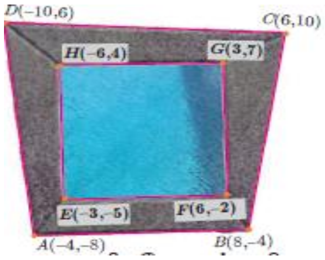
⇒ (-4k + 6 + 9 - 4) – (6 +3k - 4 -12) = 56

⇒ (11 – 4k) – (3k – 10) = 56

$\Rightarrow 11 - 4k - 3k - +10 = 56$

⇒ 21 – 7k = 56 ⇒ 7k = -35 ⇒ k = -5

5. **In the figure, the quadrilateral swimming pool shown is surrounded by concrete patio. Find the area of the patio.**



Solution:

Area of the patio = Area of ABCD – Area of EFGH

$= \frac{1}{2}\begin{pmatrix}-4 & 8 & 6 & -10 & -4\\ -8 & -4 & 10 & 6 & -8\end{pmatrix} - \frac{1}{2}\begin{pmatrix}-3 & 6 & 3 & -6 & -3\\ -5 & -2 & 7 & 4 & -5\end{pmatrix}$

= $\frac{1}{2}$ [(16+80+36 + 80)–(– 64 - 24 - 100 - 24)] - $\frac{1}{2}$ [(6+42+12+30) – (–30-6-42-12)]

= $\frac{1}{2}$ [212 – (-212)] - $\frac{1}{2}$ [90- (-90)]

= $\frac{1}{2}$ [424] - $\frac{1}{2}$ [180]

= 212 – 90 = 122 square units.

6. **A triangular shaped glass with vertices at A(-5, -4), B(1, 6) and C(7, -4) has to be painted. If one bucket of paint covers 6 square feet, how many buckets of paint will be required to paint the whole glass, if only one coat of paint is applied.**

Solution:

The required number of buckets = $\frac{\text{Area of the } \Delta\text{ABC}}{\text{Area of the paint covered by one bucket}}$

$$\therefore \text{Area of the } \Delta ABC = \frac{1}{2}\begin{vmatrix}x_1 & y_1\\ x_2 & y_2\\ x_3 & y_3\\ x_1 & y_1\end{vmatrix} = \frac{1}{2}\begin{vmatrix}-5 & -4\\ 1 & 6\\ 7 & -4\\ -5 & -4\end{vmatrix}$$

= $\frac{1}{2}$ [(-30 – 4 - 28)- (-4 + 42 + 20)]

$= \frac{1}{2}$ [-62 - 58]

$= \frac{1}{2}$ [-120]

= 60 sq.units

∴ The required number of buckets = $\frac{60}{6}$ = 10sq.units

7. **Show that the given points form a parallelogram A (2.5, 3.5), B(10, -4) , C (2.5, -2.5) and D(-5, 5)**

Solution A(2.5, 3.5) B = (10, -4) , Slope of AB = $\frac{y_2-y_1}{x_2-x_1} = \frac{-4-3.5}{10-2.5} = -\ \frac{7.5}{7.5} = -1$

C (2.5, - 2.5), D (-5, 5) , Slope of CD = $\frac{5-(-2.5)}{-5-2.5} = \frac{5+2.5}{-7.5} = \frac{7.5}{-7.5} = -1$

∴ Slope of AB = Slope of CD. So AB ∥ CD.

B (10, -4), C (2.5, -2.5) , Slope of BC = $\frac{-2.5-(-4)}{2.5-10} = \frac{-2.5+4}{-7.5} = \frac{1.5}{-7.5} \times \frac{10}{10} = \frac{15}{-75} = -\frac{1}{5}$

A (2.5, 3.5), D (-5, 5) , Slope of AD = $\frac{5-(3.5)}{-5-2.5} = \frac{1.5}{-7.5} = \frac{1.5}{-7.5} \times \frac{10}{10} = \frac{15}{-75} = -\frac{1}{5}$

∴ Slope of BC = Slope of AD. So BC ∥ AD .

∴The given points form a parallelogram.

8. **If the points A (2, 2), B(-2, -3) , C(1, -3) and D(x, y) form a parallelogram then find the value of x and y.**

Solution

Given points A (2, 2), B(-2, -3) , C (1, −3) and D(x, y) are form a parallelogram.

Then AB ∥ CD and BC ∥AD

∴ Slope of AD = Slope of BC

$\Rightarrow \frac{y-2}{x-2} = \frac{-3-(-3)}{1-(-2)} \Rightarrow \frac{y-2}{x-2} = 0$

$y - 2 = 0 \Rightarrow y = 2$

Slope of CD = Slope of AB

$\Rightarrow \frac{y-(-3)}{x-1} = \frac{-3-2}{-2-2} \Rightarrow \frac{y+3}{x-1} = \frac{-5}{-4}$

$$\Rightarrow \frac{5}{x-1} = \frac{5}{4} \Rightarrow x - 1 = 4 \Rightarrow x = 5$$

$\therefore x = 5, y = 2$

CHAPTER - 6

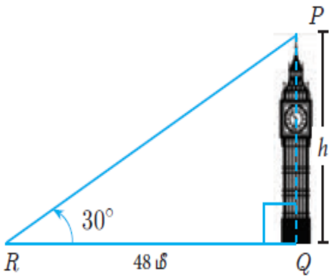
TRIGNOMETRY

1 MARKS

1. If the ratio of the height of a tower and the length of its shadow is$\sqrt{3}$: 1, then the angle of elevation of the sun has measure (1) 90° **(2) 60°** (3) 45° (4) 30°
2. The electric pole subtends an angle of 30°at point on the same level as its foot. at asecond point "b metres above the first ,the depression of the foot of the foot of the pole is 60°.The height of the pole (in metres) is equal to **(1) $\frac{b}{3}$** (2) $\frac{b}{\sqrt{3}}$ (3) $\sqrt{3}$ b (4) $\frac{b}{2}$
3. A tower is 60°m height. its shadow is x metres shorter when the sun s"altitude is 45° than when it has been 30°, then x is equal to (1) 43m (2) 41.92m **(3) 43.92m** (4) 45.6m
4. The angle of depression of the top and bottom of 20 m tall building from the top of a multistoried builing are 30° and 60° respectively. The height of the multistoried building and the distance between two building(in metres) **(1) 30,10$\sqrt{3}$** (2) 30,5$\sqrt{3}$ (3) 20,10 (4) 20,10$\sqrt{3}$
5. Two persons are standing "x" metres apart from each other and the height of the first person is double that of other, if from the middle point of the line joining their feet an observer find the angular elevation of their tops to be complementary, then the height of the shorter person(in metres) is (1)2x (2) $\sqrt{2x}$ (3) $\frac{x}{\sqrt{2}}$ **(4) $\frac{x}{2\sqrt{2}}$**
6. The angle of elevtion of a colud from a point h metres above a lake is β. The angle of depression of its reflection in the lake is 45°.The height of location of the cloud from the lake is (1) $\frac{h(1-tan\beta)}{1+tan\beta}$ **(2)$\frac{h(1+tan\beta)}{1-tan\beta}$** (3) h $\tan(45° - \beta)$ (4) none of these

2 MARKS

1. **A tower stands vertically on the ground. From a point on the ground, which is 48m away from the foot of the tower, the angle of elevation of the top of the tower 30°. Find the height of the tower.**



Solution In ΔPQR $\tan\theta = \frac{PQ}{QR}$

$\tan 30° = \frac{h}{48}$

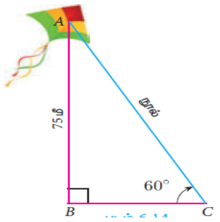
$\frac{1}{\sqrt{3}} = = \frac{h}{48}$

$h = \frac{48}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{48\sqrt{3}}{\sqrt{3}}$

Therefore the height of the tower is, h = 16$\sqrt{3}$ m

2. **A kite is flying at a height of 75m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the lengthof the string, assuming that there is no slack in the string.**

Solution $In\ \Delta ABC\ \sin\theta = \frac{AB}{AC}$



$\sin 60° = \frac{75}{AC}$

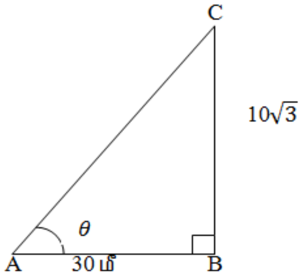
$\frac{\sqrt{3}}{2} = = \frac{75}{AC}$

$AC = \frac{75 \times 2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{150\sqrt{3}}{3}$

$AC = 50\sqrt{3}$ m

∴Hence, the length of the string is **50** $\sqrt{3}$ m.

3. **Find the angle of elevation of the top of a tower from a point on the ground, which is 30m away from the foot of a tower of height $10\sqrt{3}$ m.**



தீர்வு **In** ΔABC , $\tan\theta = \frac{\text{opposite Side}}{\text{Adjacent Side}}$

$\tan\theta = \frac{10\sqrt{3}}{30}$

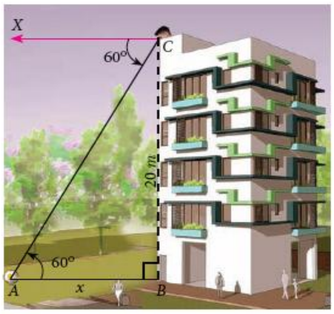
$\tan\theta = \frac{\sqrt{3}}{3}$

$\tan\theta = \frac{\sqrt{3}}{\sqrt{3}\sqrt{3}}$

$\tan\theta = \frac{1}{\sqrt{3}}$ $\theta = 30°$

4. **A player sitting on the top of a tower of height 20m observes the angle of depression of a ball lying on the ground as 60°. Find the distance between the foot of the tower and the ball.($\sqrt{3}$ = 1.732)**

தீர்வு Height of the tower = 20 m



In ΔABC , $\tan\theta = \frac{AB}{BC}$

$\tan 60° = \frac{20}{AB}$

$\sqrt{3} = \frac{20}{AB}$

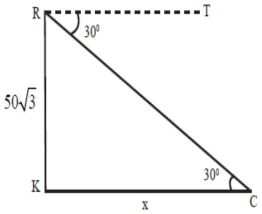
$AB = \frac{20}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$BC = \frac{20\sqrt{3}}{3} = \frac{20 \times 1.732}{3} = \frac{34.640}{3} = 11.54$ cm

5. **From the top of a rock 50 $\sqrt{3}$ high, the angle of depression of a car on the ground is observed to be 30° . Find the distance of the car from the rock.**

Solution In Figure RK – a rock , C – a car on the ground

From the figure RK = $50\sqrt{3}$ m, KC = x



$In\ \Delta RKC$

$\tan 30° = \frac{50\sqrt{3}}{KC}$

$\frac{1}{\sqrt{3}} = \frac{50\sqrt{3}}{KC}$

$KC = 50\sqrt{3} \times \sqrt{3}$

$= 50\ (3) = 150$m

CHAPTER - 7

MENSURATION

1 MARKS

1. The curved surface area of a right circular cone of height 15cm and base diameter 16 cm
   (1) 68π cm^2 (2) $60\pi\ cm^2$ **(3) 136π cm^2** (4) 120π cm^2
2. If two solid hemispheres of same base radius r units are joined together along their bases, then curved surface area of this new soild is
   (1) $3\pi r^2$Sq.units **(2) $4\pi r^2$Sq.units** (3) $6\pi r^2$Sq.units (4)$8\pi r^2$Sq.units
3. The height of a right circular cone whose radius is 5cm and slant height is 13 cm will be
   (1) 5cm (2) 10cm **(3) 12 cm** (4) 13cm
4. If the radius of the base of a right circular cylinder is halved keeping the same height, then the ratio of the volume of the cylinder thus obtained to the volume of original cylinder is
   (1) 1:6 (2) 1:8 (3) 1:2 **(4) 1:4**
5. The total surface area of a cylinder whose radius is$\frac{1}{3}$ of its height is
   (1)$\frac{8\pi h^2}{9}$ sq.units (2) $\frac{9\pi h^2}{8}$ sq. units (3) $\frac{56\pi h^2}{9}$ sq. units (4)$24\pi h^2 sq.units$
6. In a hollow cylinder, the sum of the external and internal radii is 14 cm and the width is 4 cm . If its height is 20 cm , the volume of the material in it is
   (1) 56πcm^3 (2) 3600π cm^3 (3)5600π cm^3 **(4) 11200π cm^3**
7. If the radius of the base of a cone is tripled and the height is doubled then the volume is?
   (1)Made 6 times (2) made12times **(3)made 18 times** (4) unchanged
8. The total surface area of a hemi-sphere is how much times the square of its radius..
   (1) 4π **(2) $3\ \pi$** (3) 2π (4) π
9. A solid sphere of radius xcm is melted and cast into ashape of a soild cone of same radius. The height of the cone is- **(1) 4x cm** (2) 3x cm (3) 2x cm (4) x cm
10. A frustum of a right circular cone is of height 16 cm with radii of its ends as 8cm and 20cm. Then the volume of the frustum is (1) 3228π cm^3 (2) 3240 π cm^3 **(3) 3328π cm^3** (4) 3340π
11. A shuttle cock used for playing badminton has the shape of the combination of
    (1) a cylinder and a sphere (2) a hemisphere and acone
    (3) a sphere and a cone **(4) frustum of a cone and a hemisphere**
12. A spherical ball of radius r_1 units is melted to make 8new identical balls each of radius r_2 units. Then $r_1 : r_2$ is (1) 1 : 4 (2) 4 : 1 (3) 1:2 **(4) 2:1**
13. The volume (in cm^3) of the greatest sphere that can be cut off from a cylindrical log of wood of base radius 1cm andheight 5cm is (1) 5π **(2) $\frac{4}{3}\pi$** (3)$\frac{10}{3}\pi$ (4) $\frac{20}{3}\pi$

14. The height and radius of the cone of which the frustum is a part are h_1 units and r_1 units respectively. Height of the frustum is h_2 units and radius of the smaller base is r_2 units. If $h_2 : h_1 = 1 : 2$ then $r_2 : r_1$ is **(1) 1:2** (2) 2:1 (3) 1:3 (4) 3:1

15. The ratio of the volumes of a cylinder , a cone and a sphere, if each has the same diameter and same height is (1) 1:2:3 **(2) 3:1:2** (3)2:1:3 (4)1:3:2

CHAPTER - 8
PROBABILITY
1 MARKS

1. Which of the following is incorrect?(1) P (A) +P($\bar{A}$) =1 (2) P(∅) =0 (3) 0≤ $P(A) \leq 1$ **(4) P(A)> 1**
2. The probability a red marble selected at random from a jar containing p red , q blue and r green marbles is (1) $\frac{P+R}{P+Q+R}$ (2) $\frac{P+Q}{P+Q+R}$ **(3)** $\boldsymbol{\frac{P}{P+Q+R}}$ (4) $\frac{Q}{P+Q+R}$
3. A page is selected at random from a book. The probability that the digit at units place of the page number chosen is less than 7 is

   (1) $\frac{3}{9}$ (2) $\frac{7}{9}$ (3) $\frac{3}{10}$ **(4)** $\boldsymbol{\frac{7}{10}}$
4. The probability of getting a job for a person is $\frac{x}{3}$.If the probability of not qetting the job is $\frac{2}{3}$ than value of x is **(1)1** (2) 1.5 (3) 2 (4)3
5. Kamalam went to play a lucky draw contest. 135 tickets of the lucky draw were sold .If the probability of Kamalam winning is $\frac{1}{9}$, then the number of tickets bought by Kamalam is

   (1) 20 **(2)15** (3)10 (4)5
6. If a letter is chosen at random form the English alphabets (a,b,…,z) then the probability that the letter chosen precedes x?

   (1) $\frac{1}{13}$ (2) $\frac{12}{13}$ (3) $\frac{3}{26}$ **(4)** $\boldsymbol{\frac{23}{26}}$
7. A purse contains 10 notes of ₹2000, 15 notes of ₹500, and 25 notes of ₹200. one note is drawn at random. what is the probability that the note is either a ₹500 note or ₹200 note?

   (1)$\boldsymbol{\frac{4}{5}}$ (2) $\frac{2}{3}$ (3) $\frac{3}{10}$ (4) $\frac{1}{5}$

2 MARKS

1. **A bag contains 5 blue balls and 4 green balls. A ball is drawn at random from the bag. Find the probability that the ball drawn is (i) blue (ii) not blue.**

   Solution

   Total Number of possible outcomes n(S) = 5 + 4 = 9

   (i) Let A be the event of getting a blue ball. Number of favourable outcomes for the event A. = n(A) = 5.

   Probability that the ball drawn is blue = P(A) = $\frac{n(A)}{n(S)} = \frac{5}{9}$

   (ii) $\bar{A}$ will be the event of not getting a blue ball. So P($\bar{A}$) = 1 - P(A) = 1 - $\frac{5}{9} = \frac{4}{9}$

2. **Two coins are tossed together. What is the probability of getting different faces on the coins?**

Solution

When two coins are tossed together, the sample space is

S = {HH, HT, TH, TT}; n(S) = 4

Let A be the event of getting difference face on the coins.

A = { HT, TH }; n(A) = 2

Probability of getting difference face on the coins is P(A) = $\frac{n(A)}{n(S)} = \frac{2}{4} = \frac{1}{2}$

3. **What is the probability that a leap year selected at random will contain 53 Saturdays. (Hint 366 = 52 x 7 + 2)**

Solution

A leap year has 366 days. So it has 52 full weeks and 2 days. 52 Saturdays must be in 52 full weeks.

S = {(Sun – Mon, Mon – Tue, Tue – Wed, Wed – Thu, Thu – Fri, Fri – Sat, Sat - Sun) }

n(S) = 7

Let A be the event of getting 53rd Saturday.

The A = {Fri – Sat, Sat – Sun} n(A) = 2

Probability of getting 53 Saturdays in a leap year is P(A) = $\frac{n(A)}{n(S)} = \frac{2}{7}$

4. **A die is rolled and a coin is tossed simultaneously. Find the probability that the die shows an odd number and the coin shows a head.**

Solution Sample Space, S = **{** 1H, 1T, 2H, 2T, 3H, 3T, 4H, 4T, 5H, 5T, 6H, 6T} n(S) = 12

Let A be the event of getting an odd number and a head.

A = **{** 1H, 3H, 5H} ; n(A) = 3

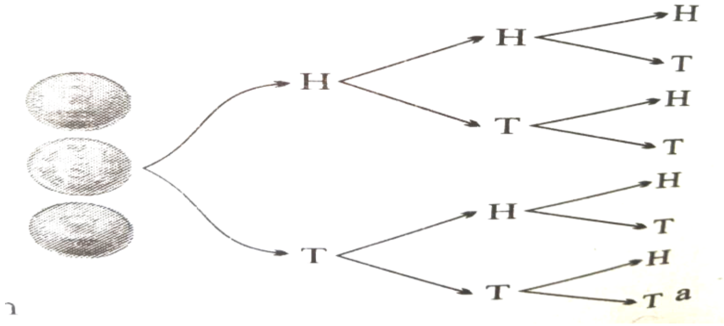
P(A) = $\frac{n(A)}{n(S)} = \frac{3}{12} = \frac{1}{4}$

5. **Write the sample space for tossing three coins using tree diagram**

Solution

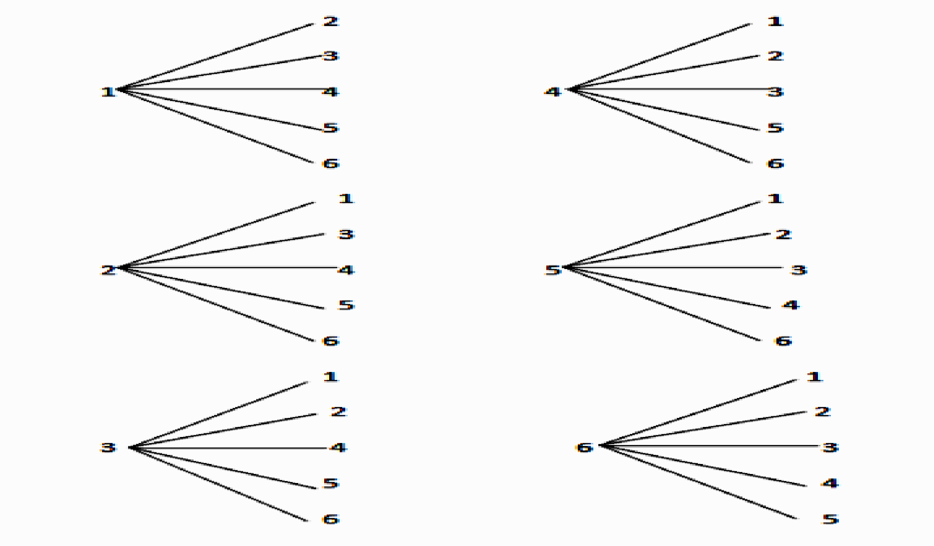
From the tree diagram, Sample space for three coins

={ (HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}



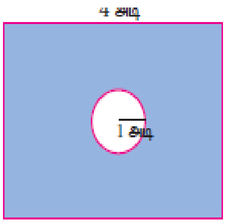
6. **Write the sample space for selecting two balls from balls from a bag containing 6 balls numbered 1 to 6 using tree diagram**

<u>Solution</u>



Sample Space **"S"** = { (1,2), (1,3), (1,4), (1,5), (1,6)
(2,1), (2,3), (2,4), (2,5), (2,6)
(3,1), (3,2), (3,4), (3,5), (3,6)
(4,1), (4,2), (4,3), (4,5), (4,6)
(5,1), (5,2), (5,3), (5,4), (5,6)
(6,1), (6,2), (6,3), (6,4), (6,5),}

7. **Some boys are playing a game, in which the stone thrown by them, landing in a circular region(given in the figure) is considered as win and landing other than the circular region is considered as loss. What is the probability to win the game?**



<u>Solution</u>

Total Region = 4 x 3 = 12 sq. ft

∴n(S) = 12

Winning Region = Area of Circle $\pi r^2 = \pi(1)^2$

$= \pi$ = 3.14 sq. ft

P(Win the game) = $\frac{3.14}{12} = \frac{314}{1200} = \frac{157}{600}$

5 MARKS

1. **Two dice are rolled. Find the probability that the sum of outcomes is (i) equal to 4 (ii) greater than 10 (iii) less than 13.**

<u>Solution</u> When we roll two dice, the sample space is given by

S = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)

(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)

(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}; n(S) =36

(i) Let A be the event of getting the sum of outcome values equal to 4.

A = {(1,3), (2,2), (3,1)}; n(A) = 3 P(A) = $\frac{n(A)}{n(S)} = \frac{3}{36} = \frac{1}{12}$

(ii) Let B be the event of getting the sum of outcome values greater than 10.

Then B = {(5,6), (6,5), (6,6)}; n(B) = 3 P(B) = $\frac{n(B)}{n(S)} = \frac{3}{36} = \frac{1}{12}$

(iii) Let C be the event of getting the sum of outcomes less that 13. Here all the outcomes have the sum value less than 13. Hence C =S **.**

n(C) = n(S) = 36. P(C) = $\frac{n(C)}{n(S)} = \frac{36}{36} = 1$

2. From a well shuffled pack of 52 cards, one card is drawn at random. Find the probability of getting (i) red card (ii) heart card (iii) red king (iv) face card (v) number card.

<u>Solution</u> n(S) = 52

(i) Let A be the event of getting a red card. n(A) = 26

Probability of getting a red card is P(A) = $\frac{26}{52} = \frac{1}{2}$

(ii) Let B be the event of getting a heart card. n(B) = 13

Probability of getting a heart card is P(B) = $\frac{n(B)}{n(S)} = \frac{13}{52} = \frac{1}{4}$

(iii) Let C be the event of getting a red king card. A red king card can be either a diamond king or a heart king. n(C) = 2

Probability of getting a red king card is P(C) = $\frac{n(C)}{n(S)} = \frac{2}{52} = \frac{1}{26}$

(iv) Let D be the event of getting a face card. The face cards are Jack (J), Queen(Q), and King(K). n(D) = 4 x 3 = 12

Probability of getting a face card is P(D) = $\frac{n(D)}{n(S)} = \frac{12}{52} = \frac{3}{13}$

(v) Let E be the event of getting a number card. The number cards are 2, 3, 4, 5,6, 7, 8, 9 and 10. n(E) = 4 x 9 = 36

Probability of getting a number card is P(E) = $\frac{n(E)}{n(S)} = \frac{36}{52} = \frac{9}{13}$

3. A bag contains 6 green balls, some black and red balls. Number of black balls is as twice as the number of red balls. Probability of getting a green ball is thrice the probability of getting a red ball. find (i) Number of black balls (ii) total number of balls.

<u>Solution</u> Number of green balls n(G) = 6

Let number of red balls be n(R) = x

Therefore, number of black balls is n(B) = 2x

Total number of balls n(S) = 6 +x + 2x = 6 +3x

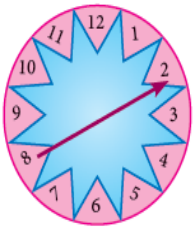
It is given that, P(G) = 3 x P(R)

$$\frac{6}{6+3x} = 3 \times \frac{x}{6+3x}$$

3x = 6 gives, x = 2

(i) Number of black balls = 2 x 2 =4

(ii)Total Number of balls = 6 + (3x 2) = 12



4. A game of chance consists of spinning an arrow which is equally likely to come to rest pointing to one of the numbers 1, 2, 3, …12. What is the probability that it will point to (i) 7 (ii) a prime number (iii) a composite number?

Solution

Sample space, S= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

n (S) = 12

(i) Let A be the event of resting in 7. n(A) = 1

$P(A) = \frac{n(A)}{n(S)} = \frac{1}{12}$

(ii) Let B be the event that the arrow will come to rest in a prime number.

B = {2, 3, 5, 7, 11} ; n(B) = 5

$P(B) = \frac{n(B)}{n(S)} = \frac{5}{12}$

(iii) Let C be the event that arrow will come to rest in a composite number.

C = {4, 6, 8, 9, 10, 12}; n(C) = 6

$P(C) = \frac{n(C)}{n(S)} = \frac{6}{12} = \frac{1}{2}$

5. If A is an event of an random experiment such that P(A) : P($\bar{A}$) = 17: 15 and n(S) = 640 then find (i) P($\bar{A}$) (ii) n(A) .

Solution $\frac{P(A)}{P(\bar{A})} = \frac{17}{15}$

$\frac{1-P(\bar{A})}{P(\bar{A})} = \frac{17}{15}$

15 [1- P($\bar{A}$)] = 17 P($\bar{A}$) $\Rightarrow$ 15 - 15 P($\bar{A}$) = 17 P($\bar{A}$)

15 = 15 P($\bar{A}$) + 17 P($\bar{A}$) $\Rightarrow$ 32 P($\bar{A}$) = 15

$P(\bar{A}) = \frac{15}{32}$ $\Rightarrow$ P(A) = 1 - P($\bar{A}$)

$= 1 - \frac{15}{32} = \frac{32-15}{32} = \frac{17}{32}$

$P(A) = \frac{n(A)}{n(S)}$; $\frac{17}{32} = \frac{n(A)}{640}$ $\Rightarrow n(A) = \frac{17 \times 640}{32}$ n(A) =340

6. At a fete, cards bearing numbers 1 to 1000, one number on one card areput in a box. Each player selects one card at random and that card is not replaced. If the selected card has a perfect square number greater than 500, the player wins a prize. What is the probability that (i) the first player wins a prize, (ii) the second player wins a prize, if the first has won?

Solution

From the given, n(S) = 1000

Let E be the event of selected card has perfect square number greater than 500.

$\Rightarrow E = \{x : (\sqrt{500})^2 < x < (\sqrt{1000})^2\}$ $(\because \sqrt{500} = 22.36. \sqrt{1000} = 31.62)$

$\therefore E = \{23^2, 24^2, 25^2, \ldots, 31^2\}$

$n(E) = 9$

$P(E) = \frac{n(E)}{n(S)} = \frac{9}{1000}$

(i) P(First player wins a prize) = $\frac{9}{1000}$

(ii) P(Second player win if fist has won) = $\frac{8}{999}$. $(\therefore n(s) = 999)$

7. A bag contains 12 blue balls and x red balls. If one ball is drawn at random (i) What is the probability that it will be a red ball? (ii) If 8 more red balls are put in the bag, and if the probability of drawing a red ball will be twice that of the probability in (i), then find x.

Solution

n(B) = 12; n(R) = x; n(S) = 12 + x. $(\because B -$ Blue Balls , R – Red Balls)

(i) $P(R) = \frac{x}{12+x}$ -----(1)

(ii) 8 red balls are put in the same bag

$\therefore n(S) = 20 + x$; n(R) = x + 8

Now, $P(R) = \frac{x+8}{20+x}$ -----(2)

But (2) is twice of (1)

$\Rightarrow \frac{x+8}{20+x} = \frac{2x}{12+x}$

$\Rightarrow (12 + x)(x + 8) = (20 + x) 2x$

$\Rightarrow 12x + 96 + x^2 + 8x = 40x + 2x^2$

$\Rightarrow x^2 + 20x - 96 = 0 \Rightarrow (x - 4)(x + 24) = 0$

$\therefore x = 4$ $(\because x = -24$ is not need, negative is not possible)

(1) $\Rightarrow P(R) = \frac{4}{16} = \frac{1}{4}$

8. Two unbiased dice are rolled once. Find the probability of getting.

(i) **a doublet (equal numbers on bothdice) (ii)the product of the prime number (iii) the sum of the prime number (iv) the sum as 1**

Solution n(S) = 36

(i) A = Probability of getting Doublets(Equal numbers on both dice)

A = {(1,1), (2,2), (3,3),(4,4),(5,5),(6,6)}

n(A) = 6 ; $P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{36}$

(ii) B = Probability of getting the product of the prime number

B = {(1,2), (1,3), (1,5), (2,1),(3,1), (5,1)} n(B) = 6 ; P(B) = $\frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{36}$

(iii) C = Probability of getting sum of the prime number.

C ={(1,1), (2,1), (1,2),(1,4), (4,1), (1,6), (6,1), (2,3), (2,5), (3,2), (3,4), (4,3), (5,2), (5,6), (6,5) }

n(C) = 15 ; P(C) = $\frac{n(C)}{n(S)} = \frac{15}{36} = \frac{5}{12}$

(iv) D = Probability of getting the sum as 1

n(D) = 0 ; P(D) = $\frac{n(D)}{n(S)} = 0$

9. Three fair coins are tossed together. Find the probability of getting (i) all heads (ii) atleast one tail (iii) atmost one head (iv) atmost two tails.

Solution Possible Outcomes = { HHH, HHT, HTH, THH, TTT, TTH, THT, HTT}

No.of possible outcomes, n(S) = 2 x 2 x 2 = 8

(i) A = Probability of getting all heads

A = { HHH} n(A) = 1 P(A) = $\frac{n(A)}{n(S)} = \frac{1}{8}$

(ii) B = Probability of getting atleast one tail

B = { HHT, HTH, THH, TTT, TTH, THT, HTT} n(B) = 7 P(B) = $\frac{n(B)}{n(S)} = \frac{7}{8}$

(iii) C = Probability of getting atmost one head.

C = { TTT, TTH, THT, HTT} n(C) = 4 P(C) = $\frac{n(C)}{n(S)} = \frac{4}{8} = \frac{1}{2}$

(iv) D = Probability of getting atmost two tails.

D = { TTH, THT, HTT, HHT, HTH, THH, HHH} n(D) = 7 P(D) = $\frac{n(D)}{n(S)} = \frac{7}{8}$

10. A bag contains 5 red balls, 6 white balls, 7 green balls, 8 black balls. One ball is drawn at random from the bag. Find the probability that the ball drawn is (i) White (ii) Black or Red (iii) Not white (iv) Neither white nor black

Solution S = {5 Red, 6 White, 7 Green, 8Black} n(S) = 26

i) A – probability of getting White Balls

n(A) = 6 ; P(A) = $\frac{6}{26} = \frac{3}{13}$

ii) B - Probability of getting Black (or) Red Balls

n(B) = 5 + 8 =13 ; P(B) = $\frac{13}{26} = \frac{1}{2}$

iii) C- Probability of not getting White Balls

n(C) = 20 ; P(C) = $\frac{20}{26} = \frac{10}{13}$

iv) D – Probability of getting of Neither White nor Black

n(D) = 12 ; P(D) = $\frac{12}{26} = \frac{6}{13}$

11. **In a box there are 20 non – defective and some defective bulbs. If the probability that a bulb selected at random from the box found to be defective is $\frac{3}{8}$ then, find the number of defective bulbs.**

Solution

Defective Bulbs	Non – Defective Bulbs
x	20

Total Number of Bulbs = n(S) = x + 20

A : Selection of a Defective bulbs. P(A) = $\frac{n(A)}{n(S)} \Rightarrow \frac{3}{8} = \frac{x}{20+x}$

3(20 + x) = 8x

60 + 3x = 8x

0 = 8x – 3x

60 = 5x

x = $\frac{60}{5} = 12$

12. **The king and queen of diamonds, queen and jack of hearts, and king of spades are removed form a deck of 52 playing cards and then well shuffled. Now one cared is drawn at random form the remaining cards. Determine the probability that the card is (i)a clavor (ii) a queen of red card (iii) a king of black card**

Solution

Removed cards: The king and queen of diamonds , queen and jack of hearts, and king of spades

(i.e) remaining number of cards =52-6=46

n(S) = 46

(i) A is probability of getting Clavor Cards

n(A) = 13 P(A) = $\frac{n(A)}{n(S)} = \frac{13}{46}$

(ii) B is probability of getting a queen of red card.

n(B) = 0 P(B) = $\frac{n(B)}{n(S)} = \frac{0}{46}$ = 0

(iii) C is probability of getting King of black card.

n(C) = 1 P(C) = $\frac{n(C)}{n(S)} = \frac{1}{46}$

13. **Two customers Priya and Amuthan are visiting a particular shop in the same week (Monday to Saturday). Each is equally likely to visit the shop on any one day as on another day. What is the probability that both will visit the shop on (i) the Same day (ii) Different days (iii) Consecutive days?**

Solution

i) S = { S, M, T, W, T, F, S}

n(S) = 6

A : Event of Visiting in the same day

n(A) = 1

∴ P(A) = $\frac{n(A)}{n(S)} = \frac{1}{6}$

ii) B : Event of Visiting in Different days

n(B) = 5

∴ P(B) = $\frac{n(B)}{n(S)} = \frac{5}{6}$

iii) C : Event of visiting in Consecutive days

Consecutive Days "S" = { (M,T), (T,W), (W,T), (T,F),(F,S), (S,M)}

n(S) = 6

We have n(C)= 5

∴ P(C) = $\frac{n(C)}{n(S)} = \frac{5}{6}$

14. In a game, the entry free is ₹150. The game consists of tossing a coin 3 times. Dhana bought a ticket for entry. If one or two heads show, she gets her entry fee back. If she throws 3 heads, she received double the entry fees. Otherwise she will lose. Find the probability that she(i) gets double entry fee, (ii) just gets her entry fee, (iii) loses the entry fee.

தீர்வு

Sample Space "S" = {HHH, HHT, HTH, THH, TTT, TTH, THT, HTT}

n(S) = 8

i) A : Event of getting Double Entry Fee (3 Head)

n(A) = 1

∴ P(A) = $\frac{n(A)}{n(S)} = \frac{1}{8}$

ii) B : Event of getting Just the Entry Fee (1 Head or 2 Heads)

n(B) = 6

∴ P(B) = $\frac{n(B)}{n(S)} = \frac{6}{8} = \frac{3}{4}$

iii) C : Event of Losing the Entry Fee (No Heads)

n(C) = 1

∴ P(C) = $\frac{n(C)}{n(S)} = \frac{1}{8}$

STAGE - 2

CHAPTER – 2

NUMBERS AND SEQUENCES

2 MARKS

1. **We have 34 cakes. Each box can hold 5 cakes only. How many boxes we need to pack and how many cakes are unpacked?**

Solution

We see that 6 boxes are required to pack 30 cakes with 4 cakes left over. This distribution of cakes can be understood as follows:

34	=	5	X	6	+	4
Total Number of Cakes	=	Number of Cakes in each box		Number of boxes	+	Number of Cakes left over
↓		↓		↓		↓
Dividend, a	=	Divisor, b		Quotient, q	+	Remainder R

2. **Show that the square of an odd interger is of the form 4q + 1, for some integer q.**

Solution

Let x be any odd integer, since any odd integer is one more than an even integer, we have x = 2k + 1 for some integer k.

$$x^2 = (2k+1)^2$$
$$= 4k^2 + 4k + 1$$
$$= 4k(k+1) + 1$$
$$= 4q + 1. \text{ Here } q = k(k+1) \text{ is some integer}$$

3. **If the Highest Common Factor of 210 and 55 is expressible in the form 55x – 325. Find x.**

Solution

Using Euclid’s Division Algorithm, let us find the HCF of given numbers

210 = 55 x 3 + 45

55 = 45 x 1 + 10

45 = 10 x 4 + 5

10 = 5 x 2 + 0

Remainder = 0

So, the last divisor 5 is the Highest Common Factor (HCF) of 210 and 55.

Since, HCF is expressible in the form 55x = 330

Hence, x = 6

4. **Find the greatest number that will divide 445 and 572 leaving remainders 4 and 5 respectively.**

Solution

Since the remainders are 4, 5 respectively the required number is the HCF of the number 415 – 4 = 441, 572 – 5 = 507.

Hence, we will determine the HCF of 441 and 567. Using Euclid's Division Algorithm, we have

567 = 441 x 1 + 126

441 = 126 x 3 + 63

126 = 63 x 2 + 0

Therefore HCF of 441, 567 = 63 and so the required number is 63.

5. **A man has 532 flower pots. He wants to arrange them in rows such that each row contains 21 flower pots. Find the number of completed rows and how many flower pots are left over**

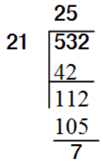
Solution Using Euclid's Division Algorith

$a = bq + r$

$532 = 21\,q + r \Rightarrow 532 = 21 \times 25 + 7$

The remainder is 7.

No. of completed rows = 25,



6. **Prove that the product of two consecutive positive integers is divisible by 2.**

Solution Let x and x - 1 be two consecutive positive integers. Then their product is (x-1) x.

$x\,(x - 1) = x^2 - x$

case 1 : when x = 2k

$x^2 - x = (2k)^2 – 2k = 4k^2 - 2k = 2k(2k – 1) = 2r$, where $r = k\,(2k -1)$

$x^2 – x$ is divisible by 2.

case 2 : when x = 2k + 1. In this case, we have

$x^2 - x = (2k+1)^2 - (2k+1) = (2k+1)\,(2k+1 - 1)=2k(2k+1)$

$x^2 - x = 2r$, where $r = k(2k + 1)$

$x^2 – x$ is divisible by 2.

Hence, $x^2 - x$ is divisible by 2 for every positive integer x. Hence it is proved.

7. **Prove that square of any integer leaves the remainder either 0 or 1 when divided by 4.**

Solution

Let x be any integer

Case 1 : x be an odd integer

x = 2k + 1 for some integer k

$x^2 = (2k + 1.)^2$

$x^2 = 4k^2 + 4k + 1$

$x^2 = 4k\,(k + 1) + 1 = 4q + 1$, where $q = k(k + 1)$,remainder r = 1.

Case 2 : x be an even integer

x = 2k, for some integer k

$x^2 = (2k)^2$

$x^2 = 4k^2 = 4q$

$x^2 = 4q + 0$, where $q = k^2$, remainder $r = 0$

8. Use Euclid's Division Algorithm to find the Highest Common Factor(HCF) of

(i) 340 and 412 (ii) 867 and 255 (iii) 10224 and 9648 (iv) 84, 90 and 120

Solution: By Euclid's Division Algorithm a = bq + r

(i) To find HCF of 340 and 412

412 = 340(1) + 72
340 = 72(4) + 52
72 = 52(1) + 20
52 = 20(2) + 12
20 = 12(1) + 8
12 = 8(1) + 4
8 = 4(2) + 0

The remainder is 0, when the last divisor is 4. ∴HCF of 340 and 412 is 4

(i) To find HCF of 867 and 255

867 = 255 (3) + 102
255 = 102(2) + 51
102 = 51(2) + 0. ∴HCF of 867 and 255 is 51

(ii) To find HCF of 10224 and 9648

10224 = 9648 (1) + 576
9648 = 576(16) + 432
576 = 432 (1) + 144
432 = 144(3) + 0. ∴ HCF of 10224 and 9648 is 144

(iii) To find HCF of 84, 90 and 120

First to find HCF of 84 and 90
90 = 84q + r (b≠0)
90 = 84x1+ 6
84 = 6 x14 + 0
∴ HCF of 84, 90 = 6.
Then to find HCF of 6 and 120
120 = 6 x 20 + 0
∴ HCF of 84, 90,120 is 6

9. Find the largest number which divides 1230 and 1926 leaving remainder 12 in each case.

Solution: Given: Remainder 12 in both 1230 and 1926

Required Number = HCF of 1230 – 12 = 1218 and 1926 – 12 = 1914

⟹ 1914 = 1218 (1) + 696

1218 = 696 (1) + 522

696 = 522(1) + 174

522 = 174(3) + 0.

∴ The required number = 174

10. If d is the Highest Common Factor of 32 and 60, find x and y satisfying d = 32x + 60y.

Solution Applying Euclid's Division Lemma, a = bq+r

60 = 32 x1 + 28 $\Longrightarrow$ 32 = 28 x1 +4

28 = 4 x 7 +0 $\therefore$ H.C.F. of 32 and 60 is 4

That is d=4. d = 32x + 60y $\Longrightarrow$ 4 = 32x + 60y

4 = 32(2) + 60(-1) $\Longrightarrow$ $\therefore$x = 2, y = -1.

11. Prove that two consecutive positive integers are always coprime.

Solution Two consecutive positive integers be x+1 , x

H.C.F of (a,b) = H.C.F of(a-b,b)

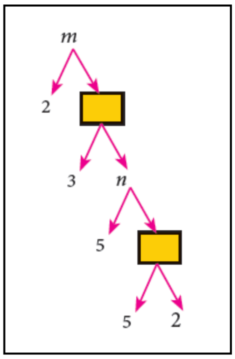
H.C.F of (x+1,x) = H.C.F of (x + 1 – x, x)

H.C.F of (x+1,x) = H.C.F of (1, x)

H.C.F of (x+1,x) = 1 $\therefore$x+1, x are co-prime numbers

12. In the give factor tree, find the numbers m and n

Solution



Value of the first box from bottom = 5 x 2 = 10

Value of n = **5** x 10 = 50

Value of the second box from bottom = **3** x 50 = 150

Value of m = **2** x 150 = 300

Thus, the required numbers are m = 300, n = 50.

13. Can the number 6^n, n being a natural number end with the digit 5? Give reason for your answer.

Solution

Since $6^n = (2 \times 3)^n = 2^n \times 3^n$

2 is a facor of 6^n . So 6^n is always even. But any number whose last digit is 5 is always odd. Hence, 6^n cannot end with the digit 5.

14. Is 7 x 5 x 3 x 2 + 3, a compsite number? Justify your answer.

Solution 7 x 5 x 3 x 2 + 3 = 213

Sum of the digits in 213 = 2 + 1 + 3 = 6

6 is divisible by 3.

$\therefore$213 is divisible by " 3".

$\therefore$213 is not a prime number and it is a composite number.

15. For what values of natural number n, 4^n can end with the digit 6?

Solution

$4^1 = 4$, $4^2 = 16$,$4^3 = 64$, $4^4 = 256$, $4^5 = 1024$, $4^6 = 4096$

So, n is an even number, 4^n can end with the digit 6. n is odd number, 4^n can end with the digit 4.

16. If m, n are natural numbers, for what values of m, does 2^n x 5^m ends in 5?

Solution

2 is a facor of (2^n x 5^m). So, 2^n x 5^m is always even.

But any number whose last digit is 5 is always odd.

Hence, 2^n x 5^m cannot end with the digit 5.

17. Find the HCF of 252525 and 363636.

Solution

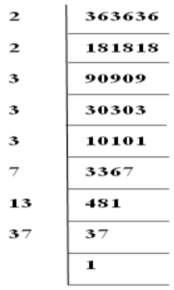
252525 = 3 x 5^2 x 7 x 13 x 37

363636 = 2^2 x 3^3 x 7 x 13 x 37

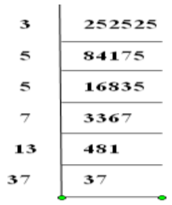
H.C.F of 252525 and 363636

= 3 x 7 x 13 x 37

= 10101.



2	363636
2	181818
3	90909
3	30303
3	10101
7	3367
13	481
37	37
	1



3	252525
5	84175
5	16835
7	3367
13	481
37	37

18. Find the LCM and HCF of 408 and 170 by applying the fundamental theorem of arithmetic.

Solution

408 = 2^3 x 3 x 17

170 = 2 x 5 x 17

H.C.F. of 408 &170 = 2 x 17 = 34

L.C.M. of 408 &170 = 2^3 x 3 x 5 x 17 = 2040

2	408
2	204
2	102
3	51
	17

2	170
5	85
	17

19. Find the least number that is divisible by the first ten natural numbers

Solution

The required number = LCM of 1,2,3,4,5,6,7,8,9,10

2 = 2 x 1

3 = 3 x 1

4 = 2 x 2 = 2^2

5 = 5 x 1

6 = 2 x 3

7 = 7 x 1

8 = 2 x 2 x 2 = 2^3

9 = 3 x 3 = 3^2

10 = 2 x 5

∴ LCM = 2^3 x 3^2 x 5 x 7 = 8 x 9 x 5 x 7 = 2520

20. Find the next three terms of the sequences $\frac{1}{2}, \frac{1}{6}, \frac{1}{10}, \frac{1}{14}, \ldots$

Solution

$\frac{1}{2}, \frac{1}{6}, \frac{1}{10}, \frac{1}{14}, \ldots$

In the above sequence the numerators are same and the denominator is increased by 4.

So the next three terms are

a$_5$ = $\frac{1}{14+4} = \frac{1}{18}$

a$_6$ = $\frac{1}{18+4} = \frac{1}{22}$

a$_7$ = $\frac{1}{22+4} = \frac{1}{26}$

21. **Find the next three terms of the sequences 5, 2, -1, -4, …**

Solution

5, 2, -1, -4, …

Here each term is decreased by 3. So the next three terms are -7, -10, -13.

22. Find the next three terms of the sequences 1,0.1,0.01,…

Solution

1,0.1,0.01,…

Here each term is divided by 10. Hence, the next three terms are

a$_4$ = $\frac{0.01}{10} = 0.001$

a$_5$ = $\frac{0.001}{10} = 0.0001$

a$_6$ = $\frac{0.0001}{10} = 0.00001$

23. Find the general term for the following sequence : 3,6,9,…

Solution

Here the terms are multiples of 3. So the general term is $a_n = 3n$, $n \in \mathbb{N}$

24. **Find the general term for the following sequence : $\frac{1}{2}, \frac{2}{3}, \frac{3}{4},$**

Solution

a$_1$ = $\frac{1}{2}$; a$_2$ = $\frac{2}{3}$; a$_3$ = $\frac{3}{4}$;

We see that the numerator of nth terms is n, and the denominator is one more than the numerator. Hence $a_n = \frac{n}{n+1}$, $n \in \mathbb{N}$

25. Find the general term for the following sequence : 5, -25, 125,…

Solution

The terms of the sequences have + and – sign alternatively and also they are in powers of 5. So the general term $a_n = (-1)^{n+1} 5^n$, $n \in \mathbb{N}$

26. The general term of a sequence is defined as $a_n = \begin{cases} n(n+3); n \in \mathbb{N} \text{ is odd} \\ n^2+1 \;;\; n \in \mathbb{N} \text{ is even} \end{cases}$.

Find the eleventh and eighteenth terms.

Solution

To find a_{11} since 11 is odd , we put n = 11 in $a_n = n(n + 3)$

Thus the Eleventh Term $a_{11} = 11(11 + 3) = 154$.

To find a_{18} since 18 is even , we put n = 18 in $a_n = n^2 + 1$

Thus the Eighteenth Term $a_{18} = 18^2 + 1 = 325$.

27. Find the first five terms of the following sequences $a_1 = 1$, $a_2 = 1$, $a_n = \frac{a_{n-1}}{a_{n-2}+3}$; $n \geq 3, n \in \mathbb{N}$

Solution

The first two terms of this sequence are given by a_1=1, a_2=1. The third terms $a_1$depends on the first and second terms

$$a_3 = \frac{a_{3-1}}{a_{3-2}+3} = \frac{a_2}{a_1+3} = \frac{1}{1+3} = \frac{1}{4}$$

Similarly, the fourth term a_4 depends upon a_2 and a_3 **.**

$$a_4 = \frac{a_{4-1}}{a_{4-2}+3} = \frac{a_3}{a_2+3} = \frac{\frac{1}{4}}{1+3} = \frac{\frac{1}{4}}{4} = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$$

In the same way, the fifth term a_5 can be calculated as

$$a_5 = \frac{a_{5-1}}{a_{5-2}+3} = \frac{a_4}{a_3+3} = \frac{\frac{1}{16}}{\frac{1}{4}+3} = \frac{1}{16} \times \frac{4}{13} = \frac{1}{52}$$

Therefore, the first five terms of the sequence are 1, 1, $\frac{1}{4}, \frac{1}{16}$ and $\frac{1}{52}$

28. Find the next three terms of the following sequence. 8, 24, 72,…

Solution

8, 24, 72,…

Each terms are multiplied by 3

∴ The next three terms

72 x 3 = 216 ; 216 x 3 = 648; 648 x 3 = 1944

29. Find the next three terms of the following sequence. 5,1,-3

Solution

5,1,-3

Each term is subtracted by 4

∴ The next three terms are

-3 - 4 = -7; -7 - 4 = -11; -11 - 4 = -15;

30. Find the next three terms of the following sequence. $\frac{1}{4}, \frac{2}{9}, \frac{3}{16}, \ldots$

Solution

$\frac{1}{4}, \frac{2}{9}, \frac{3}{16}, \ldots$

$\Rightarrow \frac{1}{2^2}, \frac{2}{3^2}, \frac{3}{4^2}, \ldots.$

∴ The next three terms are

$$\frac{4}{5^2} = \frac{4}{25}, \frac{5}{6^2} = \frac{5}{36} \text{ and } \frac{6}{7^2} = \frac{6}{49}$$

31. Find the first four terms of the sequences whose nth terms are given by $a_n = n^3 - 2$.

Solution

Given : $a_n = n^3 - 2$

$a_1 = -1$, $a_2 = 6$, $a_3 = 25$, $a_4 = 62$

32. Find the first four terms of the sequences whose nth terms are given by $a_n = (-1)^{n+1}$

Solution

Given : $a_n = (-1)^{n+1}$

$a_1 = (-1)^{1+1} = 1$ $a_2 = (-1)^{2+1} = -1$

$a_3 = (-1)^{3+1} = 1$ $a_4 = (-1)^{4+1} = -1$

33. Find the first four terms of the sequences whose nth terms are given by $a_n = 2n^2 - 6$

Solution

Given : $a_n = 2n^2 - 6$

$a_1 = 2(1) - 6 = -4$, $a_2 = 2(4) - 6 = 2$

$a_3 = 2(9) - 6 = 12$, $a_4 = 2(16) - 6 = 26$

34. Find the nth terms of the following sequences: 2, 5, 10, 17, …

Solution

Given : 2, 5, 10, 17, …

$\Rightarrow 1 + 1, 4 + 1, 9 + 1, 16 + 1$ ∴ $a_n = n^3 + 1, n \epsilon \mathbb{N}$

35. Find the nth terms of the following sequences: $0, \frac{1}{2}, \frac{2}{3}, \ldots$

Solution

Given : $0, \frac{1}{2}, \frac{2}{3}, \ldots$

$\Rightarrow \frac{1-1}{1}, \frac{2-1}{2}, \frac{3-1}{3}, \ldots.$ ∴ $a_n = \frac{n-1}{n}, n \epsilon \mathbb{N}$

36. Find the nth terms of the following sequences: 3, 8, 13, 18,….

Solution

Given : 3, 8, 13, 18,….

$\Rightarrow 5 - 2, 10 - 2, 15 - 2, 20 - 2,, \ldots.$

$\Rightarrow 5(1) - 2, 5(2) - 2, 5(3) - 2, 5(4) - 2,, \ldots.$

∴ $a_n = 5n - 2, n \epsilon \mathbb{N}.$

37. Find the number of terms in the A.P.3,6, 9, 12, ... 111

Solution

First term a = 3, Common Difference d = 6 – 3 = 3, Last term = l = 111

We know that n = $\left(\frac{l-a}{d}\right)$+1

n = $\left(\frac{111-3}{3}\right)$ +1 = 37. Thus, the A.P contains 37 terms

38. Determine the general term of an A.P. whose 7th term is -1 and 16th term is 17.

Solution

Let the A.P. be $t_1, t_2, t_3, t_4, \ldots$

Given : $t_7 = -1$ and $t_{16} = 17$

a + (7 – 1) d = -1 and a + (16 – 1) d = 17

a + 6d = -1 …..(1)

a + 15 d = 17 …..(2)

subtracting equation (1) from equation (2), we get 9d = 18 gives d = 2

Putting d = 2 in equation (1), we get a + 12 = -1 so a = -13

Hence, General Term t_n = a + (n -1) d = -13 + (n -1) x 2 = 2n - 15

39. Find the first term and common difference of the Arithmetic Progressions whose nth terms are given below (i) $t_n = 3 + 2n$ (ii) $t_n = 4 – 7n$

Solution

(i) **$t_n = 3 + 2n$**

$t_1 = 5$, $t_2 = 7$ ∴a = 5, d = 2

(ii) **$t_n = 4 – 7n$**

$t_1 = -3$, $t_2 = -10$

d = $t_2 - t_1$ = - 7

∴a = -3, d = -7

40. **Find the middle term(s) of an A.P. 9, 15, 21, 27,.......183**

Solution

A.P. ⇒ 9, 15, 21, 27,.......183

Here a = 9, d = 6, l = 183

n = $\frac{l-a}{d} + 1$

= $\frac{183-9}{6} + 1$

= $\mathbf{\frac{174}{6} + 1}$

= 29 + 1 = 30

If n = 30 then middle terms are t_{15} and t_{16}

$t_n = a + (n-1)d$

∴ $t_{15} = a + 14d$ $\qquad$ $t_{16} = a + 15d$

$= 9 + 14(6)$ $\qquad$ $= 9 + 15(6)$

$= 9 + 84$ $\qquad$ $= 9 + 90$

$= 93$ $\qquad$ $= 99$

∴ 93, 99 are the middle terms of the given A.P.

41. If nine times ninth term is equal to the fifteen times fifteenth term, show that six times twenty fourth term is zero.

Solution

$9t_9 = 15\,t_{12}$ .....(1)

To show that $6t_{24} = 0$

(1) $\Rightarrow 9(a + 8d) = 15(a+14d)$; $9a + 72d = 15a + 210d$

$\Rightarrow 6a + 138d = 0$

$\Rightarrow 6(a + 23d) = 0$

$\Rightarrow 6\,t_{24} = 0$. Hence Proved.

42. The sum of three consecutive terms that are in A.P. is 27 and their product is 288. Find the three terms

Solution Let the 3 consecutive terms in an A.P. be a-d, a, a+d என்க

Sum of three terms $a-d + a + a+d = 27$

$3a = 27,$

$a = \frac{27}{3}$

$a = 9$

Product of three terms $(a-d)(a)(a+d) = 288$

$a(a^2 - d^2) = 288$

$9(9^2 - d^2) = 288$

$81 - d^2 = \frac{288}{9}$

$81 - d^2 = 32$

$49 = d^2 \quad \therefore d = \pm 7$

∴ The three terms of A.P are2, 9,16 (or) 16, 9, 2

5 MARKS

1. Find the HCF of 396, 504, 636

Solution

504 = 396 x 1 + 108

396=108 x 3 +72

108= 72 x 1 +36

72= 36 x 2 + 0

H.C.F(396, 504) = 36

636 = 36 x 17 + 24

36=24 x 1 +12

24= 12 x 2 +0

H.C.F(636, 36) = 12

Therefore, Highest Common Factor of 396, 504 and 636 is 12.

2. **In an A.P., sum of four consecutive terms is 28 and their sum of their squares is 276. Find the four numbers.**

Solution

Let us take the four terms in the form (a - 3d),(a – d),(a+d) and (a +3d)

Since sum of the four terms is 28

$a - 3d + a – d + a + d + a + 3d = 28$

$4a = 28,$

$a =7$

Similarly, since sum of their squares is 276,

$(a - 3d)^2 + (a – d)^2 + (a + d)^2 + (a + 3d)^2 = 276$

$a^2 –6ad + 9d^2 + a^2 – 2ad + d^2 + a^2 + 2ad + d^2 + a^2 + 6ad + 9d^2= 276$

$4a^2 +20d^2 = 276 \Rightarrow 4(7)^2 + 20d^2 = 276$

$d^2 =4 \Rightarrow d = \pm\sqrt{4}$

$d =\pm 2$

If a = 7, d = 2 , then the four numbers are 7 -3(2), 7 – 2, 7 +2, 7 + 3(2)/

That is the four numbers are 1, 5, 9, and 13.

If a = 7, d= -2 then the four numbers are 13, 9, 5 and 1.

Therefore, the four consecutive terms of the A.P are 1, 5, 9, and 13

3. **A mother divides ₹207 into three parts such that the amount are in A.P. and gives it to her three children. The product of the two least amounts that the children had ₹4623. Find the amount received by each child.**

Solution

Let the amount received by the three children be in the form of A.P. is given by a - d, a, a + d.

Since, sum of the amount is ₹**207**

We have (a – d) + a + (a + d) = 207.

3a = 207 gives a = 69.

It is given that product of the two least amounts is 4623.

(a – d)a = 4623

(69 – d)69 = 4623 ; d = 2

Therefore, amount given by the mother to her three children are ₹(69 – 2), ₹ 69 , ₹(69 + 2)

That is ₹67, ₹69 and ₹71.

4. **The ratio of 6th and 8th term of an A.P. is 7: 9. Find the ratio of 9th term to 13th term.**

Solution

$t_6 : t_8 = 7 : 9$

$\Rightarrow \frac{t_6}{t_8} = \frac{7}{9} \Rightarrow \frac{a+5d}{a+7d} = \frac{7}{9}$

$\Rightarrow 9a + 45d = 7a + 49d \Rightarrow a = 2d$

To find $t_9 : t_{13} \Rightarrow \frac{t_9}{t_{13}} = \frac{a+8d}{a+12d} = \frac{10d}{14d} = \frac{5}{7}$

The required ratio is 5 : 7.

5. **In a winter season let us take the temperature of Ooty from Monday to Friday to be in A.P. The sum of temperatures from Monday to Wednesday is 0°C and the sum of the temperatures from Wednesday to Friday is 18°C. Find the temperature on each of the five days.**

Solution T_1, T_2, T_3, T_4, T_5 are in A.P. Also given

$T_1 + T_2 + T_3 = 0°C$ ----------------(1)

$T_3 + T_4 + T_5 = 18°C$ ----------------(2)

(1) $\Rightarrow a + a + d + a + 2d = 0$

$3a + 3d = 0; a + d = 0$ ----------------(3)$\because (a = -d)$

(2) $\Rightarrow a + 2d + a + 3d + a + 4d = 18$

$\Rightarrow 3a + 9d = 18; 6d = 18 \ (a = -d)$

$d = 3 \therefore a = -3$

$T_1 = -3°C, T_2 = 0°C, T_3 = 3°C, T_4 = 6°C, T_5 = 9°C,$

6. **Priya earned ₹15,000 in the first month. Thereafter her salary increased by ₹1500 per year. Her expenses are ₹13,000 during the first year and the expenses increases by ₹900 per year. How long will it take for her to save ₹20,000 per month.**

Solution	1 year	**2 year**
Income	₹15,000	₹16,500
Expenses	₹13,000	₹13,900
Savings	₹2,000	₹2,600

$\therefore$ **Annual Savings** ₹2,000, ₹2,600, ₹3,200…….a = 2,000, d = 600, t_n = 20,000

$a + (n - 1) d = 20,000$

$\Rightarrow 2000+(n-1)600 = 20,000$

$\Rightarrow 600n - 600 = 18,000$

$\Rightarrow 600n = 18,600$

$\Rightarrow n = \frac{18600}{6} = \frac{186}{6} = 31$

n= 31 years

The savings of Priya after 31 years is ₹ 20,000.

CHAPTER – 3

ALGEBRA

2 MARKS

1. **Simplify $\frac{b^2+3b-28}{b^2+4b+4} \div \frac{b^2-49}{b^2-5b-14}$**

Solution

$$\frac{b^2+3b-28}{b^2+4b+4} \div \frac{b^2-49}{b^2-5b-14} = \frac{(b-4)\,(b+7)}{(b+2)(b+2)} \times \frac{(b-7)(b+2)}{(b+7)(\,b-7)} = \frac{b-4}{b+2}$$

2. **If a polynomial P(x) = x^2 – 5x – 14 is divided by another polynomial q(x) we get $\frac{x-7}{x+2}$, find q(x) .**

Solution

Given P(x) = $x^2 - 5x - 14$

$$\frac{P(x)}{q(x)} = \frac{x-7}{x+2}$$

$$\frac{x^2-5x-14}{q(x)} = \frac{x-7}{x+2}$$

$$\frac{(x-7)(x+2)}{q(x)} = \frac{x-7}{x+2}$$

$\therefore$ q(x) = $(x + 2)^2$

3. **Solve $2m^2 + 19m + 30 = 0$.**

Solution

$2m^2 + 19m + 30 = 2m^2 + 4m + 15m + 30 = 2m(m+2) + 15(m + 2)$

$= (m + 2)(2m + 15)$

Now equating the factors to zero we get,

(m + 2) (2m + 15) = 0

m + 2 = 0 ⇒ m = -2 and 2m + 15 = 0 ⇒ $m = \frac{-15}{2}$

Therefore the roots are -2, $\frac{-15}{2}$

4. **Find $\frac{x^2+20x+36}{x^2-3x-28} - \frac{x^2+12x+4}{x^2-3x-28}$**

Solution

$$\frac{x^2+20x+36}{x^2-3x-28} - \frac{x^2+12x+4}{x^2-3x-28} = \frac{(x^2+20x+36)-(x^2+12x+4)}{x^2-3x-28}$$

$$= \frac{8x+32}{x^2-3x-28} = \frac{8(x+4)}{(x-7)(\,x+4)} = \frac{8}{x-7}$$

5. **Solve the quadratic equation by factorization method $4x^2$ -7x – 2=0**

Solution

$4x^2$ -7x – 2=0

$4x^2 - 7x - 2 = 0$

$(x-2)(4x+1) = 0$

$x - 2 = 0$ (or) $4x + 1 = 0$

$x = +2, x = -\frac{1}{4}$

6. Solve the quadratic equation by factorization method $3(p^2 - 6) = p(p+5)$

Solution

$\mathbf{3(p^2 - 6) = p(p+5)}$

$3p^2 - 18 = p^2 + 5p$

$2p^2 - 5p - 18 = 0$

$(2p - 9)(p + 2) = 0$

$2p - 9 = 0$ (or) $p + 2 = 0$

$p = +\frac{9}{2}$, $p = -2$

7. Solve the quadratic equation by factorization method $\sqrt{a(a-7)} = 3\sqrt{2}$

Solution

$\sqrt{a(a-7)} = 3\sqrt{2}$

$a(a-7) = \left(3\sqrt{2}\right)^2$

$a^2 - 7a = (3\sqrt{2})^2$

$a^2 - 7a - 18 = 0$

$(a - 9)(a + 2) = 0$

$\therefore a = 9, a = -2$

8. Solve the quadratic equation by factorization method $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

Solution

$\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$(\sqrt{2}x + 5)\left(x + \sqrt{2}\right) = 0$

$\therefore x = -\frac{5}{\sqrt{2}}$ and $x = -\sqrt{2}$

9. Solve the quadratic equation by factorization method $2x^2 - x + \frac{1}{8} = 0$

Solution

$16x^2 - 8x + 1 = 0$

$(4x - 1)(4x - 1) = 0$

$\therefore x = \frac{1}{4}, x = \frac{1}{4}$

10. Solve the quadratic equation by formula method $2x^2 - 5x + 2 = 0$

Solution

$2x^2 - 5x + 2 = 0$

Here a = 2, b = -5, c = 2

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{5 \pm \sqrt{25-16}}{4} = \frac{5 \pm 3}{4}$$

$$x = \frac{5+3}{4} \text{ or } x = \frac{5-3}{4}$$

$$\therefore x = 2 \quad (or)\ x = \frac{1}{2}$$

11. Solve the quadratic equation by formula method $3y^2 - 20y - 23 = 0$

$3y^2 - 20y - 23 = 0$

Here a =3, b = - 20, c = -23

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{20 \pm \sqrt{400+276}}{6} = \frac{20 \pm \sqrt{676}}{6}$$

$$= \frac{20 \pm 26}{6} = \Rightarrow x = \frac{20+26}{6} \text{ (or) } \frac{20-26}{6}$$

$$x = \frac{23}{3}, -1$$

12. The product of Kumaran's age (in years) two years ago and his age four years from now is one more than twice his present age. What is his present age?

Solution Let the present age of Kumaran be x years.

Two years ago, his age is (x – 2) years.

Four years from now, his age = (x + 4) years

Given (x – 2) (x + 4) = 1 + 2x

$x^2 + 2x - 8 = 1 + 2x$ gives (x-3) (x + 3) = 0

Then, x = ± 3 (Rejecting – 3 as age cannot be negative).

Kumaran's present age is 3 years.

13. If the difference between a number and its reciprocal is $\frac{24}{5}$ find the number.

Solution

Let x be the required number. From the given, $x - \frac{1}{x} = \frac{24}{5}$.

$$\Rightarrow \frac{x^2 - 1}{x} = \frac{24}{5} \Rightarrow 5x^2 - 24x - 5 = 0$$

$$\Rightarrow (x - 5)(5x + 1) = 0$$

$\therefore x = 5$ or $x = \frac{-1}{5}$

14. If the difference between the roots of the equation $x^2 - 13x + k = 0$ is 17 . Find k

Solution

$x^2 - 13x + k = 0$ Here, a = 1, b = -13, c = k

Let α and β be the roots of the equation. Then

$\alpha + \beta = \frac{-b}{a} = \frac{-(-13)}{1} = 13$ ……(1)

Also $\alpha - \beta = 17$ ……(2)

(1) + (2) we get, $2\alpha = 30$ gives $\alpha = 15$.

Therefore, $15 + \beta = 13$(from (1)) gives, $\beta = -2$

But $\alpha\beta = \frac{c}{a} = \frac{k}{1}$ $gives$ $15 \times (-2) = k$ we get, $k = -30$

15. If α and β are the roots of the equation $3x^2 + 7x - 2 = 0$, find the values of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

Solution

Given $3x^2 + 7x - 2 = 0$, Here , a = 3, b = 7, c = -2

Since α and β are the roots of the equation.

$\alpha + \beta = \frac{-b}{a} = \frac{-7}{3}$

$\alpha\beta = \frac{c}{a} = \frac{-2}{3}$

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{\left(\frac{-7}{3}\right)^2 - 2\left(\frac{-2}{3}\right)}{\frac{-2}{3}}$$

$$= \frac{\frac{49}{9} + \frac{4}{3}}{\frac{-2}{3}} = \frac{\frac{49+12}{9}}{\frac{-2}{3}} = \frac{61}{9} \times \frac{3}{-2} = \frac{-61}{6}$$

16. If α and β are the roots of the equation $3x^2 + 7x - 2 = 0$ find the values of $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

Solution

Given $3x^2 + 7x - 2 = 0$, Here , a = 3, b = 7, c = -2

Since α and β are the roots of the equation

$$\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3+\beta^3}{\alpha\beta} = \frac{(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)}{\alpha\beta} = \frac{\left(\frac{-7}{3}\right)^3 - 3\left(\frac{-2}{3}\right)\left(\frac{-7}{3}\right)}{\frac{-2}{3}}$$

$$= \frac{\frac{-343}{27} - \frac{42}{9}}{\frac{-2}{3}} = \frac{\frac{-343-126}{27}}{\frac{-2}{3}} = \frac{-469}{27} \times \frac{3}{-2} = \frac{469}{18}$$

17. Find the values of k for which the roots of the following equations are real and equal $(5k - 6)x^2 + 2kx + 1 = 0$

Solution

Given $(5K - 6)x^2 + 2kx + 1 = 0$. The roots are real and equal

a = 5k – 6, b = 2k, c = 1

The roots are real and equal. So

$b^2 - 4ac = 0$

$(2k)^2 - 4(5k\text{-}6)1 = 0 \Longrightarrow 4k^2 - 20k + 24 = 0 \quad \div 4$

$k^2 - 5k + 6 = 0 \quad \Longrightarrow (k - 2)(k - 3) = 0$

k =2 or k =3

18. **Find the values of k for which the roots of the equation are real and equal**

$kx^2 + (6k+2)x + 16 = 0$

<u>Solution</u>

$kx^2 + (6k+2)x + 16 = 0$

a = k, b = 6k+2, c = 16

The roots are real and equal. So

$b^2 - 4ac = 0$

$\Longrightarrow (6k+2)^2 - 4(k)16 = 0$

$\Longrightarrow 36k^2 + 24k + 4 - 64k = 0$

$\Longrightarrow 36k^2 - 40k + 4 = 0 \ (\div 4)$

$\Longrightarrow 9k^2 - 10k + 1 = 0$

$\Longrightarrow (k - 1)(9k - 1) = 0$

$\therefore k = 1$ (or) $k = \frac{1}{9}$

19. **If one root of the equation $3x^2 + kx + 81 = 0$ (having real roots) is the square of the other then find k.**

<u>Solution</u> Let α, β be the roots of $3x^2 + kx + 81 = 0$

$\alpha + \beta = -\frac{k}{3}$ ------ (1)

$\alpha\beta = 27$ ------ (2)

Given $\alpha = \beta^2$

From (1)

$\beta^3 = 27$

$\beta = 3$

$\therefore \alpha = 9$

(1) $\Longrightarrow 9 + 3 = -\frac{k}{3}$

$12 = -\frac{k}{3}$

$k = -36$

5 MARKS

1. **Solve the following system of linear equation in three variables 3x–2y+ z = 2, 2x+3y–z = 5, x + y + z = 6.**

Solution

3x–2y+ z = 2 ……(1)

2x+3y–z = 5 ……(2)

x + y + z = 6 ……(3)

Adding (1) and (2)

$$\begin{array}{l} 3x-2y+z=2 \\ \underline{2x+3y-z=5} \quad (+) \\ \underline{5x+y \quad =7} \quad \ldots\ldots(4) \end{array}$$

Adding (2) and (3)

$$\begin{array}{l} 2x+3y-z=5 \\ \underline{x+y+z=6} \quad (+) \\ \underline{3x+4y=11} \quad \ldots\ldots(5) \end{array}$$

$(4) \times 4 - (5)$

$$\begin{array}{l} 20x+4y=28 \\ \underline{3x+4y=11} \; (-) \\ \underline{17x \quad =17} \quad \Rightarrow x=1 \end{array}$$

Substituting x = 1 in (4), 5 + y = 7 $\Rightarrow y = 2$

Substituting x = 1, $y = 2$ in (3) – , 1 + 2 + z = 6 $\Rightarrow z = 3$

Therefore, $x = 1, y = 2, z = 3$

2. **Vani, her father and her grand father have an average age of 53. One - half of her grand father's age plus one-third of her father's age plus one fourth of Vani's age is 65. Four years ago if Vani's grandfather was four times as old as Vani then how old are they all now?**

Solution Let the present age of Vani, her father, grand father be x, y, z respectively.

By data given,

$\frac{x+y+z}{3} = 53 \Rightarrow x + y + z = 159$ ------- (1)

$\frac{1}{2}z + \frac{1}{3}y + \frac{1}{4}x = 65$

$\frac{6z+4y+3x}{12} = 65$

3x + 4y + 6z = 780 ------- (2)

(z -4) = 4(x – 4) ⇒ 4x – z = 12 ------- (3)

From (1) & (2)

(1) x (4) ⇒ 4x + 4y + 4z = 636

(2) ⇒ $\underline{3x + 4y + 6z = 780}$

(subtracting) x – 2z = - 144 ------- (4)

From (3) & (4)

(3) x (2) $\Rightarrow 8x - 2z = \quad 24$

(4) $\Rightarrow \underline{x - 2z = - 144}$

(**subtracting**) $7x = \quad 168$ ------- (5)

$x = \frac{168}{7} = 24$

Substitute x = 24 in(3)

$96 – z = 12$

$z = 84$

(1) $\Rightarrow 24 + y + 84 = 159$

$\Rightarrow y = 51$

∴Vani's Present Age = 24 years

Father's Present Age = 51years

Grand father's Age = 84 years

3. Find the G.C.D of the polynomials $x^3 +x^2 - x +2$ and $2x^3 -5x^2 + 5x – 3$.

<u>Solution</u>

Let $f(x) = 2x^3 -5x^2 + 5x – 3$ and $g(x) = x^3 +x^2 - x +2$

$$
\begin{array}{r|l}
 & 2 \\
\hline
x^3 +x^2 - x +2 & 2x^3 -5x^2 + 5x – 3 \\
 & 2x^3 +2x^2 - 2x + 4 \quad (-) \\
\hline
 & \qquad -7x^2 + 7x - 7 \\
\hline
\end{array}
$$

$= -7(x^2 - x + 1)$

$7(x^2 - x + 1) \neq 0$, note that – 7 is not divisor of g(x). Now dividing $g(x) = x^3 +x^2 - x +2$ by the new remainder $x^2 - x + 1$ (leaving the constant facor), we get

$$
\begin{array}{r|l}
 & x + 2 \\
\hline
x^2 - x + 1 & x^3 +x^2 - x +2 \\
 & x^3 -x^2 + x \qquad\qquad (-) \\
\hline
 & \quad 2x^2 – 2x + 2 \\
 & \quad 2x^2 – 2x + 2 \qquad (-) \\
\hline
 & \qquad 0 \\
\hline
\end{array}
$$

Here, we get zero remainder. Therefore, GCD of$(2x^3 -5x^2 + 5x – 3$ and $x^3 +x^2 - x +2) = x^2 - x + 1$

4. Find the G.C.D of the polynomial $x^4 + 3x^3 - x-3, x^3 + x^2 – 5x + 3$

Solution

$f(x) = x^4 + 3x^3 - x-3$ and $g(x) = x^3 + x^2 - 5x + 3$

$$\begin{array}{r|l}
 & \qquad x+2 \\ \cline{2-2}
x^3+x^2-5x+3 & x^4+3x^3+0 \quad -x \ -3 \\
 & x^4+\ x^3\ -5x^2\ +3x \qquad\qquad (-) \\ \cline{2-2}
 & \qquad 2x^3+5x^2-4x\ -3 \\
 & \qquad 2x^3+2x^2-10x+6 \qquad (-) \\ \cline{2-2}
 & \qquad\qquad 3x^2+6x-9 \\ \cline{2-2}
\end{array}$$

$= 3(x^2 + 2x - 3)$

$$\begin{array}{r|l}
 & \qquad x-1 \\ \cline{2-2}
x^2+2x-3 & x^3+x^2\ -5x+3 \\
 & x^3\ +2x^2-3x \qquad\qquad (-) \\ \cline{2-2}
 & \qquad -x^2-2x+3 \\
 & \qquad -x^2-2x+3 \qquad (-) \\ \cline{2-2}
 & \qquad\qquad 0 \\ \cline{2-2}
\end{array}$$

$\therefore$G.C.D. of (f(x), g(x)) $= x^2 + 2x - 3$

5. **Find the G.C.D of the polynomial $x^4 - 1, x^3-11x^2+x - 11$**

Solution $f(x) = x^4 - 1$ and $g(x) = x^3-11x^2+x - 11$

$$\begin{array}{r|l}
 & \qquad\quad x+11 \\ \cline{2-2}
x^3-11x^2+x-11 & x^4+0x^3+0\,x^2-0x\ -1 \\
 & x^4-11x^3\ +x^2\ -11x \qquad\qquad (-) \\ \cline{2-2}
 & \quad 11x^3\ -x^2\ \ +11x\ -1 \\
 & \quad 11x^3-121x^2+11x\ -121 \qquad (-) \\ \cline{2-2}
 & \qquad\qquad 120x^2+120 \\ \cline{2-2}
\end{array}$$

$= 120(x^2 + 1)$

$$\begin{array}{r|l}
 & \quad x-11 \\ \cline{2-2}
x^2-0x+1 & x^3-11x^2\ +x-11 \\
 & x^3\ +0x^2+x \qquad\qquad (-) \\ \cline{2-2}
 & \qquad -11x^2+0x-11 \\
 & \qquad -11x^2+0x-11 \qquad (-) \\ \cline{2-2}
 & \qquad\qquad 0 \\ \cline{2-2}
\end{array}$$

$\therefore$G.C.D. of (f(x), g(x)) $= x^2 + 1$

6. Find the G.C.D of the polynomial $3x^4 + 6x^3 - 12x^2 - 24x, 4x^4 + 14x^3 + 8x^2 - 8x$

Solution

$f(x) = 3x^4 + 6x^3 - 12x^2 - 24x = 3x(x^3 + 2x^2 - 4x - 8)$
$g(x) = 4x^4 + 14x^3 + 8x^2 - 8x = 2x(2x^3 + 7x^2 + 4x - 4)$
G.C.D.of (3x,2x) is x

$$\begin{array}{r|l}
 & \qquad 2 \\
\hline
x^3 + 2x^2 - 4x - 8 & 2x^3 + 7x^2 + 4x - 4 \\
 & 2x^3 + 4x^2 - 8x - 1 \qquad (-) \\
\hline
 & \qquad 3x^2 + 12x + 12 \\
\hline
\end{array}$$

$= 3(x^2 + 4x + 4)$

$$\begin{array}{r|l}
 & \qquad x - 2 \\
\hline
x^2 + 4x + 4 & x^3 + 2x^2 - 4x - 8 \\
 & x^3 + 4x^2 + 4x \qquad (-) \\
\hline
 & \qquad -2x^2 - 8x - 8 \\
 & \qquad -2x^2 - 8x - 8 \qquad (-) \\
\hline
 & \qquad\qquad 0 \\
\hline
\end{array}$$

$\therefore$G.C.D. of (f(x), g(x)) = $x(x^2 + 4x + 4)$

7. Find the G.C.D of the polynomial $3x^3 + 3x^2 + 3x + 3, 6x^3 + 12x^2 + 6x + 12$

Solution

$f(x) = 3x^3 + 3x^2 + 3x + 3 = 3(x^3 + x^2 + x + 1)$
$g(x) = 6x^3 + 12x^2 + 6x + 12 = 6(x^3 + 2x^2 + x + 2)$
G.C.D of (3,6) is 3

$$\begin{array}{r|l}
 & \qquad 1 \\
\hline
x^3 + x^2 + x + 1 & x^3 + 2x^2 + x + 2 \\
 & x^3 + x^2 + x + 1 \qquad (-) \\
\hline
 & \qquad x^2 + 0x + 1 \\
\hline
\end{array}$$

$$\begin{array}{r|l}
 & \qquad x + 1 \\
\hline
x^2 + 0x + 1 & x^3 + x^2 + x + 1 \\
 & x^3 + 0x^2 + x \qquad (-) \\
\hline
 & \qquad x^2 + 0x + 1 \\
 & \qquad x^2 + 0x + 1 \qquad (-) \\
\hline
 & \qquad\qquad 0 \\
\hline
\end{array}$$

$\therefore$G.C.D. of (f(x), g(x)) = $3(x^2 + 1)$

8. Given the LCM and GCD of the two polynomials p(x) and q(x) find the unknown polynomial in the following table

LCM	GCD	p(x)	q(x)
$a^3 - 10a^2 + 11a + 70$	a - 7	$a^2 - 12a + 35$	

Solution

LCM = $a^3 - 10a^2 + 11a + 70$, GCD = (a-7) and

p(x) = $a^2 - 12a + 35$

$$q(x) = \frac{\text{LCM X GCD}}{p(x)} = \frac{(a^3 - 10a^2 + 11a + 70)(a-7)}{a^2 - 12a + 35}$$

$$\begin{array}{r|l} & \quad a+2 \\ \hline a^2 - 12a + 35 & a^3 - 10a^2 + 11a + 70 \\ & a^3 - 12a^2 + 35a \qquad (-) \\ \hline & \quad 2a^2 - 24a + 70 \\ & \quad 2a^2 - 24a + 70 \qquad (-) \\ \hline & \qquad 0 \end{array}$$

∴q(x) = (a + 2) (a - 7)

9. Simplify $\frac{1}{x^2-5x+6} + \frac{1}{x^2-3x+2} - \frac{1}{x^2-8x+15}$

Solution

$$\frac{1}{x^2-5x+6} + \frac{1}{x^2-3x+2} - \frac{1}{x^2-8x+15}$$

$$= \frac{1}{(x-2)(x-3)} + \frac{1}{(x-2)(x-1)} - \frac{1}{(x-5)(x-3)}$$

$$= \frac{(x-1)(x-5)+(x-3)(x-5)-(x-1)(x-2)}{(x-1)(x-2)(x-3)(x-5)}$$

$$= \frac{(x^2-6x+5)+(x^2-8x+15)-(x^2-3x+2)}{(x-1)(x-2)(x-3)(x-5)}$$

$$= \frac{x^2-11x+18}{(x-1)(x-2)(x-3)(x-5)}$$

$$= \frac{(x-9)(x-2)}{(x-1)(x-2)(x-3)(x-5)}$$

$$= \frac{x-9}{(x-1)(x-3)(x-5)}$$

10. If A = $\frac{2x+1}{2x-1}$**, B =** $\frac{2x-1}{2x+1}$ **find** $\frac{1}{A-B} - \frac{2B}{A^2-B^2}$

Solution

$$\frac{1}{A-B} - \frac{2B}{A^2-B^2} = \frac{1}{A-B} - \frac{2B}{(A+B)(A-B)}$$

$$= \frac{A+B-2B}{(A+B)(A-B)} = \frac{(A-B)}{(A+B)(A-B)}$$

$$= \frac{1}{A+B} = \frac{1}{\frac{2x+1}{2x-1} + \frac{2x-1}{2x+1}}$$

$$= \frac{1}{\frac{(2x+1)^2+(2x-1)^2}{(2x+1)(2x-1)}} \quad = \frac{(2x+1)(2x-1)}{(2x+1)^2+(2x-1)^2}$$

$$= \frac{[2x]^2 - 1^2}{4x^2+1+4x+4x^2+1-4x}$$

$$= \frac{4x^2-1}{8x^2+2}$$

$$= \frac{4x^2-1}{2(4x^2+1)}$$

11. If A = $\frac{x}{x+1}$ **and** $B = \frac{1}{x+1}$**,Prove that** $\frac{(A+B)^2+(A-B)^2}{A\div B} = \frac{2(x^2+1)}{x(x+1)^2}$

Solution

Let A = $\frac{x}{x+1}$, $B = \frac{1}{x+1}$

$$\frac{(A+B)^2+(A-B)^2}{A\div B} = \frac{2(A^2+B^2)}{A\div B}$$

$A^2 + B^2 = \frac{x^2}{(x+1)^2} + \frac{1}{(x+1)^2} = \frac{x^2+1}{(x+1)^2}$

$A \div B = \frac{x}{x+1} \times \frac{x+1}{1} = x$

$$\frac{2(A^2+B^2)}{A\div B} = (2)\left(\frac{x^2+1}{(x+1)^2}\right)\left(\frac{1}{x}\right) = \frac{2(x^2+1)}{x(x+1)^2}$$

12. Solve $x^2 + 2x - 2 = 0$ by formula method

Solution

Compare $x^2 + 2x - 2 = 0$ with the standard form $ax^2 + bx + c = 0$

a = 1, b = 2, c = -2

Substituting the values of a, b, c in the formula we get,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-2)}}{2(1)} = \frac{-2 \pm \sqrt{12}}{2} = -1 \pm \sqrt{3}$$

Therefore, $x = -1 + \sqrt{3}, x = -1 - \sqrt{3}$

13. Solve $2x^2 - 3x - 3 = 0$ by formula method

Solution

Compare $2x^2 - 3x - 3 = 0$ with the standard form $ax^2 + bx + c = 0$

a = 2, b = -3, c = -3

Substituting the values of a, b, c in the formula we get,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-3)}}{2(2)} = \frac{3 \pm \sqrt{33}}{4}$$

Therefore, $x = \frac{3+\sqrt{33}}{4}, x = \frac{3-\sqrt{33}}{4}$

14. A passenger train takes 1 hr more than an express train to travel a distance of 240km from Chennai to Virudhachalam. The speed of passenger train is less than that of an express train by 20 km per hour. Find the average of speed of both the trains.

Solution

Let the average speed of passenger train be x km/ hr. Then the average speed of express train will be (x + 20) km /hr

Time taken by the passenger train to cover distance of 240 km = $\frac{240}{x}\ hr$

Time taken by the express train to cover distance of 240 km = $\frac{240}{x+20}\ hr$

Given, $\frac{240}{x} = \frac{240}{x+20} + 1$

$240\left[\frac{1}{x} - \frac{1}{x+20}\right] = 1 \Rightarrow 240\left[\frac{x+20-x}{x(x+20)}\right] = 1 \Rightarrow 4800 = (x^2 + 20x)$

$x^2 + 20x - 4800 = 0 \Rightarrow (x+80)(x-60) = 0 \quad \Rightarrow x = -80$ or 60

Therefore x = 60(Rejecting -80 as speed cannot be negative)

Average speed of the passenger train is 60 km / hr.

Average speed of the express train is 80 km / hr.

15. The hypotenuse of a right angled triangle is 25cm and its perimeter 56cm. Find the length of the smallest side.

Solution

From the given, hypotenuse is 25cm, other two sides are x and y

Perimeter = 56cm

25 + x + y = 56

$\therefore\ x + y = 31$

$y = 31 - x$

In right angled triangle, by Pythagoras theorem,

$$x^2 + y^2 = 25^2$$

$$x^2 + (31-x)^2 = 25^2$$

$\Rightarrow x^2 + 961 + x^2 - 62x = 625$

$\Rightarrow 2x^2 - 62x + 336 = 0$

$\Rightarrow x^2 - 31x + 168 = 0$

$\Rightarrow (x-24)(x-7) = 0$

$\therefore x = 24\ (Or)\ x = 7$

Hence, the length of smallest side of the given right angled triangle is 7 cm.

16. If the roots of the equation $(c^2 – ab)x^2 – 2(a^2 – bc)x + b^2 – ac = 0$ are real and equal prove that either $a = 0$ (or) $a^3 + b^3 + c^3 = 3abc$

Solution $(c^2 – ab)x^2 – 2(a^2 – bc)x + b^2 – ac = 0$

$\mathbf{A} = c^2 – ab$, $B = – 2(a^2 – bc)$, $C = b^2 – ac$

$\Delta = B^2 – 4AC$

Since the roots are real and equal $\Delta = \mathbf{0}$

$[-2(a^2 – bc)]^2 – 4[c^2 – ab] [b^2 – ac] = 0$

$4(a^2 – bc)^2 – 4 (c^2 – ab) (b^2 – ac) = 0$

$4[a^4+ b^2c^2 – 2a^2bc] – 4 [b^2c^2 – ac^3 – ab^3 + a^2bc] = 0$

$4a^4+ 4b^2c^2 – 8a^2bc – 4 b^2c^2 +4ac^3 + 4ab^3 - 4a^2bc = 0$

$4a^4+ 4ab^3 + 4ac^3 - 12a^2bc = 0$

$4a [a^3+b^3+c^3 – 3abc] = 0$

$a = 0$ OR $a^3+b^3+c^3 – 3abc = 0$

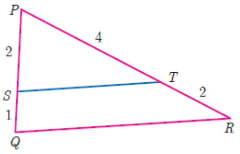
$\Rightarrow a^3+b^3+c^3 = 3abc$

CHAPTER – 4

GEOMETRY

2 MARKS

1. **Show that $\Delta PST \sim \Delta PQR$**



Solution

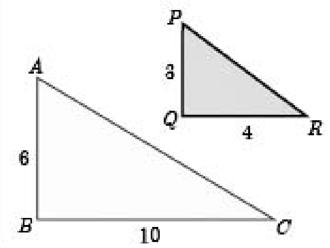
In ΔPST and ΔPQR

$$\frac{PS}{PQ} = \frac{2}{2+1} = \frac{2}{3}, \frac{PT}{PR} = \frac{4}{4+2} = \frac{2}{3}$$

Thus, $\frac{PS}{PQ} = \frac{PT}{PR}$ and $\angle P$ is common.

Therefore, by SAS similarity $\Delta \boldsymbol{PST} \sim \Delta \boldsymbol{PQR}$

2. **Is ΔABC ~ ΔPQR?**



தீர்வு

In ΔABC and ΔPQR

$\frac{PQ}{AB} = \frac{3}{6} = \frac{1}{2}, \frac{QR}{BC} = \frac{4}{10} = \frac{2}{5}$

Since $\frac{1}{2} \neq \frac{2}{5}$, $\frac{PQ}{AB} \neq \frac{QR}{BC}$

The corresponding sides are not proportional. Therefore ΔABC is not similar to ΔPQR

3. **A boy of height 90cm is walking away from the base of a lamp post at a speed of 1.2 m/sec. If the lamp post is 3.6m above the ground, find the length of his shadow cast after 4 seconds.**

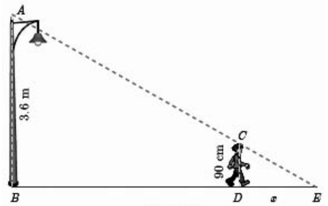


Fig. 4.19

Solution

Speed = 1. 2 m/ s
Time = 4 seconds

Distance = Speed x Time = 1.2 × 4 = 4.8 m = BD
Let x be the length of the shadow after 4 seconds

Since, $\Delta ABE \sim \Delta CDE, \frac{BE}{DE} = \frac{AB}{CD}$ gives $\frac{4.8+x}{x} = \frac{3.6}{0.9} = 4$

$4.8 + x = 4x$ gives $3x = 4.8$, so, $x = 1.6$ m.

The length of his shadow DE = 1.6m

4. **If ΔABC is similar to DEF such that BC = 3cm, EF = 4cm and area of ΔABC = 54cm^2. Find the area of ΔDEF.**

Solution Since the ratio of area of two similar triangles is equal to the ratio of the squares of any two corresponding sides, we have

$\frac{Area\ (\Delta ABC)}{Area\ (\Delta DEF)} = \frac{BC^2}{EF^2}$ gives $\frac{54}{Area\ (\Delta DEF)} = \frac{3^2}{4^2}$

Area $(\Delta DEF) = \frac{16 \times 54}{9} = 96$cm^2

5. **If ΔABC ~ ΔDEF such that area of ΔABC is 9cm^2 and the area of ΔDEF is 16cm^2 and BC = 2.1 cm. Find the length of EF.**

Solution

Given ΔABC~ ΔDEF

$$\frac{Area\ of\ (\Delta ABC)}{\text{Area of }(\Delta DEF)} = \frac{BC^2}{EF^2}$$

$$\Rightarrow \frac{9}{16} = \frac{(2.1)^2}{EF^2}$$

$$\Rightarrow EF^2 = (2.1)^2 \times \frac{16}{9}$$

$$\Rightarrow \text{EF} = 2.1 \times \frac{4}{3} = 2.8\text{cm}$$

6. **In ΔABC, D and E are points on the sides AB and AC respectively. For each of the following cases show that DE∥ BC.**

(i) AB = 12cm, AD = 8cm, AE = 12cm and AC = 18cm.

(ii) AB = 5.6cm, AD = 1.4cm, AC = 7.2 cm and AE = 1.8cm

Solution

To prove that DE∥ BC

(i) AB = 12 cm, AD = 8 cm AE = 12cm and AC = 18cm

$$\frac{\text{AD}}{\text{BD}} = \frac{8}{4} = 2 \text{ ……………….. (1) ∵ BD = AB - AD}$$

$$\frac{\text{AE}}{\text{EC}} = \frac{12}{6} = 2 \text{……………….. (2) ∵ EC = AC - AE}$$

(1) , (2) ⇒ $\frac{\text{AD}}{\text{BD}} = \frac{\text{AE}}{\text{EC}}$ ∴ DE∥ BC

(ii) AB = 5.6 cm, AD = 1.4 cm, AC = 7.2 cm and AE = 1.8 cm

$$\frac{AD}{BD} = \frac{1.4}{4.2} = \frac{1}{3} \text{ ………………(1) ∵BD = AB - AD}$$

$$\frac{AE}{EC} = \frac{1.8}{5.4} = \frac{1}{3} \text{ ……………..(2) ∵ EC = AC - AE}$$

(1), (2) ⇒ $\frac{AD}{BD} = \frac{AE}{EC}$ ∴ DE ∥BC

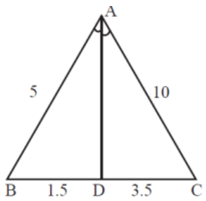
7. **Check whether AD is bisector of ∠A of ΔABC in each of the following:**

(i) AB = 5cm, AC = 10cm, BD = 1.5cm and CD = 3.5cm

(ii) AB = 4cm, AC = 6cm, BD = 1.6cm and CD = 2.4cm

தீர்வு

(i) AB = 5cm , AC =10 cm, BD =1.5 cm and CD = 3.5 cm



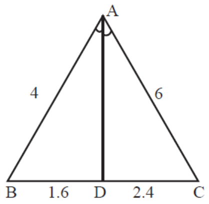
$\frac{AB}{AC} = \frac{5}{10} = \frac{1}{2}$ -------------(1)

$\frac{BD}{CD} = \frac{1.5}{3.5} = \frac{3}{7}$ -------------(2)

(1) , (2) ⇒ $\frac{AB}{AC} \neq \frac{BD}{CD}$(∵By ABT)

AD is not a bisector of $\angle A$ in ΔABC

(ii) AB = 4 cm, AC =6 cm, BD =1.6 cm and CD= 2.4 cm



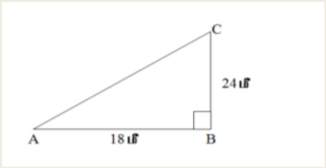
$\frac{AB}{AC} = \frac{4}{6} = \frac{2}{3}$ -------------(1)

$\frac{BD}{CD} = \frac{1.6}{2.4} = \frac{2}{3}$ -------------(2)

(1) , (2) ⇒ $\frac{AB}{AC} = \frac{BD}{CD}$(∵By ABT)

AD is a bisector of $\angle A$ in ΔABC

8. **A man goes 18 m due east and then 24 m due north. Find the distance of his current position from the starting point.**



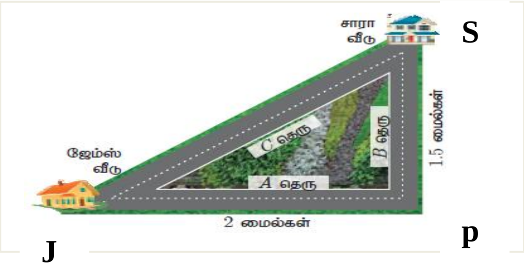
Solution $AC^2 = AB^2 + BC^2$

$AC^2 = (18)^2 + (24)^2 = 324 + 576$

$AC^2 = 900$ $AC = \sqrt{900}$ ⟹AC = 30 m

∴ The distance from the starting point is 30m

9. **There are two paths that one can choose to go from Sarah's house to James house. One way is to take C street, and the other way requires to take A street and then B street. How much shorter is the direct path along C street (Using Figure)**



Solution SJ $= \sqrt{(1.5)^2 + (2)^2}$

$= \sqrt{2.25 + 4}$ **=** $\sqrt{6.25}$ = **2.5** miles

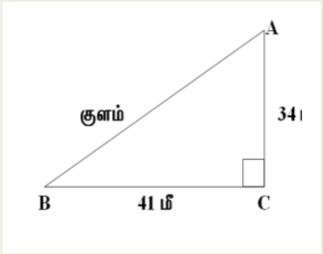
If one chooses A street and B street he has to go

SP + PJ = 1.5 + 2 = 3.5 miles

If one chooses C street the distance from James house to Sarah's house is 2.5 miles

∴ **1** km less

10. **To get from point A to point B you must avoid walking through a pond. You must walk 34m south and 41 m east. To the nearest meter, how many meters would be saved if it were possible to make a way through the pond?**



Solution To make a Straight way through the pond

$AB^2 = AC^2 + BC^2$

$= (34)^2 + (41)^2 = 1156 + 1681$

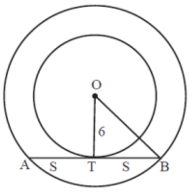
$AB^2 = 2837$ ⟹$AB = \sqrt{2837} = 53.26$ m

Through Cone must walk AB = AC + BC = 34 + 41 = 75 m walking through a pond one must come only 53.2m. The difference is (75 – 53.26)m = 21.74m.

To the nearest, one can save 21.74m

11. In two concentric circles, a chord of length 16cm of larger circle becomes tangent to the smaller circle whose radius is 6cm. Find the radius of the larger circle.

Solution



AB = 16 cm and OC = 6 cm

But OC ⊥ AB and C is divided two equal parts (∵ by circles theorem)

then , AC = CB = 8cm

To find OB.(OB is radius of larger circle)

By Pythagoras, OB = $\sqrt{OC^2 + BC^2} = \sqrt{6^2 + 8^2} = \sqrt{36 + 64}$

OB = 10 cm

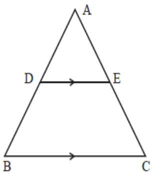
5 MARKS

1. In ΔABC , D and E are the points on the sides AB and AC respectively such that DE∥ BC. (i) $\frac{AD}{DB} = \frac{3}{4}$ and AC = 15 cm. Find AE. (ii) If AD = 8x – 7, DB = 5x – 3, AE = 4 x- 3 and EC =3x – 1, find the value of x.

Solution

(i) If $\frac{AD}{DB} = \frac{3}{4}$, AC = 15cm , DE ∥ BC then by basic proportionality theorem.

$\frac{AD}{AB} = \frac{AE}{AC}$



$\Rightarrow \frac{3}{7} = \frac{AE}{15} \Rightarrow AE = \frac{3}{7} \times 15 = 6.43$cm

(ii) Given AD = 8x – 7, DB = 5x – 3, AE = 4x- 3 and EC = 3x – 1

By basic proportionality theorem

$\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{8x-7}{5x-3} = \frac{4x-3}{3x-1}$

$\Rightarrow (8x-7)(3x-1) = (5x-3)(4x-3)$

$\Rightarrow 24x^2 - 29x + 7 = 20x^2 - 27x + 9$

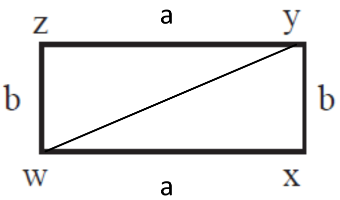
$\Rightarrow 4x^2 - 2x - 2 = 0$

$\Rightarrow 2x^2 - x - 1 = 0$

$(2x+1)(x-1) = 0$

$x = 1, x = -\frac{1}{2}. \quad \therefore x = 1.$

2. In the rectangle WXYZ, XY + YZ = 17 cm, and XZ + YW = 26 cm. Calculate the length and breadth.



Solution

XY + YZ = 17cm −−−−−−−−−(1)

XZ + YW = 26cm −−−−−−−−−(2)

XZ = YW (∵ Diagonals are equal in rectangle)

∴ XZ = YW = 13cm

(1) ⇒ a + b = 17 ---------(3)

By Pythagoras theorem

$a^2 + b^2 = d^2 \Rightarrow a^2 + b^2 = 169$ ---------(4)

(3) $\Rightarrow (a + b)^2 = (17)^2 \Rightarrow a^2 + b^2 + 2ab = 289$

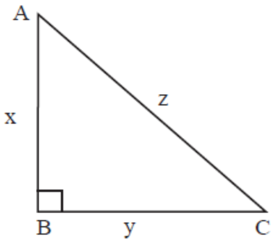
$\Rightarrow \quad ab = \frac{289-169}{2} = \frac{120}{2} \Rightarrow ab = 60$

If a + b = 17 and ab = 60 then a = 12 and b = 5.

. ∴ The length is 12 cm and the breadth is 5 cm.

3. The hypotenuse of a right triangle is 6m more than twice of the shortest side. If the third side is 2m less than the hypotenuse, find the sides of the triangle.

Solution From the given, (In figure z – hypotenuse and x > y)



$z = 2y + 6$ ……(1)

$x = z - 2$ ……(2)

(2) ⇒ $x = 2y + 6 - 2 = 2y + 4$

$x = 2y + 4$ ……(3)

By Pythagoras Theorem, $x^2 + y^2 = z^2$

$\Rightarrow (2y + 4)^2 + y^2 = (2y + 6)^2$ ∵ From (1) and (3)

$4y^2 + 16y + 16 + y^2 = 4y^2 + 24y + 36$

$y^2 - 8y - 20 = 0$

$(y - 10)(y + 2) = 0$

y = 10, -2 (∵ $y = -2$ is not possible)

∴ y = 10 in (1) and (3), We get x = 24, z= 26.

Hence, the sides of the required right triangle are 24cm, 26cm and 10cm.

4. 5m long ladder is placed leaning towards a vertical wall such that it reaches the wall at a point 4m high. If the foot of the ladder is moved 1.6m towards the wall, then find the distance by which the top of the ladder would slide upwards on the wall.

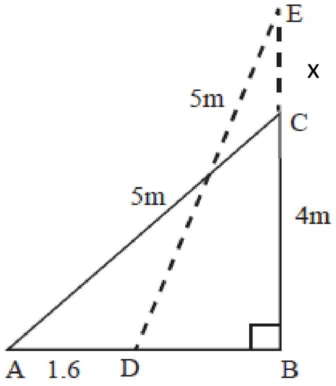
Solution From ΔABC , By Pythagorous theorem

$AB = \sqrt{AC^2 - BC^2} = \sqrt{25 - 16}$

AB = 3m

From the figure we have, AB = AD + BD ⇒ $BD = 1.4$m

In ΔDBE, By Pythagorous theorem



$(BE)^2 = (DE)^2 - (BD)^2$

$(4 + x)^2 = 5^2 - (1.4)^2$

$(4 + x)^2 = 23.04$

$4 + x = \sqrt{23.04} = 4.8$

$\therefore x = 0.8$

The distance by which top of the slide moves upwards is 0.8m.

CHAPTER – 5

CO-ORDINATE GEOMETRY

2 MARKS

1. **What is the slope of a line whose inclination is 30°?**

Solution

Here $\theta = 30°$

Slope, m = tanθ

Therefore, Slope m = tan$30°$ = $\frac{1}{\sqrt{3}}$

2. **What is the inclination of a line whose slope is $\sqrt{3}$?**

Solution

Given Slope, m = $\sqrt{3},$ Let θ be the inclination of the line.

tanθ = $\sqrt{3}$

We get, $\theta = 60°$

3. **The line r passes through the points (-2,2) and (5,8) and the line s passes through the points (-8,7) and (-2,0). Is the line r perpendicular to s?**

Solution

The slope of line r is $m_1 = \frac{8-2}{5+2} = \frac{6}{7}$

The slope of line s is $m_2 = \frac{0-7}{-2+8} = \frac{-7}{6}$

The product of the slopes = $\frac{6}{7} \times \frac{-7}{6} = -1$

That is , $m_1 m_2 = -1$

Therefore, the line r is perpendicular to line s.

4. **The line p passes through the points (3, -2), (12, 4) and the line q passes through the points (6, -2) and (12,2). Is p parallel to q?**

Solution

The slope of line p is $m_1 = \frac{4+2}{12-3} = \frac{6}{9} = \frac{2}{3}$

The slope of line q is , $m_2 = \frac{2+2}{12-6} = \frac{4}{6} = \frac{2}{3}$

Thus, slope of line p = slope of line q. Therefore, the line p is parallel to the line q.

5. **What is the slope of a line whose inclination with positive direction of x-axis (i) 90° (ii)0°**

Solution

(i) Given $\theta = 90°$

slope = m = tan $\theta = m = tan 90° =$ undefined

(ii) Given $\theta = 0°$

Slope = m = tan $\theta = m = tan 0° = \mathbf{0}$

6. **The line through the points (-2,a) and (9,3) has slope $-\frac{1}{2}$. Find the value of a.**

Solution Slope = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{3-a}{9+2} = \frac{3-a}{11}$

Given Slope = $-\frac{1}{2}$

$\therefore \frac{3-a}{11} = -\frac{1}{2}$

6 – 2a = - 11

2a = 17

a = $\frac{17}{2}$

7. **The line through the points (-2,6) and (4,8) is perpendicular to the line through the points (8,12) and (x,24). Find the value of x**

Solution Slope of line joining (-2,6), (4,8)

$m_1 = \frac{8-6}{4+2} = \frac{2}{6} = \frac{1}{3}$

Slope of line joining (8, 12) (x,24)

$m_2 = \frac{24-12}{x-8} = \frac{12}{x-8}$

Since two lines are perpendicular m_1 x m_2 = -1

$\Rightarrow \frac{1}{3} \text{ x } \frac{12}{x-8} = -1$

$\Rightarrow \frac{4}{x-8} = -1$

$\Rightarrow$ x- 8 = -4

$\Rightarrow$ x = 4

8. **Find the equation of a line passing through the point A(1,4) and perpendicular to the line joining points (2,5) and (4, 7).**

Solution

Let the given points be A(1,4), B(2,5), C(4,7)

Slope of line BC = $\frac{7-5}{4-2} = \frac{2}{2} = 1$

Let m be the slope of the required line.

Since the required line is perpendicular to BC,

m x 1 = -1

m= -1

The required line also pass through thepoint A(1,4)

The equation of the required straight line is $y - y_1 = m(x - x_1)$

y – 4 = -1(x -1)

y – 4 = -x + 1

We get, x + y – 5 = 0.

9. Find the equation of a straight line whose slope is 5 and y-intercept is -9.

Solution Slope, m = 5, y- intercept, c = -9

Equation of a straight line , y = mx + c

$y = 5x - 9$

$0 = 5x - y - 9$

Equation is $5x - y - 9 = 0$

10. Find the equation of a straight line whose inclination is 45° and y intercept is 11.

Solution Inclination, $\theta = 45°$

Slope, M = tan θ

M = tan 45°

Slope, M = 1

Y - Intercept, C = 11

Equation of the straight line, y = mx + C

$y = 1x + 11$

$0 = x + 11 - y$

∴ Equation is $x - y + 11 = 0$

11. Calculate the slope and y-intercept of the straight line 8x – 7y + 6 = 0.

Solution $8x - 7y + 6 = 0$

$8x + 6 = 7y$ ÷by 7

$\frac{8}{7}x + \frac{6}{7} = \frac{7}{7}y$

$\frac{8}{7}x + \frac{6}{7} = y$

Compare with mx+c=y

Slope,m = $\frac{8}{7}$

Y - Intercept, C = $\frac{6}{7}$

12. Find the equation of the line passing through the point (3,-4) and having slope $\frac{-5}{7}$.

Solution $(x_1 , y_1) = (3, -4)$

Slope,m = $-\frac{5}{7}$

Equation of the straight line $y - y_1 = m(x - x_1)$

$y - (-4) = -\frac{5}{7}(x - 3)$

$7(y + 4) = -5(x - 3)$

$7y + 28 = -5x + 15$

$5x + 7y + 28 - 15 = 0$

$5x + 7y + 13 = 0$

13. Find the equation of a straight line passing through (5,-3) and (7, -4)

Solution The equation of a straight line passing through the two points

x_1 , y_1 x_2 , y_2

(5, -3) (7, -4)

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y + 3}{-4 - (-3)} = \frac{x - 5}{7 - 5}$$

$$\frac{y + 3}{-4 + 3} = \frac{x - 5}{2}$$

2(y + 3) = -1(x – 5)

2y + 6 = -x +5

x +2y +6 – 5 =0

Therefore x +2y +1 =0

14. Find the equation of a straight line which has slope $\frac{-5}{4}$ and passing through the point(-1, 2)

Solution Given Point (−1,2), Slope $\frac{-5}{4}$

The required equation, $y - y_1 = m(x - x_1)$

$\Rightarrow$ $\mathbf{y - 2} = \frac{-5}{4}\left(x-(-1)\right) \Rightarrow 4y - 8 = -5x - 5$

$\Rightarrow$ $5x + 4y - 3 = 0$

15. Find the intercepts made by the line 3x – 2y – 6 = 0 on the coordinate axes.

Solution

Intercepts form: $\frac{x}{a} + \frac{y}{b} = 1$

$\because a-$x intercept, b- y intercept

$\Rightarrow 3x - 2y = 6 \Rightarrow \frac{3x}{6} - \frac{2y}{6} = 1$

$\Rightarrow \frac{x}{2} + \frac{y}{-3} = 1 \Rightarrow \therefore a = 2, b = -3$

16. Find the intercepts made by the line 4x + 3y +12 = 0 on the coordinate axes.

Solution

Intercepts form: $\frac{x}{a} + \frac{y}{b} = 1$

$\because a-$x intercept, b- y intercept

4x + 3y = - 12 (÷-12)

$\Rightarrow \frac{4x}{-12} + \frac{3y}{-12} = 1$

$\Rightarrow \frac{x}{-3} + \frac{y}{-4} = 1 \Rightarrow \therefore a = -3, b = -4$

17. Find the equation of a straight line Passing through (1,-4) and has intercepts which are in the ratio 2:5

Solution

Given, Ratio of intercepts , $\frac{a}{b} = \frac{2}{5}$

$\therefore a = \frac{2b}{5}$ ---------(1)

The required equation $\frac{x}{a} + \frac{y}{b} = 1$

$\Rightarrow \frac{x}{\frac{2b}{5}} + \frac{y}{b} = 1 \Rightarrow \frac{5x}{2b} + \frac{y}{b} = 1$

$\frac{5(1)}{2b} + \frac{(-4)}{b} = 1$ ($\because (1,-4)$ passing through)

$\Rightarrow \frac{5}{2b} + \frac{-8}{2b} = 1 \Rightarrow -3 = 2b \Rightarrow b = -\frac{3}{2}$

$\therefore a = \frac{2}{5} \times \left(\frac{-3}{2}\right) = \frac{-3}{5}$ From (1)

The required equation $\frac{x}{\frac{-3}{5}} + \frac{y}{\frac{-3}{2}} = 1$

$\Rightarrow 5x + 2y = -3 \Rightarrow 5x + 2y + 3 = 0$

18. Find the equation of a line whose intercepts on the x and y axes are given below.(4,-6)

Solution x intercept, a = **4**, y intercept, b =**-6**

Equation of the line in intercept form, $\frac{x}{a} + \frac{y}{b} = 1$

$\frac{x}{4} + \frac{y}{-6} = 1$

$\frac{x}{4} - \frac{y}{6} = 1$

$\frac{3x-2y}{12} = 1$

3x – 2y = 12

3x – 2y – 12 = 0

19. Find the equation of a line whose intercepts on the x and y axes are given below.-5, $\frac{3}{4}$

Solution

x intercept, a = -5 , y intercept, b =$\frac{3}{4}$

Equation of the line in intercept form, $\frac{x}{a} + \frac{y}{b} = 1$

$\frac{x}{-5} + \frac{y}{\frac{3}{4}} = 1$

$\frac{x}{-5} + \frac{4y}{3} = 1$

$\frac{3x-20y}{-15} = 1$

3x – 20y = -15

3x – 20y + 15 = 0

20. A cat is located at the point (-6, -4) in xy plane. A bottle of milk is kept at (5, 11). The cat wish to consume the milk travelling through shortest possible distance. Find the equation of the path it needs to take milk.

Solution The required equation of the line joining the points (-6, -4) and (5, 11)

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$\Rightarrow \frac{\boldsymbol{y}-(-\mathbf{4})}{\mathbf{11}-(-\mathbf{4})} = \frac{\boldsymbol{x}-(-\mathbf{6})}{\mathbf{5}-(-\mathbf{6})} \quad \Rightarrow \frac{\boldsymbol{y}+\mathbf{4}}{\mathbf{15}} = \frac{\boldsymbol{x}+\mathbf{6}}{\mathbf{11}}$

⇒ 11y + 44 = 15x + 90

$\Rightarrow 15x - 11y + 90 - 44 = 0$

$\Rightarrow 15x - 11y + 46 = 0$

21. Find the equation of a straight line Passing through (-8,-4) and making equal intercepts on the coordinate axes.

Solution Given Intercepts a = b

∴ Required Equation $\frac{x}{a} + \frac{y}{b} = 1$

⇒ x + y = a ($\because (1) is\ lies\ on\ (-8, 4)$))

⇒ −8 + 4 = a ⇒ a = −4

∴ $(1) \Rightarrow x + y = -4$ ⇒ x + y + 4 = 0

5 MARKS

1. If the points P(-1,-4), Q(b,c) and R(5,-1) are collinear and if 2b + c = 4, then find the values of b and c.

Solution

Since the three points P(-1, -4), Q(b,c) and R(5, -1) are collinear

Area of ΔPQR = **0**

$$\frac{1}{2}\begin{vmatrix} -1 & -4 \\ b & c \\ 5 & -1 \\ -1 & -4 \end{vmatrix} = 0$$

$\frac{1}{2}$ {(-c – b - 20) – (-4b + 5c + 1)} = 0

-c – b – 20 + 4b – 5c – 1 = 0

b – 2c = 7

Also, 2b + c = 4(from the given information)

Solving (1) and (2) we get b = 3, c = -2

2. Determine whether the sets of points are collinear (a, b+c), (b, c + a) and (c, a + b)

Solution Area of Δ $= \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} a & b+c \\ b & c+a \\ c & a+b \\ a & b+c \end{vmatrix}$

$= \frac{1}{2}$ [(ac +a^2 + ab + b^2 +bc + c^2)– (b^2 + bc + c^2 + ca + a^2 + ab]

$= \frac{1}{2}$ [ac +a^2 + ab + b^2 +bc + c^2 – b^2 – bc – c^2 – ca – a^2 – ab] = $\frac{1}{2}$ [0] = 0 sq.units

∴The given points are collinear.

3. Find the value of 'a' for which the given points are collinear (a, 2 - 2a) ,(-a+1, 2a) and (-4- a, 6-2a)

Solution Δ = 0

$$\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = 0 \implies \begin{vmatrix} a & 2-2a \\ -a+1 & 2a \\ -4-a & 6-2a \\ a & 2-2a \end{vmatrix} = 0$$

(2a^2 -6a+2a^2+6 – 2a –8+8a–2a+2a^2) – (-2a+2a^2+2-2a-8a-2a^2+6a-2a^2) = 0

= (6a^2 - 2a – 2) – (- 2a^2 -6a + 2) = 0 ⟹ 8a^2 + 4a – 4 =0 ÷ 4

2a^2 + a – 1 =0 ⟹ a = $\frac{1}{2}$, -1

4. If the points A(-3,9), B(a,b) and C(4, -5) are collinear and if a +b = 1, then find "a" and "b".

Solution Given A (-3,9), B(a,b), C(4, -5) are collinear and a +b = 1 → (1)

Area of the triangle formed by 3 points = 0

(i.e.) $\frac{1}{2}\begin{pmatrix} -3 & a & 4 & -3 \\ 9 & b & -5 & 9 \end{pmatrix} = 0$

⇒ (-3b – 5a +36) – (9a + 4b + 15) = 0 ⇒ – 5a – 3b +36 – 9a - 4b - 15 = 0

⇒ – 14a – 7b +21 = 0 (÷ 7) ⇒ 2a + b = 3 → (2)

a + b = 1 → (1)

(1) – (2) ⇒ $a = 2$ b= -1

5. Let P(11,7), Q (13.5, 4) and R (9.5, 4) be the mid-points of the sides AB, BC and AC respectively of ΔABC. Find the coordinates of the vertices A, B and C. Hence find the area of ΔABC and compare this with area of ΔPQR.

Solution

P = Midpoint of AB

$\Rightarrow \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = (11, 7)$

$\Rightarrow \frac{x_1+x_2}{2} = 11 \Rightarrow x_1 + x_2 = 22$ ……….(1)

$\Rightarrow \frac{y_1+y_2}{2} = 7 \Rightarrow y_1 + y_2 = 14$ ……….(2)

Q = Midpoint of BC

$\Rightarrow \left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right) = (13.5, 4)$

$\Rightarrow \frac{x_2 + x_3}{2} = 13.5 \quad \Rightarrow x_2 + x_3 = 27$ ……….(3)

$\Rightarrow \frac{y_2 + y_3}{2} = 4 \quad \Rightarrow y_2 + y_3 = 8$ ……….(4)

R = Midpoint of AC

$\Rightarrow \left(\frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2}\right) = (9.5, 4)$

$\Rightarrow \frac{x_1 + x_3}{2} = 9.5 \quad \Rightarrow x_1 + x_3 = 19$ ……….(5)

$\Rightarrow \frac{y_1 + y_3}{2} = 4 \quad \Rightarrow y_1 + y_3 = 8$ ……….(6)

$(3) - (1) \Rightarrow x_3 - x_1 = 5$ ……….(7)

$(5) + (7) \Rightarrow 2x_3 = 24 \quad \Rightarrow x_3 = 12$

Substitute $x_3 = 12$ in (5)

$\Rightarrow \quad x_1 = 19 - 12 = 7$

$\Rightarrow \quad x_1 = 7$

Substitute $x_1 = 7\ in$ (1)

$\Rightarrow \quad x_2 = 22 - 7 = 15$

$\Rightarrow \quad x_2 = 15$

$(2) - (4) \Rightarrow \quad y_{1-} y_3 = 6$ ……….(8)

$(6) + (8) \Rightarrow \quad 2y_1 = 14$

$\Rightarrow \quad y_1 = 7$

$\Rightarrow$ Substitute $y_1 = 7\ -$ in (6)

$\Rightarrow \quad y_3 = 8 - 7 = 1$

$\Rightarrow \quad y_3 = 1$

Substitute $y_3 = 1\ -$ in (4)

$\Rightarrow y_2 + 1 = 8$

$\Rightarrow y_2 = 7$

A(7,7), B(15,7) and C(12,1)

Area of ΔABC $= \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 7 & 7 \\ 15 & 7 \\ 12 & 1 \\ 7 & 7 \end{vmatrix}$

$= \frac{1}{2}$ [(49 + 15 + 84) – (105 +84 +7)]

$= \frac{1}{2}$ [148 - 196] $= \frac{1}{2}$ [-48]= 24 sq.units (∵Area cannot be –ve)

Area of $\Delta PQR = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 11 & 7 \\ 13.5 & 4 \\ 9.5 & 4 \\ 11 & 7 \end{vmatrix}$

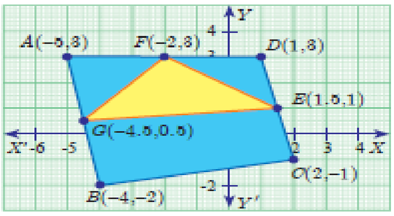
$= \frac{1}{2}$ [(44 + 54 + 66.5) – (94.5 +38 +44)]

$= \frac{1}{2}$ [164.5 – 176.5] $= \frac{1}{2}$ [-12] = 6 sq.units   (∵Area cannot be –ve)

Now, Area of ΔPQR = 6 sq.units, Area of ΔABC = 24 sq.units

∴ Area of ΔABC = **4** × Area of ΔPQR

6. In the figure, find the area of (i) triangle AGF (ii) triangle FED (iii) Quadrilateral BCEG.



Solution

(i) The Area of ΔAGF $= \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} -5 & 3 \\ -4.5 & 0.5 \\ -2 & 3 \\ -5 & 3 \end{vmatrix}$

$= \frac{1}{2}$ [(-2.5 – 13.5 - 6)- (-13.5 - 1 - 15)]

$= \frac{1}{2}$ [-22+29.5]

$= \frac{1}{2}$ [7.5]

= **3.75** sq. units

(ii) The Area of ΔFED $= \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} -2 & 3 \\ 1.5 & 1 \\ 1 & 3 \\ -2 & 3 \end{vmatrix}$

$= \frac{1}{2}$ [(-2 + 4.5 + 3)- (4.5 + 1 -6)]

$= \frac{1}{2}$ [5.5 + 0.5]

$= \frac{1}{2}$ [6]

= **3** sq. units

(iii) Area of Quadrilateral BCEG $= \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_4 & y_4 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} -4 & -2 \\ 2 & -1 \\ 1.5 & 1 \\ -4.5 & 0.5 \\ -4 & -2 \end{vmatrix}$

$= \frac{1}{2}$ [(4 + 2 + 0.75 + 9)- (- 4 - 1.5 – 4.5 - 2)]

$= \frac{1}{2}$ [15.75 + 12]

$= \frac{1}{2}$ [27.75]

= **13.875 ச.அ**

7. Without using Pythagoras theorem, show that the points (1, -4), (2, -3) and (4, -7) form a right angle triangle.

Solution

Let the points be A(1, -4) , B(2,-3) andC(4,-7).

The Slope of AB = $\frac{-3+4}{2-1} = \frac{1}{1} = 1$

The Slope of BC = $\frac{-7+3}{4-2} = \frac{-4}{2} = -2$

The Slope of AC = = $\frac{-7+4}{4-1} = \frac{-3}{3} = -1$

Slope of AB × Slope of AC = (1) (-1) = -1

AB is perpendicular to AC. $\angle A = 90°$

Therefore, ΔABC is a right angle triangle.

8. Show that the points (1,-4), (2,-3) and (4,-7) form a right angled triangle and check whether they satisfies pythagoras theorem

(i) A(1,-4), B(2,-3) and C(4,-7) (ii) L(0,5), M(9,12) and N(3,14)

Solution

(i) A(1,-4), B(2,-3) and C(4,-7)

Slope of AB = $\frac{-3-(-4)}{2-1} = \frac{1}{1} = 1$

Slope of BC = $\frac{-7-(-3)}{4-2} = \frac{-4}{2} = \mathbf{-2}$

Slope of AC = $\frac{-7+4}{4-(+1)} = \frac{-3}{3} = -1$

(Slope of AB) ×(Slope of AC) = $\mathbf{1} \times (-1) = -1$

$\therefore \Delta ABC$ is a right angled triangle ($\because AB \perp AC$)

Using Pythagoras theorem, $AB^2 + AC^2 = BC^2$ ($\because d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$AB^2 = (2-1)^2 + (-3 + 4)^2 = (1)^2 + (1)^2 = 2$

$AC^2 = (4-1)^2 + (-7 + 4)^2 = (3)^2 + (-3)^2 = 18$

$BC^2 = (4-2)^2 + (-7 + 3)^2 = (2)^2 + (-4)^2 = 4 + 16 = 20$

$AB^2 + AC^2 = 2 + 18 = 20 = BC^2$. Hence Proved

(ii) **L(0,5), M(9,12) and N(3,14)**

Slope of LM = $\frac{12-5}{9-0} = \frac{7}{9}$

Slope of MN = $\frac{14-12}{3-9} = \frac{2}{-6} = -\frac{1}{3}$

Slope of LN = $\frac{14-5}{3-0} = \frac{9}{3} = 3$

(Slope of MN) × (Slope of LN) = $\left(-\frac{1}{3}\right) \times (3) = -1$

$\therefore$ MN $\perp$ rLN. Δ LMN is a right angled triangle.

By Pythagoras theorem , $MN^2 + LN^2 = LM^2$

$MN^2 = (3-9)^2 + (14 -12)^2 = (-6)^2 + (2)^2 = 36 + 4 = 40$

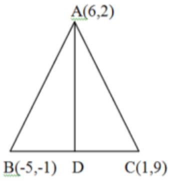
$LN^2 = (3-0)^2 + (14 -5)^2 = (3)^2 + (9)^2 = 9 + 81 = 90$

$LM^2 = (9-0)^2 + (12 -5)^2 = (9)^2 + (7)^2 = 81 + 49 = 130$

Here $MN^2 + LN^2 = LM^2$

9. Find the equation of the median and altitude of ΔABC through A where the vertices are A(6,2), B(-5, -1) and C(1,9)

Solution



Equation of the median through A.

Midpoint of BC = $\left(\frac{-5+1}{2}, \frac{-1+9}{2}\right)$

= D(-2, 4)

Equation of AD is A(6,2), D(-2,4)

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$\Rightarrow \frac{y-2}{4-2} = \frac{x-6}{-2-6}$

$\Rightarrow \frac{y-2}{2} = \frac{x-6}{-8}$

$\Rightarrow \frac{y-2}{1} = \frac{x-6}{-4}$

$\Rightarrow x - 6 = -4y + 8$

$\Rightarrow x + 4y - 14 = 0$

Equation of altitude through ‘A’

Slope of BC = $\frac{9+1}{1+5} = \frac{10}{6} = \frac{5}{3}$

Since AD ⊥ BC, Slope of AD = $\frac{-3}{5}$ and A is (6,2)

Equation of altitude AD is

$y - y_1 = m(x - x_1)$

$\Rightarrow y - 2 = \frac{-3}{5}(x - 6)$

$\Rightarrow 5y - 10 = -3x + 18$

$\Rightarrow 3x + 5y - 28 = 0$

CHAPTER– 6

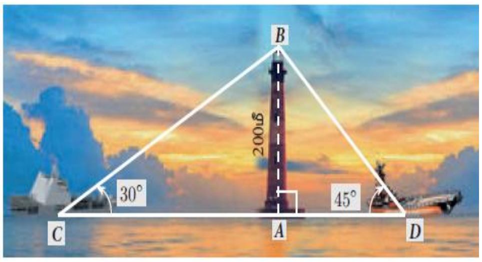
TRIGNOMETRY

5 MARKS

1. **Two ships are sailing in the sea on either sides of a light house. The angle of elevation of the top of the lighthouse as observed from the ships are 30° and 45° respectively. If the lighthouse is 200m high, find the distance between the two ships. ($\sqrt{3} = 1.732$)**

Solution Let AB be the lighthouse. Let C and D be the positions of the two ships.

Then, AB = 200மீ



$\angle ACB = 30°, \angle ADB = 45°$

In right triangle BAC, $\tan 30° = \frac{AB}{AC}$

$\frac{1}{\sqrt{3}} = \frac{200}{AC}$ gives, AC = 200 $\sqrt{3}$

In right triangle BAD, $\tan 45° = \frac{AB}{AD}$

1 = $\frac{200}{AD}$ gives AD = 200

Now, CD = AC + AD = 200$\sqrt{3}$ + 200 [by (1) and (2)]

CD =200 ($\sqrt{3} + 1$) = 200 x 2.732 = 546.4

Distance between two ships is **546.4**m

2. **From a point on the ground, the angles of elevation of the bottom and top of a tower fixed at the top of a 30m high building are 45° and 60° respectively. Find the height of the tower ($\sqrt{3} = 1.732$)**

Solution In ΔAPB $\tan\theta$ = $\frac{\text{opposite Side}}{\text{Adjacent Side}}$

tan 45° = $\frac{30}{BP}$

1 = $\frac{30}{BP}$

BP = 30m

In ΔBPC tan 60° = $\frac{BC}{BP}$

$\sqrt{3}$ = $\frac{h+30}{30}$

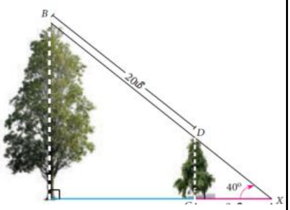
30 $\sqrt{3}$ = h + 30

h = 30 $\sqrt{3}$ - 30

= 30(1.732 – 1) = 30(0.732)

= 21. 960

3. **Two trees are standing on flat ground. The angle of elevation of the top of both the trees from a point X on the ground is 40°. If the horizontal distance between X and the smaller tree is 8m and the distance of the top of the two trees is 20m, calculate**



(i) the distance between the point X and top of the smaller tree.

(ii) the horizontal distance between the two trees. (Cos 40° = 0.7660)

Solution

Let AB be the height of the bigger tree and CD be the height of the smaller tree and X is the point on the ground

(i) In Right Angle ΔXCD ,

$\cos \mathbf{40}^\circ = \frac{CX}{XD}$

XD = $\frac{8}{0,7660} = 10.44$ m

∴The distance between X and top of the smaller tree = XD= 10.44m

XD = 10.44 m

(ii) In Right Angle ΔXAB

$\cos \mathbf{40}^\circ = \frac{AX}{BX} = \frac{AC+CX}{BD+DX}$

$0.7660 = \frac{AC+8}{20+10.44} \Rightarrow AC = 23.32 - 8 = 15.32$m

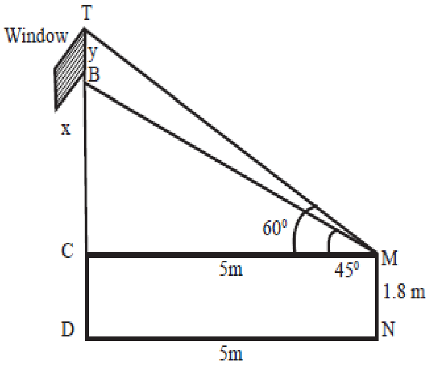
Therefore the horizontal distance between two trees AC= 15.32m

4. To a man standing outside his house, the angles of elevation of the top and bottom of a window are 60° and 45° respectively. If the height of the man is 180 cm and if he is 5m away from the wall, what is the height of the window? ($\sqrt{3}$= 1.732).

Solution

In Figure AB – a man Standing, EF – Window, CF – House

From the figure, EF = h, ED = x, DEF = x + h.



In ΔADE $\tan 45^\circ = \frac{DE}{AD} \Rightarrow 1 = \frac{x}{500} \Rightarrow x = 500$

In ΔADF $\tan 60^\circ = \frac{DF}{AD} \Rightarrow \sqrt{3} = \frac{h+x}{500}$

$\Rightarrow h + x = \sqrt{3}(500)$

$\Rightarrow h = (500 \times \sqrt{3}) - 500$

$= 500[\sqrt{3} - 1]$

$= 500[1.732 - 1]$

$= 500[0.732]$

= 366 cm

= 3.66 cm

5. The top of a 15m high tower makes an angle of elevation of 60° with the bottom of an electronic pole and angle of elevation of 30° with the top of the pole. What is the height of the electric pole?

Solution

AB – Electric Pole , CE – Tower

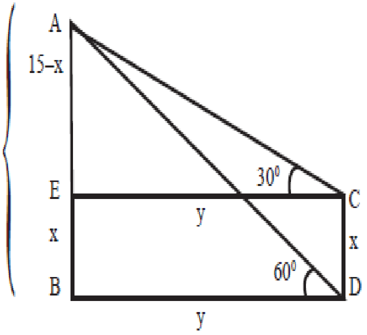
From the figure, AB = h, CE = 15 மீ, DE = 15 - h, BC = AD = x

In ΔADE tan 30° = $\frac{DE}{AD}$ ⇒ $\frac{1}{\sqrt{3}} = \frac{15-h}{x}$

$x = (15-h)\sqrt{3}$ ….(1)

In ΔBCE tan 60° = $\frac{CE}{BC}$ ⇒ $\sqrt{3} = \frac{15}{x}$

$x = \frac{15}{\sqrt{3}} = 5\sqrt{3}$ ….(2)



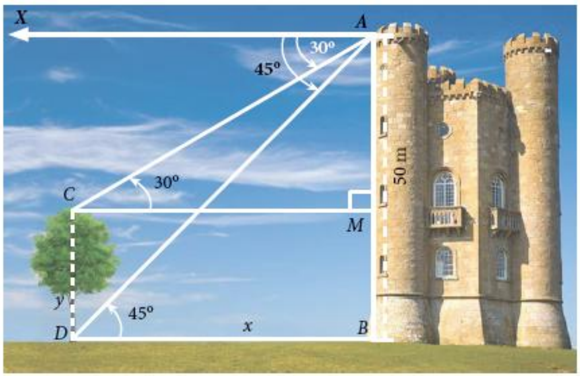
$(1) = (2) \Rightarrow (15 - h)\sqrt{3} = 5\sqrt{3}$

15 – h = 5 ⇒ h = 10 மீ

Height of the electric pole = 10m

6. **From the top of a tower 50m high, the angles of depression of the top and bottom of a tree are observed to be $30°$ and $45°$respectively. Find the height of the tree. ($\sqrt{3} = 1.732$)**

Solution Height of the tower AB = H = 50 மீ



Let the height of the tree = CD = y

∠ACM = α = 30° ∠ADB = β = 45°

Height of the tree = y = $\frac{H[tan\beta - tan\alpha]}{tan\,\beta}$

$= \frac{50[tan\,45° - tan\,30°]}{tan\,45°}$

$= \frac{50[1-\frac{1}{\sqrt{3}}]}{1}$

$= 50[1 - \frac{1}{\sqrt{3}}]$

$= 50 - \frac{50}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= 50 - \frac{50\sqrt{3}}{3}$

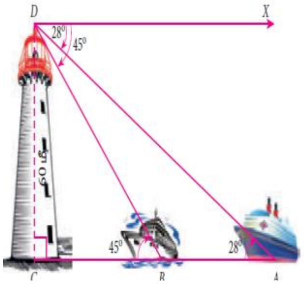
$= 50 - \frac{50 \times 1.732}{3}$

= 50 − **28.85**

Height of the tree = y = 21.15 m

7. **As observed from the top of a 60m high light house from the sea level, the angles of depression of two ships are 28° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships. (tan28° = 0.5317).**

Solution



Height of the Light house = CD = 60m

Position of the observer = D

From the Diagram $\angle XDA$ = 28° = ∠ DAC and

∠ XDB = 45° = ∠ DBC

From the Triangle DCB, We have

tan 45° = $\frac{DC}{BC}$

1= $\frac{60}{BC}$

BC = 60m

From the Triangle DCA, we have

tan 28° = $\frac{DC}{AC}$

0.5317= $\frac{60}{AC}$

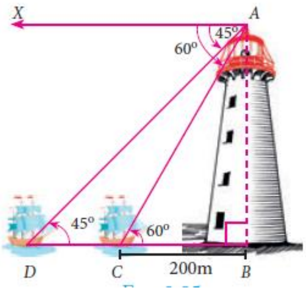
AC = $\frac{60}{0.5317}$

AC = 112.85m

Distance between two ships AB = AC – BC = 52.85m

8. **A man is watching a boat speeding away from the top of a tower. The boat makes an angle of depression of 60° with the man's eye when at a distance of 200m from the tower. After 10 seconds, the angle of depression becomes 45°. What is the approximate speed of the boat(in km/hr), assuming that it is sailing in still water? ($\sqrt{3} = 1.732$)**

Solution Let AB be the tower.



Let C and D be the positions of the boat

∠ XAC = 60° = ∠ACB and

∠ XAD = 45° = ∠ADB, BC = 200m

In right triangle, ABC, tan60° =$\frac{AB}{BC}$

gives $\sqrt{3}$ = $\frac{AB}{200}$

BC = 200 $\sqrt{3}$ -------- (1)

In right triangle, ABD tan45° = $\frac{AB}{BD}$

gives 1 = $\frac{200\sqrt{3}}{BD}$ [by (1)]

We get, BD= 200 $\sqrt{3}$

Now, CD= 200 $\sqrt{3}$ - 200

= 200 ($\sqrt{3}$ - 1) = 146.4

It is given that the distance CD is covered in 10 seconds.

That is, the distance of 146.4m is covered in10 seconds.

Therefore , speed of the boat = $\frac{distance}{time}$ = $\frac{146.4}{10}$

= 14.64 m/s gives 14.64 x $\frac{3600}{1000}$ km / hr

= 52.704 km / hr

9. **An aeroplane at an altitude of1800m finds that two boats are sailing towards it in the same direction. The angles of depression of the boats as observed from the aeroplane are 60° and 30° respectively. Find the distance between the two boats. ($\sqrt{3}$= 1.732).**

Solution

In figure A – An Aeroplance, C, D are Two Boats

From the figure, AB = 1800m , BC = y, CD = x, BD = x + y

In ΔABC tan 60° = $\frac{AB}{BC}$

$\Rightarrow \sqrt{3} = \frac{1800}{y}$

$\Rightarrow y = \frac{1800}{\sqrt{3}} = 600\sqrt{3}$ m

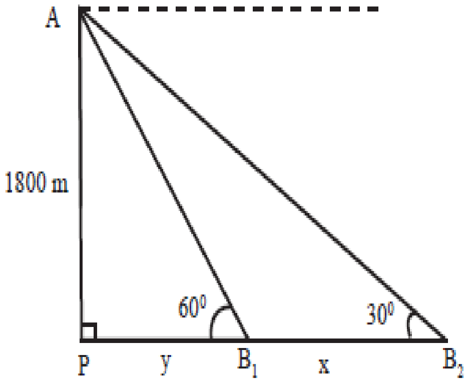
In ΔABD tan 30° = $\frac{AB}{BD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{1800}{x+y}$

$\Rightarrow x + y = 1800\sqrt{3}$

$\Rightarrow x = 1800\sqrt{3}$ - $600\sqrt{3}$

$\Rightarrow 1200\sqrt{3}$ m = **1200** x 1.732



Hence, the distance between the boats = **2078.4** m

10. From the top of a lighthouse, the angle of depression of two ships on the opposite sides of it are observed to be 30° and 60°. If the height of the lighthouse is h meters and the line joining the ships passes through the foot of the lighthouse, show that the distance between the ships is $\frac{4h}{\sqrt{3}}$ m.

Solution

C, D – Positions of the two ships

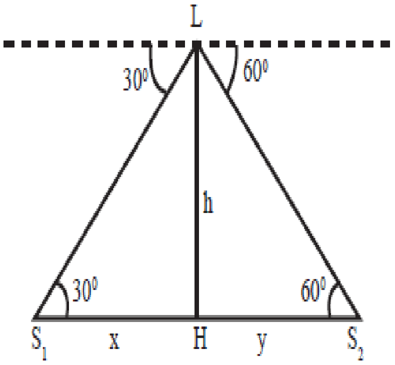
Height of the Light House AB = h m

In Δ ABC tanθ = $\frac{\text{opposite Side}}{\text{Adjacent Side}}$

tan30° = $\frac{h}{x}$

$\frac{1}{\sqrt{3}} = \frac{h}{x}$

$x = h\sqrt{3}$



In Δ ABD

tan60° = $\frac{h}{y}$

$\sqrt{3} = \frac{h}{y}$

$y = \frac{h}{\sqrt{3}}$

Distance between two ships = (x + y) = $h\sqrt{3} + \frac{h}{\sqrt{3}}$

$$d = \frac{3h}{\sqrt{3}} + \frac{h}{\sqrt{3}} = \frac{3h+h}{\sqrt{3}} = \frac{4h}{\sqrt{3}} \text{ m}$$

CHAPTER - 7

MENSURATION

2 MARKS

1. **A cylindrical drum has a height of 20cm and base radius of 14 cm. Find its curved surface area and the total surface area.**

Solution Given that, height of the cylinder h = 20cm; radius r = 14cm.

Now, C.S.A of the cylinder = $2\pi r$h sq.units

T.S.A of the cylinder = $2\pi r$ (h + r) sq.units

$= 2 \times \frac{22}{7} \times 14\ (20+14)$

$= 2992$ cm^2

C.S.A of the cylinder = $2\pi r$h = $2 \times \frac{22}{7} \times 14 \times 20 = 1760$ cm^2.

Therefore, C.S.A = 1760 cm^2 and T.S.A = 2992 cm^2

2. **The curved surface area of a right circular cylinder of height 14 cm is 88cm^2. Find the diameter of the cylinder**

Solution Given that C.S.A of the cylinder = $2\pi r$h sq.units

$2\pi r$h = 88

$2 \times \frac{22}{7} \times$ r $\times$ 14 = 88 (Given h = 14cm) $\Longrightarrow$ 2r = $\frac{88}{14} \times \frac{7}{22}$ $\Longrightarrow$ 2r = 2

Therefore, diameter = 2cm.

3. **A garden roller whose length is 3m long and whose diameter is 2.8m is rolled to level a garden. How much area will it cover in 8 revolutions.**

Solution Diameter d = 2.8m and height = 3 m

radius,r = 1.4 m,

Area covered in one revolution = curved surface area of the cylinder.

= $2\pi r$ h sq.units. = $2 \times \frac{22}{7} \times 1.4 \times 3$

Area covered in 1 revolution = 26.4 m^2

Area covered in 8 revolutions = 8 x 26.4 = 211.2

Therefore, area covered is 211.2 m^2

4. **If the total surface area of a cone of radius 7cm is 704 cm^2, then find its slant height.**

Solution Total Surface Area = 704 ச.செ.மீ

$\pi r\ (l + r)$ = 704

$\frac{22}{7} \times 7\ (l + 7)$ = 704

$l + 7$ = $\frac{704}{22} = \frac{64}{2} = 32$

$l + 7$ = 32, l = 32 – 7 = 25 cm.

Therefore, slant height of the cone is 25 cm.

5. **Find the diameter of a sphere whose surface area is 154 m^2**

Solution Surface area of sphere = 154m^2

$4\pi r^2$ = 154 $\Longrightarrow$ 4x $\frac{22}{7}$ x r^2 = 154

gives $r^2 = \frac{154}{4}$ x $\frac{7}{22} \Longrightarrow r^2 = \frac{49}{4} \Longrightarrow$ r = $\frac{7}{2}$

Radius of sphere r = $\frac{7}{2}m$; Diameter of sphere d = 7m

6. **The radius of a spherical balloon increase from 12cm to 16 cm as air being pumped into it. Find the ratio of the surface area of the balloons in the two cases,**

Solution Let r_1 and r_2 be the radii of the balloons.

Given that, $\frac{r_1}{r_2} = \frac{12}{16} = \frac{3}{4}$

Ratio of C.S.A.of balloons $= \frac{4\pi r_1^2}{4\pi r_2^2} = \frac{r_1^2}{r_2^2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$

Therefore, ratio of C.S.A of balloons is 9 : 16

7. **If the base area of a hemispherical solid is 1386 sq.metres, then find its total surface area?**

Solution

Base area of a hemispherical solid, πr^2 =1386 sq.m

Total Surface of Area = 3 πr^2 =3 x1386 = 4158

Therefore, T.S.A of the hemispherical solid is 4158 m^2

8. **The internal and external radii of a hollow hemispherical shell are 3m and 5m respectively. Find the T.S.A and C.S.A. of the shell.**

Solution Let the internal and external radii of the hemispherical shell be r and R

Given that, R = 5 m, r = 3 m

CSA of the Shell = $2\pi \, (R^2 + r^2)$sq.units

$= 2\times \frac{22}{7} \, (25 + 9) = 213.71$

TSA of the Shell = $\pi(3R^2 + r^2)$ sq.units

$= \frac{22}{7} \, (75 + 9) =$ **213.71**

Therefore, CSA = 213.71m^2 and TSA = 264 m^2

9. **A sphere, a cylinder and a cone(see figure) are of the same radius, where as cone and cylinder are of same height. Find the ratio of their curved surface areas.**

Solution Required Ratio

= CSA of the sphere : CSA of the cylinder : CSA of the Cone

$= 4\pi r^2 : 2\pi rh : \pi rl$ $(l = \sqrt{r^2 + h^2} = \sqrt{2r^2} = \sqrt{2}r$

$= 4\pi r^2 : 2\pi r^2 : \pi r\sqrt{2}r$

$= 4\pi r^2 : 2\pi r^2 : \sqrt{2}\,\pi r^2$

$= 4 : 2 : \sqrt{2} = 2\sqrt{2} : \sqrt{2} : 1$

10. **The slant height of a frustum of a cone is 5cm and the radii of its ends are 4cm and 1 cm. Find its curved surface area.**

Solution l = 5 **cm**, R = 4 **cm,** r = 1 **cm**

C.S.A of the frustum = π (R + r) l sq.units

$= \frac{22}{7}\ (4+1)$ x 5 $= \frac{22\times5\times5}{7} = \frac{550}{7}$ = 78.57 m^2

11. **The radius and height of a cylinder are in the ratio 5 : 7 and its curved surface area is 5500 sq.cm. Find its radius and height**

Solution

r: h = 5 : 7 ⇒ r = 5$x$cm, h= 7 $x$cm

CSA = 5500 sq. cm

$2\pi rh = 5500 \Rightarrow 2 \times \frac{22}{7} \times 5x \times 7x = 5500$

$x^2 = \frac{5500}{2\times22\times5} = 25 \Rightarrow x = 5$

Hence, Radius = $5 \times 5 = 25$cm, height = $7\times 5 = 35$ cm

12. **Find the volume of a cylinder whose height is 2m and whose base area is 250m^2.**

Solution

Let r and h be the radius and height of the cylinder respectively

Given that, height, h = 2m, base area = 250 m^2

Now, Volume of the cylinder = $\pi r^2 h$ cu.units = base area x h

$= 250 \times 2 = 500$ m^3

Therefore, volume of t he cylinder $= 500$ m^3

13. **Find the volume of the iron used to make a hollow cylinder of height 9cm and whose internal and external radii are 21cm and 28cm respectively.**

Solution

Let r, R and h be the internal radis, external radius and height of the hollow cylinder respectively.

Given that, r = 21cm. R = 28cm, h = 9cm

Now, Volume of hollow cylinder $= \pi(R^2 - r^2)h$ cu.units

$= \frac{22}{7}(28^2 - 21^2) \times 9 = \frac{22}{7}(784 - 441) \times 9 = 9702$

Therefore, Volume of iron used = 9702 cm^3

14. **The volume of a solid right circular cone is 11088 cm^3. If its height is 24cm then find the radius of the cone.**

Solution

Let r and h be the radius and height of the cone respectively.

Given that, Volume of the cone = 11088 cm^3

$$\frac{1}{3}\pi r^2 h = 11088$$

$$\frac{1}{3} \times \frac{22}{7} \times r^2 \times 24 = 11088$$

$$r^2 = 441$$

Therefore, the radius of the cone, r = 21cm

15. **The ratio of the volumes of two cones is 2 : 3. Find the ratio of their radii if the height of second cone is double the height of the first.**

Solution

Let r_1, h_1 be the radius and height of the Cone – I and let r_2, h_2 be the radius and height of the cone – II.

Given, $h_2 = 2h_1$ and $\frac{\text{Volume of Cone I}}{\text{Volume of Cone II}} = = \frac{2}{3}$

$$\frac{\frac{1}{3}\pi {r_1}^2 h_1}{\frac{1}{3}\pi {r_2}^2 h_2} = \frac{2}{3} \Rightarrow \frac{{r_1}^2}{{r_2}^2} \times \frac{h_1}{2h_1} = \frac{2}{3}$$

$\frac{{r_1}^2}{{r_2}^2} = \frac{4}{3}$ gives $\frac{r_1}{r_2} = \frac{2}{\sqrt{3}}$

Therefore, ratio of their radii = **2 :** $\sqrt{3}$

16. **If the radii of the circular ends of a frustum which is 45cm high are 28cm and 7cm, find the volume of the frustum.**

Solution height of the frustum, h=45cm, Bottom Radius, R=28 cm,Top Radius, r = 7 cm

Volume of the frustum $= \frac{1}{3}\pi h[\ R^2 + Rr + r^2\ ]$cu.units

$$= \frac{1}{3} \times \frac{22}{7} \times 45[\ 28^2 + 28 \times 7 + 7^2\ ] = \frac{1}{3} \times \frac{22}{7} \times 45[\ 784 + 196 + 49\ ]$$

$= \frac{1}{3} \times \frac{22}{7} \times 45 \times 1029$ = 22 x 15 x 147 = 48510 cm^3

17. **The volumes of two cones of same base raius are 3600 cm^3 and 5040 cm^3. Find the ratio of heights.**

Solution Ratio of the volumes of two cones $= \frac{1}{3}\pi r^2 h_1 : \frac{1}{3}\pi r^2 h_2$

= $h_1 : h_2$

= 3600 : 5040

= 360 : 504

= 40 : 56

= 5 : 7

18. A conical container is fully filled with petrol. The radius is 10 m and the height is 15m. If the container can release the petrol through its bottom at the rate of 25 cu. meter per minute, In how many minutes, the container will be emptied. Round off your answer to the nearest minute

Solution

In conical container, r = 10m, h = 15 m

Volume = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 10 \times 10 \times 15$

= 1571.43 cu.metre

The petrol in the container is release at the rate of 25 cu. metre per minute.

The required time for the container to be emptied = $= \frac{1571.43}{25} = 62.8$ minutes

Hence, the required time = 63minutes.

19. If the ratio of radii of rwo spheres is 4:7, find the ratio of their volumes.

Solution

The ratio of radii of two spheres is 4 : 7 = $\frac{4}{7}$

Let radius of first sphere is 4x , that is $r_1 = 4x$

Let radius of second sphere is 7x, that is $r_2 = 7x$

Ratio of their volumes $= \frac{\frac{4}{3}\pi r_1{}^3}{\frac{4}{3}\pi r_2{}^3} = \frac{r_1{}^3}{r_2{}^3} = \frac{(4x)^3}{(7x)^3} = \frac{4^3 \times x^3}{7^3 \times x^3}$

$= \frac{4^3}{7^3} = \frac{64}{343}$

Hence, the Ratio of their volumes 64 : 343

20. A Vessel is in the form of a hemispherical bowl mounted by a hollow cylinder. The diameter is 14cm and the height of the vessel is 13cm. Find the capacity of the vessel.

Solution

In hemisphere, r = 7cm , In cylinder, r = 7cm, h = 6cm.

Volume of the Vessel = Volume of the cylinder + Volume of hemisphere

$= \pi r^2 h + \frac{2}{3}\pi r^3 = \pi r^2 \left(\mathrm{h} + \frac{2}{3}\mathrm{r}\right)$

$= \frac{22}{7} \times 7 \times 7 \times \left(6 + \frac{2}{3} \times 7\right) = 22 \times 7 \times \frac{32}{3}$

= 1642.67 cm^3

Hence, the Capacity of the vessel is 1642.67 cm^3

21. 4 persons live in a conical tent whose slant height is 19cm. If each person require 22cm^2 of the floor area, then find the height of the tent.

<u>Solution</u> Base area of the cone $= \pi r^2 = 22$ sq.units.

4 persons living area = 4 x 22 = 88 cm^2

$\pi r^2 = 88 \Rightarrow \frac{22}{7} \times r^2 = 88$

$r^2 = 88 \times \frac{7}{22} = 28$ cm^2

$l = 19$ cm $l^2 = 361$

$h = \sqrt{l^2 - r^2} = \sqrt{361 - 28} = \sqrt{333}$

Height of the tent = 18.25 cm.

5 MARKS

1. If one litre of paint covers 10m^2, how many litres of paint is required to paint the internal and external surface areas of a cylindrical tunnel whose thickness is 2m, internal radius is 6m and height is 25m.

<u>Solution</u>

Given that, height h = 25cm,thickness = 2m, internal radius r = 6m
Now, External Radius R = 6 + 2 = 8m
CSA of the cylindrical Tunnel = CSA of the hollow cylinder

$= 2\pi(R + r)h$ sq. units.

$= 2 \times \frac{22}{7}(8 + 6) \times 25$

Hence, CSA of the cylindrical tunnel = 2200 m^2

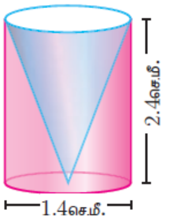
Area of covered by one litre of paint = 10 m^2

Number of litres required to paint the tunnel $= \frac{2200}{10} = 220$

Therefore, 220 litres of paint is needed to pain the tunnel

2. From a solid cylinder whoses height is 2.4cm and the diameter 1.4cm, a cone of the same height and same diameter is carved out. Find the volume of the remaining solid to the nearest cm^3 .

<u>Solution</u> h = 2.4 cm, d = 1.4 cm, $\Rightarrow r = 0.7$ cm



Total Surface Area of the remaining Solid

= CSA of the cylinder +CSA of the cone + Area of the bottom

$= (2\pi rh + \pi rl + \pi r^2)$ sq. units.

$= \pi r$ (2h + l + r) sq. units.

$l = \sqrt{r^2 + h^2} = \sqrt{(0.7)^2 + (2.4)^2} = \sqrt{0.49 + 5.76} = \sqrt{6.25} = 2.5$ cm

Total Surface Area of the remaining Solid $= \pi r(2h + l + r)$ sq. units.

$= \frac{22}{7} \times 0.7[2(2.4) + 2.5 + 0.7)]$

= 22 x 0.1 (4.8 + 2.5 + 0.7)

= 22 x 0.1 x 8.0 = 2.2 x 8 = 17.6 cm^2

3. **An industrial metallic bucket is in the shape of the frustum of a right circular cone whose top and bottom diameters are 10m and 4 m and whose height is 4m. Find the curved and total surface area of the bucket.**

<u>Solution</u> Diameter of the top = 10m ; Radius of the top, R = 5m

Diameter of the bottom = 4m ; Radius of the bottom, r = 2m, height h = 4m

Now, l = $\sqrt{h^2 + (R-r)^2}$ = $\sqrt{4^2 + (5-2)^2}$ = $\sqrt{16+9}$

= $\sqrt{25}$ = 5m ⟹ l = 5m

C.S.A = π (R + r) l sq.units

= $\frac{22}{7}$ $(5+2)$ x 5

= $\frac{22}{7} \times 7 \times 5$ = 110 m^2

T.S.A = [π (R + r) l + $\pi R^2 + \pi r^2$]sq. units

= π [(R + r) l + R^2 + r^2]

= $\frac{22}{7}$ [(5 + 2) 5 + 5^2 + 2^2]

= $\frac{22}{7}$ (35 + 25 + 4) = $\frac{1408}{7}$ = 201.14 m^2

Therefore, C.S.A = 110 m^2 and T.S.A = 201.14 m^2

4. **A solid iron cylinder has total surface area of 1848 sq.m. Its curved surface area is five – sixth of its total surface area. Find the radius and height of the iron cylinder.**

<u>Solution</u>

In Cylinder, CSA = $\frac{5}{6}$ TSA

⇒$2\pi rh$ = $\frac{5}{6} \times 308 = 1540$ (Given TSA = 1848) …..(1)

But, TSA == 1848

$2\pi r(h+r) = 1848$

$2\pi rh + 2\pi r^2 = 1848$

$2\pi r^2 = 1848 - 1540$

$2\pi r^2 = 308$

$2 \times \frac{22}{7} \times r^2$ = 308

r^2 = 308$\times \frac{1}{2} \times \frac{7}{22}$

r^2 = 49 ⇒ r = 7m

(1) ⇒ $2 \times \frac{22}{7} \times 7 \times h$ = $\frac{5}{6} \times 1848$

h = 35 m

Hence, Radius = 7 m, Height = 35m

5. **The external radius and the length of a hollow wooden log are 16cm and 13cm respectively. If the thickness is 4cm then find its T.S.A.**

Solution

In hollow cylinder, R = 16 cm, h = 13 cm and r = 16 - 4 = 12cm

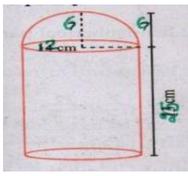
TSA = $2\pi(R+r)\,(R-r+h)$ sq.units

$= 2 \times \frac{22}{7} \times (16+12)(\,16-12+13)$

$= 2 \times \frac{22}{7} \times 28 \times 17$

TSA = 2992 cm^2

6. **A toy is in the shape of a cylinder surmounted by a hemisphere. The height of the toy is 25cm. Find the total surface area of the toy if its common diameter is 12 cm.**



Solution Diameter, d = 12 cm, radius, r = 6 cm

Height of the cylindrical portion, h = 25 – 6 = 19 cm

T.S.A of the toy = C.S.A of the cylinder + C.S.A of the hemisphere+ Base Area of the cyliner

$= 2\pi rh + 2\pi r^2 + \pi r^2$ sq.units

$= 2\pi rh + 3\pi r^2 = \pi r\,(2h + 3r)$

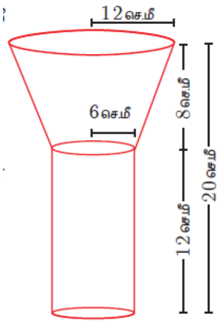
$= \frac{22}{7} \times 6 \times (38+18) = \frac{22}{7} \times 6 \times 56 =$ =1056cm^2

7. **A funnel consists of a frustrum of a cone attached to a cylindrical portion 12cm long attached at the bottom. If the total height be 20cm, diameter of the cylindrical portion be 12cm and the diameter of the top of the funnel be 24cm. Find the outer surface area of the funnel.**

Solution Let h_1, h_2 be the heights of the frustum and cylinder respectively.

Let R, r be the top and bottom radii of the frustum.

Given that, R = 12 cm, r = 6 cm, h_2 = 12 cm, h_1 = 20 – 12 = 8 cm



Slant height of the frustum l = $\sqrt{(R-r)^2 + h_1^2}$ units

$= \sqrt{36+64}$

l = 10 cm

Outer Surface Area = $2\pi r h_2 + \pi(R+r)l$ sq.units

$= \pi(2rh_2 + (R+r)l)$

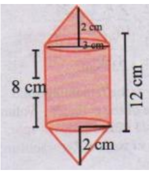
$= \pi(2 \times 6 \times 12) + (18 \times 10))$

$= \pi(144+180)$

$= \frac{22}{7} \times 324 = 1018.28$

Therefore, outer surface area of the funnel is 1018.28 cm^2

8. **Nathan, an engineering student was asked to make a model shaped like a cylinder with two cones attached at its two ends. The diameter of the model is 3 cm and its length is 12cm. If each cone has a height of 2cm, find the volume of the model that Nathan Made.**



<u>Solution</u> <u>Cylinder</u> Diameter d =3 cm, Radius, r=1.5 cm

Height h_1 = 12 – (2+2) = 8 cm

<u>Cone</u> Radius, r = $\frac{3}{2}$ cm, height h_1 = 2 cm

Volume of the model = Volume of the cylinder + Volume of 2 cones

$$= \pi r^2 h_1 + 2\frac{1}{3}\pi r^2 h_2 = \pi r^2 [h_1 + 2\frac{1}{3}h_2]$$

$$= \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2}[8 + \frac{2}{3} \times 2] = \frac{22}{7} \times \frac{9}{4}[8 + \frac{4}{3}]$$

$$= \frac{99}{14}[\frac{28}{3}] = 66 \text{ cm}$$

9. **A capsule is in the shape of a cylinder with two hemisphere stuck to each of its ends. If the length of the entire capsule is 12mm and diameter of the capsule is 3 mm, how much medicine it can hold?**

<u>Solution</u> In cylindrical part, Radius = $\frac{3}{2}$ mm, Height = 9mm

The required volume = Volume of cylinder + (2 x Volume of hemisphere)

$$= \pi r^2 h + 2 \times \frac{2}{3}\pi r^3 = \pi r^2\left(h + \frac{2 \times 2}{3} \times r\right)$$

$$= \frac{22}{7} \times \frac{3^2}{2^2} \times \left(9 + \frac{2 \times 2}{3} \times \frac{3}{2}\right)$$

$$= \frac{22 \times 9 \times 11}{7 \times 4} = 77.785\text{mm}^3$$

Hence, the volume of the medicine the capsule can hold is 77.785 mm^3

10. **A girl wishes to prepare birthday caps in the form of right circular cones for her birthday party, using a sheet of paper whose area is 5720 cm^2, how many caps can be made with radius 5 cm and height 12 cm.**

<u>Solution</u> In cone shaped caps $r = 5$ cm, h = 12cm

$$\therefore l = \sqrt{h^2 + r^2} = \sqrt{12^2 + 5^2} = \sqrt{144 + 25} = \sqrt{169} = 13 \text{ cm}$$

Let the total number of caps = n

n x CSA = 5720

$$n = \frac{5720}{CSA} = n = \frac{5720}{\pi r l}$$

$$= \frac{5720}{\frac{22}{7} \times 5 \times 13}$$

$$n = \frac{5720 \times 7}{22 \times 5 \times 13} = 28$$

Hence, the required number of caps is 28.